On the Selection of Grids for Semi-Implicit Schemes

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ABSTRACT

It is shown that semi-implicit time differencing on a nonstaggered grid using centered time and space derivatives leads to a decoupling into *four* separate solutions on different subgrids. This deficiency may be successfully overcome by combining weighted averages of Laplacian operators on the subgrids. However, a far more satisfactory approach is shown to be the use of a staggered grid which appears to be a natural choice for the semi-implicit scheme.

1. Description of the problem

It has been known for some time that the use of explicit time and space centered differences in the primitive equations leads to a decoupling of the gravity wave modes on two different elementary subgrids (e.g., Gerrity and McPherson, 1970). Usually, the noise generated is removed by smoothing or filtering. However, Janjić (1974) proposed an alternative approach in which he imposes an interaction between the two elementary subgrids.

In the case of a semi-implicit scheme the problem is compounded by the fact that the Helmholtz equation system which must be solved at each time step separates into four independent solutions when applied to a nonstaggered grid. Although our attention was drawn to the problem during development of a multi-level semi-implicit model (Gauntlett *et al.*, 1976), it is sufficient to consider the free surface equations. When these equations of motion are expressed in semi-implicit form on a nonstaggered grid, the key diagnostic equation is a Helmholtz equation of the form

$$\overline{P_{xx}}^{xx}+\overline{P_{yy}}^{yy}-\alpha P=F, \qquad (1)$$

where $P=\bar{\phi}^{2t}$ (ϕ is geopotential), α is independent of time and F consists of quantities known from times t and $t-\Delta t$. The operators $\bar{P}^x$, P_x are defined by

$$\left.\begin{aligned}\bar{P}^x&=0.5[P(x+\tfrac{1}{2}\Delta x,y,t)+P(x-\tfrac{1}{2}\Delta x,y,t)]\\ P_x&=\frac{1}{\Delta x}[P(x+\tfrac{1}{2}\Delta x,y,t)-P(x-\tfrac{1}{2}\Delta x,y,t)]\end{aligned}\right\}.$$

Eq. (1) has been derived in full [Kwizak and Robert, 1971, Eq. (47)].

For a grid size $\Delta x=\Delta y=d$, it is noted that the Laplacian $\overline{P_{xx}}^{xx}+\overline{P_{yy}}^{yy}$ is of the form

$$L_{2+}(P)=\frac{1}{4d^2}(P_{i+2,j}+P_{i-2,j}+P_{i,j+2}+P_{i,j-2}-4P_{ij}), \qquad (2)$$

which clearly separates into four independent solutions in the lattices (marked by □, Δ, ×, ○ on Fig. 1). It was found that this separation could be controlled simply by combining a weighted average of the two Laplacian operators, i.e., $\beta_1L_{2+}(P)+\beta_2L_+(P)$, where

$$L_+(P)=\frac{1}{d^2}(P_{i+1,j}+P_{i-1,j}+P_{i,j+1}+P_{i,j-1}-4P_{i,j}), \qquad (3)$$

and $\beta_1=0.8$, $\beta_2=0.2$. Unless this procedure was adopted, noise was generated in the forecast, particularly if forcing (for example, by mountains) was present.

Despite the fact that the procedure works satisfactorily, there are important disadvantages. Because

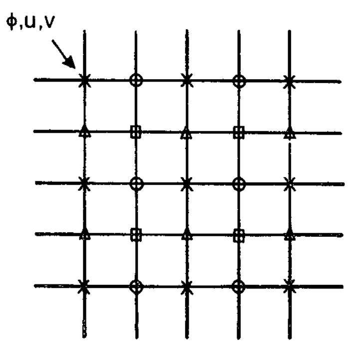

FIG. 1. Nonstaggered grid formulation. Velocity components and geopotential are located at all points. Solution to Eq. (1) separates on four independent lattices as marked.

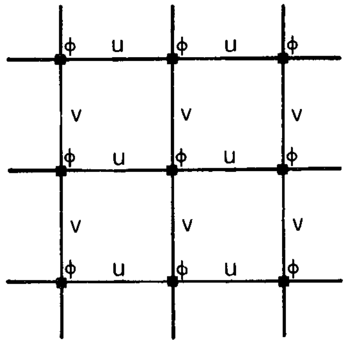

FIG. 2. Staggered grid configuration. Velocity components and geopotential are located as shown. Solution decoupling of Eq. (1) does not occur.

the combination of operators is not the mathematically correct choice, a small reduction of allowable time step occurs, and there is a slight violation of the conservationtion properties of the difference equations. A better approach is available and will be described in the next section.

2. Staggered grid reformation

If the semi-implicit equations are redefined on the staggered grid shown in Fig. 2, it is seen immediately that the operator (2) no longer leads to solution independence, and might be referred to as a "natural" grid for the centered space and time semi-implicit scheme. Gerrity and McPherson (1971) employed this staggered grid in their free surface model, and they correctly described its advantages over the nonstaggered grid in terms of the large reduction in computation points and in avoiding problems near the boundary associated with the L_{2+} operator. However, we have pointed to a further advantage of this staggered grid, namely, that it prevents solution separation of the type described above. This grid has now been tested in a modified version of the semi-implicit model described by Gauntlett *et al.* (1976). It has been found to have significant superiority over the combined operators approach described in Section 1 in three important respects. First, the time step allowed was found to be greater. Specifically for an equivalent resolution, a time step of 40 min was allowed, compared with 30 min for the nonstaggered grid. Second, domain integrals of surface pressure, kinetic and potential energy were conserved to a higher degree, improving by about 5%. Finally, the simplicity of the single Laplacian operator used in the staggered grid scheme enabled a faster solution to the Helmholtz equations.

3. Concluding remarks

It has been shown that for semi-implicit schemes the use of a nonstaggered grid with the usual time and space centered finite-difference approximations leads to a decoupling into four separate solutions on different elementary subgrids. This decoupling may be overcome, in a pragmatic way, by using a weighted combination of Laplacian operators. However, a preferable procedure is to use a particular staggered grid which has no such solution separation.

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