Suggested Revisions to Certain Boundary Layer Parameterization Schemes Used in Atmospheric Circulation Models[1]

S. P. S. ARYA[2]

Department of Atmospheric Sciences, University of Washington, Seattle 98195

(Manuscript received 9 June 1976, in revised form 11 November 1976)

ABSTRACT

The results from some of the latest experimental and theoretical studies of the planetary boundary layer (PBL) are used to revise certain better known schemes (Clarke, 1970b; Deardorff, 1972b) of parameterizing the boundary layer in general circulation models (GCM's). For high-resolution models, our proposed scheme is based on the surface layer similarity theory relations derived by Businger *et al.* (1971) from the Kansas tower data. For low-resolution GCM's, the proposed scheme is based on the generalized PBL-similarity theory with the various similarity functions evaluated from the best available numerical models of the boundary layer. The usual practice of specifying a single value for the roughness parameter (z_0) for all the land surfaces in a GCM is questioned. The specified z_0 value at each grid point should actually reflect the roughness characteristics of the surface represented by it.

1. Introduction

With the advent of modern high-speed computers, there has been a great upsurge in the development of numerical general circulation models (GCM's) of the atmosphere for the purpose of predicting or simulating weather and climate [see GARP Publication Series No. 14, WMO, Geneva (1974)]. The horizontal and vertical resolutions of such models, limited as they are by the available computer capacity, are too coarse to resolve the small-scale motions in the boundary layer which are responsible for the transfer of momentum, heat and moisture between earth's surface and the atmosphere. These subgrid-scale transport processes may not affect short-range forecasts over a day or two, but are considered to become quite important for long-range predictions of weather and climate. Their effects on large-scale motions have to be parameterized in a GCM.

The most important elements of the interaction between the boundary layer and the free atmosphere are the vertical fluxes of momentum, heat and moisture, the vertical velocity at the top of the boundary layer and the frictional dissipation of kinetic energy in the boundary layer. Since the last two can easily be expressed in terms of the surface stress and the geostrophic wind field (Lettau, 1962; Wiin-Nielsen, 1974), the problem of boundary layer parameterization essentially boils down to the parameterization of fluxes only. Furthermore, if the vertical resolution of a GCM is so coarse that the whole boundary layer is placed below the first interior GCM level, only the surface fluxes need to be parameterized.

Several different schemes are being used for parameterizing the boundary layer in current GCM's [for a detailed review, see Bhumralkar (1976)]. The simplest ones employ the usual bulk transfer relations with all the transfer coefficients assumed equal and prescribed *a priori*. In some cases different values are assigned for land and ocean surfaces and also some allowance is made for different stability conditions. But, by and large, these parameterization schemes are very crude and hardly reflect the much improved knowledge of the atmospheric boundary layer (ABL) obtained during the last decade or so. Better schemes have been formulated from similarity considerations of the ABL, and our discussion will be confined only to such parameterizations (e.g., Clarke, 1970b; Deardorff, 1972b).

2. Parameterizations based on the surface layer similarity theory

A detailed and critical review of the various similarity theories proposed for the atmospheric boundary layer has been given elsewhere (Arya and Sundararajan, 1976). For use in global models, the parameterizations based on rather broad and generalized similarity theories should be preferable to those based on highly restrictive or specialized ones (e.g., free convection similarity theory). Therefore, the latter will be omitted from the discussion here.

Whenever the first grid level (z_1) of a GCM can be placed within the constant flux or surface layer, one can use the Monin-Obukhov similarity theory relations

[1] Contribution Number 390, Department of Atmospheric Sciences, University of Washington.

[2] Present affiliation: Department of Geosciences, North Carolina State University, Raleigh 27607.

for parameterizing surface fluxes. It is easy to show [for details see Deardorff (1968) and Clarke (1970b)] that the parametric relations can be expressed in the form

$$\left.\begin{aligned} F_0 &= -C_{D1}\left(\frac{z_1}{z_0},\mathrm{Ri}_{b1}\right)V_1^2 \\ Q_{v0} &= -C_{H1}\left(\frac{z_1}{z_0},\mathrm{Ri}_{b1}\right)V_1(\theta_{v1}-\theta_{v0}) \\ E_0 &= -C_{E1}\left(\frac{z_1}{z_0},\mathrm{Ri}_{b1}\right)V_1(q_1-q_0) \end{aligned}\right\}. \tag{1}$$

Here $F_0 \equiv -\tau_0/\rho$, Q_{v0} and E_0 represent the surface fluxes of momentum, virtual heat and moisture (all expressed in kinematic units here) and V, θ_v and q denote mean wind speed, virtual potential temperature and specific humidity, respectively (the subscript 0 denotes the value at z_0 and 1 the value at z_1). The transfer coefficients C_{D1}, C_{H1} and C_{E1}, according to the predictions of similarity theory, are some universal functions of the ratio z_1/z_0 (z_0 being the roughness parameter) and a bulk Richardson number

$$\mathrm{Ri}_{b1} = \beta z_1(\theta_{v1}-\theta_{v0})/V_1^2, \tag{2}$$

where $\beta \equiv g/T$ is the buoyancy parameter. The so-called virtual heat flux and virtual potential temperature are used here to consider the buoyancy effects of water vapor in a simple manner.

The actual forms of the functions C_{D1} (z_1/z_0, Ri_{b1}), etc., depend on the particular forms of the flux-profile relations adopted for the surface layer. Deardorff (1968) has given the ratios of the various transfer coefficients to their expected neutral values (for the same z_1/z_0) as functions of the bulk Richardson number. As expected, his results showed that all transfer coefficients decrease monotonically with increasing stability, approaching to zero as Ri_{b1} reaches its critical value of about 0.20. Similar trends are shown in Figs. 1 and 2, which are based on what may now be regarded as the most widely accepted flux–profile relations derived by Businger *et al.* (1971). For unstable conditions, these are not much different from the results of Deardorff (1968) or Clarke (1970b). But for very stable conditions, our results differ considerably from Clarke's nomograms which indicate no finite value of the bulk Richardson number at which C_{D1}, C_{H1} or C_{E1} vanish. The log-linear profile relations used here give $(\mathrm{Ri}_{b1})_{\mathrm{crit}} \approx 0.21$, but it is doubtful that these will remain valid close to critical conditions.

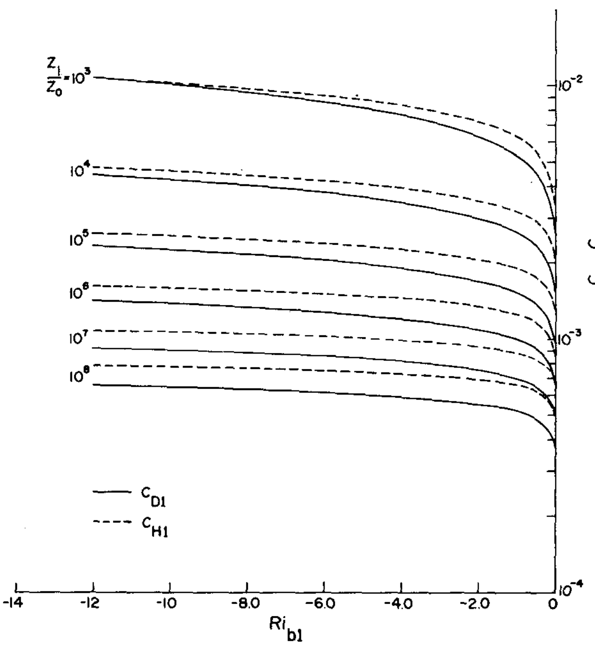

FIG. 1. The surface layer drag and heat transfer coefficients as functions of surface roughness and stability for unstable stratification.

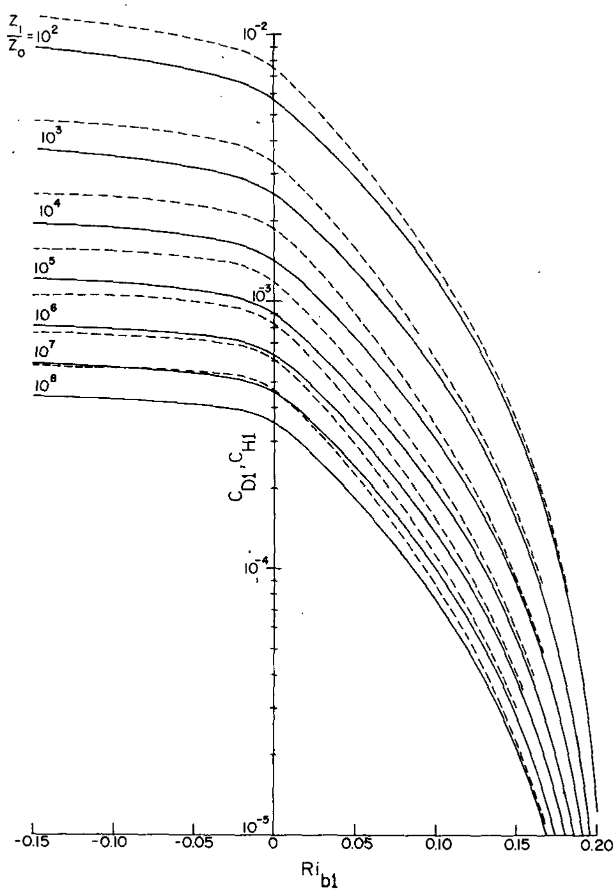

FIG. 2. As in Fig. 1 except for near-neutral and stable stratification.

3. Parameterizations based on the matching of surface and outer layer similarity theories

The method of flux parameterization discussed in Section 2 cannot be used in large-scale models with such poor vertical resolution that the first interior grid level is well above the top of the surface layer and, in some cases, even above the top of the PBL. For these one must parameterize the whole boundary layer. The best forms of parametric relations for this have been derived by the matching of mean profiles predicted by surface and outer layer similarity theories (Blackadar and Tennekes, 1968; Zilitinkevich, 1969, 1975).

The basic assumptions underlying all similarity theories are that the ABL flow is horizontally homogeneous and quasi-stationary, which apparently are very restrictive when these theories are applied to many real atmospheric situations. In a GCM, however, the variables are considered to be averaged horizontally over a fairly large grid area, and the assumption of horizontal homogeneity is probably well-justified.

The general form of the drag and other transfer relations obtained from similarity matching arguments is

$$\left.\begin{aligned} ku_h/u_* &= -(\ln\hat{z}_0+A) \\ kv_h/u_* &= -B \operatorname{sign} f \\ k(\theta_{vh}-\theta_{v0})/\theta_{v*} &= -(\ln\hat{z}_0+C) \\ k(q_h-q_0)/q_* &= -(\ln\hat{z}_0+D) \end{aligned}\right\}, \tag{3}$$

in which u and v are the horizontal components (in the direction of surface shear and perpendicular to it, respectively) of mean velocity vector $\mathbf{V}$ (the subscript h refers to the variables at the top of the boundary layer), $\hat{z}_0$ is the roughness parameter normalized by the scale height (h or u_*/f) of the boundary layer, and A, B, C, D are some universal functions of the dimensionless similarity parameters which depend on the particular similarity hypothesis that is used.

In some formulations, u_h and v_h are replaced by the surface geostrophic wind components u_{g0} and v_{g0}, so that the alternative drag relations are

$$\left.\begin{aligned} ku_{g0}/u_* &= -(\ln\hat{z}_0+A_0) \\ kv_{g0}/u_* &= -B_0 \operatorname{sign} f \end{aligned}\right\}. \tag{4}$$

The similarity functions A_0 and B_0 are expected to differ from A and B due to the presence of baroclinity and also in low latitudes, where the actual winds at the top of the boundary layer may differ considerably from the surface geostrophic winds.

Another alternative form of the parametric relations originally proposed by Deardorff (1972b) uses the layer-averaged (denoted by the subscript m) wind components, virtual potential temperature and humidity, i.e.,

$$\left.\begin{aligned} ku_m/u_* &= -(\ln\hat{z}_0+A_m) \\ kv_m/u_* &= -B_m \operatorname{sign} f \\ k(\theta_{vm}-\theta_{v0})/\theta_* &= -(\ln\hat{z}_0+C_m) \\ k(q_m-q_0)/q_* &= -(\ln\hat{z}_0+D_m) \end{aligned}\right\}. \tag{5}$$

There are some good reasons for preferring Eqs. (5) to Eqs. (3) or (4). The layer-averaged variables are less affected by sampling errors than their local values. Also, since any GCM computed variable actually represents the average over the grid size, which is quite likely to be of the order of or larger than h in the vertical, Eqs. (5) are more appropriate for boundary layer parameterization in a GCM. Furthermore it is shown in a separate paper (Arya, 1977) that the functions A_m and B_m are less sensitive to baroclinity than are A and B or A_0 and B_0.

TABLE 1. The list of similarity parameters affecting A, B, etc. according to two different similarity theories.

Parameter representing		Kazanski-Monin theory	Generalized theory
Buoyancy		$\mu_*\equiv\dfrac{u_*}{fL}$	$\dfrac{h}{L}$
Scale-height ratio		—	$\dfrac{fh}{u^*}$
Baroclinity	Magnitude	$S_0\equiv\dfrac{1}{f}\left\lvert\dfrac{\partial\mathbf{V}_g}{\partial z}\right\rvert_0$	$M_0\equiv\dfrac{h}{u_*}\left\lvert\dfrac{\partial\mathbf{V}_g}{\partial z}\right\rvert_0$
	Direction	β_0	β_0

The similarity parameters on which A, B, etc., presumably depend are listed in Table 1 for the two competitive similarity theories: 1) the Kazanski-Monin (1960) theory, also known as the Rossby number similarity theory, and 2) the generalized similarity theory (Deardorff, 1972a, b; Wyngaard *et al.*, 1974a, b; Arya and Wyngaard, 1975).

The basic difference between these two theories is that while the former assumes that the boundary layer height is uniquely determined by u_*/f and L (Obukhov length) only, in the generalized version h is considered as an independent variable (Zilitinkevich and Deardorff, 1974). Therefore, in the latter the effects of such complicated factors as nonstationarity, diurnal heating, large-scale advections of heat and moisture, large-scale subsidence, etc., on which h depends in the real atmosphere, can be considered indirectly by specifying h, for example, through a separate rate equation (see Deardorff, 1974; Stull, 1975).

Note that in the definitions of most parameters of Kazanski-Monin similarity theory, f appears in the denominator. For this reason, as discussed, for example, by Clarke (1970b), the parameterizations based on this theory become highly inaccurate and misleading in low latitudes and completely meaningless at the equator. In global models the parameterizations based on the generalized similarity theory appear to be more appropriate (Deardorff, 1972b).

a. Determination of the various similarity functions

Similarity theories tell nothing about the actual forms of the similarity functions A, B, etc. There have been many attempts toward a satisfactory empirical determination of these functions. In principle they can be evaluated from boundary layer observations from some homogeneous sites, if the rest of the quantities in their defining relations are measured covering a wide range of the parameters involved. This requires rather extensive and carefully executed field experiments in different latitudes over land as well as oceans. Much of the data used for this purpose come from the experiments carried out in the Great Plains (Lettau and Davidson, 1957) and Southern Australia (Clarke, 1970a,b; Clarke *et al.*, 1971), i.e., from mid-latitude land sites. Since the baroclinity parameters M_0 (or S_0) and β_0 are not well determined from these experiments, A, B, etc., have been plotted as functions of stability parameter only (Zilitinkevich, 1969; Clarke 1970a; Clarke and Hess, 1974; Melgarejo and Deardorff, 1974; Arya, 1975, Yamada, 1976). The resulting scatter in data is so large that one can only see the approximate trend of the various similarity functions with stability rather than be able to determine their functional forms with any degree of confidence. The main causes of the scatter are the noise and errors in observations, which are often incomplete and inadequately averaged, the natural variability of the atmosphere and horizontal variations of surface roughness, temperature and other variables.

The parameterization scheme proposed by Clarke (1970b), which is adopted in several GCM's, was in fact based on rather preliminary, and what now appears to be erroneous, evaluations of the A, B and C functions from the Wangara data. A somewhat peculiar and physically unrealistic feature of his nomograms of the geostrophic drag and other transfer coefficients is their nonmonotonic behavior with increasing stability. More recent evaluations of the same data have shown that A, B, etc., and hence the transfer coefficients, are indeed monotonic functions (Arya, 1975; Clarke and Hess, 1974, Yamada, 1976), although the data scatter still renders these empirical determinations very uncertain.

In the absence of a satisfactory experimental determination of A, B, etc., numerical models of the atmospheric boundary layer have been used for the same purpose. In these one has the advantage of studying the effects of one parameter at a time, while keeping others fixed. However, these also suffer from difficulties, especially in stable stratification.

A very sophisticated model of the ABL based on three-dimensional numerical integrations of the Navier-Stokes equations has been proposed by Deardorff (1972a). His results for neutral and unstable barotropic conditions, converted to a value of von Kármán constant $k=0.35$ (0.40 was used in Deardorff's original publications), are compared in Figs. 3–5 with the results obtained from the second-order closure models of Wyngaard *et al.* (1974a, b) and Sundararajan (1975a). Here the overbar denotes that the functions $\bar{A}_0$, $\bar{B}_0$, etc., are for the barotropic case ($M_0=0$). Note that in spite of completely different approaches used in these two types of models, their results are in good agreement and can be approximated by the following relations over a fairly wide range of the parameters $fh/u_* \gtrsim 0.1$ and $-h/L \gtrsim 2$:

$$\left.\begin{aligned} \bar{A}_0 &= \ln(-h/L)+\ln(fh/u_*)+1.5 \\ \bar{B}_0 &= k(fh/u_*)^{-1}+1.8(fh/u_*)\exp(0.2h/L) \\ \bar{C} &= \ln(-h/L)+3.7 \end{aligned}\right\}. \quad (6)$$

The dependence of $\bar{A}_0$ and $\bar{B}_0$ on fh/u_* has been particularly emphasized by Wyngaard *et al.* (1974a), Csanady (1974) and Sundararajan (1975a, b). It is easy to see this for the barotropic convective conditions

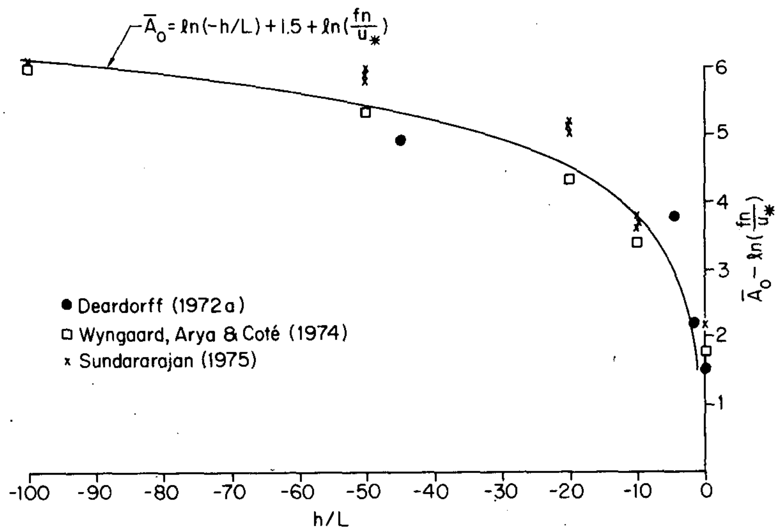

FIG. 3. The similarity function $\bar{A}_0$ determined from numerical models of the unstable barotropic ABL.

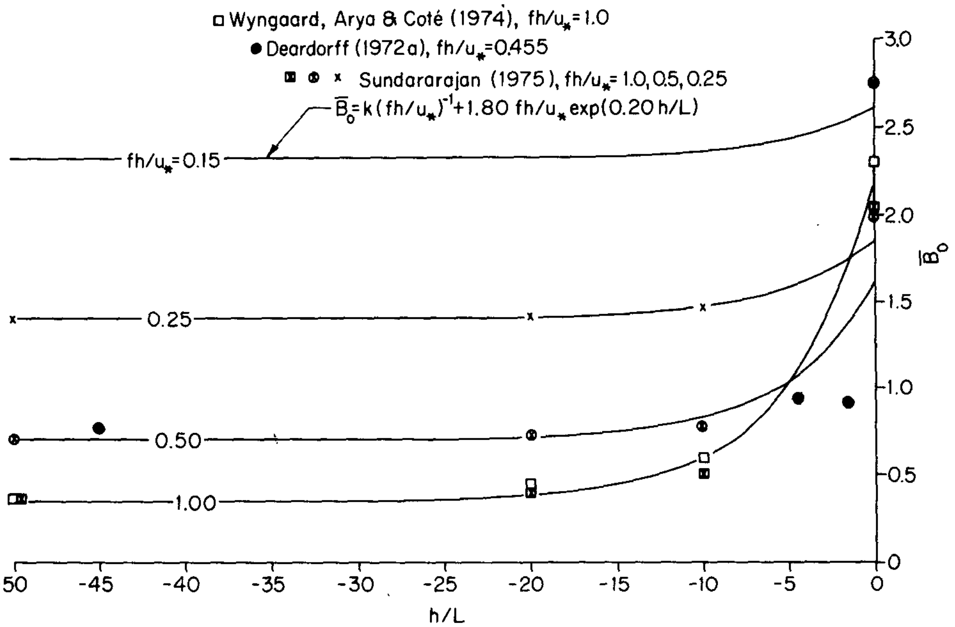

FIG. 4. As in Fig. 3 except for the similarity function $\bar{B}_0$.

$(-h/L \gg 1)$, in which the momentum flux profile becomes linear and the lateral wind component vanishes throughout the depth of the mixed layer, so that the equation of motion in the direction of surface shear simply yields

$$f|v_{g0}| = u_*^2/h$$

or

$$\bar{B}_0 \equiv k|v_{g0}|/u_* = k(fh/u_*)^{-1}. \tag{7}$$

It is interesting to note from the second of Eqs. (6) that it should approach (7) when $fh/u_* \ll 1$, even for small values of $-h/L$. This is indeed confirmed by our numerical model results (not presented here) of a neutral boundary layer capped by an inversion. The condition $fh/u_* \ll 1$ is often satisfied in low latitudes, especially in the trade wind regime (see Pennell and Lemone, 1974; Augstein *et al.*, 1974). Eq. (7) implies that the surface cross-isobar angle (α_0) should be larger in these regions as compared to that in middle or higher latitudes, other factors (surface roughness, boundary layer height, stability, etc.) remaining the same. Some experimental evidence in support of this is given by Gordon (1952) and Brummer *et al.* (1974). These observations, taken over different parts of the oceans, indicate an increasing trend in α_0 in going toward the

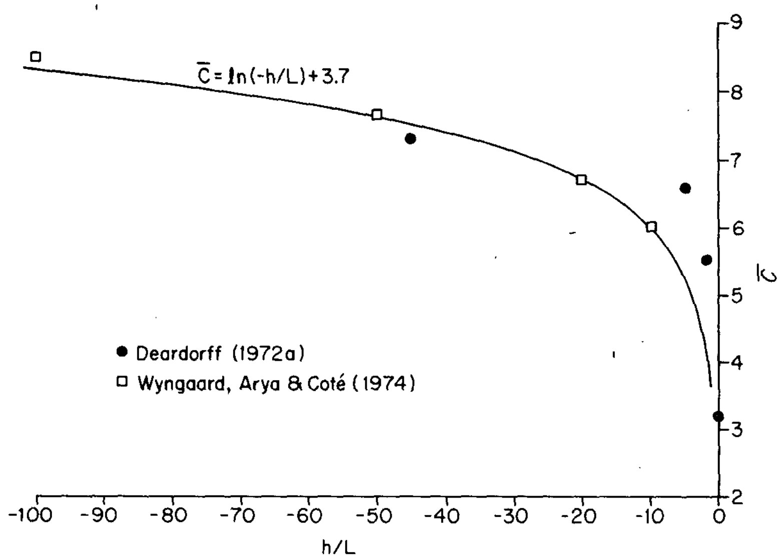

FIG. 5. As in Fig. 3 except for the similarity function $\bar{C}$.

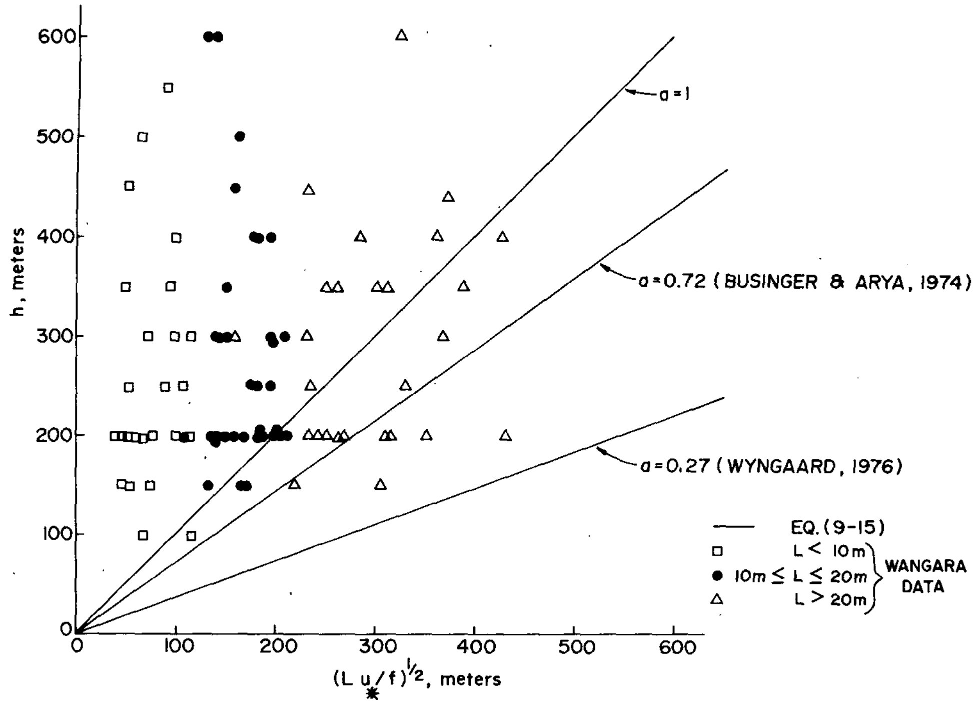

FIG. 6. Comparison of theoretically predicted boundary layer heights with observations of the stable ABL.

equator, part of which is due to the north-south horizontal temperature gradient or baroclinity (Clarke and Hess, 1975). The comparative effects of baroclinity and earth's rotation (the ratio fh/u_* is actually a measure of rotational effects in the ABL) on α_0 and other drag relations are investigated in a separate paper (Arya, 1977).

For the stably stratified steady state ABL, the boundary layer height is not so well-defined from the mean potential temperature and humidity profiles as it is for the unstable case. For the former, one can perhaps make a stronger case for h being uniquely determined by wind shear, earth's rotation and gravitational stability only. Even if it is influenced by some large-scale factors as well, we do not know their effects well enough to obtain a rate equation comparable to that for the unstable case. For these reasons, we shall consider fh/u_* for a steady barotropic stable ABL as some universal function of the stability parameter μ_* only, as implied by the Kazanski-Monin similarity theory. Several investigators using widely different theoretical approaches have come out with a simple form for this function (Zilitinkevich, 1972; Businger and Arya, 1974; Wyngaard, 1976), i.e.,

$$\frac{fh}{u_*}=a\mu_*^{-\frac{1}{2}}, \quad \mu_*\gg 1, \tag{8a}$$

or

$$\frac{h}{L}=a\mu_*^{\frac{1}{2}}, \tag{8b}$$

where a is a proportionality constant of the order unity.

Note that in the dimensional form the above expression implies that $h=a(Lu_*/f)^{\frac{1}{2}}$, i.e., the boundary layer height is proportional to the geometric mean of the length scales u_*/f and L. It provides a rather simple parameterization of the boundary layer thickness under stably stratified conditions. There are some difficulties, however, in verifying the above relation against the type of observations we have available so far, and in establishing the value of a. The turbulence is observed to be very weak and intermittent (both in space and time) in the upper portion of the nighttime boundary layer and the boundary layer top is not so well-defined as it is for the daytime boundary layer. Both u_* and L are also difficult to determine in very stable conditions. It is not surprising then to find a large scatter of data in Fig. 6 based on the Wangara observations. Also represented in the same figure are the predictions of several theoretical models of the stable ABL (Zilitinkevich, 1972; Businger and Arya, 1974; Wyngaard, 1976). In these, h is defined as the height where the momentum flux reduces to 1% of its surface value, while the Wangara h represents the height of the surface inversion layer as indicated by the radiosonde measured θ profile. Because of the radiative effects, the latter may differ from the height to which turbulence extends. The large difference in the value of a given by different models indicates that h is perhaps more critically dependent on some of the model assumptions than are the computed similarity functions $\bar{A}$ and $\bar{B}$ shown in Figs. 7 and 8. Here we have used $\mu_*^{\frac{1}{2}}$ instead of h/L mainly because of the large uncertainty in determining h. The agreement between the results of the two widely

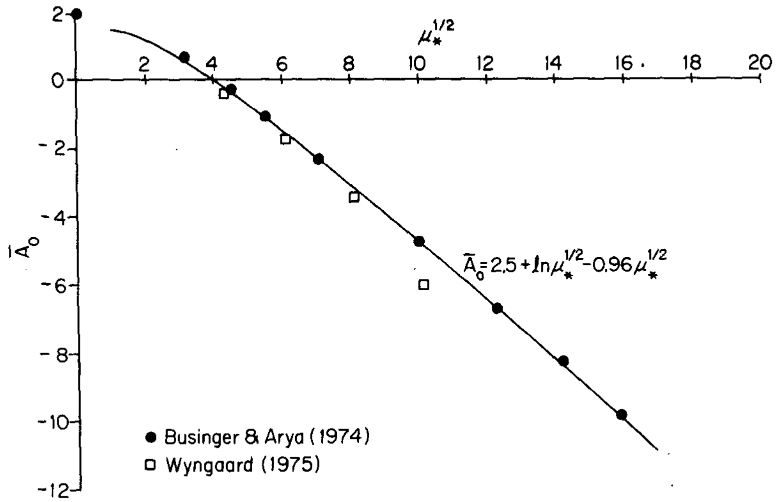

FIG. 7. The similarity function determined from numerical models of the stable barotropic ABL.

different models is very good for $\bar{A}_0$ but not so good for $\bar{B}_0$. After comparing these with the results of the Wangara data analyzed by Clarke and Hess (1974) and Arya (1975), we adopt the following forms closely fitting the results of the model proposed by Businger and Arya (1974) for $\mu_* \gg 1$:

$$\left.\begin{aligned} \bar{A}_0 &= \ln\mu_*^{\frac{1}{2}} - 0.96\mu_*^{\frac{1}{2}} + 2.5 \\ \bar{B}_0 &= 1.15\mu_*^{\frac{1}{2}} + 1.1 \end{aligned}\right\}. \tag{9}$$

Note that the above forms without the specific constants are also predicted by similarity matching arguments, if one makes some reasonable assumptions about the turbulence structure in a stably stratified boundary layer (see, e.g., Zilitinkevich, 1975). The function $\bar{C}$ is expected to have a form very similar to that of $\bar{A}_0$ or $\bar{A}$, although it has not been evaluated in any numerical model. The radiation effects, so far ignored in modeling the stable ABL, may have considerable effect on $\bar{C}$, which is perhaps one of the reasons for very large scatter in its empirical evaluations (Clarke, 1972; Arya, 1975). In the absence of any better information about $\bar{C}$, we adopt the expression

$$\bar{C} = \ln\mu_*^{\frac{1}{2}} - C_1\mu_*^{\frac{1}{2}} + C_2, \tag{9c}$$

with the constants $C_1 = 3.0$ and $C_2 = 7.0$ determined from the best fit of Eq. (9c) to the Wangara data analyzed by Arya (1975). The uncertainty in the above estimate of $\bar{C}$ may be as large as $\pm 50\%$ under very stable conditions. Because of the logarithmic terms in the expressions for $\bar{A}_0$ and $\bar{C}$, they are assumed to be valid only for $\mu_* \gtrsim 4$.

The definitions of $\bar{A}_0$ and $\bar{C}$ in (9) differ from those in (6) due to different scale heights (u_*/f and h) used in the two similarity formulations. If for the sake of uniformity we use the definitions based on the generalized similarity theory for stable conditions also, instead of (9) we would have

$$\left.\begin{aligned} \bar{A}_0 &= -0.96(h/L) + 2.5 \\ \bar{B}_0 &= 1.15(h/L) + 1.1 \\ \bar{C} &= -3.0(h/L) + 7.0 \end{aligned}\right\}. \tag{10}$$

Here we have also used Eqs. (8) with $a=1$.

We have so far discussed mainly the similarity functions $\bar{A}_0$ and $\bar{B}_0$, which relate the surface stress vector to the surface geostrophic wind through Eqs. (4). These are, of course, related to the corresponding functions appearing in Eqs. (5), through the integrated (with

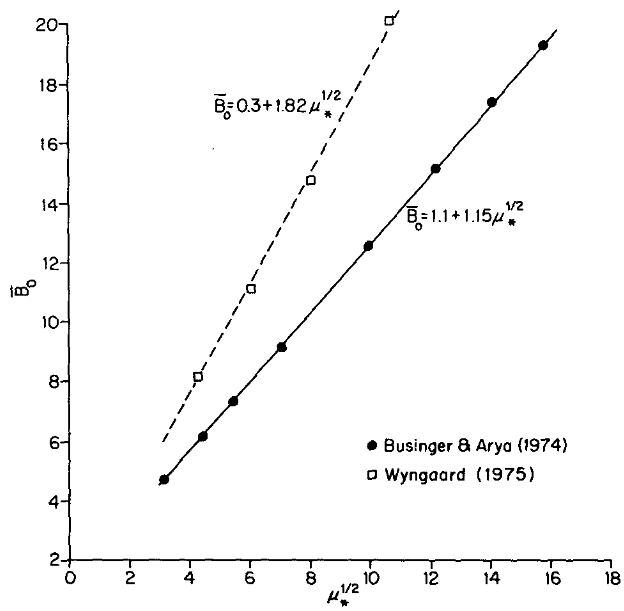

FIG. 8. As in Fig. 7 except for the similarity function $\bar{B}_0$.

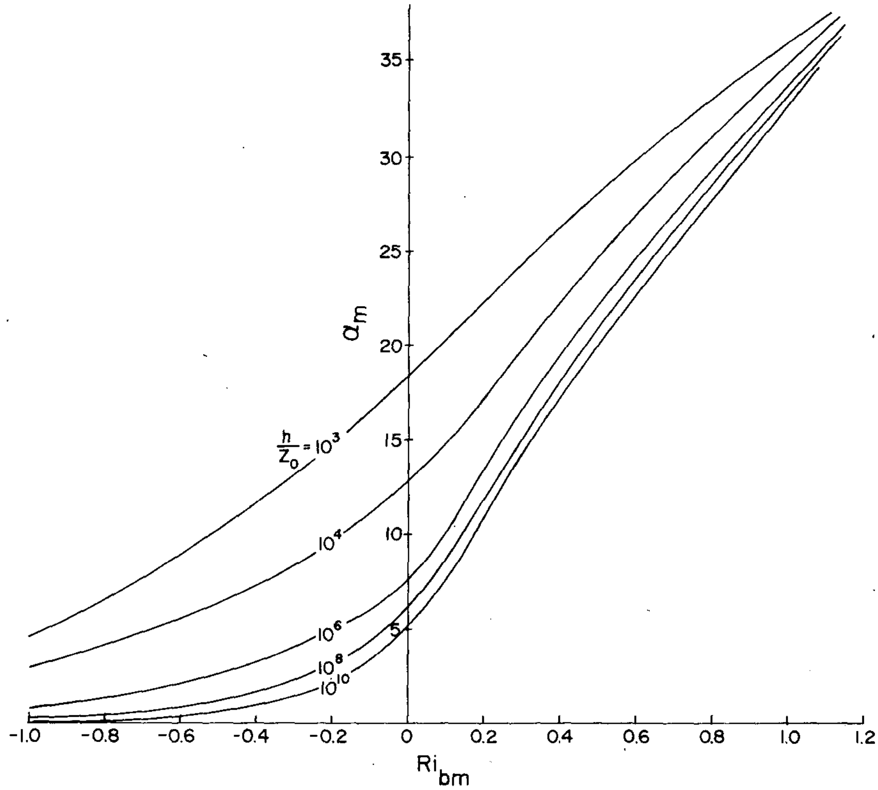

FIG. 9. The computed angle α_m as a function of h/z_0 and stability.

respect to z) forms of the equations of mean motion, as

$$\left.\begin{aligned}\bar{A}_m&=\bar{A}_0\\ \bar{B}_m&=\bar{B}_0-k\left(\frac{fh}{u_*}\right)^{-1}\end{aligned}\right\}. \qquad (11)$$

The above relations are valid for a barotropic ABL [for the baroclinic case, see Arya (1977)], irrespective of stability, provided that momentum fluxes vanish at $z=h$.

The ratio $\bar{C}_m/\bar{C}$ as given by Eqs. (3) and (5) is expected to be close to unity for unstable and convective conditions because $\theta_{vm}\approx\theta_{vh}$ (Deardorff, 1972a). It may become as low as one-half under extremely stable conditions when the θ profile is linear throughout the ABL. However, from the observed profiles under most stable conditions, the ratio $\bar{C}_m/\bar{C}$ is roughly estimated to about two-thirds (the uncertainty of this estimate is less than that of $\bar{C}$). Thus, our adopted relations are

$$\left.\begin{aligned}\bar{C}_m&=\bar{C}, &\quad -h/L&\geqslant 2\\ \bar{C}_m&=\tfrac{2}{3}\bar{C}, &\quad h/L&\geqslant 2\end{aligned}\right\}. \qquad (11c)$$

Eqs. (6), (10) and (11) give the following forms for $\bar{A}_m$, $\bar{B}_m$ and $\bar{C}_m$:

$$\left.\begin{aligned}\bar{A}_m&=\ln(-h/L)+\ln\frac{fh}{u_*}+1.5\\ \bar{B}_m&=1.8\frac{fh}{u_*}\exp(0.2h/L)\\ \bar{C}_m&=\ln(-h/L)+3.7\end{aligned}\right\}, \quad \frac{h}{L}\leqslant -2, \qquad (12)$$

$$\left.\begin{aligned}\bar{A}_m&=-0.96(h/L)+2.5\\ \bar{B}_m&=0.80(h/L)+1.1\\ \bar{C}_m&=-2.0(h/L)+4.7\end{aligned}\right\}, \quad \frac{h}{L}\equiv\mu_*^{\frac{1}{2}}\geqslant 2. \qquad (13)$$

For near-neutral conditions $(-2<h/L<2)$, $\bar{A}_m$, $\bar{B}_m$, etc., are assumed to be given by the linear interpolation of the above computed values at $h/L=\pm2$. Thus for $h/L=0$ and $fh/u_*=1$ our adopted values are $\bar{A}_m=1.39$, $\bar{B}_m=1.95$ and $\bar{C}_m=2.55$.

The above relations may be compared with the empirical formulas obtained by Yamada (1976) from a best fit to the selected Wangara data. The differences on the unstable side are rather insignificant as compared to the expected error bars on the empirically fitted curves. However, the two sets of relations differ considerably on the stable side. Yamada's coefficients for the linear terms in h/L are roughly 0.4 times those in

(13). This may be primarily attributed to the difference in the estimated height of the surface inversion which Yamada used for the boundary layer height and our theoretical h given by Eq. (8a). If one substitutes for $h/L=\mu_*^{\frac{1}{2}}$ in (13) and then compares these with the empirically determined functions $\bar{A}_m(\mu_*)$, $\bar{B}_m(\mu_*)$ and $\bar{C}_m(\mu_*)$, Eqs. (13) give a reasonable fit to the data.

b. Inversion of parametric relations

Since Eqs. (3), (4) or (5) implicitly involve the internal characteristics of the boundary layer, *viz.*, u_*, θ_{v*} and q_*, it is desirable to invert them so as to express the surface fluxes in terms of only the "external" characteristics which are either prescribed or otherwise computed in a GCM. For this we particularly choose Eqs. (5) mainly because the layer-averaged variables are more readily available in a GCM (see Deardorff, 1972b). Also, since A_m and B_m are less sensitive to the effects of baroclinity than are the corresponding functions in Eqs. (3) or (4), neglecting such complicating effects, or assuming $A_m \approx \bar{A}_m$, $B_m \approx \bar{B}_m$, etc., for the sake of simplicity would be more justified in the former (Arya, 1977). When such a simplification is indeed permissible or desired, the inversion of (5) yields:

$$\left.\begin{aligned} F_0 &= -C_{Dm}(h/z_0, \mathrm{Ri}_{\mathrm{bm}})V_m^2 \\ Q_{v0} &= -C_{Hm}(h/z_0, \mathrm{Ri}_{\mathrm{bm}})V_m(\theta_{vm}-\theta_{v0}) \\ E_0 &= -C_{Em}(h/z_0, \mathrm{Ri}_{\mathrm{bm}})V_m(q_m-q_0) \end{aligned}\right\}, \qquad (14)$$

in which the drag and other transfer coefficients are some universal functions of the ratio h/z_0 and the bulk Richardson number

$$\mathrm{Ri}_{\mathrm{bm}} = \beta h(\theta_{vm}-\theta_{v0})/V_m^2. \qquad (15)$$

The angle between $\boldsymbol{\tau}_0$ and $\mathbf{V}_m$ is equal to $\alpha_m(h/z_0, \mathrm{Ri}_{\mathrm{bm}})$.

Corresponding to our chosen forms (12) and (13) for the various similarity functions, the computed α_m, C_{Dm} and C_{Hm} (it is assumed that $C_{Em}=C_{Hm}$) are represented in Figs. 9, 10 and 11 as functions of h/z_0 and $\mathrm{Ri}_{\mathrm{bm}}$. These may be considered as revisions to the similar results derived by Deardorff (1972b) following a different procedure. As expected, α_m increases and the transfer coefficients decrease with increasing bulk Richardson number. Over the expected ranges of h/z_0 and $\mathrm{Ri}_{\mathrm{bm}}$, the drag and heat transfer coefficients may vary over more than a hundred-fold range. On the unstable side, the above computed nomograms are only valid for $fh/u_* \approx 1$. In low latitudes, fh/u_* may become considerably smaller than unity so that it would be advisable to compute α_m, C_{Dm}, etc., directly from Eqs. (5) and (12). This may require a few iterations since fh/u_* is not known to begin with. For more details of how to use the various results in specific cases, the reader is referred to Deardorff (1972b).

4. Specification of boundary layer height *h*

In order to use the parameterization scheme discussed in Section 3, the height h of the boundary layer must be

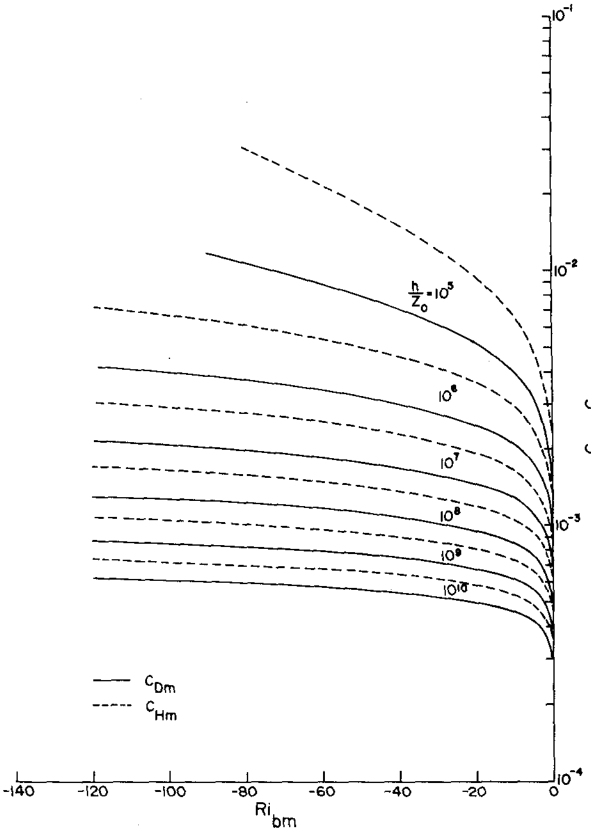

FIG. 10. The computed drag coefficient C_{Dm} as a function of h/z_0 and stability.

specified or parameterized in a GCM in terms of other known or calculated variables. For an unstable ABL capped by inversion, h is determined by the height of the inversion base z_i (Deardorff, 1972a; Wyngaard *et al.*, 1974b), for which prognostic equations have been derived by several investigators [for a good review of these, see Bhumralkar (1975) and Stull (1975)]. Considering many factors, the following rate equation suggested by Deardorff (1974) can be recommended for use in a GCM[1]:

$$\frac{dh}{dt}-w_h=\frac{1.8Q_0\left[1+1.1\left(\dfrac{u_*}{w_*}\right)^3\left(5-3\dfrac{fh}{u_*}\right)\right]}{\gamma^+h+\dfrac{9w_*^2T}{gh}\left[1+0.8\left(\dfrac{u_*}{w_*}\right)^2\right]}, \qquad (16)$$

where $w_*=(Q_0hg/T)^{\frac{1}{3}}$ is the convective velocity scale, w_h the large-scale vertical velocity (subsidence) at the top of the boundary layer and γ^+ is the lapse rate of potential temperature in the inversion layer.

Unlike many other proposed rate equations, (16) prevents the development of a singularity when $Q_0 \rightarrow 0$ and/or $\gamma^+ \rightarrow 0$. It has been tested favorably against observations, as well as against the simulated

[1] But, see also Zeman (1975).

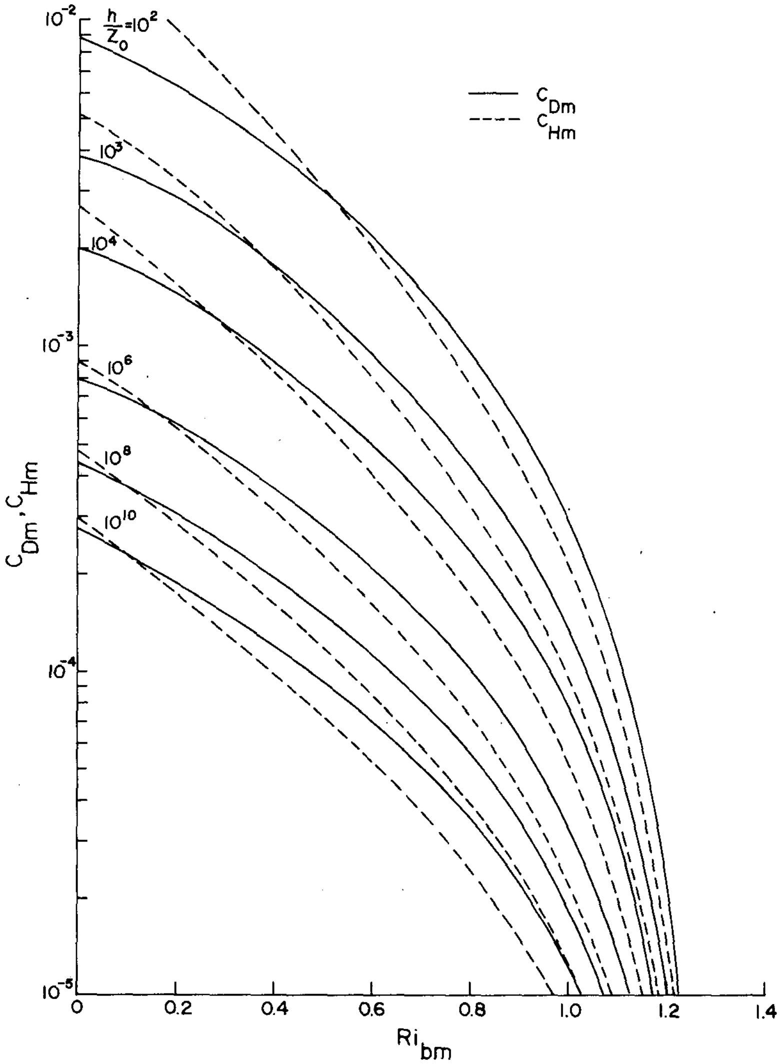

FIG. 11. The computed heat transfer coefficient C_{Hm} as a function of h/z_0 and stability.

evolution of an unstable ABL using a very comprehensive three-dimensional model (Deardorff, 1974). Eq. (16), however, is valid only under dry convection conditions when cloud-induced local circulations and radiative transfers near the top of the boundary layer do not influence h, although this may not be a serious limitation in most GCM's having little or no capability of predicting cloud cover. Tennekes (1973) has suggested that since the radiative heat flux tends to decrease the net heating rate of air in the boundary layer, it can be parameterized in a model by making a small reduction in the surface heat flux.

The stably stratified boundary layer also displays evolutionary characteristics, although variations in h are not as pronounced as under daytime convective conditions over land. A satisfactory rate equation for h in stable conditions is not available. It is not clear whether such an equation would indeed be appropriate (Zilitinkevich and Deardorff, 1974). Therefore, h may be specified through a diagnostic relation such as (8) proposed here, or that suggested by Deardorff (1972b), i.e.,

$$h=\left(\frac{1}{30L}+\frac{f}{0.25u_*}+\frac{1}{z_T}\right)^{-1}, \tag{17}$$

in which z_T denotes the height of the troposphere. Eq. (17) is essentially an interpolation formula, which assumes that $h\approx 30L$ in very stable conditions and/or in low latitudes. The inclusion of z_T ensures that h would not exceed the height of the tropopause even if exactly neutral conditions were to occur near or at the equator. For this reason (17) may be preferred to (8); they both give comparable results in middle or high latitudes.

Other diagnostic relations for the stable boundary layer height have been proposed in the literature [for a review, see Hanna (1969)]. But most of these are less satisfactory, as they seem to imply (or assume) that the bulk Richardson number is a fixed constant, irrespective of stability, and that h adjusts until this criterion is met. Observations, on the contrary, indicate that the bulk Richardson number increases with increasing stratification and may approach a constant (critical) value only under extremely stable conditions (Deardorff, 1972b). This is indeed the whole basis of considering the bulk Richardson number as an external stability parameter in the similarity forms of drag and heat transfer relations such as those given by Eqs. (14).

5. Specification of the roughness parameter (z_0)

The effects of small-scale surface irregularities on the transfer processes in the boundary layer are incorporated only through the roughness parameter z_0. As a result z_0 appears in all the parametric relations discussed in Sections 2 and 3 and has to be specified for each grid point before such parameterization schemes can be used. The usual practice of arbitrarily specifying one value for all the land surfaces and another (usually much smaller) one for all the ocean areas is highly questionable in view of the high sensitivity of the parametric relations to variations in z_0. It completely ignores the large differences in the roughness characteristics of different land surfaces (from flat sand or snow-covered surfaces to intensely forested and hilly areas), as well as of the ocean surface under calm and stormy conditions.

Over the oceans, z_0 is determined by very complex air-sea interactions giving rise to a wave field which depends on the air stress, fetch, precipitation, etc. For very calm conditions ($V_1 \leqslant 1$ m s^{-1}), the surface may be aerodynamically smooth, so that $z_0 \approx 0.1\nu/u_*$ (ν being the kinematic viscosity of air). More often though, the sea surface is an aerodynamically rough surface with z_0 in general depending on the wind stress, phase speed and the root-mean-square height of waves, and some molecular properties of air and water (Kitaygorodskiy, 1969; Hsu, 1974). For equilibrium wave conditions, Charnock (1955) has proposed, on dimensional grounds, a relation $z_0 = bu_*^2/g$, where the constant $b \approx 0.02$ (estimated values by different investigators range from 0.01 to 0.08). Wippermann (1972) has suggested the following interpolation formula to be used for smooth as well as rough conditions:

$$z_0 = 0.1\nu/u_* + bu_*^2/g. \tag{18}$$

Note that Charnock's relation or Eq. (18) implies a considerable increasing trend of z_0 or the drag coefficient C_{D1} with increasing wind speed. On the contrary, most recent measurements indicate very little or no such trend, especially in the range of wind speeds at 10 m level of $6 \lesssim V_{10} \lesssim 12$ m s^{-1}, and imply a more or less constant value of $z_0 \approx 0.02$ cm (Stewart, 1974). Whether the same could be said for the much higher wind speeds typical of hurricanes and other severe storms is not clear.

The range of variation of z_0 over different land surfaces is even larger (Kung, 1963). Determining z_0 in terms of some measurable physical characteristics of an underlying surface is a very difficult problem, especially when these characteristics vary considerably in space and time. The roughness parameter determined from profile measurements from masts or towers is a local parameter. In the parametric relations for large-scale models, on the other hand, z_0 represents the average roughness characteristics over a large area, the typical grid size being several hundred kilometers. Since the land surface is rarely uniform over such large areas, except perhaps over large desert regions, z_0 in Eqs. (1) and (14) must be considered as an effective roughness parameter, and determined separately for each grid area.

A practical method of determining z_0 for densely packed vegetation of different types has been demonstrated by Kung (1963). The two basic assumptions used in his analysis are 1) that the effective roughness for a large region is the area-weighted average of log z_0 values for different vegetation types in the region, and 2) for a uniform vegetation cover, that the local roughness parameter is determined only by the average height h_0 of roughness elements. Then using an empirical relation between z_0 and h_0, and the available data on land use and vegetation heights for different months and seasons, Kung (1963) has also estimated the meridional distributions of the zonally averaged z_0 over three continental areas. The results indicate that the effective roughness over different land areas may vary over 3 or 4 orders of magnitude. Inclusion of topography in the determination of z_0 could extend this range even further.

6. Conclusions

Some of the proposed schemes of boundary layer parameterization in atmospheric general circulation models are revised in the light of our improved knowledge about the atmospheric boundary layer in recent years. If a GCM has enough vertical resolution to place its first grid level in the surface layer ($\lesssim 50$ m), the parameterizations based on the Monin-Obukhov similarity theory appear to be the best. Our results for this may be considered as revisions to the scheme proposed by Clarke (1970b).

In most GCM's, the vertical resolution is too coarse to permit the placing of the first grid level within the surface layer and, in some cases, even within the whole boundary layer. For these we recommend the parameterization scheme proposed by Deardorff (1972b); it is revised here on the basis of more recent studies of the ABL. The alternative schemes proposed by Zilitinkevich (1969), Clarke (1970b) and others become highly inaccurate in low latitudes and are very sensitive to baroclinic effects (which are usually neglected for the sake of simplicity). The comparative effects of baroclinity and the earth's rotation on the drag laws are investigated in a separate paper (Arya, 1977).

The usual practice of specifying a single fixed value of z_0 for all the land or ocean surfaces is questioned. Over the oceans an interpolation formula such as (18) might give an adequate parameterization for z_0. For land surfaces it is suggested that the effective roughness parameter be determined for each grid point using an approach similar to that adopted by Kung (1963).

Acknowledgments. I am very thankful to Drs. J. W. Deardorff and J. C. Wyngaard for providing me with some of the numerical model results used in this study. The financial support by the Atmospheric Research Section, National Science Foundation, under Grant ATM76-00855 is gratefully acknowledged.

REFERENCES

Arya, S. P. S., 1975: Geostrophic drag and heat transfer relations for the atmospheric boundary layer. *Quart. J. Roy. Meteor. Soc.*, **101**, 147–161.

——, 1977: Comparative effects of baroclinicity and rotation on drag laws for the atmospheric boundary layer. Submitted to *J. Atmos. Sci.*

——, and J. C. Wyngaard, 1975: Effect of baroclinicity on wind profiles and the geostrophic drag law for the convective planetary boundary layer. *J. Atmos. Sci.*, **32**, 767–778.

——, and A. Sundararajan, 1976: A critical review of the proposed similarity theories for the atmospheric boundary layer. *Bound. Layer Meteor.* (in press).

Augstein, E., H. Schmidt and F. Ostapoff, 1974: The vertical structure of the atmospheric planetary boundary layer in undisturbed trade winds over the Atlantic ocean. *Bound. Layer Meteor.*, **6**, 129–150.

Bhumralkar, C. M., 1975: A survey of parameterization techniques for the planetary boundary layer in atmospheric circulation models. Rep. R-1653-ARPA, The Rand Corp., Santa Monica, Calif., 84 pp.

——, 1976: Parameterization of the planetary boundary layer in atmospheric general circulation models—A review. *Rev. Geophys. Space Phys.*, **14**, 215–226.

Blackadar, A. K., and H. Tennekes, 1968: Asymptotic similarity in neutral barotropic planetary boundary layers. *J. Atmos. Sci.*, **25**, 1015–1020.

Brummer, B., E. Augstein, and H. Riehl, 1974: On the low-level wind structure in the Atlantic trade. *Quart. J. Roy. Meteor. Soc.*, **100**, 109–121.

Businger, J. A., and S. P. S. Arya, 1974: The height of the mixed layer in the stably stratified planetary boundary layer. *Advances in Geophysics*, Vol. 18A, Academic Press, 73–92.

Businger, J. A., J. C. Wyngaard, Y. Izumi and E. F. Bradley, 1971: Flux-profile relationships in the atmospheric surface layer. *J. Atmos. Sci.*, **28**, 181–189.

Charnock, H., 1955: Wind stress on a water surface. *Quart. J. Roy. Meteor. Soc.*, **81**, 639–642.

Clarke, R. H., 1970a: Observational studies in the atmospheric boundary layer. *Quart. J. Roy. Meteor. Soc.*, **96**, 91–114.

——, 1970b: Recommended methods for the treatment of boundary layer in numerical models. *Aust. Meteor. Mag.*, **18**, 51–73.

——, 1972: Discussion of "Observational studies in the atmospheric boundary layer." *Quart. J. Roy. Meteor. Soc.*, **98**, 234–235.

——, and G. D. Hess, 1974: Geostrophic departure and the functions *A* and *B* of Rossby-number similarity theory. *Bound. Layer Meteor.*, **7**, 267–287.

——, and ——, 1975: On the relation between surface wind and pressure gradient, especially in lower latitudes. *Bound. Layer Meteor.*, **9**, 325–339.

——, A. J. Dyer, R. R. Brook, D. G. Reid and A. J. Troup, 1971: The Wangara experiment: Boundary layer data. Tech. Pap. No. 19, CSIRO, Div. Meteor. Phys., Melbourne, Australia, 362 pp.

Csanady, G. T., 1974: Equilibrium theory of the planetary boundary layer with an inversion lid. *Bound. Layer Meteor.*, **6**, 63–79.

Deardorff, J. W., 1968: Dependence of air-sea transfer coefficients on bulk stability. *J. Geophys. Res.*, **73**, 2549–2557.

——, 1972a: Numerical investigation of neutral and unstable planetary boundary layers. *J. Atmos. Sci.*, **29**, 91–115.

——, 1972b: Parameterization of the planetary boundary layer for use in general circulation models. *Mon. Wea. Rev.*, **100**, 93–106.

——, 1974: Three-dimensional numerical study of the height and mean structure of a heated planetary boundary layer. *Bound. Layer Meteor.*, **7**, 81–106.

Gordon, A. H., 1952: The relation between the mean vector surface wind and the mean vector pressure gradient over the oceans. *Geofis. Pura Appl.*, **21**, 49–51.

Hanna, S. R., 1969: The thickness of the planetary boundary layer. *Atmos. Environ.*, **3**, 519–536.

Hsu, S. A., 1974: A dynamic roughness equation and its application to wind stress determination at the air-sea interface. *J. Phys. Oceanogr.*, **4**, 116–120.

Kazanski, A. B., and A. S. Monin, 1960: A turbulent regime above the ground atmospheric layer. *Izv. Akad. Sci. USSR Geophys. Ser.*, **1**, 110–112.

Kitaygorodskiy, S. A., 1969: Small-scale atmosphere-ocean interactions. *Izv. Atmos. Ocean. Phys.*, **5**, 641–650.

Kung, E. C., 1963: Climatology of aerodynamic roughness parameter and energy dissipation in the planetary boundary layer of the northern hemisphere. Annual Rep., Contract DA-36-039-AMC-00878, Dept. of Meteorology, University of Wisconsin, 37–96.

Lettau, H. H., 1962: Theoretical wind spirals in the boundary layer of a barotropic atmosphere. *Beitr. Phys. Atmos.*, **35**, 195–212.

——, and B. Davidson, 1957: *Exploring the Atmosphere's First Mile*, Vol. 1 and 2. Pergamon Press.

Malgarejo, J. W., and J. W. Deardorff, 1974: Stability functions for the boundary-layer resistance laws based upon observed boundary layer heights. *J. Atmos.*, **31**, 1324–1333.

Pennell, W. T., and M. A. Lemone, 1974: An experimental study of turbulence structure in the fair weather trade wind boundary layer. *J. Atmos. Sci.*, **31**, 1308–1323.

Stewart, R. W., 1974: The air-sea momentum exchange. *Bound. Layer Meteor.*, **6**, 151–167.

Stull, R. B., 1975: Temperature inversions capping atmospheric boundary layers. Ph.D. dissertation, University of Washington, 218 pp.

Sundararajan, A., 1975a: Mean Reynolds stress modeling of the atmospheric boundary layer. Ph.D. dissertation, University of Washington, 150 pp.

——, 1975b: Significance of the neutral height scale for the convective, barotropic planetary boundary layer. *J. Atmos. Sci.*, **32**, 2285–2287.

Tennekes, H., 1973: A model for the dynamics of the inversion above a convective boundary layer. *J. Atmos. Sci.*, **30**, 558–567.

Wiin-Nielsen, A., 1974: Vorticity, divergence and vertical velocity in a baroclinic boundary layer with a linear variation of geostrophic wind. *Bound. Layer Meteor.*, **6**, 459–476.

Wippermann, F., 1972: A note on parameterization of large-scale wind stress at the sea-surface. *Beitr. Phys. Atmos.*, **45**, 260–266.

Wyngaard, J. C., 1976: Modeling the planetary layer extension to the stable case. *Bound. Layer Meteor.*, **9**, 441–460.

——, S. P. S. Arya and O. R. Coté, 1974a: Some aspects of the structure of convective planetary boundary layers. *J. Atmos. Sci.*, **31**, 747–754.

——, O. R. Coté and K. S. Rao, 1974b: Modeling the atmospheric boundary layer. *Advances in Geophysics*, Vol. 18A, Academic Press, 193–211.

Yamada, T., 1976: On the similarity functions *A*, *B* and *C* of the planetary boundary layer. *J. Atmos. Sci.*, **33**, 781–793.

Zeman, O., 1975: The dynamics of entrainment in the planetary boundary layer: A study in turbulence modeling and parameterization. Ph.D. thesis, The Pennsylvania State University.

Zilitinkevich, S. S., 1969: On the computation of the basic parameters of the interaction between the atmosphere and the ocean. *Tellus*, **21**, 17–24.

——, 1972: On the determination of the height of the Ekman boundary layer. *Bound. Layer Meteor.*, **3**, 141–145.

——, 1975: Resistance laws and prediction equations for the depth of the planetary boundary layer. *J. Atmos. Sci.*, **32**, 741–752.

——, and J. W. Deardorff, 1974: Similarity theory for the planetary boundary layer of time-dependent height. *J. Atmos. Sci.*, **31**, 1449–1452.