A *Z-R* Relationship for the GATE B-Scale Array

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ABSTRACT

A better understanding of how the precipitation budget operates in tropical convective systems is a prime objective of the GATE research effort. Measurement of rainfall rate with shipboard radar is the principal method by which precipitation from tropical clouds that develop within the GATE B-scale array will be determined. Knowledge of the relationship between radar reflectivity (Z) and rainfall rate (R) is essential for an accurate interpretation of precipitation data derived through the use of radar technology. The Z-R relationship is determined through application of a least-squares linear regression to data points derived by appropriate integration of the third and sixth moments of a series of raindrop size spectra.

Drop spectra measurements were obtained during GATE by means of a foil impactor operated at cloud-base level on board the NOAA DC-6 aircraft. A total of 107 Z-R data points are available, representing showers occurring on 12 days. The best-fit Z-R relationship for the cloud-base aircraft foil data showed little variability from day to day or on the basis of stratification by rain rate. For all foil data combined, the best-fit Z-R relationship was found to have the form $Z=170\,R^{1.52}$, which gives, for example, rain rates of 66, 15 and 3 mm h^{-1} for Z values of 50, 40 and 30 dBZ, respectively.

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1. Introduction

The manner in which cloud systems process available moisture plays a major role in the energetics of the tropical atmosphere. The release of latent heat of condensation is one of the primary energy sources available to drive the tropical weather machine. The efficiency of the water budget (i.e., ratio of precipitation out to moisture flux in) operating within mesoscale cloud systems provides a measure of how energy is being converted and distributed. A knowledge of how the water budget operates is an essential prerequisite to a further understanding of energy exchange processes which exercise such a controlling influence on the dynamics and kinematics of the atmosphere. An accurate measurement of precipitation from cloud systems is an obvious fundamental necessity for the derivation of such water budgets and is a prime objective of research carried out within the B-scale array of the GARP Atlantic Tropical Experiment (GATE).

As the GATE B-scale array encompassed about 75 000 km^2 of ocean well removed (500–1000 km) from any major land mass (Fig. 1), shipboard radar formed the primary tool to obtain information about hydrometeor water contents and precipitation rates. It has been found by many workers that radar reflectivity (Z) and rainfall rate (R) are related through an expression of the form

$$Z=aR^b, \tag{1}$$

where the coefficient a and exponent b have been found to vary widely as a function of geographical location and meteorological conditions. Battan (1973) provides an extensive bibliography of published Z-R relationships for many different areas. Since for the C-band (5.6 cm wavelength) radar used on GATE ships, the Rayleigh scattering approximation is valid for all but the largest of raindrops, the rainfall rate and radar reflectivity can be calculated from an integration of the third and sixth moments, respectively, of the drop size spectrum. Radar reflectivity can be computed from

$$Z=\int N(D)D^6dD, \tag{2}$$

where $N(D)$ is the drop concentration per unit volume, D the drop diameter, and Z is expressed in the units mm^6 m^{-3}. The rainfall rate can be calculated from

$$R=(\pi/6)\int N(D)D^3\rho V_T dD, \tag{3}$$

where ρ is the density of water and V_T the terminal velocity of raindrops of diameter D. An understanding of the manner in which Z and R are related for precipitation occurring within the GATE research area is prerequisite to a realistic determination of rainfall rates from radars located on board ships in the region.

This paper discusses drop size distributions obtained at cloud base level by means of an airborne foil impactor and provides an analysis of the relationship between Z and R within the GATE B-scale array.

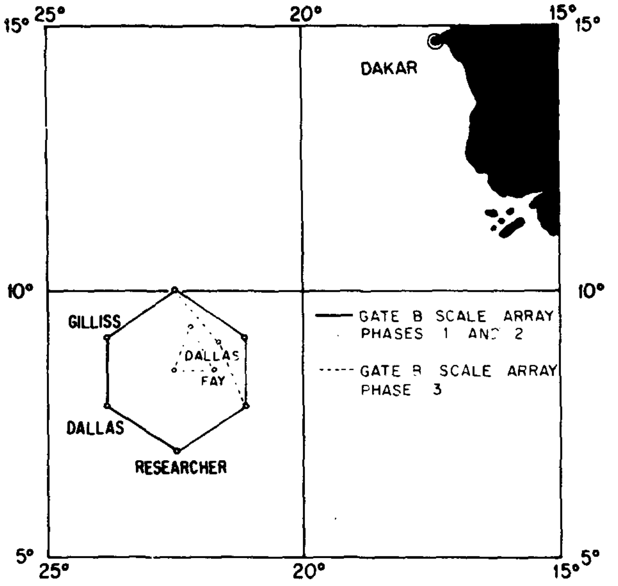

Fig. 1. Location of GATE B-scale array.

2. Measurement program and data analysis technique

The calculation of a Z-R relationship that would be appropriate for the oceanic region encompassed by the GATE B-scale array requires that the size distribution of raindrops be measured on shipboard and/or from aircraft operating within that area. Since spectra measurements contain high natural spatial and temporal variability (primarily because of sampling volume limitations of instrumentation currently in use), it is important to compensate for possible error by calculating ensemble averages from a sample size that is large enough for statistical significance. Furthermore, it is absolutely necessary that the shower core, not just the periphery, be sampled for the computation of a meaningful Z-R relationship.

Garstang (1969) showed that during the 1968 Barbados Experiment the NOAA ship *Discoverer*, stationed in the eastern tropical North Atlantic near Barbados from 4–27 August, was in a significant shower (greater than 2.5 mm h^{-1} of rainfall) on only 10% of the experimental days. Holle (1968) found, however, that precipitation occurred from at least one cloud within a 30 km radius of the R/V *Crawford*, stationed in the tropical Atlantic near Barbados in the summer of 1963, during 60% of all hours having supporting photographic data. This indicates that, although the occurrence of a shower directly overhead at a given location in the tropical ocean is a rather rare event, the occurrence of precipitation within a surrounding extended area is a frequent event. Thus, measurement from a highly mobile instrumented platform, such as an aircraft, is desirable to obtain a large ensemble average of drop size distributions over an oceanic area.

A foil impactor of MRI design (Ruskin and Scott, 1974) was carried on the NOAA DC-6 aircraft during all three phases of GATE. The aircraft was operated mainly at an altitude near cloud base (typically 1 km or less) and penetrations into the core regions of rainshafts were carried out whenever possible. The foil impactor, located on one wingtip of the DC-6 (Fig. 2), is operated manually whenever the aircraft traverses a precipitation shaft. When the impactor is activated, a motor mechanism moves a strip of aluminum foil (width of 3 inches and thickness of 0.001 inch) over a grooved (0.25 mm) drum. A striated pattern remains at the point where the drop impacts on the grooved drum. A metal shutter (3.8 cm×3.8 cm) operates synchronously with the transport mechanism to expose the foil to precipitation. As configured for GATE, the shutter is open and the foil exposed for ~0.2 s. During the next 0.8 s, the shutter is closed and the foil is advanced. The shutter then opens again and the sampling is repeated in this mode until the aircraft passes completely through the precipitation core. The cross-sectional area of the shutter opening is 14.5 cm^2; thus, for an exposure of 0.2 s on an aircraft moving at 100 m s^{-1}, the sampling volume is approximately 30 ℓ. It has been found from experience with heavy tropical rain that sampling volumes much in excess of 30 ℓ per frame of foil result in a serious overlapping of drops. The foil impactor mounted on the DC-6 aircraft was, therefore, able to sample about 1 m^3 of air during a traverse through an

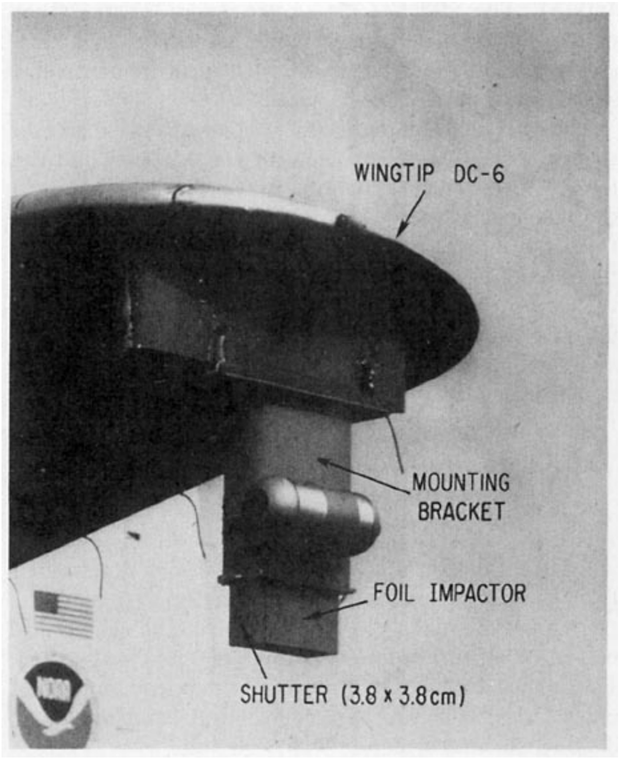

Fig. 2. Foil impactor of MRI design mounted on the wingtip of the NOAA DC-6 aircraft.

TABLE 1. Cloud-base drop spectra measurements made during GATE.

Date (1974)	Amount of foil analyzed (cm)	Number of rainshaft penetrations	GATE mission type	DC-6 flight altitude (ft)	Date (1974)	Amount of foil analyzed (cm)	Number of rainshaft penetrations	GATE mission type	DC-6 flight altitude (ft)
3 July	586	4	2 (ITCZ crossing)	500	17 August*	4572	3	2B (box)	500
12 July*	7112	7	5A4 (railroad)	1300	30 August*	3048	5	5A1 (box)	300
16 July	1500	7	5A/8A1 (aborted)		2 September	1543	5	1C2 (line)	850–2000
29 July*	5080	9	1A (butterfly)	500	3 September	615	4	5B2 (line)	500
3 August*	7620	15	1C2 (line)	5000	6 September*	5080	10	5B2 (line)	300–800
5 August*	8890	19	1C2 (line)	300–2000	7 September	421	2	5B1/6A (L's)	1200
8 August	1800	8	1A (butterfly)	4000	9 September	1100	4	1C2 Box (aborted)	300
10 August*	7620	12	1A (butterfly)	500	14 September*	5080	7	1C2 (line)	1000
11 August*	5588	5	2A-mod (line)	500	17 September	402	2	7A2 (line)	3000
13 August*	8890	12	1A (butterfly)	500	18 September	1570	4	1 (box, line)	3500
14 August*	2286	3	5B2 (line)	3000–5000					

* Dates analyzed with more than 2000 cm of foil.

isolated medium-sized convective element (~3 km in diameter). When fully loaded, the impactor can carry about 9000 cm of foil, enough to obtain measurements from about 10–20 rain shafts of the sizes typically encountered within the area covered by the GATE B-scale array.

Upon completion of each flight day, the foil from the impactor device was dried to prevent corrosion. The data were checked to determine whether the impactor was operating properly and were cataloged for delivery to the United States. At the conclusion of GATE, all foil data were inch-marked for time and length continuity and analyzed on a frame-by-frame basis with a Bausch and Lomb Quantitative Metallurgical System (QMS). The QMS is an electro-optical system that employs a vidicon camera and associated logic circuitry to allow for fast, automated measurements of a spectrum of particle sizes.

The analyzed foil data were sorted into ten 0.50 mm size categories, the first being $0<D\leqslant 0.50$ mm and the tenth 4.5 mm$<D\leqslant 5.0$ mm. A correction, similar to that used by Schecter and Russ (1970), was applied to the drop imprint diameters to account for spreading of the liquid upon impact with the foil. The terminal velocity expression used in the calculation of the rainfall rate (2) for the aircraft data was derived by Foote and DuToit (1969) to fit the empirical data of Gunn and Kinzer (1949) and has the form

$$V_0=\sum_{j=0}^{N} a_j D^j, \tag{4}$$

where the subscript zero refers to conditions at 20°C and 1013 mb, D is drop diameter, and a_j is determined from a least-squares curve fitting technique. A ninth-degree polynomial ($N=9$) was used in the calculation of a_j. An altitude correction for $V(D)$ as discussed by Foote and DuToit (1969) was also applied, but in the case of the DC-6 foil data collected at or below about 3000 ft, such a correction was negligible.

A single drop size distribution is derived for each aircraft penetration through a precipitation core region. To provide a valid statistical representation, each drop size distribution typically has an integrated sample size of at least several hundred drops. Finally, from the number concentration of drops in each of the 10 size categories, a value of radar reflectivity (2), rainfall rate (3) and other related variables such as liquid water content and slope (λ) of the best-fit curve to the drop size spectra are calculated for each aircraft traverse through a precipitation core region.

The accuracy of the foil impactor technique to obtain drop size distributions has been addressed by Schecter and Russ (1970), who used a whirling arm device to investigate effects of drop spreading. Through calibration with standard dye techniques, they determined errors of no more than 10% for drops as large as 5 mm in diameter. Ruskin and Scott (1974) suggest a possible 10% error as a result of sizing procedures due to interpretation problems with shadowy images at the edges of impressions. This would be applicable to the QMS system, since it depends upon sharp contrasts for accurate sizing. Ruskin and Scott (1974) present a series of collection efficiency curves for 10 cm cylindrical objects imbedded in a 100 m s^{-1} airflow, which indicates that drops larger than 50 μm in diameter should be collected with an efficiency approaching 100%. All drops of precipitation size ($D>0.1$ mm) should be collected by the impactor if the flow field is relatively undisturbed, a condition which should be met given the wingtip location of the device. The most serious possibility of error is related to the sampling volume problems discussed in detail by Cornford (1968), who concluded from Poisson statistics that at least 23 drops must be present in each size category if the distribution is to be measured to ±50% on 95% of occasions. This would suggest that the small and middle range of the raindrop size spectrum can be defined with a fair degree of accuracy, but the large end, where the number density of drops can fall below 10 m^{-3} cannot be realistically represented. This problem exists with all devices that sample a finite volume for a finite time, and it is this consideration which prevents us from par-

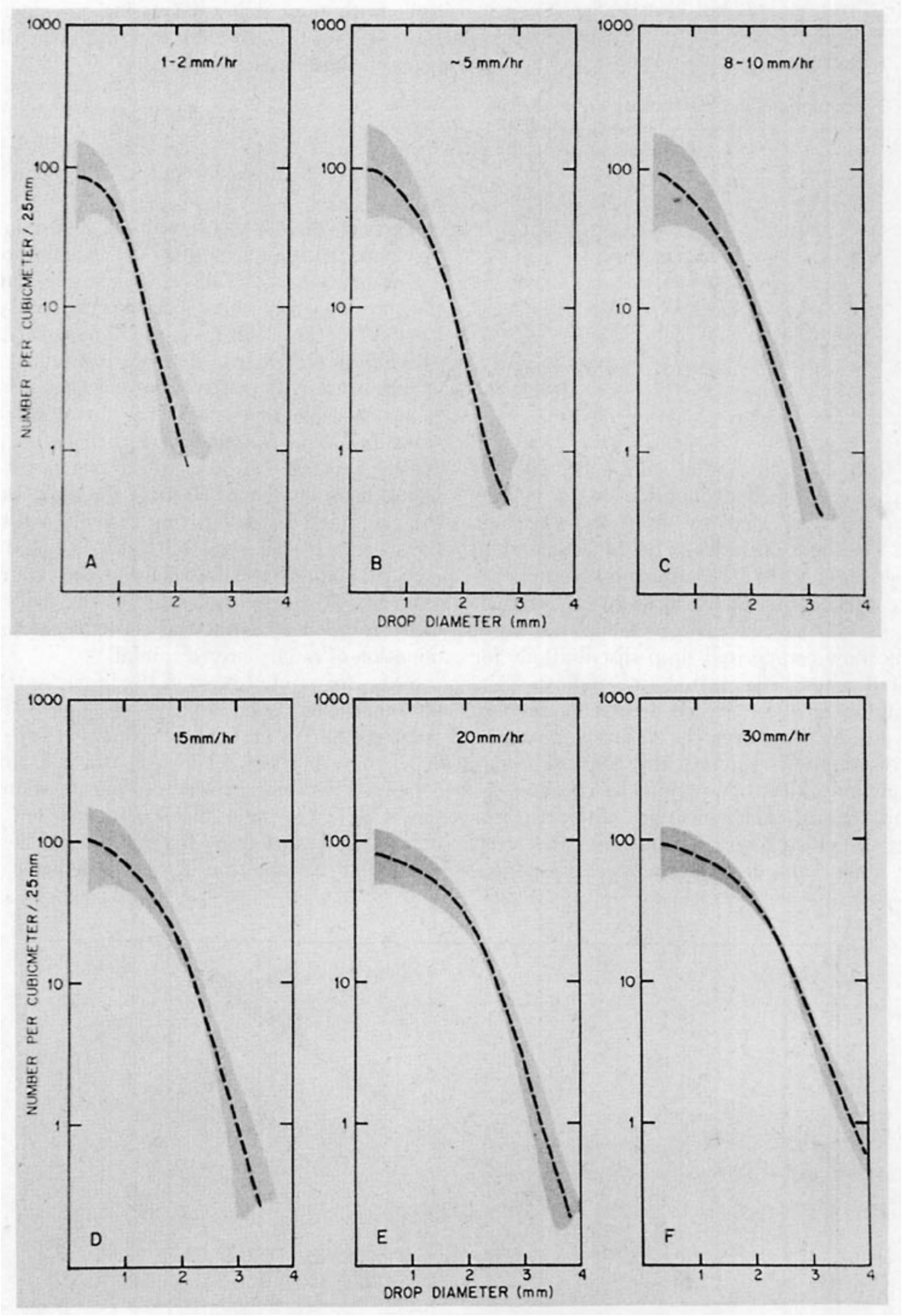

FIG. 3. Composites of drop size distributions for various rainfall rates obtained from DC-6 foil data during GATE.

titioning the data from rainshaft traverses into multiple sectional increments to examine spatial variability.

3. Results

Measurements of drop size spectra were obtained during each phase of GATE with the majority of data collected during the second phase. Table 1 gives the dates, duration, altitudes and aircraft mission type from which data were collected. As can be seen from Table 1, a large amount of data, totaling in excess of 76 000 cm of foil, was collected on 21 days during GATE. The days selected for analysis were those on

TABLE 2. Daily cloud-base *Z-R* relationships.

Date (1974)	Number of rainshaft penetrations	Calculated *Z-R* relationship	Correlation coefficient
12 July	7	$Z=164\ R^{1.54}$	0.99
29 July	9	$Z=146\ R^{1.55}$	0.99
3 August	15	$Z=261\ R^{1.34}$	0.97
5 August	19	$Z=176\ R^{1.45}$	0.99
10 August	12	$Z=211\ R^{1.47}$	0.98
11 August	5	$Z=163\ R^{1.58}$	0.97
13 August	12	$Z=291\ R^{1.38}$	0.98
14 August	3	$Z=150\ R^{1.44}$	0.99
17 August	3	$Z=116\ R^{1.63}$	0.99
30 August	5	$Z=179\ R^{1.38}$	0.99
6 September	10	$Z=164\ R^{1.54}$	0.99
14 September	7	$Z=118\ R^{1.66}$	0.99

which more than 2000 cm of foil were collected. For the 12 days that met the 2000 cm criterion, more than 70 000 cm of foil were analyzed from 107 rainshaft penetrations. The flight altitudes of the DC-6 generally varied between 300 and 1500 ft, although several penetrations were carried out at an altitude higher than 3000 ft.

Figs. 3a–3f show composited drop distributions for various rainfall rates. The dotted line indicates the mean distribution and the shaded area indicates two standard deviations away from the mean. None of the composited distributions showed any marked differences as a function of altitude within the range of observations and, for ease of classification, will be categorized together as cloud-base distributions. For drop diameters $\gtrsim$ 1 mm, the distributions decay exponentially with their slopes decreasing with increasing rain rate. A best-fit curve to the drop size distributions given in Fig. 3 assumes the form

$$N(D)=N_0e^{-\lambda D}, \tag{5}$$

where N_0 is the intercept and λ is the slope of the best-fit line.

Through the use of (2) and (3), *Z-R* data points can be computed from each drop size distribution to derive a *Z-R* relationship. Table 2 summarizes the *Z-R* relationships on a day-to-day basis for the 12 days analyzed for GATE. The best-fit curves for eight days during which drop size distributions were obtained from seven or more rainshaft penetrations are shown in Fig. 4. It is not possible to discern any significant day-to-day variation as a function of either flight altitude or mission type. It can be seen that, for radar reflectivity values between 35 and 50 dBZ, the daily variability of the computed rainfall is quite small. A divergence of the calculated daily rainfall rates is noted for values <35 dBZ and >50 dBZ. Only in the latter case, that of extremely heavy rain rates, is the daily variability likely to be of any practical consequence to the determination of radar-derived rainfall.

Since, for rainfall rates in the important 10–100 mm h^{-1} range, there is no indication of high *Z-R* variability on a day-to-day basis or from mission type to mission type, the total data set for the 107 *Z-R* points can be combined to derive an averaged *Z-R* relationship. Fig. 5 shows all of the individual *Z-R* points and the best-fit line to the points derived from a regression of *Z*, the dependent variable, on *R*, the independent variable.

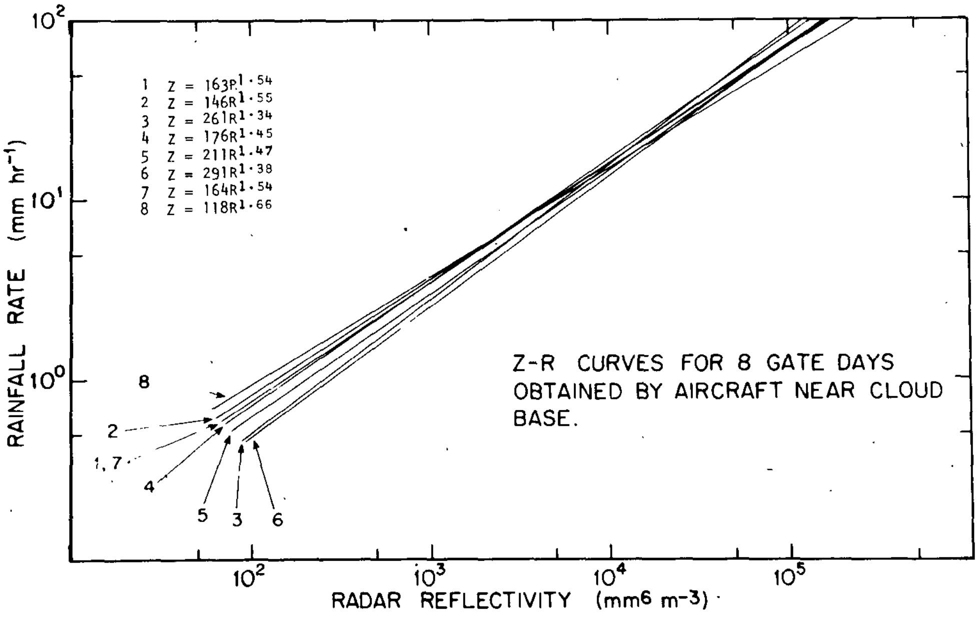

FIG. 4. Daily best-fit *Z-R* relationship curves obtained from an analysis of DC-6 foil data collected near cloud base on eight days during GATE.

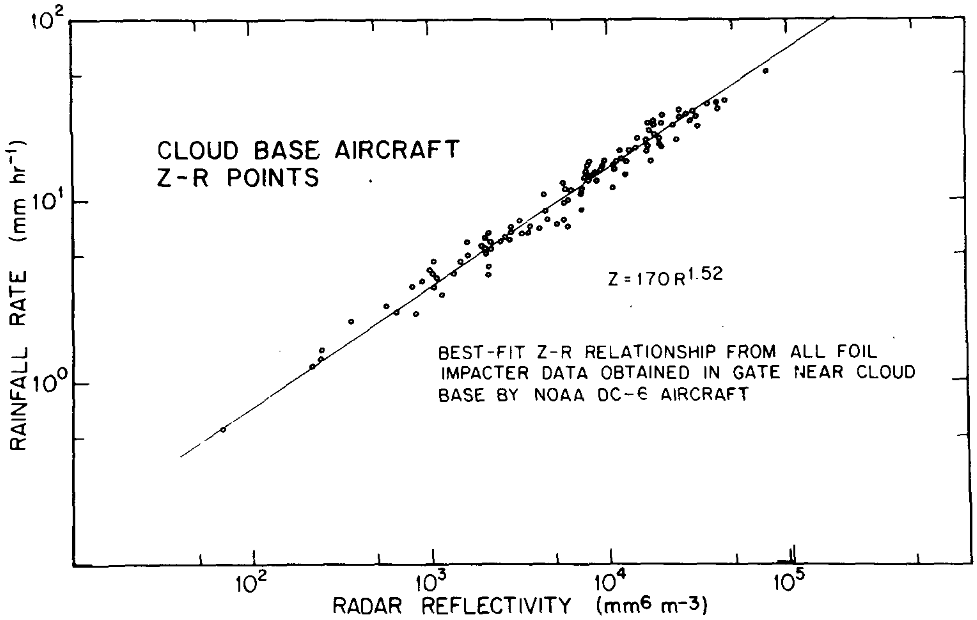

Fig. 5. Best-fit *Z-R* relationship drawn to all data points obtained through an analysis of DC-6 foil data collected near cloud base within the GATE B-scale array.

The *Z-R* relationship

$$Z = 170\,R^{1.52} \tag{6}$$

has a correlation coefficient of 0.986, and we suggest that this relationship be used in the calculation of radar-derived rainfall within the GATE B-scale array.

Table 3 provides an additional stratification of the total set of *Z-R* points as a function of rainfall rate. Here, all the *Z-R* points greater than the *R* values indicated in the first column are used to calculate a *Z-R* relationship. It can be seen that the rainfall rates computed for 50, 45, 40 and 35 dBZ are, for all practical purposes, completely unaffected by the stratification by rainfall rate. This indicates that the best-fit *Z-R* relationship is not unduly weighted by a predominance of data points at any given rainfall intensity.

4. Discussion

Table 4 gives values of rain rate (mm h^{-1}) as a function of radar reflectivity (dBZ) for four different *Z-R* relationships (A–D). Relationship A, discussed in this paper, is that derived from the foil data without any transformation. Relationship B is that developed by Marshall and Palmer (1948), using drop distribution data from various locations. Relationship C is that proposed by Austin *et al.* (1976) from a combination of airborne foil and shipboard disdrometer data collected during GATE. Relationship D is based upon a limited sample of shipboard disdrometer data and has been used by EDS/CEDDA to average and convert polar coordinate radar reflectivities into Cartesian coordinate rainfall rates. The averaged values of *R* were subsequently reconverted back to values of *Z* through use of the same relationship (D) and stored on tape for future GATE rain map processing.

It can be seen from Table 4 that rather substantial differences in derived rainfall rates exist between relationship A and relationships C and D for radar reflectivities $\gtrsim$35 dBZ, with the use of C resulting in about a factor of 2 higher estimation of rainfall relative to the

Table 3. Cloud-base best-fit *Z-R* relationships as a function of rainfall rate.

Stratification	Number of data points	*Z-R* relationship	Correlation coefficient	*R* at 50 dbZ (mm h^{-1})	*R* at 45 dbZ (mm h^{-1})	*R* at 40 dbZ (mm h^{-1})	*R* at 35 dbZ (mm h^{-1})
All data	107	$Z=170\,R^{1.52}$	0.986	66	31	15	7
$R>$ 2.5 mm h^{-1}	100	$Z=173\,R^{1.51}$	0.980	67	31	15	7
$R>$ 5.0 mm h^{-1}	86	$Z=186\,R^{1.49}$	0.974	68	31	15	7
$R>$ 7.5 mm h^{-1}	63	$Z=192\,R^{1.48}$	0.949	68	31	14	7
$R>$10.0 mm h^{-1}	56	$Z=138\,R^{1.58}$	0.941	65	31	15	7
$R>$12.5 mm h^{-1}	49	$Z=140\,R^{1.58}$	0.937	64	31	15	7

TABLE 4. Rainfall rate R (mm h^{-1}) as a function of radar reflectivity (Z) for four different Z-R relationships. See text for identification of Z-R relations.

Z (dbZ)	A $170R^{1.52}$	B $220R^{1.60}$	C $230R^{1.25}$	D $300R^{1.3}$
60	302	193	814	513
55	142	94	324	211
50	66	46	129	87
45	31	22	51	36
40	15	11	20	15
35	7	5	8	6
30	3	3	3	3
25	2	1	1	1

use of A. Although a discussion of the drop distribution data obtained by the shipboard raindrop disdrometers is beyond the scope of this paper, it should be mentioned that considerable corrections had to be applied to the small end ($D<2$ mm) of the spectrum, and the day-to-day variability was large relative to the foil data set (Cunning and Sax, 1977). Because of the day-to-day consistency of the foil data (Fig. 5) and the exponential shape of the foil-derived drop distributions from diameters of about 1 mm (Fig. 3), we place more confidence in the representativeness of relationship A. Note, though, that the Z-R relationship A obtained from the foil data is based on a drop distribution spatially integrated through the entire shower, including the core region, and, as such, may not be strictly valid for peripheral areas of the echo (e.g., anvil rain) and does not address temporal rainfall variability (e.g., size sorting) within the shower. This should not introduce serious errors unless the results are applied for very short time intervals (<10 min) over cloud-scale spatial areas (100 km^2).

5. Summary

The drop spectra measurements obtained during GATE with the foil impactor on the NOAA DC-6 aircraft provide a consistent set of derived Z-R relationships that have only minor, day-to-day variability. The practical consequence of this is shown in Fig. 4, where it can be seen that, within the range of the highest[1] reflectivity values (30–50 dBZ) that are most likely to occur, similar rainfall rates are computed from each of the daily Z-R relationships. Stratification of the foil-derived set of Z-R data points on the basis of rainfall rate (Table 4) makes no practical difference to the calculated rainfall rates for values of radar reflectivity <50 dBZ. The foil data provide drop distributions that are, in most cases, monomodal and feature an expected exponential decay in concentration for drop sizes >1 mm in diameter. The mass integration of the drop size spectrum obtained through the foil analysis usually gives shower-averaged rainfall contents of order 1 g m^{-3}, a value which appears to be quite reasonable.

For the 107 rainshaft penetrations carried out near cloud base by the NOAA DC-6 during 12 days of GATE, the best-fit relationship to the Z and R points derived from an analysis of the individual drop size distributions was found to be $Z=170\ R^{1.52}$.

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[1] Geotis (personal communication) has calculated a mean radar reflectivity value of 26 dBZ with an 8 dB standard deviation for observations taken on board the *Gilliss* during Phase 3 of GATE. This would indicate, if we assume a normal distribution of reflectivity values, that Z would be expected to exceed 39 dBZ on only about 5% of all occasions.