The Energy Budget in a Clear Air Turbulence Zone as Observed by Aircraft

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(Manuscript received 20 December 1974; in revised form 11 March 1975)

ABSTRACT

High sample rate aircraft data in a zone of moderate turbulence are analyzed to determine a turbulent energy budget. Combination of the Reynolds stress, buoyancy, and frictional dissipation terms produces a good balance. A further look at the Reynolds stress component shows the relation of the turbulent zone growth and decay with the synoptic situation. The need is demonstrated for including turbulent mixing processes in modeling fronts.

1. Introduction

In a previous study by Shapiro (1974a), the three-dimensional structure of a multiple frontal zone-jet stream system was described using a composite of conventional aerological data and meteorologically instrumented research aircraft data. Here we present the results of utilizing more detailed three-dimensional air motion and temperature data taken within two patches of clear air turbulence situated within one of the zones of the previously cited study. As a review, Fig. 1 shows the two aircraft flight tracks and wind and potential temperature analysis for this case. The original traces as measured by the NCAR Buffalo aircraft at four vertical levels are presented in Fig. 2.

2. The energy budget

The quality and high sample rate of the data, coupled with the judicious and fortuitous placement of the airplanes within the turbulent zone, suggested that a turbulent energy budget could be computed. Two penetrations by the NCAR Buffalo (for details on Buffalo instrumentation, see Lenschow, 1972) into the strongest turbulence were chosen for this more detailed analysis: the 470 mb segment from 23:33:00 to 23:38:00 GMT, and the 420 mb segment from 23:43:30 to 23:48:00 GMT (see Figs. 1 and 2). Only ten minutes and a kilometer, vertically, separate the two observations.

Following Dutton (1969), the turbulent energy equation is simplified by assuming nondivergent motion, horizontal homogeneity, and a vanishing mean vertical velocity within the turbulent patch. We obtain then

\[
\frac{\partial \langle E \rangle}{\partial t} = -\langle uw \rangle \frac{\partial U}{\partial z} - \langle vw \rangle \frac{\partial V}{\partial z} \frac{g}{\theta_0} \frac{\partial \theta}{\partial z} - \epsilon \frac{\partial}{\partial \theta} \left( \langle \theta' \omega \rangle + \langle \omega E \rangle \right). \tag{1}
\]

Here, the brackets denote an average within the turbulent patch. The lower case \( u, v, \) and \( w \) are perturbations from the mean flow, \( U \) and \( V \). The specific turbulent kinetic energy is \( E = \frac{1}{2} (u^2 + v^2 + w^2) \). The quantities \( \theta' \) and \( \theta_0 \) are the perturbation and mean potential temperatures; \( \theta' \) and \( \theta_0 \) represent the perturbation pressure and the mean density, respectively, in the turbulent patch.

Each term of Eq. (1) represents an easily identifiable, physically meaningful quantity. The term on the left is the local rate of change of the turbulent kinetic energy. Since we did not fly through any one turbulent patch twice, we cannot say what \( \partial \langle E \rangle / \partial t \) is. We suspect, however, it is relatively small since several passes through the turbulent zone at different altitudes at different times (see Figs. 1 and 2) showed the zone to be somewhat long-lived.

The terms \( -\langle uw \rangle \partial U / \partial z \) and \( -\langle vw \rangle \partial V / \partial z \) show to what extent the shear of the mean wind is reduced by Reynolds stresses to enhance the turbulence. With our positive shear (see Fig. 1), such enhancement can occur only if the covariances \( \langle uw \rangle \) and \( \langle vw \rangle \) are negative. The term \( (g/\theta_0)\langle \theta' \omega \rangle \) represents the loss of energy by buoyant forces if the covariance is negative. The frictional dissipation rate of energy is \( \epsilon \). The last term is the vertical transport of energy by the turbulence itself; it is ignored since it is believed small, and is difficult to compute from aircraft data.
In Table 1 we present the computation of the energy budget for the two turbulent patches at 470 and 420 mb. For all calculations \( U \) represents the longitudinal wind component, that is, the wind parallel to the flight track (257°). The lateral wind component is \( V \) (167°) and the vertical component is \( W \). In this turbulent patch \( U \) is always much less than \( V \) and \( \partial U/\partial z \) is negligibly small. The quantities \( \partial V/\partial z \) and \( \partial W/\partial z \) were computed in the turbulent patches on the analyzed cross section (Fig. 1) using a height differential \( \partial z \) of about 1 km. The covariances in Table 1 were computed after linear trends were removed.

The theory of the inertial subrange (see Chapters 2.9 and 5.2, Lumley and Panofsky, 1964) is used in calculating the dissipation rate, \( \epsilon \). In the turbulent patch, if the power spectral density of a velocity component falls off with the \(-5/3\) power of wavenumber, one can get an estimate of \( \epsilon \) by solving

\[
E(k) = A k^{-5/3},
\]

where \( E \) is the power spectral density, \( k \) is wavenumber expressed in radians per unit length, and \( A \) is a constant equal to about 0.5 for longitudinal (along flight track) perturbations and about 0.67 for lateral and vertical perturbations. A recent experimental determination of \( A \) and discussion of previous attempts was done by Gibson et al. (1970). Figures 3 and 4 show the spectra of the three wind components for the 470 and 420 mb patches, where it is clear that the \(-5/3\) power law is approximated.

Equation (2) can be solved for \( \epsilon \) by extending a \(-5/3\) slope line drawn through the spectrum to the point where \( k = 1 \) and reading the associated \( E(k) \). (The oblique lines in Figs. 3 and 4 have \(-5/3\) slope but are
Fig. 2. Original traces of 1 s averaged Buffalo meteorological data as taken at 470, 420, 370, and 400 mb along the flight paths shown in Fig. 1. Direction of aircraft motion shown by arrows at bottom.

Fig. 3. Power spectral density of the three wind components ($U$, longitudinal; $V$, lateral; $w$, vertical) in the 470 mb turbulent zone, from the NCAR Buffalo aircraft. The oblique line has $-5/3$ slope; when positioned through the spectrum, such a line can be used to determine the frictional dissipation rate, $\epsilon$. In this paper an alternative integral method, summing only the last decade of wavenumbers, was used to determine $\epsilon$. The wavenumber axis is obtained by multiplying the frequency axis by $2\pi$ and dividing by the aircraft ground speed ($\sim$100 m/s).

Fig. 4. As in Fig. 3, but for the 420 mb zone.
Table 1. Energy budget computation for Buffalo penetration of a turbulent zone, 19 April 1971.

<table>
<thead>
<tr>
<th></th>
<th>470 mb</th>
<th>420 mb</th>
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<tbody>
<tr>
<td><strong>Energy production</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Shear (\partial V/\partial z) ((107'))</td>
<td>(1.7 \times 10^{-8} \text{ s}^{-1})</td>
<td>(1.7 \times 10^{-8} \text{ s}^{-1})</td>
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<tr>
<td>B. Reynolds stresses (\langle uv \rangle)</td>
<td>(-1.1 \text{ m}^2 \text{ s}^{-2})</td>
<td>(-1.3 \text{ m}^2 \text{ s}^{-2})</td>
</tr>
<tr>
<td>C. Total production (-\langle uv \partial V/\partial z\rangle)</td>
<td>(1.8 \times 10^{-7} \text{ m}^3 \text{ s}^{-2})</td>
<td>(2.6 \times 10^{-7} \text{ m}^3 \text{ s}^{-2})</td>
</tr>
<tr>
<td><strong>Energy losses</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>A. Temperature flux (\langle u' \theta' \rangle)</td>
<td>(-0.22 \text{ m} \text{ s}^{-1} \text{ K})</td>
<td>(-0.15 \text{ m} \text{ s}^{-1} \text{ K})</td>
</tr>
<tr>
<td>B. Buoyancy (\langle \varepsilon \theta' \rangle \langle u' \theta' \rangle)</td>
<td>(-0.69 \times 10^{-3} \text{ m} \text{ s}^{-1})</td>
<td>(-0.47 \times 10^{-3} \text{ m} \text{ s}^{-1})</td>
</tr>
<tr>
<td>C. Frictional dissipation (-\varepsilon)</td>
<td>(-1.0 \times 10^{-4} \text{ m}^2 \text{ s}^{-2})</td>
<td>(-1.7 \times 10^{-4} \text{ m}^2 \text{ s}^{-2})</td>
</tr>
<tr>
<td>D. Total losses (\langle u' \theta' \rangle \langle u' \theta' \rangle - \varepsilon)</td>
<td>(-1.7 \times 10^{-3} \text{ m}^2 \text{ s}^{-2})</td>
<td>(-2.2 \times 10^{-3} \text{ m}^2 \text{ s}^{-2})</td>
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<tr>
<td><strong>Residual</strong></td>
<td>(+0.1 \times 10^{-2} \text{ m}^2 \text{ s}^{-2})</td>
<td>(+0.4 \times 10^{-2} \text{ m}^2 \text{ s}^{-2})</td>
</tr>
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</table>

Additional quantities

|                |        |        |
| A. Stability \(\bar{\varepsilon} \theta' \) | \(1.0 \times 10^{-3} \text{ s}^{-2}\) | \(1.0 \times 10^{-4} \text{ s}^{-2}\) |
| B. Richardson number \(\langle \varepsilon \theta' \rangle / \langle \partial V/\partial z \rangle^2\) | 0.35    | 0.35   |
| C. Flux Richardson number \(\langle u' \theta' \rangle / \langle uv \rangle \partial V/\partial z\) | 0.38    | 0.18   |

within the inertial subrange:

\[
\int_{k_1}^{k_2} E(k) dk = \frac{3}{4} A \varepsilon (k_1^{-1} - k_2^{-1}).
\]  

(3)

We performed this integration for each of the three wind components and then averaged the \(\varepsilon\) so obtained. It is interesting that \(\varepsilon\) as estimated with the longitudinal component was, for both the 470 and 420 mb patches, about 35% larger than the lateral. In turn, the laterally-determined \(\varepsilon\) was, for both patches, somewhat larger than the vertical. This same pattern, as yet unexplained, was observed by Lilly et al. (1974) for a large number of HICAT flights. It is not clear which estimate of \(\varepsilon\) is the most accurate.

In Figs. 3–5, each of the points on the spectral curves is a centered average of the adjacent eleven spectral estimates, except at either end where 1-, 3-, 5-, 7-, and 9-point averages are taken. A fast Fourier transform technique was used on the series with linear trends removed initially. The wavenumber axis is obtained by multiplying the frequency axis by \(2\pi\) and dividing by the average ground speed of the aircraft (\(\sim 100 \text{ m/s}\)).

The residual shown in Table 1 is reasonably small, showing that we have accounted for the significant components of the budget. In addition to the simplifying assumptions made at the beginning, errors might be traced to gravity waves generated by the Rocky Mountains, which lie directly beneath the two turbulent patches analyzed. However, since the southerly flow is roughly parallel to, and not traversing, the Rockies, we do not expect the flow to be greatly disturbed by topography.

It is interesting to note what wavenumbers are contributing to the covariances \(\langle uv \rangle\) and \(\langle w \theta' \rangle\) in the budget. The covariances might result from a spurious residual of large positive and negative contributions from a few wavenumbers, or contributions from only a limited band of wavenumbers. However, in Fig. 5 the cospectra of the lateral wind component and \(w\) for the two turbulent patches show that nearly all wavenumbers contribute to the net negative covariance.

3. Predicted decay of the zone

Given the above momentum flux calculations, it is possible to obtain an estimate of the rate at which turbulent-scale vertical mixing is destroying the mean vertical wind shear within the center of the frontal zone.

Consider the along-wind component of the equation for mean horizontal motion in which the local tendency

![Fig. 5. Cospectra of the lateral and vertical winds (V and w) in the 470 and 420 mb turbulent zones from the NCAR Buffalo aircraft. Note that virtually all wavenumbers contribute to the net negative covariance.](image-url)
term $\partial V/\partial t$ and vertical Reynolds stress term $\langle vw \rangle$ are retained, with all other terms dropped for the sake of argument,

$$\frac{\partial V}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial z} \rho \langle vw \rangle. \tag{4}$$

Differentiation of Eq. (4), neglecting vertical variations of density, $\rho$, with height over the shallow depth of the frontal zone, yields

$$\frac{\partial}{\partial t} \left( \frac{\partial V}{\partial z} \right) = -\frac{\partial^2}{\partial z^2} \langle vw \rangle. \tag{5}$$

Thus, from Eq. (5), the rate of dissipation of mean vertical wind shear is given by the second derivative of the vertical Reynolds stress.

Since $\langle vw \rangle$ of Eq. (5) was evaluated from continuous horizontal measurements at discrete vertical levels which were separated in time, we can make use of vertical extrapolation of horizontal measurement techniques to obtain an approximation to the second vertical derivative of the Reynolds stress. Let us assume that the turbulent motions are confined to the interior of the frontal layer and taper to a minimum toward the lateral boundaries and in the synoptic environment on either side of both shearing zones. Then the inferred vertical profile of the Reynolds stress is one of near zero at the top and bottom of the zone with a maximum (negative) value in between in the center. Using a second order finite difference approximation to the second derivative for a linear profile of $\langle vw \rangle$ of the form shown in Fig. 6, we obtain for a frontal layer 1.5 km thick,

$$\frac{\partial}{\partial z} \left( \frac{\partial V}{\partial z} \right) \approx -7 \text{ (m$\,$s$^{-1}$ km$^{-1}$)} h^{-1}. \tag{6}$$

Given the initial mean shear of 15 m s$^{-1}$ km$^{-1}$, persistent turbulent motions of the kind observed would nearly destroy the shear of the zone in less than three hours. Comparison of the Eq. (6) estimation with the frontogenesis calculations of Newton (1954) shows this estimate to be of the same order of magnitude as the rate at which mean vertical shear is created by large-scale frontogenetic processes. Thus, the implication is that numerical prediction and simulation models which include frontal-scale development (see Shapiro, 1974b) will require the incorporation of a parameterization for such turbulent mixing processes, the role of which would be to limit the development of subcritical Richardson numbers, vertical mean shear, and the minimum vertical scale of atmospheric fronts.

4. Conclusion

With a well-instrumented, well-positioned aircraft capable of recording data at several samples per second, it is possible to determine a reasonably balanced energy budget in a turbulence zone. A further analysis of the Reynolds stress component of the budget shows that wind shear is destroyed in the turbulence zone at roughly the rate that large-scale frontogenetic processes create that shear. A proper accounting for turbulent mixing processes appears important for models which include frontogenesis.

Acknowledgments. The authors are grateful to Drs. D. K. Lilly and J. W. Deardorff for their helpful comments. Bonnie Gancik of the NCAR Computing Facility competently transformed the raw aircraft data tapes into useable formats.

REFERENCES


