A Case Study of Apparent Gravity Wave Initiation of Severe Convective Storms

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ABSTRACT

A detailed study of an outbreak of severe convective storms is presented which investigates the interaction between subsynoptic scale gravity waves and the convective activity. The gravity waves were isolated by passing digital forms of nearly 130 National Weather Service and FAA barograph traces through a normal weighted band pass filter and then analyzing pressure perturbation \( p' \) fields for the Midwest region at 15-minute intervals. The waves had periods of about 3 h, trace speeds between 35 and 45 m s\(^{-1}\) and amplitudes between 0.5 and 2.5 mb. Analyses of surface weather reports, radar data, surface wind convergence, and surface \( p' \) fields revealed that the intensity of the convective systems pulsed with periods ranging from 2 to 4 h and that the gravity waves were a precursor to storm development in Iowa and Wisconsin and appeared to initiate convection in those areas. Reinforcement of preexisting storm cells or the development of new cells generally followed the passage of the wave trough, with maximum rainfall intensity coinciding with the passage of the ridge. The cycle is completed with a general weakening of the convective storms as the next trough approaches. To substantiate the proposed causal relationship, the observations were found to be consistent with a theoretical model of subsynoptic scale gravity waves.

1. Introduction

A number of case studies have revealed the existence of large amplitude, subsynoptic gravity waves which have a significant effect upon mesoscale weather features such as surface wind gusts, mid-level cloud distribution, and convective storm development. Brunk (1949) investigated a singular pressure pulsation that was associated with or originated from thunderstorm activity, and which was marked by 6 mb pressure falls and 25 m s\(^{-1}\) surface wind gusts. Bosart and Cussons (1973) and Eom (1975) found that gravity waves with amplitudes ranging from 2 to 6 mb and trace speeds of up to 50 m s\(^{-1}\) were attended by surface wind gusts between 20 and 30 m s\(^{-1}\). In both of these cases a 2 to 4 h period between the passage of each wave trough was noted. Tepper (1951) traced a gravity wave from Indiana to the East Coast and found that it desiccated a mid-level cloud bank along its track.

Large amplitude gravity waves have also been linked to the onset of convective storm systems. Tepper (1950, 1954) proposed that pressure jump lines effectively lifted the lowest layers of the atmosphere and appeared to initiate squall line development in convectively unstable air. Matsumoto and Akiyama (1969); Matsumoto and Tsuneoka (1969); and Matsumoto et al. (1967a, 1967b) contend that internal gravity waves were responsible for a pulsating tendency of winter and summer convective storms in western Japan. The intensity of the convective systems pulsed with a 3 h period as the mesoscale area of surface convergence associated with the gravity waves activated the intense storms.

In this report, an analysis of the outbreak of severe convective storms in the Midwest on 18 May 1971 is presented. The purpose of the paper is to further document the existence of large amplitude gravity waves that are marked by a 3 h period, and to show that these waves were an important mechanism for the initiation of the severe convection in this case. The diagnostic study reveals that a cause and effect relationship between gravity waves and thunderstorm development is credible. To further substantiate this contention, the observations are compared to, and found to be consistent with, a theoretical model of subsynoptic scale gravity waves derived by Eom (1972). The model is also utilized to demonstrate that the observed gravity waves were capable of releasing convective instability.

2. Case study: 18 May 1971

From the late afternoon of 17 May through 18 May 1971, a developing cyclone in the Midwest was the dominant feature on the synoptic charts (Fig. 1). A stationary front stretched from Kansas, across Iowa, and into Wisconsin for a 30 h period, with the cyclone developing in northern Missouri by 0000 GMT 19 May. An abundant amount of moisture moved northward from the Gulf of Mexico and strong jet streaks at
Fig. 1. Surface and 500 mb maps from 1200 GMT 18 May 1971 through 1200 GMT 19 May 1971. Isobars are drawn for every 4 mb. Heights (thin solid) are at 6 dam intervals and isotachs (thick solid) are at 10 m s⁻¹ intervals.

Fig. 2. ATS III satellite photographs at 2205 GMT and 2258 GMT 18 May 1971
the 500 mb level moved over the frontal zone from the southwest. These features are signs of impending severe convective activity (Miller, 1959) and led meteorologists in the National Severe Storms Forecast Center at Kansas City to issue numerous severe thunderstorm and tornado watches along nearly the entire frontal zone.

ATS III satellite pictures (Fig. 2) show the convective cloud clusters in a line from northeast Oklahoma to southwest Wisconsin which developed during the afternoon of 18 May. The thunderstorms in Missouri, Iowa, Wisconsin, and Michigan developed along the stationary front, moved slowly northeastward, and were responsible for several tornado outbreaks, wind and hail storms, and heavy rainfall (U. S. Department of Commerce, 1971). A preliminary study (a more detailed analysis is presented in Section 4) of the convective activity in Iowa and Wisconsin indicated that these storms displayed a pulsating tendency, as the intensity of the storm systems fluctuated with periods ranging from two to four hours.

An analysis of the pressure field using hourly altimeter settings from FAA and National Weather Service stations revealed that subsynoptic scale depressions were
moving through Iowa and seemed to be associated with the periodic behavior of the thunderstorms. Figure 3 illustrates one such disturbance which moved through Iowa and Wisconsin at a speed nearing 45 m s⁻¹. The depression exhibited many characteristics quite similar to those observed by Eom (1975), as did the response of the surface wind field with confluent winds existing behind the depression and divergent winds ahead of it. Eom found these depressions correspond to simple gravity wave concepts and they will thus be referred to as gravity waves in the remainder of this paper.

3. Isolating the gravity waves

Although the gravity wave noted above was quite apparent, other waves could be obscured by the synoptic scale systems and associative pressure tendencies and thus go unnoticed in this type of analysis. To isolate the waves, pressure traces from nearly 130 weather stations throughout the Midwest (Fig. 4) were first digitized (the time interval between data points equal to 15 minutes) and then filtered through a normal weighted statistical band pass filter, derived in a manner thoroughly described by Holloway (1958) and briefly discussed in Appendix A. The filter was designed after a survey of the barograph traces from stations in Iowa and Wisconsin revealed distinct periodic fluctuations in the pressure, with periods about 3 h.

Figure 5 illustrates the response function, \( R(f) \), for the band pass filter. \( R(f) \) rapidly approaches zero as the frequency, \( f \), decreases from 0.2 cycles h⁻¹, thus eliminating low frequency waves which represent synoptic scale trends. The waves having the observed 2 to 4 h periods are passed through the filter. \( R(f) \) gradually decreases as \( f \) increases from 0.4 to 1.0 cycles h⁻¹. Therefore high frequency waves (period less than 1.5 h) are considerably damped, removing the undesired noise from the perturbation trace and in the process lessening the effects of individual storm cells upon the pressure perturbations.

The effect of the band pass filter upon the pressure trace from Madison, Wis., shows the success of the filtering procedure (Fig. 5). The synoptic trends toward higher pressure during the first half of the period and toward lower pressure during the second half were eliminated. High frequency perturbations were also eliminated, as is reflected by the disappearance of the sharp fluctuation near 1500 GMT. The smooth perturbation trace represents the subsynoptic scale waves which passed through southern Wisconsin. The negative values represent wave troughs, while the positive values represent ridges.

Although this filtering technique offers the most objective method of analysis, some deficiencies inherent within the method should be considered before evaluating the results of this study. First the wave amplitude is reduced by 10 to 15% because of the imperfections of the filter:

a) \( R(f) \) is less than unity in the 0.2 to 0.4 frequency range, indicating that part of the gravity wave is itself being filtered out.

b) The lack of a sharp cut-off at higher frequencies causes further reduction in the wave amplitude.

Secondly, the nonuniformity of the station density could cause spatial aliasing of the perturbation values. Maximum and minimum values may pass between reporting stations and evade detection. This problem is especially acute in western Iowa, central Wisconsin, east central Illinois, and over the Great Lakes. To maintain continuity in the analysis and to more easily trace the movement of the waves through these gaps, pressure perturbation maps (\( \rho' \) maps) were plotted and analyzed for the entire Midwest region every 15 minutes. The massive job of filtering the pressure traces and plotting \( \rho' \) values on midwestern base maps was done using a computer program which can be found in the technical report by Uccellini (1973).

4. Results from the pressure perturbation analysis

Successive \( \rho' \) maps (Fig. 6) show the characteristic features of the wave train stretching from Michigan to northeast Kansas. The waves moved NNE with speeds (relative to ground) ranging from 35 to 45 m s⁻¹, having wavelengths ranging from 300 to 450 km, and amplitudes up to 2.0 mb.
Some cases of erratic wave behavior occurred in areas where thunderstorms developed. The waves seemed to split or disappear and then reappear further downstream, indicating perhaps a detrimental effect of the thunderstorms upon the continuity of the wave propagation and amplitude. In many other cases however, waves moved through areas of significant convective activity with no degeneration and were easily traced.

Evidence of an interaction between gravity waves and thunderstorms which enhanced cell development can be gained by tracing the movement of these waves and observing the response of convective storms within an area through which the wave passes. One example
is the gravity wave indicated by $L_7$ in Fig. 7. As $L_7$ moved northeastward through eastern Iowa, preexisting thunderstorm cells underwent tremendous intensification. These cells remained in Iowa and decreased in intensity as the wave propagated into southwest Wisconsin where new cells rapidly developed. This convective system was later reintensified by wave 10 as it moved through Iowa and into Wisconsin. Note that the cell velocity is nearly one-half that of the wave speed. Thus the appearance of the same area of thunderstorms moving with a particular wave is deceptive. Rather, new cells developed, or preexisting cells reintensified, slightly behind the wave trough as it moved through a region experiencing convective activity tending to extend the area of thunderstorms along its track. This example illustrates that the sub-synoptic scale gravity wave was a precursor to the severe storms, which developed in Iowa and Wisconsin after the wave trough passed through this region. Another example is wave $L_4$, which appeared to first inhibit the development of convection as it entered southwestern Minnesota (2145 and 2245 GMT) and to then initiate the redevelopment of rainshowers with its passage (2345 GMT).

The gravity waves indicated by $L_3$ and $L_4$ in Fig. 7 are important in that they offer evidence of the independence of the observed gravity waves from the existence and intensity of individual thunderstorm cells.
Both of these waves moved into regions that were more stable and less conducive to the development of convective storms (see Fig. 13). As wave 3 moved into southeastern Wisconsin, intense convection failed to develop, yet the wave amplitude remained large. Wave 8 moved from Arkansas, through southeast Missouri and south central Illinois, into Indiana. During this period the St. Louis, Mo. radar log indicated that no echoes were detected along the wave track.

The movement of the gravity waves for the 24 h period (0600 GMT 18 May through 0600 GMT 19 May 1971) seemed to be restricted to four major tracks...
(Fig. 8). To summarize the wave movement and the response of the atmosphere to the gravity waves for the entire 24 h period, time-cross sections were constructed and analyzed along the central track and across the three tracks in the Midwest. The data for these cross sections were obtained from the $\rho'$ map analyses, at the numbered points (Fig. 8) every 15 minutes.

Time-cross section #1 (Figs. 9a and 9b) was constructed along the central wave track. The continuous and alternating bands of negative and positive $\rho'$ values illustrates the movement of the wave along the general path shown in Fig. 8 and listed on the ordinate
of the cross section. The important points gleaned from this cross section are listed below:

1) Since the ordinate is the distance along the cross section and the abscissa is time, the slope of the bands represents the wave trace speeds measured relative to the ground. The speeds obtained from this cross section range between 30 and 45 m s\(^{-1}\).

2) The \(\psi\) map analyses revealed two sets of gravity waves which are depicted on this diagram. Between 0600 and 1600 GMT a set of waves moved northeastward through Texas into Oklahoma. The map analyses revealed that the wave activity ceased after this time, and that the perturbation values in Oklahoma and Texas after 2000 GMT were a reflection of the squall line and frontal passage through that area and were not due to gravity waves.

The other set of waves first appeared in Kansas and moved northeastward toward northern Michigan. It is this set that most of the analysis is focused upon.

3) This set of waves seemed to originate in east-central and northeastern Kansas, near the developing cyclone. Vertical cross sections (Fig. 10) analyzed at nearly a 30° angle to the central wave track (Fig. 8), or nearly along the direction of the upper air flow,

Fig. 8. The four major tracks of the gravity waves analyzed. Time cross section #1 constructed along central track; the numbers indicate position on the ordinate of the cross section. The dashed line from Albuquerque, N. Mex., to Flint, Mich., is the line along which the vertical cross sections (Fig. 10) are constructed.

Fig. 9a. Time cross section #1 of \(\psi\) values (see details in caption of Fig. 6) analyzed along central track (see Fig. 8).
reveal that this area was characterized by a strong jet core with significant horizontal and vertical wind shears, and also by a stable lower layer nearly 300 mb deep. The shears strengthened during the day as the jet stream winds extended downward, while at the same time the stability in the lower layers decreased.

4) The only region where the wave trace speeds are comparable with the upper level wind velocity is near the apparent source region. From northern Missouri northeastward to Michigan, waves propagated at a rate greater than any measured wind in that region.

5) The duration of each wave ranges from 6 to 10 h. A 10 h duration has been observed for similar gravity wave activity in Japan (Matsumoto et al., 1967a), and up to 15 h in the case reported on by Bosart and Cussens (1973).

6) Moving along the abscissa at point 11 (Iowa) a minimum $\rho'$ value is attained every 3 h, indicating this area experienced gravity waves with a 3 h period.

7) Figure 9b relates surface weather reports to gravity waves. A cyclical pattern is apparent as precipitation tends to occur immediately after the passage of a wave trough. For the most part, when showers developed rainfall commenced after the passage of the wave trough, maximum rainfall intensity occurred with positive $\rho'$ values, and the intensity decreased with the onset of the next wave trough.

8) In northern Missouri and Iowa (points 8 through 12) the precipitation outburst became less intense as the morning progressed (Fig. 9b):

a) Wave trough passage at 1200 GMT (point 10) was followed by significant rainfall.
encended gravity wave activity. Noteworthy are the waves in southeast Missouri and southern Illinois after 1800 GMT of which wave 8 (Fig. 7) is an example. Surface reports indicate no rainfall to be associated with these waves, although some scattered to broken mid-level clouds were reported in Missouri and Illinois after the wave trough passed through.

A time section of values for $\rho'$, subsynoptic scale surface convergence, and precipitation rates (from Hourly Precipitation Data) for east-central Iowa is presented in Fig. 11. The effects of the gravity waves upon an atmosphere primed for the development of convective storms by the synoptic circulation, in a manner described in Section 2, are quite apparent. For the most part, after the passage of the wave trough or nearly at the time of the ridge, a peak in the magnitude of surface convergence and rainfall rates was observed. The intensity of the storms as indicated by surface reports (Fig. 9b) also increased after the passage of the trough. As the wave ridge passed on, the magnitude of convergence, rainfall rates, and storm intensity all decreased, tending toward a minimum value with the onset of the next wave trough. The instantaneous reaction of the convergence field and rainfall rates could not be measured here because the rainfall and wind data are available on an hourly basis only. The important feature, however, is that in general the maximum values did occur after the passage of the wave trough and that a cyclical pattern, with a 2 to 4 h period, is quite apparent. This response of the atmosphere to the gravity waves reflects the ability of these waves to influence thunderstorm development.

5. A comparison of the observations with a theoretical model of subsynoptic scale gravity waves

A comparison was made between the observations of this case study and the results of a theoretical model of gravity waves derived by Eom (1972). The objective of this comparison is to check the time lag between the passage of the wave trough and the onset of convective growth and subsequent precipitation which appears to be associated with the ridge.

Eom derived a homogeneous, three-layer model utilizing the governing equations from Houghton and Isaacson (1968) assuming linear hydrostatic, and inviscid conditions in each layer. The lower layer has a mean depth $H_1$ equal to 3 km, no mean motion, and a constant density $\rho_l$. The middle layer has density $\rho_2$ and a constant horizontal mean motion $U$ defined in the direction of wave propagation. The third layer corresponds to the lower stratosphere and acts as a passive stable layer with density $\rho_3$ held constant. In short, the

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1 Surface convergence obtained by interpolating winds to a 1° long. by 1° lat. grid using a cubic weighting scheme and calculating the finite difference $\frac{(\Delta u/\Delta x) + (\Delta v/\Delta y)}{2}$ on this grid.
model describes the behavior of internal tropospheric gravity waves within a simplified sheared environment.

Although this model is admittedly simple, it proved to be successful in both Eom's and this case study, as the derived values for the trace speed and horizontal surface wind perturbations associated with the gravity waves were in close agreement with surface observations. The model seemed applicable to this study, given the similar synoptic scale conditions to Eom's case study. Both cases consisted of a cyclogenetic period with significant mid- and upper-level jet winds, and similar upper air soundings in the region of wave occurrence. These soundings are marked by a stable lower layer topped by a deeper layer with a nearly constant lapse rate (see Fig. 13).

The use of the simple wave form \( \exp(ik(x-ct)) \) and the expressions for the vertical motion and parcel displacement at \( H_1 \) (top of model layer 1) derived by Eom as

\[
\begin{align*}
\omega_1' &= \frac{ikH_1p'}{c\rho_0}, \\
D_1' &= \frac{H_1p'}{c^2\rho_0},
\end{align*}
\]

yield the following expressions for the surface pressure \( P_s(x) \), vertical motion \( \omega_1(x) \), and parcel displacement \( D_1(x) \) (setting \( t=0 \)):

\[
\begin{align*}
P_s(x) &= p' \cos(kx) \\
\omega_1(x) &= \frac{kH_1p'}{c\rho_0} \cos(kx - \pi/2) \\
D_1(x) &= \frac{H_1p'}{c^2\rho_0} \cos(kx).
\end{align*}
\]

Here, \( k \) is the wavenumber in the direction of propagation (x direction), \( p' \) is the amplitude of the surface pressure perturbation, \( \rho_0 \) is the density at the surface, and \( c \) is the trace speed along the x axis. The vertical motion at \( H_1 \) is 90° out of phase with the pressure oscillation, while the parcel displacement is in phase. According to this model, for positive \( c \), the maximum rising motion in the lower troposphere should occur midway between the trough and advancing ridge with maximum parcel displacement occurring at the ridge. If the atmosphere is conducive to convective storms, the development of these storms would be enhanced after the trough passage, during the period of upward vertical motion. Maximum convective intensity should coincide with the period of maximum parcel displacement, i.e., at the wave ridge. The observations fit this model very well as the most intense convection developed after the trough passage with maximum intensity found near the ridge. Note that the 90-minute period between trough and ridge lines is enough time for thunderstorm cells to go through the developing and mature precipitating phases (Byers and Braham, 1949; Schlesinger, 1973).

6. The ability of gravity waves to initiate convective storm systems

No direct evidence has yet been offered which demonstrates that vertical motion associated with gravity waves is capable of initiating severe convective storms. This is obviously due to the sparsity of upper air data, which prevents a reasonable measure of the gravity wave induced vertical velocities. The lack of upper air data on the time and space scales of the observed waves necessitates the use of a model to estimate a vertical motion profile associated with these waves. The three-layer model derived by Eom (1972) and briefly described earlier is used to approximate such a profile (Fig. 12), from which the atmospheric displacement between the wave trough and ridge can be estimated. Descriptions of the use of Eom's model to construct a vertical motion profile and of the layer lift procedure to estimate wave displacements can be found in Appendix B.

Several factors must be considered to determine the effect of the wave displacement. There is destabilization of the sounding due to the differential lifting, which is further enhanced by the vertical moisture distribution usually found in convectively and conditionally unstable air; that is, dry air overlying very moist air separated by a distinct boundary. Secondly, the displacement could sufficiently cool the air for it to become saturated. Thus if the lapse rate is increased beyond the moist adiabatic, the saturated parcels would be at their level of free convection, could accelerate upward, and therefore enhance the development of convective storms.

The change in the sounding due to the layer lift procedure was quantified by considering the change in the work done on parcels. The net work, \( W_k \), done on a parcel displaced from \( z_1 \) to \( z_2 \) is

\[
W_k = \int_{z_1}^{z_2} g \left( \frac{T' - T}{T} \right) dz,
\]

where \( g \), \( T' - T \), and \( T \) is the buoyancy force per unit mass, and \( T' \) and \( T \) are the parcel and environment temperatures, respectively. Substituting the hydrostatic equation and the equation of state into (6) yields

\[
W_k = \int_{z_1}^{z_2} T'R d\ln p - \int_{p_2}^{p_1} TR d\ln p.
\]

Positive \( W_k \) indicates that a parcel will gain kinetic energy from the environment and experience accelerating, upward vertical motion. \( W_k \) was determined for two layers: 850 to 700 mb, and 850 to 500 mb, before and after the modeled displacement to test for significant destabilization in the low and middle troposphere due solely to the modeled gravity wave.
It was also determined whether any level of free convection existed after the layer lift procedure was applied. The lowest level of free convection (LLFC) is marked on the soundings and occurs for saturated parcels situated within an environment having a moist adiabatic lapse rate.

Two soundings are presented to illustrate the effect of the modeled gravity wave. Green Bay, Wis., (0000 GMT 19 May 1971; Fig. 13) was within the area of periodic intense convective activity while Salem, Ill., (1200 GMT 18 May 1971; Fig. 13) was outside the region of convective storms, but within the area which experienced gravity waves.

1) Green Bay, Wis.: The destabilization due to the model gravity wave is a critical factor in this case. For the original sounding, \( W_b \) (850-700 mb) equals \(-6 \times 10^4\) J kg\(^{-1}\), indicating that forced lifting would be necessary for convection to originate in the lower troposphere. The feature on the sounding responsible for this is the inversion near 750 mb. After the wave displacement, \( W_b \) (850-700 mb) equals \(+8 \times 10^4\) J kg\(^{-1}\), while \( W_b \) (850-500 mb) equals \(+3 \times 10^4\) J kg\(^{-1}\). The change in the sounding reflecting this destabilization is the elimination of the inversion layer due largely to the differential lifting of dry air overlying very moist air. It is now possible for air parcels to experience a gain of kinetic energy even in the low levels and for convective storms to experience unimpeded growth. The LLFC is is found near 830 mb within a region of saturated air. Therefore, explosive development of convection could be expected and in fact did occur. The Chicago radar summary reported the rapid growth of moderate thunderstorms just south of Green Bay within one and a half hours of the balloon release, after a wave trough passed through that region.

2) Salem, Ill.: The initial conditions at Salem were not suitable for convective storms. Abundant moisture was restricted to a shallow layer near the surface and was capped by a very strong inversion. \( W_b \) remains negative and in fact becomes more negative after the displacement due to lifting the inversion from below.

**Fig. 11.** Pressure perturbation trace for east-central Iowa (solid) (point 11 on time cross section #1, Fig. 9a) along with surface wind convergence (dashed) and hourly rainfall totals (from *Hourly Precipitation Data*, published by U. S. Dept. of Commerce).

**Fig. 12.** Vertical motion profile given the parameters measured from the gravity waves moving through Iowa and Wisconsin, mean vertical motion for each layer 1 km deep, and the maximum parcel displacement using theoretical model (see text and Appendix B).
850 mb to within the bottom layer. Since the entire lifting process occurs dry adiabatically in the low to middle levels, the inversion is not significantly weakened and therefore no convection could develop even though significant gravity waves propagated through this region. The mid-level clouds which developed in this area following the passage of wave troughs are accounted for by the saturated layer near 500 mb.

This experiment indicates that gravity waves can initiate thunderstorm development in areas having the necessary amount and proper vertical distribution of moisture. The subsynoptic scale waves enhanced development by weakening the inversion that precludes convection, and by bringing saturated air parcels to their level of free convection. The ability of the modeled wave to release convective instability and the heretofore observed causal relationship (Matsumoto et al., 1967a, 1967b; Matsumoto and Akiyama, 1969; Matsumoto and Tsuneoka, 1969) indicate that subsynoptic scale gravity waves can initiate the development of severe convective storms, and apparently did in this case.

7. Summary and discussion

The analysis of the cyclogenetic period of 18 May 1971 revealed that large amplitude, subsynoptic scale gravity waves were an integral part of the entire system. More importantly, the waves appear to be an important mechanism for the initiation and/or redevelopment of the severe convective storms which occurred in the Midwest. A summary of the analysis is presented below.

1) The subsynoptic scale gravity waves had the following characteristics:
   a) duration: 6 to 10 h
   b) amplitude: 0.5 to 2.5 mb
   c) trace speed relative to the ground: 35 to 45 m s⁻¹
   d) period: 2.5 to 4.0 h
   e) wavelength: 400 to 500 km

2) The gravity waves propagated over a large region characterized by a low-level inversion and by strongly developing vertical and horizontal wind shears. All waves moved in the general direction of the upper air flow; but, for the waves along the central and southern tracks, at a speed greater than any measured wind in that region.

3) The gravity waves were a precursor to the thunderstorms that developed in Iowa, Wisconsin, and northern Michigan. The observations revealed that a cause and effect relationship between gravity waves and the development of severe convection is credible. The consistent periodic behavior of the storm intensity, as revealed by radar and surface observations, was linked to the passage of the gravity waves through the area of convective activity. The passage of the wave trough through slower moving areas of convective storms was followed by the reintensification of thunderstorm cells and/or the development of new cells. The heaviest precipitation (maximum storm intensity) occurred within the ridge, with a decrease in the storm intensity observed as the next wave trough approached. These observations were found to be consistent with a theoretical model derived by Eom.
4) By estimating the vertical motion associated with the observed waves through utilization of Eom's model and comparison with surface convergence measured in other cases, it was determined that subsynoptic scale gravity waves are capable of releasing convective instability. The waves provide the mechanical lift necessary to weaken the inversion that would otherwise inhibit thunderstorm development. The lift is also sufficient to bring saturated air parcels to their level of free convection. For regions where soundings were affected in this manner, observations show that thunderstorm development and/or redevelopment was closely associated with the passage of gravity waves.

5) The characteristics of the waves appear to be independent of convective storm systems. This is exemplified by the movement of some gravity waves in regions void of any observed convection. Also, there were cases of gravity wave initiation of convection in which there existed little correlation between the magnitude of the waves and the intensity of the convection. The passage of large amplitude, well defined gravity waves was not necessarily followed by the outbreak of severe convective storms. The proper vertical moisture distribution had to exist to support the development of such storms.

While this research adds to the increasing evidence of the existence of subsynoptic scale gravity waves and documents the consequence of these waves in convectively unstable air, many important questions are left to be answered. How often these waves may occur, what special circumstances exist which allow for such a long duration, and to what extent gravity waves initiate severe convection can only be answered with additional case studies and more theoretical analyses. One can only speculate at present as to the question of what caused the gravity waves. Organized convection in eastern Kansas might have been responsible for the development of the waves which later moved through Iowa and Wisconsin. However, other gravity waves first appeared in regions void of convection like those that moved through southwest and south central Texas on the morning of 18 May, and in part the waves which originated in central Kansas near the clear tongue (as revealed by the ATS III photographs in Fig. 2) associated with the jet stream. The existence of a strong jet stream and developing cyclone is a common feature in this and other studies of large amplitude waves (Brunk, 1949; Tepper, 1951; Bosart and Cussens, 1973; Eom, 1975). Perhaps the mutual mass and wind adjustments that accompany the development of such large scale disturbances are also responsible for the generation of these waves. More detailed diagnostic and numerical analyses should be attempted to determine if this is indeed the case.

Finally, with regard to this and future case studies: it should be readily evident that limitations of the present operational data network, both in space and in time, make it impossible to complete a conclusive study of important aspects of this problem; e.g., the vertical structure of these waves, their interaction with individual convective cells, and their possible source mechanisms. It is gratifying that a portion of the proposed Severe Environmental Storms and Mesoscale Experiment (U. S. Department of Commerce, 1974) has been designed to supplement subsynoptic and mesoscale networks with sophisticated remote sensing instruments, providing for a more complete data source to study these problems.

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APPENDIX A

The band pass filter used to isolate the gravity waves consists of a high and low pass filter. The digitized pressure traces are first passed through the high pass filter, designed to eliminate low frequency waves representing synoptic scale trends, leaving pressure perturbation (p') traces. These are then passed through the low pass filter to eliminate the high frequency waves (representing the noise) from the p' traces.

The normal curve filter function was chosen because it does not exhibit either phase shifts or polarity reversals. The weights of this function are determined by the expression

$$\omega(d) = \frac{(2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(-d^2/2\sigma^2\right)}{\int_{-\infty}^{\infty} \frac{(2\pi\sigma^2)^{-\frac{1}{2}} \exp\left(-x^2/2\sigma^2\right)}{\sqrt{2\pi}}} dx},$$  \hspace{1cm} (A1)

where \(\omega(d)\) is the filter weight as a function of the position \(d\) (e.g., setting \(d=0\) yields the principal weight, \(d=1\) yields the weight influencing the next data value on either side of the principal data point, and so on). \(\sigma\) is the standard deviation of the normal curve smoothing function and is related to the "cut-off" frequency of the filter (Holloway, 1958). For this study \(\sigma\) was set equal to 5 for the high pass filter and 1 for the low pass filter. The sum of the weights, \(S\), must equal unity so as not to alter the mean value of the series where

$$S = \omega_0 + 2 \sum_{d=1}^{N} \omega(d).$$  \hspace{1cm} (A2)

As \(\sigma\) increases, the number of weights required for \(S\) to
equal unity also increases. For \( \sigma = 5 \), 13 weights were needed and since the time increment of the series was 0.25 h, this meant that 3.25 h of data on both ends of the time series would be lost due to the filtering process. This problem was alleviated by extending the analysis well beyond the period of wave activity.

The response functions \( R(f) \) for the normal weighted low and high pass filters are

\[
R_L(f) = \exp(-2\pi \sigma^2 f^2)
\]

(\ref{eq:R_L})

\[
R_H(f) = 1 - R_L(f)
\]

(\ref{eq:R_H})

and for the band pass filter used in this study (Fig. 4)

\[
R_B(f) = R_H(f)_{(0-1)} - (1 - R_L(f)_{(0-1)}).
\]

(\ref{eq:R_B})

**APPENDIX B**

The vertical motion profile was estimated using the following equations for vertical velocity at the 3 and 12 km levels (Eom, 1972):

\[
w_1 \text{ (vertical motion at } H_1; 3 \text{ km)} = \frac{i k H_1 \rho'}{\rho_0}
\]

(\ref{eq:w1})

\[
w_2 \text{ (vertical motion at } H_2; 12 \text{ km)} = \frac{i k H_1 (\bar{U} - \bar{c})(c/H_1 - g(1-N)) \rho'}{c^2 \rho_0 (M-N)}
\]

(\ref{eq:w2})

Here \( \bar{U} \) is the average mid-layer wind component in the direction of wave propagation, \( g \) is gravity, and \( N \) and \( M \) are the density ratios \( \rho_2/\rho_1 \) and \( \rho_3/\rho_1 \). To eliminate the discontinuity at \( H_1 \) due to the discontinuous wind shear, a modified vertical velocity \( w_1^* \) is calculated,

\[
w_1^* = \frac{w_1 + (1 - \bar{U}/c) w_1}{2}
\]

(\ref{eq:w1*)}

Note that \( w_1 \) and \( (1 - \bar{U}/c) w_1 \) are the vertical velocities just below and above \( H_1 \). \( W_2 \) is modified in a similar manner.

The lateral dimension of the wave is accounted for by replacing \( k \) in such a way that

\[
k^* = \sqrt{k^2 + \ell^2},
\]

(\ref{eq:k*)}

where \( \ell \) is the wavenumber perpendicular to the axis of wave movement. Since the period, \( \tau \), is invariant, \( c \) must also be modified,

\[
c^* = \frac{2\pi}{k^* \tau}
\]

(\ref{eq:c*)}

as is \( \bar{U} \),

\[
\bar{U}^* = \bar{U} c^*.
\]

(\ref{eq:U*)}

Considering these modifications and specifying all the variables from the observations (Fig. 12), the vertical velocities at 3 and 12 km are

\[
w_1^* = 0.28 \text{ m s}^{-1}
\]

\[
w_2^* = -0.16 \text{ m s}^{-1}.
\]

Assuming \( \omega(0) = 0 \), a linear profile was constructed from \( z = 0 \) to \( z = 3 \) km, and from \( z = 3 \) km to \( z = 12 \) km. This profile represents the model vertical motion midway between the trough and advancing ridge of the wave (Fig. 12). As a check on the magnitude of \( w_1^* \), note that the 0.28 m sec\(^{-1}\) value at \( H_1 \) corresponds to a surface convergence of nearly \( 1.0 \times 10^4 \) s\(^{-1}\), the magnitude measured for subsynoptic and mesoscale gravity waves in western Japan (Matsumoto et al., 1967b). It is thus believed that the model profile should be at least fairly representative for the lower troposphere where vertical motions will have the most significant impact on convectively unstable soundings.

To determine the effect of the modeled wave on the atmosphere a layer lift procedure was applied to atmospheric soundings. First the mean vertical velocity, \( \omega(z) \), was determined for nine sub-layers, each 1 km deep. Then by integrating \( \omega(z) \) over half a wave period the maximum displacement \( D_m(Z) \) for each layer was calculated where

\[
D_m(Z) = \omega(z)^{\tau}.
\]

(\ref{eq:D_m})

The displacement values for the 9 sublayers are shown in Fig. 12.

**APPENDIX C**

**Radar Legend**

Type: RW rainshower

TRW thundershower

Intensity: (–) light

(+) moderate

(++) heavy

(++++) very heavy

Intensity tendency:

(–) decrease

(+) increase

NC no change

NEW new cells developing

Echo movement: ddvv

dd direction (24° = 240°)

vv speed (m s\(^{-1}\))

Echo coverage (tenths):

\( \odot \) 1 → 4

\( \ominus \) 5 → 8

\( \bullet \) 9 → 10
REFERENCES


