Operational Objective Temperature Forecasts at Non-MOS[1] Stations

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ABSTRACT

A method is presented for increasing the number of points for which a local forecast office may obtain objective maximum and minimum temperature forecasts twice daily. The method is simple enough that it may be both developed locally and used operationally in daily forecasts with a minimum amount of time involved. The additional objective temperature forecasts rely heavily on objective temperature forecasts produced twice daily at the National Meteorological Center, but extrapolates these forecasts through statistical methods to secondary points. Therefore, they remain strongly linked to the operational numerical models. Examples of "predictor" curves are also presented.

1. Introduction

An automated system for objective prediction of maximum and minimum temperatures for numerous points in the conterminous United States has been used operationally by the National Weather Service (NWS) since 1968. Initially these forecasts utilized the "perfect prog" (Klein, 1970; Klein and Lewis, 1970) approach, using predictions of 700 mb height and 700–1000 mb thickness from operational numerical models run twice daily at the National Meteorological Center (NMC), together with previous values of maximum and minimum temperatures at selected points, to produce objective temperature forecasts at 131 United States locations. The day of the year was also used as a climatological term in these equations.

Since 1973, however, the model output statistics (MOS) technique (Glahn and Lowry, 1972; National Weather Service, 1973; Klein and Hammons, 1975) has been used to derive the operational temperature prediction equations. This technique statistically relates maximum and minimum temperatures at given points to the output from operational NWS numerical models. The MOS technique showed a significant improvement in forecast accuracy over the perfect prog method from the outset. Subsequent refinements to the MOS program (National Weather Service, 1974, 1975a, b, c, 1976) have resulted in still further improvements to the objective temperature forecast guidance.

However, in spite of this improved accuracy in the MOS temperature forecasts as well as its greater station coverage (MOS forecasts are now produced for 228 stations (see Fig. 1) in the conterminous United States), there are still not enough forecast points to describe even the normal variations of maximum and minimum temperatures in or near mountainous terrain. It is well known that mountains can, and often do, produce unusually large variations in weather over very small distances. Indeed, these "local effects" can be quite pronounced in nearly all parameters, both in terms of daily weather and long-term climatic differences. This is especially true in the western United States where differential cloud cover, snow cover, drainage wind situations, etc., can result in simultaneous temperature differences of 30°F or more over distances of less than 50 mi, within the same air mass and at nearly the same elevation.

One solution to the above problem, to more adequately describe the temperature field, would be to increase the MOS station density. However, the addition of each new MOS station to the network would require a separate 10-term multiple linear regression equation for each projection time (four), each NMC daily run time (0000 and 1200 GMT) (two), and each season (four); or 32 additional equations per new MOS station. Further, "backup" equations would have to be developed for each primary equation which incorporated data from local synoptic observations as one or more of the predictors in the 10-term prediction equation. Thus, doubling the MOS network density could require more than 10 000 additional equations, and would still not provide the station density required in

[1] A MOS station is one for which objective model output statistics forecasts are routinely produced.

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some areas to adequately describe the temperature field. Therefore, the MOS approach does not appear to be a feasible means for significant expansion of the objective temperature forecast network. Yet the greatly improved forecast skill shown by MOS methods indicates that any alternative objective forecast scheme devised to expand this network should continue to rely on a mixture of statistics and dynamics.

2. Objective forecasting aspects

An objective forecast system, according to Allen and Vernon (1951), is "one which can produce one and only one forecast from a specific set of data." Further it "does not depend for its accuracy upon the forecasting experience or the subjective judgment of the meterorologist using it." Nowhere is such a system more desirable than in mountainous terrain where a virtually limitless number of local effects can be observed and ultimately incorporated into forecasts. However, at smaller weather service forecast offices (WSFO's) the forecaster turnover rate often precludes an individual forecaster from gaining enough experience to fully do so. Therefore, we have attempted to develop for Wyoming a simple method for obtaining twice daily objective temperature forecasts for 17 locations throughout the state in addition to the five locations for which MOS forecasts are now produced. This 22-station network (see Fig. 2) should provide consistent and more reliable temperature forecasts for the entire state regardless of forecaster experience. This method begins with the MOS temperature guidance and extrapolates these data through statistical methods to the secondary points.

3. Method

In order to extrapolate an objective MOS forecast temperature to a secondary point (the non-MOS station) it must be determined that the two points are functionally related in temperature, and then determine what mathematical expression best describes this functional relationship. For example, if the expression

$$Y=f(X) \qquad (1)$$

accurately describes the temperature relationship between a MOS station [X] and a given non-MOS station [Y], then this same expression would also be applicable as a prediction equation,

$$\hat{Y}=f(\hat{X}), \qquad (2)$$

where the caret indicates a forecast (or predicted) value. Since, for our purposes $\hat{X}$ in all cases is an actual MOS forecast, then the forecast temperature at [Y] is strongly linked to the NMC dynamic numerical models through this pseudo "perfect prog" approach.

Observed daily maximum and minimum temperatures for each month for the 20-year period (1951–70) for each of the 17 selected non-MOS stations were compared with similar data from each of three or four nearby MOS stations to determine in each case the best fitting pair. Both linear and nonlinear equations were evaluated, with the nonlinear equation in all cases taking the form

$$Y=\alpha_0+\alpha_1X+\alpha_2X^2. \qquad (3)$$

This second-order polynomial was chosen for simplicity's sake only and does not imply that some higher order polynomial, or some other nonlinear function

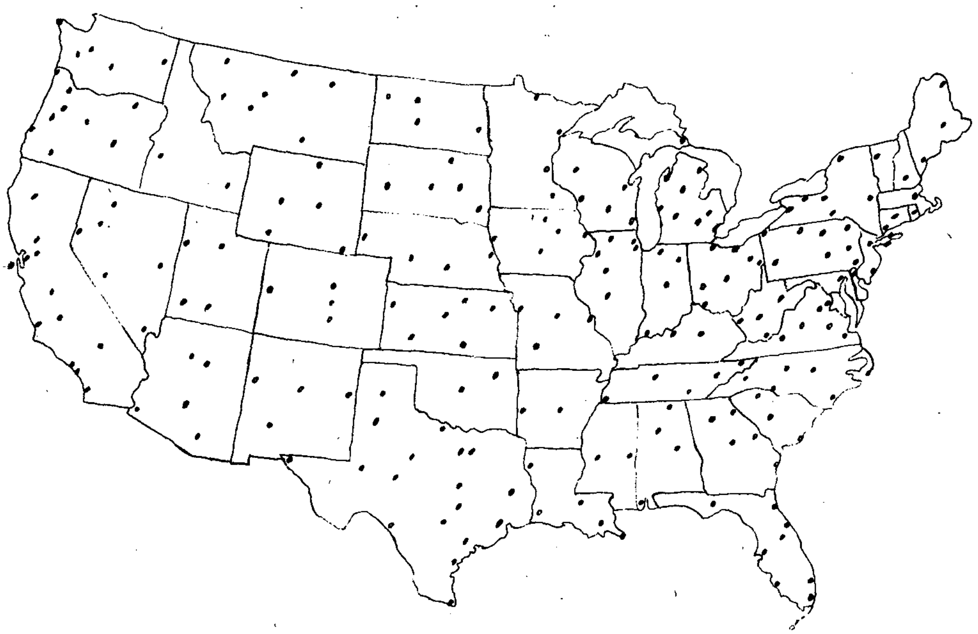

FIG. 1. Current MOS temperature forecast network of 228 stations in the conterminous United States.

might, in some cases, better describe the temperature relationship between a given MOS/non-MOS pair. This remains for future investigations.

If Eq. (3) adequately describes the climatic relationship between the daily maximum (or minimum) temperature at two points for a given month, then a prediction equation of the same form may be written, i.e.,

$$\hat{Y}=\alpha_0+\alpha_1\hat{X}+\alpha_2(\hat{X})^2, \tag{4}$$

where $\hat{X}$ again is a forecast MOS temperature. The alpha coefficients in (4) can be determined using standard statistical methods described in the Appendix.

Separate second order equations were determined for each of 52 selected MOS/non-MOS pairs, for both daily maximum and minimum temperatures, and for each month of the year; or 1248 separate equations. We simultaneously determined how well each polynomial actually fit its data set by evaluating the corresponding nonlinear correlation coefficient given by the equation

$$r=\left[1-\left(\frac{\Sigma Y^2-\alpha_0\Sigma Y-\alpha_1\Sigma XY-\alpha_2\Sigma X^2Y}{\Sigma Y^2-(\Sigma Y)^2/n}\right)\right]^{\frac{1}{2}}, \tag{5}$$

where standard mathematical symbols are used (Spiegel, 1961).

Only the MOS/non-MOS pair with the highest correlation in each case was retained. For sufficiently low values[3] of r, it was determined that the second-order polynomial did not describe a satisfactory relationship for that particular MOS/non-MOS pair, and some other curvilinear expression should be investigated. However, for a sufficiently high correlation value, the resulting polynomial equation could be expected to give a reasonably good objective forecast temperature at [Y] given an accurate MOS forecast temperature for [X].

Such "extrapolated" forecasts have all the inherent errors of the MOS forecasts (National Weather Service, 1973; Klein and Hammons, 1975) plus additional curve fit errors. However, in regions where very large climatic differences occur over short distances, and these differences also vary greatly from season to season, such extrapolated MOS temperatures should provide stability to the temperature forecasting program.

To further simplify the extrapolation process, graphs were prepared using the polynomials evaluated above as predictor curves for both maximum and minimum temperatures for each month of the year for each of the 17 non-MOS stations. (These equations may also be easily adapted to card-reading programmable calculators or to AFOS[4] minicomputers for even quicker and easier evaluation.) Once the MOS temperatures have been transmitted from NMC, either via teletype or facsimile, the extrapolated forecast temperatures

[3] $r\leqslant 0.7$ was arbitrarily selected as the cutoff value.

[4] Automation of Field Operations and Services—proposed National Weather Service communication and display network.

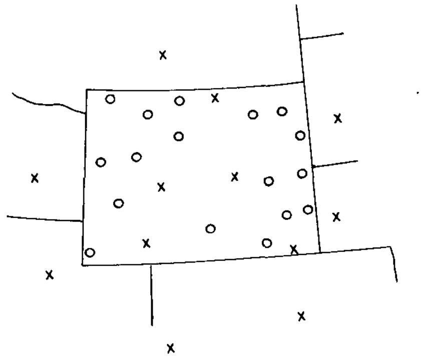

FIG. 2. Current MOS temperature forecast locations (×) in Wyoming and the immediate surrounding area, plus the 17 additional non-MOS locations (○) for which objective temperature forecasts are now available.

may be easily obtained from these graphs. The forecaster will still have to make subjective adjustments due to unusual synoptic differences such as sharp frontal zones, differential cloud cover, etc. It should be noted, however, that normal differences (i.e., seasonally varying snow cover) which are reflected in the climatology will already be incorporated in the polynomial prediction curve. Fig. 3 shows an example of this. The March minimum temperature prediction curve for Cody using MOS data for Billings, Mont. shows a very flat slope at the very cold end of the curve. This reflects the predominance of strong downslope drainage winds at Cody during the spring season which prevents many of these very cold temperature outbreaks. By the summer season (July), the shape of the minimum temperature prediction curve has changed drastically. In July the cold end of the curve shows a very steep slope reflecting the persistence of the clear, calm nights with greater radiation loss from Cody (5093 ft) than Billings (3610 ft). Therefore, differences in elevation as well as other local effects are automatically incorporated into the prediction curves.

4. Error analysis

There are two primary error sources which contribute to the total error E in objective temperature forecasts at non-MOS stations: the error due to the scatter of the data points (observed) about each regression curve, and the error contributed as a result of random errors in the MOS forecasts used to obtain the non-MOS forecasts.

A measure of the first error source is obtained by evaluating the standard error of estimate $S_{x\cdot y}$ for each predictor curve using

$$S_{x\cdot y}=\mathrm{SD}(1-r^2)^{\frac{1}{2}}, \tag{6}$$

where SD is the standard deviation of the daily maximum or minimum temperature of the non-MOS station, and r, the corresponding nonlinear correlation coefficient (Spiegel, 1961). According to Brier (1976,

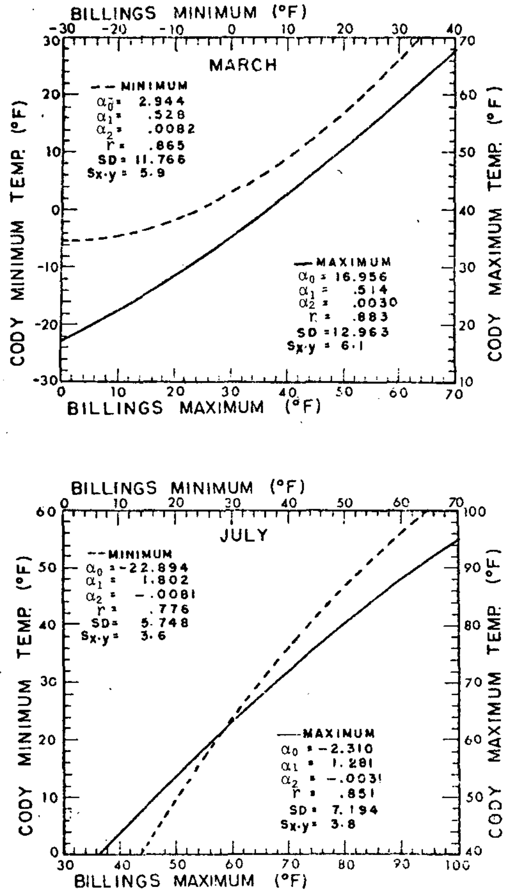

FIG. 3. Examples of predictor curves for Cody from MOS forecasts for Billings for March (top) and July (bottom). Listed for each curve are the polynomial coefficients (α_0, α_1 and α_2), the nonlinear correlation coefficient (r), the standard deviation of the daily maximum and minimum temperature (SD) of the non-MOS station (Cody), and the standard error of estimate ($S_{x \cdot y}$) of the objective non-MOS forecast temperature assuming perfect MOS forecasts.

personal communication) the second source of error (that due to MOS errors) may be expressed as

$$\sigma_{\text{mos}}^2 = \alpha_1^2\epsilon^2 + 2\alpha_1\alpha_2\epsilon^3 + \alpha_2^2\epsilon^4, \tag{7}$$

where ϵ is the root-mean-square error of the MOS forecasts and α_1 and α_2 the coefficients of the appropriate polynomial predictor curve.

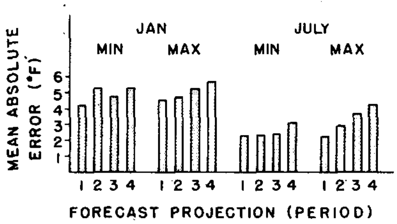

FIG. 4. Preliminary values for the mean absolute error of the MOS temperature forecasts for Lander for the years 1974, 1975 and 1976. (Courtesy National Weather Service, Techniques Development Laboratory.)

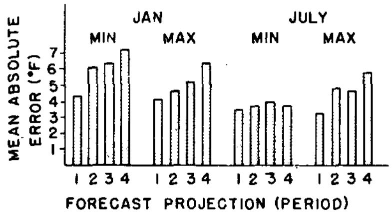

FIG. 5. As in Fig. 4 except for Rapid City.

For normally distributed errors, the third moment term ($2\alpha_1\alpha_2\epsilon^3$) goes to zero (Brier, 1976, personal communication). Also, the fourth moment ($\alpha_2^2\epsilon^4$) is always small compared to $\alpha_1^2\epsilon^2$, and may be ignored. Thus, (7) may be written

$$\sigma_{\text{mos}}^2 \approx \alpha_1^2\epsilon^2. \tag{8}$$

Then the total error E for the non-MOS forecasts may be approximated as

$$E \approx [(S_{x \cdot y})^2 + \alpha_1^2\epsilon^2]^{\frac{1}{2}}. \tag{9}$$

The mean absolute error of the MOS forecasts τ may be substituted for ϵ in (9) if τ is first multiplied by the factor 1.5. Mean absolute errors for three representative MOS stations, showing values for each of the four forecast projections (periods) for both January and July, are given in Figs. 4, 5 and 6. The data in these three figures are only preliminary, and are based on the years 1974, 1975 and 1976.

Monthly values of the standard error of estimate as well as the nonlinear correlation coefficient and the standard deviation for three non-MOS stations are shown in Figs. 7, 8 and 9. The value of the standard error of estimate for each predictor curve has been printed on each operational graph (see Fig. 3). However, the mean absolute error of the appropriate MOS station has not been printed, since this value will likely change as the data base for MOS forecasts expands, or as techniques used to obtain these forecasts change.

5. Conclusions

Because of the wide spacing of MOS temperature forecast stations in the western United States, coupled

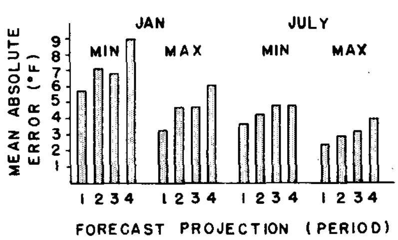

Fig. 6. As in Fig. 4 except for Casper.

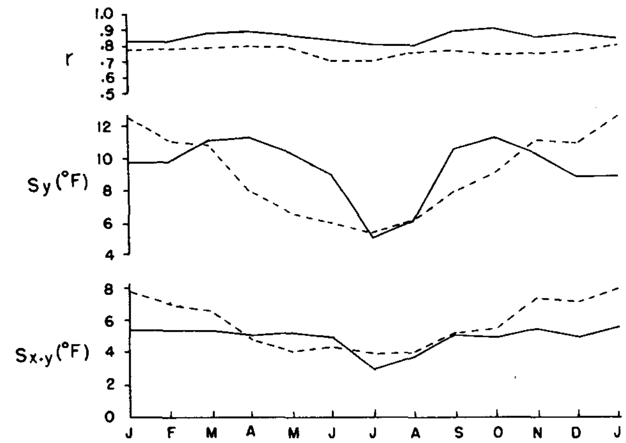

FIG. 7. Monthly values of the standard error of estimate ($S_{x\cdot y}$) and standard deviation (S_y) of the daily maximum (——) and minimum (– – –) temperature for Rawlins. Also the monthly correlation coefficient (r) between Rawlins and the appropriate MOS station.

with the difficulty of forecasting temperatures in this region, a scheme for expanding the number of objective temperature forecast points has been developed. The method developed relies strongly on the objective temperature forecast output from NMC, but it is not tied to any particular NMC method for obtaining this output. As the operational numerical models, as well as the statistical application of output from these models, continue to improve, these extrapolated temperature forecasts should also improve.

For those polynomial equations which yielded low correlation coefficients, it is planned to investigate the effects of using a higher order polynomial or other nonlinear equation to improve the correlation. Since the correlation coefficient and the standard error of estimate for each data set is printed with the resultant curve on each graph, the forecaster has a real measure of the confidence he can place in the predicted temperature obtained from that particular curve. Any subjec-

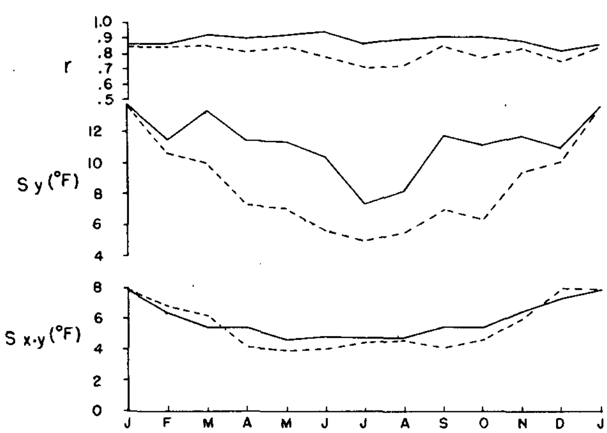

FIG. 8. As in Fig. 7 except for Lovell.

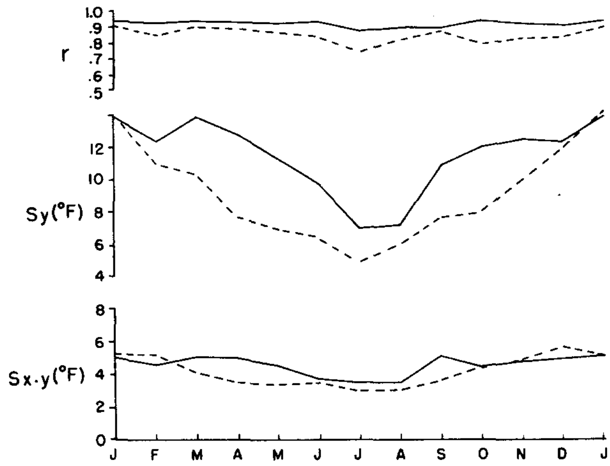

FIG. 9. As in Fig. 7 except for Torrington.

tive adjustment to objective values should be made with these statistics as well as surface synoptic and numerical model considerations in mind.

In any event, the greater density of stations for which objective temperature forecasts are now available in Wyoming should reduce the amount of experience necessary to make reasonably good forecasts for unusually difficult locations or situations.

APPENDIX

Evaluation of the Least-Squares Second-Order Polynomial

The alpha coefficients for the second order polynomial,

$$Y=\alpha_0+\alpha_1X+\alpha_2X^2, \tag{10}$$

had to be evaluated for each MOS/non-MOS station pair for both maximum and minimum temperatures for each month of the year. The method used to do so was the standard "least-squares" procedure (Spiegel, 1961). The normal equations for the least-squares parabola are given by

$$\Sigma Y=\alpha_0 n+\alpha_1\Sigma X+\alpha_2\Sigma X^2, \tag{11}$$

$$\Sigma XY=\alpha_0\Sigma X+\alpha_1\Sigma X^2+\alpha_2\Sigma X^3, \tag{12}$$

$$\Sigma X^2Y=\alpha_0\Sigma X^2+\alpha_1\Sigma X^3+\alpha_2\Sigma X^4. \tag{13}$$

X and Y represent the daily maximum (minimum) temperature at a given MOS and non-MOS station, respectively.

The summations were made on these daily maximum and minimum temperatures for each month over a 20-year period (1951–70). The alpha coefficients could then be evaluated using the following standard matrix solution for three equations and three unknowns.

$$\alpha_0=\frac{\begin{vmatrix}\Sigma Y & \Sigma X & \Sigma X^2\\ \Sigma XY & \Sigma X^2 & \Sigma X^3\\ \Sigma X^2Y & \Sigma X^3 & \Sigma X^4\end{vmatrix}}{\begin{vmatrix}n & \Sigma X & \Sigma X^2\\ \Sigma X & \Sigma X^2 & \Sigma X^3\\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4\end{vmatrix}},$$

$$\alpha_1=\frac{\begin{vmatrix}n & \Sigma Y & \Sigma X^2\\ \Sigma X & \Sigma XY & \Sigma X^3\\ \Sigma X^2 & \Sigma X^2Y & \Sigma X^4\end{vmatrix}}{\begin{vmatrix}n & \Sigma X & \Sigma X^2\\ \Sigma X & \Sigma X^2 & \Sigma X^3\\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4\end{vmatrix}}, \qquad (14)$$

$$\alpha_2=\frac{\begin{vmatrix}n & \Sigma X & \Sigma Y\\ \Sigma X & \Sigma X^2 & \Sigma XY\\ \Sigma X^2 & \Sigma X^3 & \Sigma X^2Y\end{vmatrix}}{\begin{vmatrix}n & \Sigma X & \Sigma X^2\\ \Sigma X & \Sigma X^2 & \Sigma X^3\\ \Sigma X^2 & \Sigma X^3 & \Sigma X^4\end{vmatrix}}.$$

Because of the very large numbers ($>10^{18}$) encountered in some of these situations, especially during the summer season, double precision had to be used on the Sigma 7 computer in solving the above matrices. This allowed the necessary accuracy to the fourth decimal place in the α_2 coefficient which was unobtainable otherwise.

If these summations are made on either a desk- or hand-held programmable calculator, the double precision method need not be used. Modern calculators normally have either a 10 or 12 place accuracy, either of which is sufficient for these computations.

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REFERENCES

Allen, R. A., and E. M. Vernon, 1951: Objective weather forecasting. *Compendium of Meteorology*, Thomas F. Malone, Ed., Amer. Meteor. Soc., 796–801.

Glahn, H. R., and D. A. Lowry, 1972: The use of model output statistics (MOS) in objective weather forecasting. *J. Appl. Meteor.*, **11**, 1203–1211.

Klein, W. H., 1970: The forecast research program of the techniques development laboratories. *Bull. Amer. Meteor. Soc.*, **51**, 133–142.

——, and F. Lewis, 1970: Computer forecasts of maximum and minimum temperatures. *J. Appl. Meteor.*, **9**, 350–359.

——, and G. A. Hammons, 1975: Maximum/minimum temperature forecasts based on model output statistics. *Mon. Wea. Rev.*, **103**, 796–806.

——, F. Lewis and G. A. Hammons, 1971: Recent developments in automated max/min temperature forecasting. *J. Appl. Meteor.*, **10**, 916–920.

National Weather Service, 1973: Maximum/minimum temperature forecasts based on model output statistics—No. 1. Tech. Proc. Bull., No. 94, 14 pp.

——, 1974: New MOS minimum/maximum temperature forecast equations—No. 5. Tech. Proc. Bull., No. 123, 8 pp.

——, 1975a: MOS maximum/minimum temperature forecast equations based on three-month seasons—No. 6. Tech. Proc. Bull., No. 144, 8 pp.

——, 1975b: MOS maximum/minimum temperature forecast equations based on three-month season—No. 7. Tech. Proc. Bull., No. 145, 6 pp.

——, 1975c: MOS maximum/minimum temperature forecast equations based on three-month season—No. 8. Tech Proc. Bull., No. 148, 5 pp.

——, 1976: MOS maximum/minimum temperature forecast equations based on three-month seasons—No. 9. Tech. Proc. Bull., No. 155. 5 pp.

Spiegel, M. R., 1961: *Theory and Problems of Statistics.* Schaum Publishing Co., 359 pp.