Some Non-Quasigeostrophic Effects in Linear Baroclinic Waves

Robert Gall

Institute of Atmospheric Physics, The University of Arizona, Tucson 85721

(Manuscript received 27 October 1976, in revised form 12 May 1977)

ABSTRACT

Non-quasigeostrophic effects in unstable baroclinic waves, defined as differences between solutions in primitive equation and quasigeostrophic models due to terms in the governing equations that are neglected in the quasigeostrophic model, are examined. It is determined that these non-quasigeostrophic effects produce significant asymmetries in a baroclinic wave, as compared to the symmetry that would be expected if that wave were governed by the quasigeostrophic equations, even when the Rossby number is relatively small. These asymmetries result in significant differences in some of the eddy fluxes in a primitive equation model as compared to a quasigeostrophic model. This was found to be especially true for $\overline{u'v'}$, where even the sign was different in certain regions.

1. Introduction

In the theory of baroclinic waves of middle latitudes, the quasigeostrophic equations have played a central role. This is, of course, because the quasigeostrophic equations often permit analytic solutions, while analytic solutions to the primitive equations are much more difficult to find. Furthermore, the scaling assumptions that lead to the quasigeostrophic equations are usually very good for flows resembling that of the general circulation in middle latitudes.

The scaling arguments that justify the quasi-geostrophic equations suggest only that certain terms within the primitive equations tend to be smaller than other terms in the same equation and for this reason can be neglected. However, this does not necessarily insure that solutions to the quasigeostrophic equations will not differ significantly from corresponding solutions to the primitive equations even when the quasi-geostrophic assumptions are good, i.e., when the Rossby number is small. Qualitatively the solutions to the two sets of equations should be quite similar, but there can be some important quantitative differences.

For linear baroclinic waves when the Rossby number is small, the structure of the waves in the fields of pressure, wind, temperature and vertical velocity will not differ by much between the corresponding fields in primitive and quasigeostrophic models, but there can be some small differences. Some demonstrations of these differences are given by Hoskins (1975) and by Saltzman and Tang (1972, 1975).

The differences in wave structure, however, can lead to much more significant differences in some of the eddy fluxes. This would be especially true for the eddy fluxes that result from the correlation of variables that are nearly $\pm 90°$ out of phase with one another, such as $\overline{u'v'}$ and $\overline{v'\phi'}$. Since nonzero values of these terms depend on components of the flow that are not strictly geostrophic or depend on slight east-west tilts (in the horizontal) of the pressure wave, then even slight changes in wave structure can lead to large changes in these eddy fluxes. Eddy fluxes such as $\overline{\omega'T'}$ or $\overline{v'T'}$ that depend on quantities which tend to be nearly in phase or 180° out of phase might be expected to be much less affected by slight changes in wave structure.

Consideration of the differences in the eddy fluxes between primitive equations and quasigeostrophic models is important when baroclinic instability theory is used to parameterize the eddy fluxes in simplified climate models. If the quasigeostrophic baroclinic instability models are selected to determine the parameterizations of the eddy fluxes, then, as will be shown, serious error in some of the fluxes could result, such that even the sign could be wrong in certain regions. Furthermore, the apparent differences between the eddy fluxes in quasigeostrophic and primitive equation models would support an argument against the use of the quasigeostrophic equations in simplified climate models that expressly predict the eddies, such as low-resolution general circulation models.

A recent paper by Simmons and Hoskins (1976) presented some comparisons of the differences between the linear baroclinic waves that would develop in simplified zonal flows on a sphere when the primitive equations are utilized and when the quasigeostrophic equations are utilized. They show that there are significant differences between the primitive equation solutions and the quasigeostrophic solutions, especially in the vertical and horizontal fluxes of heat and momentum that are produced by these waves. This paper will only reinforce that conclusion; however, here we shall concentrate on just the differences that are due to motions in the primitive equation models which are absent in the quasigeostrophic models and not try to include spherical effects at the same time. In this way, it will be easier to isolate some of the ways in which quasigeostrophic solutions differ from the primitive equation solutions. Furthermore, our approach is somewhat different, and we feel that it more clearly illustrates the differences between baroclinic waves in primitive equation and quasigeostrophic models.

We shall refer to these differences as non-quasigeostrophic effects. This terminology is necessary because the differences are due to certain terms which are absent in the quasigeostrophic equations but retained in the primitive equations. Although the motions for which these terms are responsible are small if the usual quasigeostrophic scaling arguments are valid, these motions can have a rather strong geostrophic component. For this reason, such motions cannot be referred to as non-geostrophic.

The reader should keep in mind that other factors such as spherical geometry of the earth (Simmons and Hoskins, 1976; Moura and Stone, 1976; Hollingsworth, 1975) or nonlinear effects (Saltzman and Tang, 1975; Gall, 1967b) can further distort the waves and their eddy fluxes from what they would be in a quasigeostrophic model on a constant f plane. The intention here is to show some of the non-quasigeostrophic effects that can appear in linear baroclinic waves in primitive equation models, even when the Rossby number is fairly small.

2. Theory

The usual way to demonstrate the differences between linear baroclinic waves in primitive equation models and quasigeostrophic models is to construct two models: one from the primitive equations and one from the quasigeostrophic equations. If everything else, including the Coriolis parameter and its variations, is the same in these two models, then in theory the differences between them should be due only to non-quasigeostrophic effects.

For isolating some of the non-quasigeostrophic effects, there exists an alternative method which considers only waves in which the quasigeostrophic assumptions are sound and is based upon certain well-known symmetry properties that baroclinic waves have in the usual quasigeostrophic models. A wave is allowed to evolve in the primitive equation model, and the symmetry of the wave in the primitive equation model is compared with the symmetry that would be expected of a wave in a similar quasigeostrophic model. Any deviation from the symmetry expected of the quasigeostrophic wave must then be due to non-quasigeostrophic effects. The method and what is meant by the symmetry of a baroclinic wave are outlined below.

Consider a model where f is constant, the zonal flow is confined to an east-west oriented channel, and the mean zonal wind is exactly symmetric about a vertical plane down the center of this channel (at $y=0$). Furthermore, assume that the zonal flow is constant with time and that the perturbations to this zonal flow are governed by the frictionless, adiabatic, linearized primitive equations. By combining the horizontal equations of motion to form a vorticity and a divergence equation, we can write exactly, for the above system, the following equations:

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\zeta'+\frac{\partial\omega'}{\partial p}f-\left\{\frac{\partial\omega'}{\partial y}\frac{\partial\bar{u}}{\partial p}\right\}+\left\{\frac{\partial\omega'}{\partial p'}\frac{\partial\bar{u}}{\partial y}\right\}-v'\frac{\partial^2\bar{u}}{\partial y^2}-\left\{\omega'\frac{\partial^2\bar{u}}{\partial y^2}\right\}=0 \quad (1)$$

$$\left\{\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial\omega'}{\partial p}-\frac{\partial\omega'}{\partial x}\frac{\partial\bar{u}}{\partial p}\right\}+f\zeta'-\nabla^2\phi'-\left\{2\frac{\partial\bar{u}}{\partial y}\frac{\partial v'}{\partial x}\right\}=0 \quad (2)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial\phi'}{\partial p}-\omega'\bar{\sigma}+v'\frac{\partial\bar{u}}{\partial p}f=0 \quad (3)$$

$$\left\{\nabla^2 v'=\frac{\partial\zeta'}{\partial x}-\frac{\partial}{\partial p}\left(\frac{\partial\omega'}{\partial y}\right)\right\} \quad (4)$$

$$\bar{\sigma}=-\frac{\bar{\alpha}}{\bar{\theta}}\frac{\partial\bar{\theta}}{\partial p}. \quad (5)$$

If, as outlined above, we were to use the quasigeostrophic equations to describe linear motions in the channel instead of the primitive equations, the equations would be

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\zeta'+\frac{\partial\omega'}{\partial p}f-v'\frac{\partial^2\bar{u}}{\partial y^2}=0 \quad (6)$$

$$f\zeta'=\nabla^2\phi' \quad \text{or} \quad v'=\frac{1}{f}\frac{\partial\phi'}{\partial x} \quad (7)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial\phi'}{\partial p}-\omega'\bar{\sigma}+v'\frac{\partial\bar{u}}{\partial p}f=0. \quad (8)$$

Note that (6)–(8) are obtained by eliminating the terms enclosed in braces in (1)–(5). In most applications of (6)–(8), the static stability parameter $\bar{\sigma}$ is taken to be constant. See the Appendix for a list of symbols.

Any variable in the above channel can be broken up into two parts: one that is symmetric about a vertical plane down the center of the channel, i.e., $\gamma(y)=\gamma(-y)$, and another that is antisymmetric about this plane, i.e., $\gamma(y)=-\gamma(-y)$. We define symmetric and antisymmetric perturbations to the quasigeostrophic equations [Eqs. (6)–(8)] as in Table 1,[1] where it is assumed that u', v' and ζ' are calculated from ϕ' using the geostrophic approximation; hence, for a given symmetry of ϕ', the symmetry of the remaining variables is immediately determined if (8) is used to determine the symmetry of ω'. The zonal wind $\bar{u}$ has been chosen to be symmetric about the center of the channel; therefore, since (6)–(8) are linear and if the initial perturbation is symmetric (as defined in Table 1), then that perturbation in a quasigeostrophic model will remain symmetric for all time, even if the perturbation is unstable. Similarily, an initial perturbation that is antisymmetric about the channel center will remain antisymmetric in the quasigeostrophic model.

Furthermore, it is known from previous baroclinic instability studies (Song, 1971) that the symmetric perturbation (as defined in Table 1) of a quasigeostrophic baroclinic wave is much more unstable than the antisymmetric perturbation if the zonal wind is symmetric. Therefore, even if the initial perturbations in a baroclinic zonal flow consist of a collection of both symmetric and antisymmetric modes, as these modes grow to larger amplitude at least one of the symmetric modes will quickly become much larger than all antisymmetric modes, and eventually the perturbations will appear to be completely symmetric (as defined in Table 1).

The same is not true of the primitive equations. For example, let (1), (2) and (3) determined the time rate of change of ζ', ω', and ϕ', respectively, and use (4) to evaluate v'. The primitive equations that determine the time rate of change of the symmetric parts of these variables are

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\zeta_s'-\frac{\partial \omega_s'}{\partial p}f-\left\{\frac{\partial \omega_{as}'}{\partial y}\frac{\partial \bar{u}}{\partial p}\right\}^*+\left\{\frac{\partial \omega_{as}'}{\partial p}\frac{\partial \bar{u}}{\partial y}\right\}^*-v_s'\frac{\partial^2\bar{u}}{\partial y^2}-\left\{\omega_s'\frac{\partial^2 \bar{u}}{\partial y^2}\right\}=0, \quad (9)$$

$$\left\{\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial \omega_s'}{\partial p}-\frac{\partial \omega_s'}{\partial x}\frac{\partial \bar{u}}{\partial p}\right\}+f\zeta_s'-\nabla^2\phi_s'-\left\{2\frac{\partial \bar{u}}{\partial y}\frac{\partial v_{as}'}{\partial x}\right\}^*=0, \quad (10)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial \phi_s'}{\partial p}-\omega_s'\bar{\sigma}_s-\{\omega_{as}'\bar{\sigma}_{as}\}^*+v_s'\frac{\partial \bar{u}}{\partial p}f=0, \quad (11)$$

$$\left\{\nabla^2 v_s'=-\frac{\partial \zeta_s'}{\partial x}-\left(\frac{\partial^2\omega_{as}'}{\partial p\partial y}\right)^*\right\}, \quad (12)$$

and the equations for the antisymmetric parts of these variables are

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\zeta_{as}'-\frac{\partial \omega_{as}'}{\partial p}f-\left\{\frac{\partial \omega_s'}{\partial y}\frac{\partial \bar{u}}{\partial p}\right\}^*+\left\{\frac{\partial \omega_s'}{\partial p}\frac{\partial \bar{u}}{\partial y}\right\}^*-v_{as}'\frac{\partial^2\bar{u}}{\partial y^2}-\left\{\omega_{as}'\frac{\partial^2 \bar{u}}{\partial y^2}\right\}=0, \quad (13)$$

$$\left\{\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial \omega_{as}'}{\partial p}-\frac{\partial \omega_{as}'}{\partial x}\frac{\partial \bar{u}}{\partial p}\right\}+f\zeta_{as}'-\nabla^2\phi_{as}'-\left\{2\frac{\partial \bar{u}}{\partial y}\frac{\partial v_s'}{\partial x}\right\}^*=0, \quad (14)$$

$$\left(\frac{\partial}{\partial t}+\bar{u}\frac{\partial}{\partial x}\right)\frac{\partial \phi_{as}'}{\partial p}-\omega_{as}'\bar{\sigma}_s-\{\omega_s'\bar{\sigma}_{as}\}^*+v_{as}'\frac{\partial \bar{u}}{\partial p}f=0, \quad (15)$$

$$\left\{\nabla^2 v_{as}'=-\frac{\partial \zeta_{as}'}{\partial x}-\left(\frac{\partial^2\omega_s'}{\partial p\partial y}\right)^*\right\}. \quad (16)$$

In the above, the symmetry of a term is determined by noticing that first derivatives in x, p and t and second derivatives in y of a quantity do not change the symmetry of the term from that of the quantity itself, while a first derivative in y does. The product of two quantities that are both symmetric or antisymmetric is symmetric, while a product is antisymmetric if one quantity is symmetric and the other antisymmetric. For reference, the terms that are dropped to form the quasigeostrophic equations are enclosed in braces.

Note that, even if the initial perturbations were given as purely symmetric (as defined in Table 1), then as the wave grows in the primitive equation model, there will be a gradual development of an antisymmetric part through the terms that are identified by asterisks in (9)–(12) and (13)–(16). Since the terms that result in the

TABLE 1. Definition of the symmetric and antisymmetric perturbations.

Symmetric perturbation	Antisymmetric perturbation
ϕ' symmetric	ϕ' antisymmetric
u' antisymmetric	u' symmetric
v' symmetric	v' antisymmetric
ζ' symmetric	ζ' antisymmetric
ω' symmetric	ω' antisymmetric

[1] Hereinafter, the terms *symmetric perturbation* and *antisymmetric perturbation* will refer to the definitions in Table 1 for both the quasigeostrophic model and the primitive equation model. In the primitive equation model, u', v' and ζ' are not calculated directly from ϕ' as in the quasigeostrophic model; however, the definitions in Table 1 will be retained for the purposes of this discussion.

TABLE 2. The normalized pressure and approximate height of the levels used in the linear model.

Level	p/p_*	Height (km)
1	0.017	27.51
2	0.036	22.61
3	0.064	18.85
4	0.106	15.72
5	0.162	13.02
6	0.235	10.64
7	0.323	8.56
8	0.421	6.72
9	0.525	5.11
10	0.631	3.72
11	0.732	2.55
12	0.824	1.60
13	0.902	0.86
14	0.960	0.36
15	0.991	0.08

development of the antisymmetric part of the perturbation are not present in the quasigeostrophic equations, then the antisymmetric part that develops can be identified directly as a non-quasigeostrophic effect. Likewise, a purely antisymmetric initial perturbation will eventually develop a symmetric part through non-quasigeostrophic processes.

If the initial perturbation contains both symmetric and antisymmetric perturbations, we would still expect that the symmetric perturbation would be most unstable in a primitive equation model for a zonal flow where the zonal wind is symmetric about the channel center, if the usual quasigeostrophic approximations are good. This is because the quasigeostrophic models show the symmetric perturbation to be most unstable in such a case. Therefore, the symmetric perturbation will quickly become the dominant perturbation in any flow where the Rossby number is small ($\sim 10^{-1}$). However, due to the coupling to the antisymmetric equations through the terms identified by asterisks in (9)–(12) and (13)–(16), this dominant perturbation would have an antisymmetric perturbation associated with it. We expect that this antisymmetric perturbation would be smaller than the symmetric part if the Rossby number is small; however, as will be seen shortly, it is not necessarily insignificant.

The most unstable wave in a quasigeostrophic model with a symmetric mean zonal wind would have exactly the symmetry defined by the symmetric solution in Table 1. Therefore, any deviations from this symmetry caused by the development of the antisymmetric solution in the primitive equation model can be due only to non-quasigeostrophic effects. This does not imply that this antisymmetric part will not affect the development of the symmetric solution; it most certainly will, to some extent. Similarly, this does not imply that the antisymmetric part will be composed of non-geostrophic motions. As will be seen, it clearly is not. All that is implied is that the presence of the antisymmetric part is due to the terms that are dropped from the *usual* quasigeostrophic models.

The symmetry defined in Table 1 also defines the symmetry of the eddy fluxes $\overline{v'T'}$, $\overline{\omega'\alpha'}$, $\overline{u'v'}$ and $\overline{v'\phi'}$ that we would expect in a quasigeostrophic model. Any deviations from this symmetry can also be defined as non-quasigeostrophic effects in these fluxes. Again, we must note that these deviations are due to the presence of terms left out of the quasigeostrophic equations and are not due entirely to motions that are nongeostrophic.

Finally, note that in a primitive equation model, even if $\bar{u}$ is exactly symmetric, $\bar{\sigma}$ will have a *small* antisymmetric part, as well as a symmetric part. This gives rise to the terms enclosed in braces in (11) and (15). Since $\bar{\sigma}$ is usually taken to be constant in most quasigeostrophic models, we shall refer to the terms in (11) and (15), due to the antisymmetric part of $\bar{\sigma}$, as non-quasigeostrophic terms. However, there is no reason why antisymmetric components of $\bar{\sigma}$ could not be included in a quasigeostrophic model.

3. The model

The primitive equation linear model used in this study has been described in detail by Gall (1976a); therefore, it will not be described here. The only differences from the model of Gall (1976a) are in the vertical and horizontal resolution. In the present model 15 layers were used in the vertical as shown in Table 2. As with the model described by Gall (1976a), p/p_* is used as the vertical coordinate. In the north-south direction across the channel, 31 grid points were used with a grid spacing of 133 km. In the east-west direction 20 points described the wave.

The walls of the channel were oriented east-west, and the boundary conditions at these walls were

$$\left.\begin{aligned} v&=0\\ \frac{\partial\phi}{\partial y}&=0\end{aligned}\right\}. \qquad (17)$$

The Coriolis parameter was constant across the channel and a value of 10^{-4} s^{-1} was adopted.

The most unstable mode of a given zonal wavelength is computed for an arbitrary zonal flow. This is accomplished by introducing a small initial perturbation, then integrating the model in time, with the zonal flow held fixed, until exponential growth of the wave kinetic energy is achieved. Thus, we use the so-called initial-value technique (Brown, 1969).

The computer program of this linear model has undergone extensive testing (Gall, 1976a), including repeated checks of each line of code. Furthermore, it was constructed from the Geophysical Fluid Dynamics Laboratory GCM computer program, which itself has under-

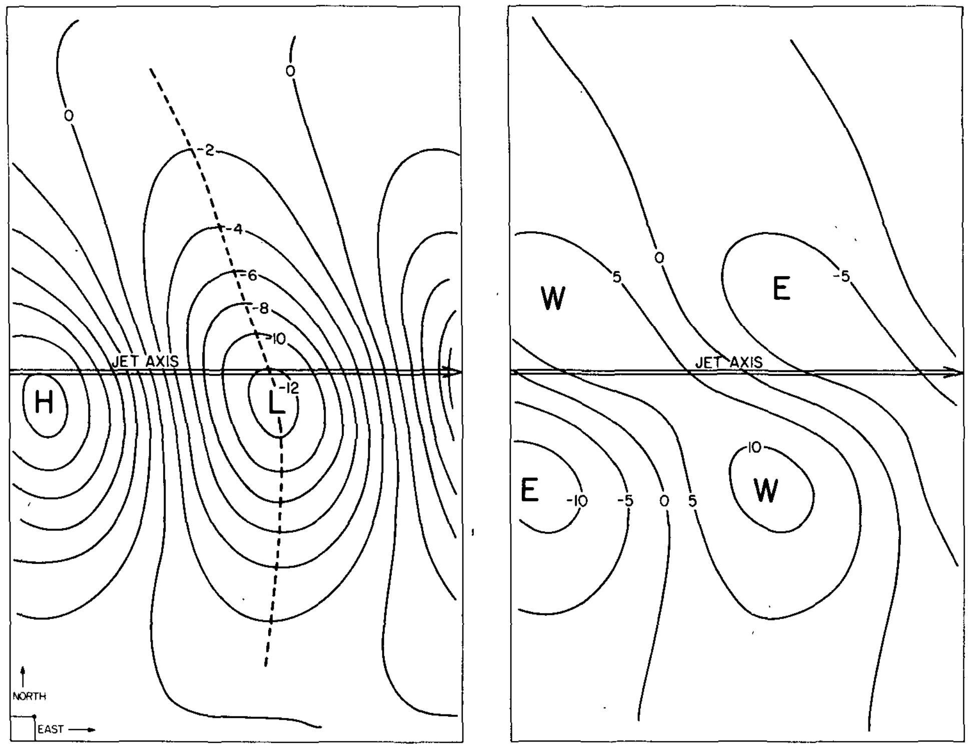

FIG. 1. Horizontal distribution of perturbation surface pressure p_* (mb), left, and zonal velocity u' (m s^{-1}), right, across the width of the channel and over one wavelength for zonal wavenumber 7 in the primitive equation model. The axis of maximum mean zonal wind is indicated at the center of the channel. The square in the left-hand corner indicates the grid spacing, which is 133 km in the north-south direction and 200 km in the east-west direction.

gone extensive tests to uncover programming errors over the years. Therefore, we feel that the asymmetries to be discussed in Section 5 have physical meaning and are not due to programming errors.

4. The zonal flow and initial conditions

The zonal mean wind is defined by

$$\bar{u}(y,p)=\tfrac{1}{2}\bar{u}_c(p)\left\{1-\cos\left[\frac{2\pi}{L}\left(\frac{L}{2}-y\right)\right]\right\}, \qquad -\frac{L}{2}\leqslant y\leqslant\frac{L}{2}, \tag{18}$$

$$\bar{u}_c(p)=\frac{\bar{p}_*^{\kappa}-\bar{p}^{\kappa}}{\bar{p}_*^{\kappa}-\bar{p}_T^{\kappa}}\bar{u}_{\max}, \qquad \bar{p}_*\geqslant\bar{p}\geqslant\bar{p}_T, \tag{19}$$

$$\bar{u}_c(p)=\bar{u}_{\max}, \qquad \bar{p}_T>\bar{p}\geqslant 0, \tag{20}$$

where $\bar{p}_T$ is 200 mb, $\bar{u}_{\max}$ is 45 m s^{-1}, $\bar{p}_*$ is 1013 mb and the channel width L is 4100 km. The zonal mean temperature is defined by the thermal wind relation and (18)–(20). As a boundary condition for integrating the thermal wind equation to obtain the zonal mean temperature,

$$\bar{T}_c=T_0(\bar{p}/\bar{p}_*)^{0.21} \tag{21}$$

is assumed at the center of the channel, where T_0 is 273 K. Hence, the zonal wind is exactly symmetric about the vertical plane passing down the center of the channel and is in geostrophic balance with the temperature field. There is no zonal mean meridional or vertical motion.

Onto this zonal mean flow is added a small perturbation in the temperature field that has a wavelength of 4000 km, an amplitude of 0.1 K and a phase angle of zero degrees. By keeping the amplitude and phase angle constant, the initial temperature perturbation is therefore exactly symmetric about the center of the channel.

The wavelength of the initial perturbation corresponds, of course, to the wavelength of the wave to be considered here. A length of 4000 km was selected to correspond with the approximate length of wavenumber 7 in middle latitudes, which is a wave that is important in determining the eddy fluxes in this region.[2] Furthermore, it is short enough that non-quasigeostrophic effects might be important, yet the Rossby number is small enough ($\sim 10^{-1}$) so that the usual

[2] Hereinafter, we shall refer to waves approximately 4000 km in length or with a wavenumber of approximately 7 as intermediate-scale waves.

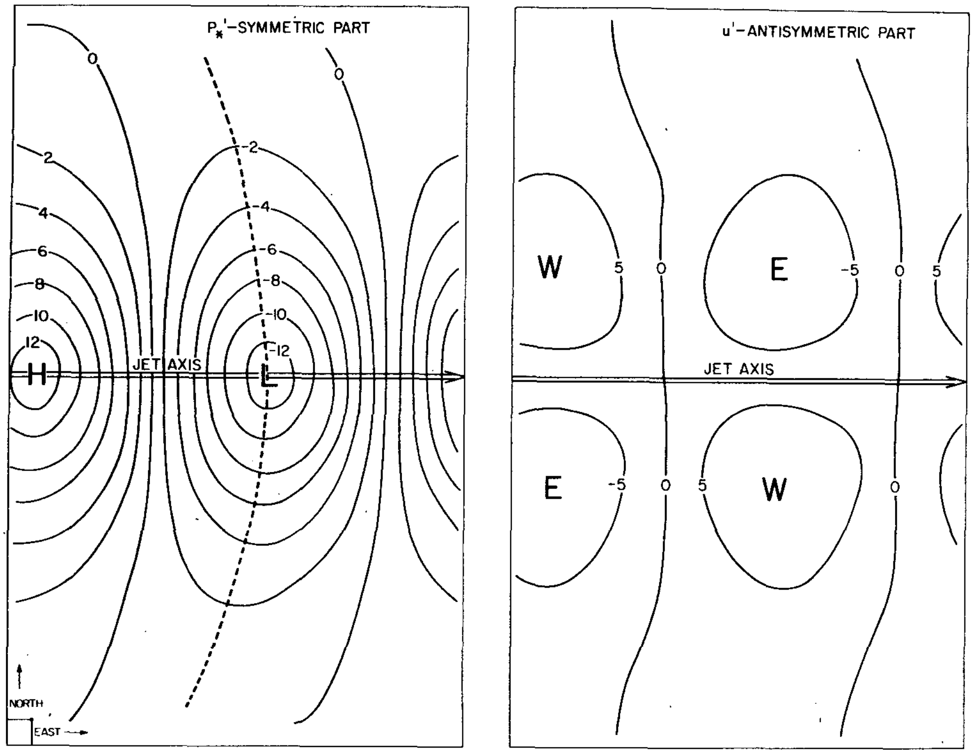

FIG. 2. As in Fig. 1 except only the symmetric part of p_*' and the antisymmetric part of u' are displayed.

quasigeostrophic approximations can still be justified with the usual scaling arguments (Holton, 1972).

5. The results

We have carefully selected the wavelength of the perturbation and the parameters of the zonal flow so that the quasigeostrophic approximations are good, and hence the symmetry arguments of Section 2 hold. That is, the perturbation, after the amplitude of the most unstable mode greatly exceeds the amplitude of all other modes in the initial perturbation, should have approximately the symmetry of the symmetric perturbation listed in Table 1.

Fig. 1 depicts the surface pressure and surface zonal wind perturbations after exponential growth of the wave kinetic energy is achieved in the model. While the expected symmetry of the wave is mostly apparent, there are a number of obvious discrepancies. For example, the tilt of the trough lines of the surface pressure perturbation away from the mean zonal jet axis is different north and south of this axis. In fact, there is virtually no tilt to the trough south of the jet axis. If the quasigeostrophic equations had been used, the tilt of the trough both north and south of the axis of the jet would have been the same. In addition, the centers of low and high pressure would have been directly under the jet axis, whereas they are located ~200 km south of the jet axis in Fig. 1. These discrepancies from the expected quasigeostrophic solutions can be identified as due to non-quasigeostrophic motions, as outlined in Section 2.

The perturbation zonal wind component would be antisymmetric about the jet axis in a quasigeostrophic model. Basically, it is in the primitive equation model, but again there are some definite deviations from the quasigeostrophic symmetry. Most noticeable is that the maximum perturbation amplitude south of the jet axis exceeds the maximum perturbation amplitude north of the jet axis by ~30%. Also, the maximum perturbation south of the jet axis is further from this axis than is the maximum perturbation that is located north of the jet axis. Again, these deviations from the expected symmetry of a quasigeostrophic model are a result of non-quasigeostrophic effects.

Figs. 2 and 3 show the symmetric and antisymmetric parts of the surface pressure and surface zonal wind perturbations that were shown in Fig. 1. Fig. 2, which shows the symmetric perturbation, gives an approximate representation of what the wave would have looked like if the quasigeostrophic approximation had been made in the model. Fig. 3 shows the antisymmetric perturbations (see Table 1) that develop in conjunction with the symmetric perturbation of Fig. 2.

Fig. 3 can be said to represent non-quasigeostrophic effects; however, as was emphasized in Section 2, this does not imply that the flow in the antisymmetric perturbation is completely non-geostrophic. In fact, it has a rather large geostrophic component that is easily recognized in Fig. 3. In addition, the symmetric perturbation (Fig. 2) will be modified by the presence of the antisymmetric perturbation so that, while the upper panel resembles the wave that would result in a quasigeostrophic model, it is possible that there are significant differences. For example, the extent of the tilt of the trough could differ in a truly quasigeostrophic model from that presented in Fig. 2. Whether this is the case or not cannot be deduced from these results. However, the correspondence of Fig. 2 to a similar solution in a quasigeostrophic model is probably good, since the Rossby number is small.

Finally, note that the antisymmetric perturbations (Fig. 3) are smaller than the symmetric perturbations (Fig. 2). The maximum surface pressure perturbation is ~75% less, and the zonal wind component is 30% less. This is to be expected since the Rossby number is small.

Fig. 4 shows the vertical distribution of the geopotential perturbation in the north-south cross section. This distribution is typical of a baroclinic wave of wavelength 4000 km in middle latitudes, including a quasigeostrophic wave, in that there are two maxima in the vertical—one at the earth's surface and a second near 200 mb (the tropopause). However, the lower maximum is located south of the jet axis, while the upper maximum is located north of the jet axis. Hence the wave tends to tilt northward with height, although by only ~500 km. A quasigeostrophic wave in the same zonal flow will not tilt northward, since the amplitude of the quasigeostrophic wave of geopotential would be symmetric about the center of the channel. The tendency for non-quasigeostrophic motions to cause a northward tilt of the linear wave with height has been discussed by Hollingsworth (1975) and Hoskins (1975). This northward tilt, however, appears to be a property unique to the linear wave, since once the wave is allowed to interact with the zonal flow a southward tilt develops (Saltzman and Tang, 1972)[3].

The amplitude of the zonal wind perturbation shows even more asymmetry in the north-south cross section than does the geopotential perturbation. This is illustrated in Fig. 4. The asymmetry that was noted at the earth's surface, where the maximum amplitude of the perturbation zonal wind component is south of the jet axis, changes with height. At 200 mb the maximum zonal wind perturbation is located north of the jet axis and is over twice as large as the perturbation south of the jet axis at the same height. Had the quasigeostrophic equations been used instead of the primitive equations, the amplitude of zonal wind perturbation would have been exactly symmetric about the center of the channel.

The differences between the primitive equation

[3] Note that if nonlinear processes are allowed, in particular the interactions of the wave with waves of other wavelengths, asymmetries also develop in the structure of the wave in the east-west direction. These asymmetries are, of course, eliminated in this linear model.

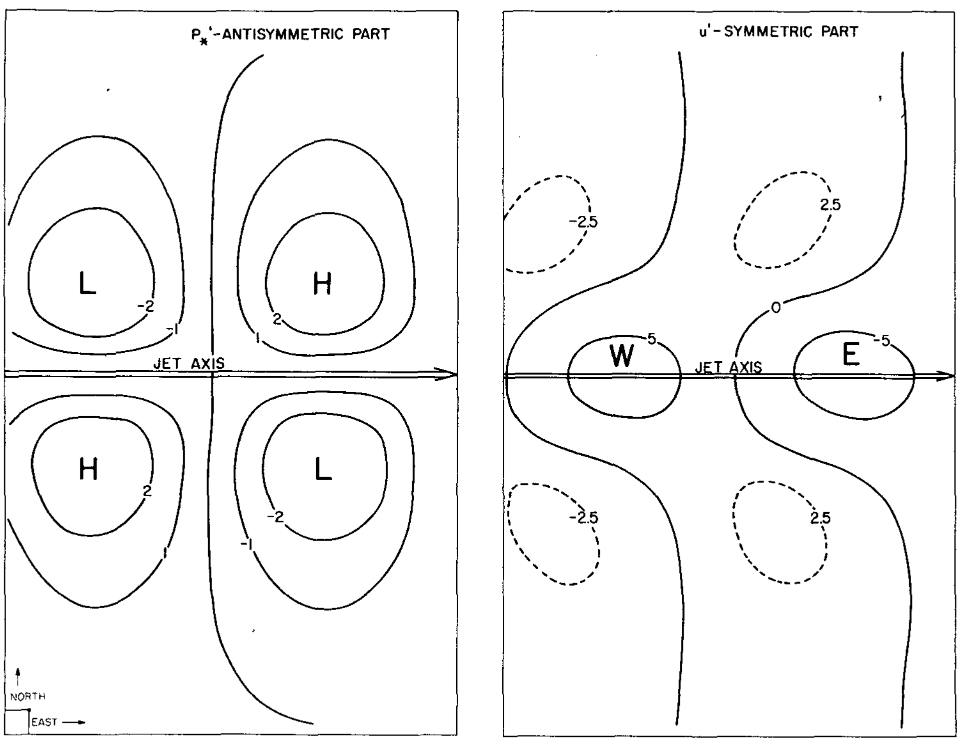

FIG. 3. As in Fig. 1 except only the antisymmetric part of p_* and the symmetric part of u' are displayed.

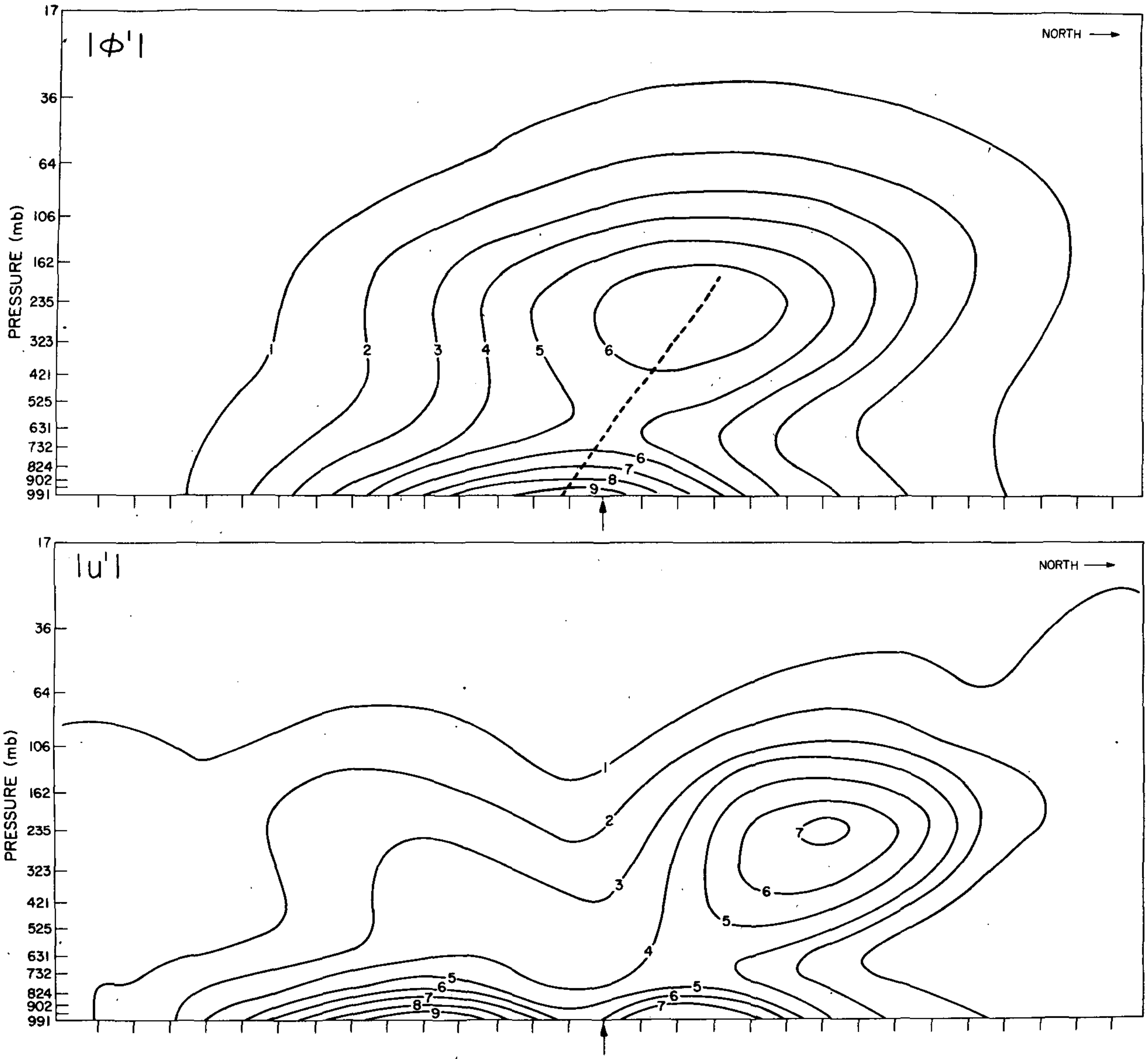

FIG. 4. North-south cross section of the amplitude of ϕ' (top) and u' (bottom) of wavenumber 7 across the width of the channel in the primitive equation model. The center of the channel is indicated by the arrow. Tick marks along the abscissa indicate the grid spacing, which is 133 km. In both figures the amplitude has been normalized so that the largest value is 10.

solutions and those that would be expected with the quasigeostrophic equations, which have been noted above, suggest the possibility that at least some of the eddy fluxes will also differ significantly. This was found to be especially true for the meridional flux of zonal momentum by the wave. The quasigeostrophic equations suggest that this flux should be exactly antisymmetric about the channel center—directed southward to the north of the jet axis and northward to the south of the jet axis. The sign of these fluxes has been suggested by previous studies (Brown, 1969; Song, 1971; McIntyre, 1970).

Fig. 5 shows the north-south cross section of $\overline{u'v'}$. Note that this flux is southward in most regions near the center of the channel where the wave is active. Rather than being antisymmetric about the center at all levels, it is actually almost symmetric, especially at the earth's surface. With height, the maximum value of $\overline{u'v'}$ on a pressure surface shifts northward, so that at 300 mb it is located ~400 km north of the jet axis.

A crude approximation of what the zonal momentum flux would have looked like if the quasigeostrophic equations have been used can be obtained by forming the correlation $\overline{u'_{as}v'}$. This assumes that the so-called symmetric perturbations have not been drastically altered by the presence of the antisymmetric perturbation in the primitive equation model.

Fig. 5 shows $\overline{u'_{as}v'_s}$, which is antisymmetric with the flux directed northward to the south of the jet axis and southward to the north. Therefore, in the region extending from the jet axis southward about 700 km, the fluxes in the upper and lower cross sections of Fig. 5 are of *opposite sign*. With this, it is possible to conclude that, in some regions, the quasigeostrophic equations *could* predict erroneous fluxes of $\overline{u'v'}$, to the extent that even the sign would be wrong.

The data in Fig. 5 have been normalized so that the largest value is 10. This is for convenience in contouring. The largest absolute value in the cross section for $\overline{u'v'}$ is 0.44×10^2 m^2 s^{-2} and for $\overline{u'_{as}v'_s}$ 0.08×10^2 m^2 s^{-2}. Note that the largest value of $\overline{u'v'}$ is nearly an order of magnitude larger than the largest value of $\overline{u'_{as}v'_s}$. This might suggest that the quasigeostrophic wave will, in addition to indicating the wrong sign in some regions, greatly underestimate the magnitude. Unfortunately, since the wave is linear, there is little that can be said about the expected amplitude of any perturbation quantity or the associated eddy fluxes.

Figs. 6 and 7 display additional eddy fluxes. Of those shown, $\overline{\omega'\alpha'}$, $\overline{v'T'}$ and $\overline{v'\phi'}$ would be symmetric about the center of the channel in a quasigeostrophic model. Figs. 6 and 7 show that this is essentially also true for $\overline{\omega'\alpha'}$ and $\overline{v'T'}$ when the primitive equations are used. The flux $\overline{v'\phi'}$, however, has a strong asymmetry, in much the same fashion that ϕ' itself is asymmetric about the center of the channel. The maximum absolute amplitude of $\overline{v'\phi'}$ is south of the center of the

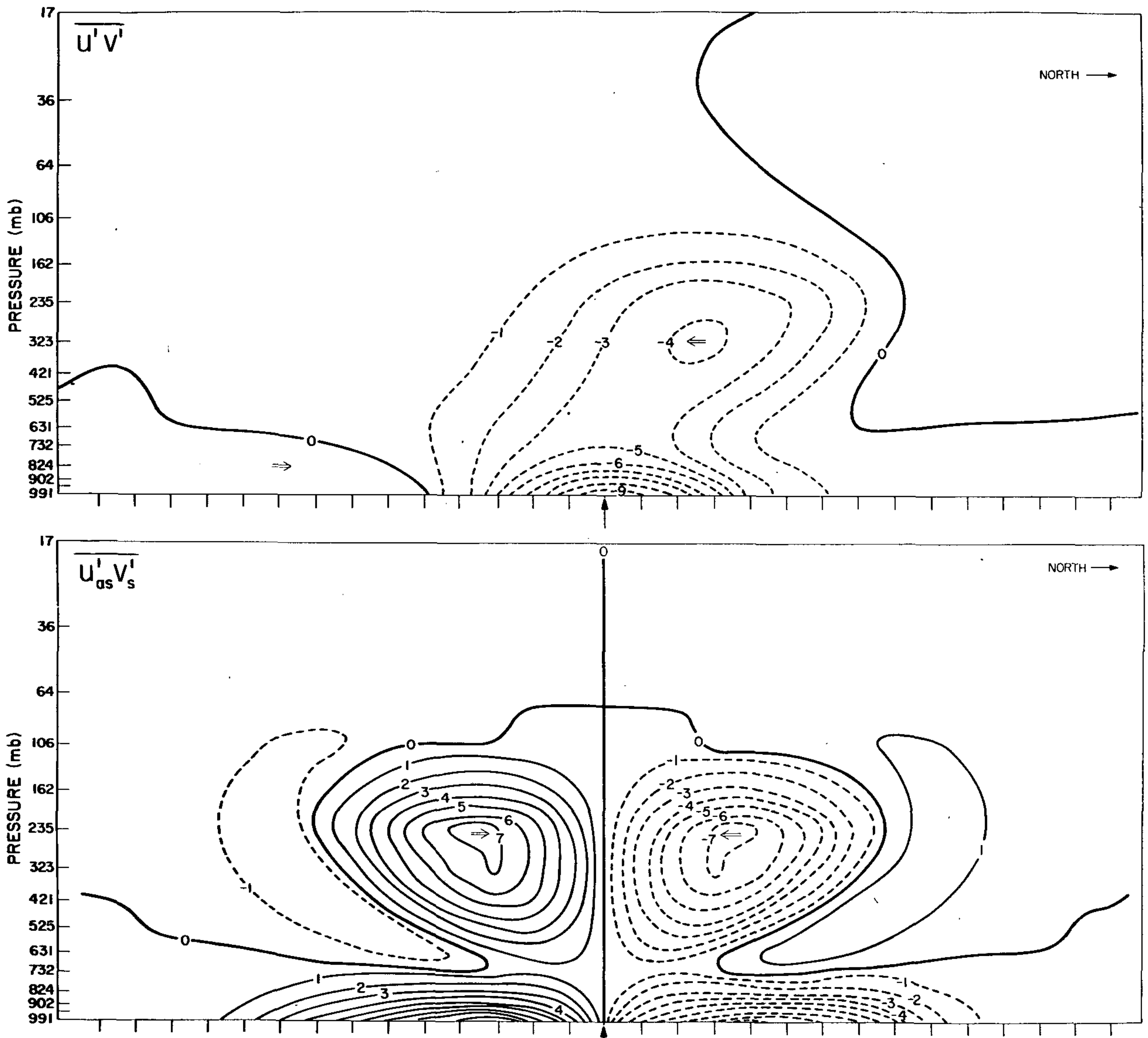

FIG. 5. As in Fig. 4 except north-south cross sections of $\overline{u'v'}$ (top) and $\overline{u'_{as}v'_s}$ (bottom).

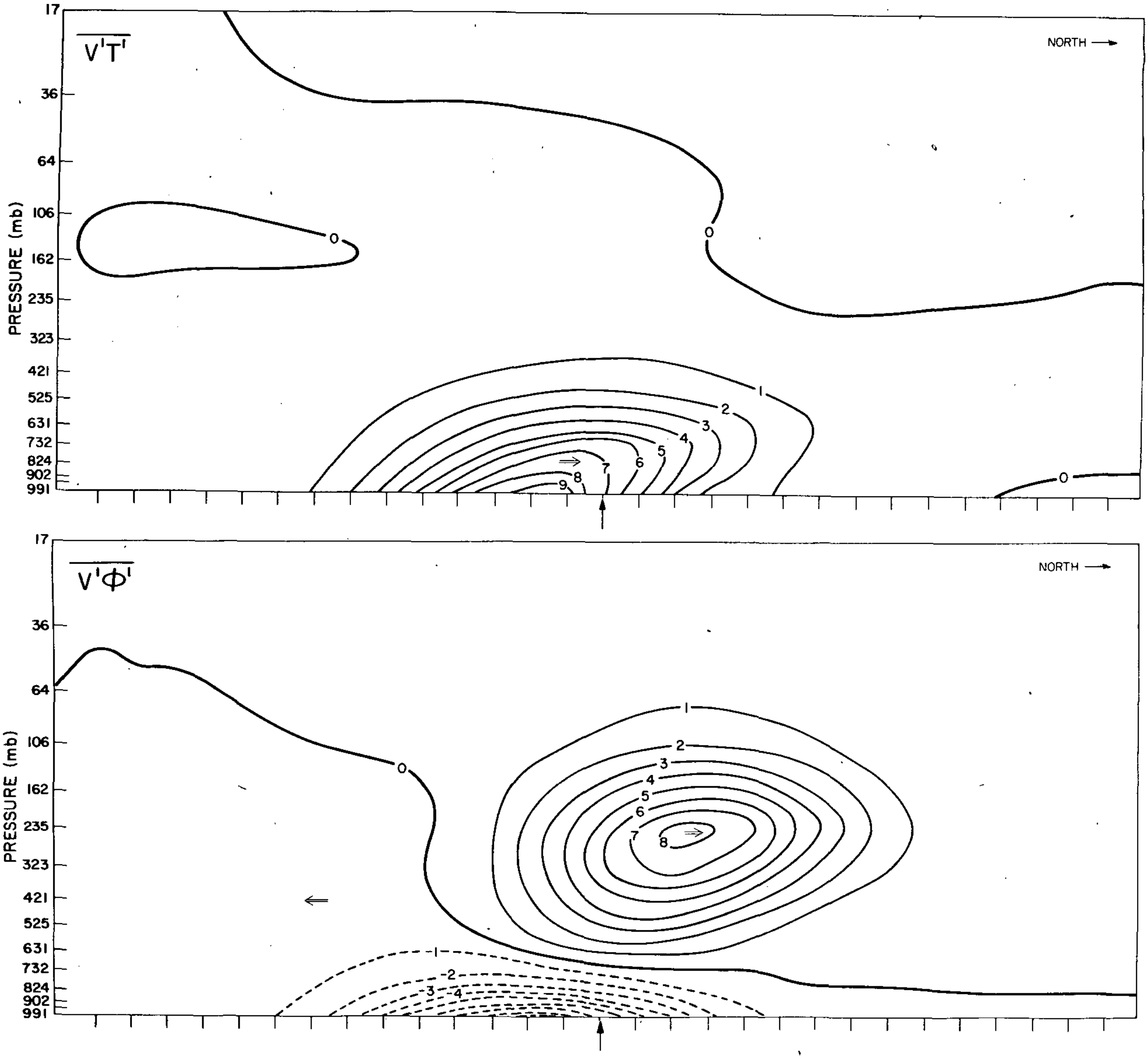

FIG. 6. As in Fig. 4 except north-south cross sections of $\overline{v'T'}$ (top) and $\overline{v'\phi'}$ (bottom).

channel at the earth's surface and shifts to north of the center by 250 mb.

The vertical fluxes of geopotential $\overline{\omega'\phi'}$ and momentum $\overline{u'\omega'}$ are absent in any model of the general circulation that uses the quasigeostrophic equations. However, the quasigeostrophic equations can be used to estimate the distribution of these fluxes within baroclinic waves, which, in turn, can be used to parameterize these fluxes in climatic models.

For example, u' could be calculated from

$$\frac{\partial u'}{\partial x}+\frac{\partial u'}{\partial y}+\frac{\partial \omega'}{\partial p}=0$$

and correlated with ω' to form $\overline{u'\omega'}$ (e.g., Kuo, 1952). If so, then $\overline{u'\omega'}$ estimated from the quasigeostrophic equations would be composed of both symmetric and antisymmetric parts. On the other hand, if u' were to be estimated in the usual fashion in quasigeostrophic models, i.e., from

$$u'=-\frac{1}{f}\frac{\partial \phi'}{\partial y},$$

then the quasigeostrophic model would indicate this flux to be entirely antisymmetric. Note that in the primitive equation model (Fig. 7), $\overline{u'\omega'}$ is mostly symmetric about the center of the channel.

Fig. 7 also indicates that $\overline{\omega'\phi'}$ is essentially symmetric in the primitive equation model. If this flux were to be estimated using ω' and ϕ' from the quasigeostrophic

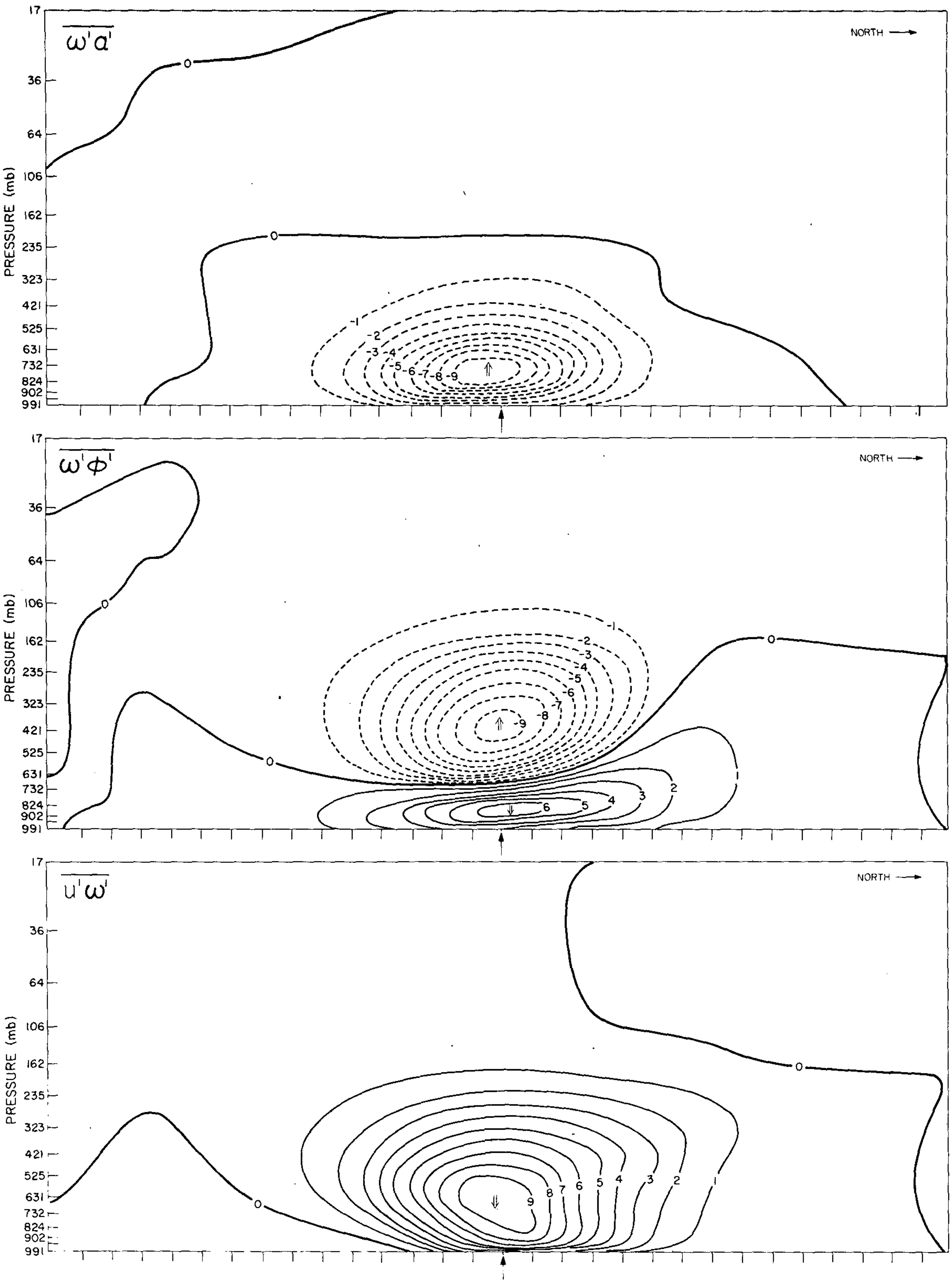

Fig. 7. As in Fig. 4 except north-south cross sections of $\overline{\omega'\alpha'}$ (top), $\overline{\omega'\phi'}$ (middle) and $\overline{u'\omega'}$ (bottom).

model, it would also be symmetric about the channel center.

6. Relevance to the general circulation

The temptation now is to use the above results to explain certain features that are found in the eddy fluxes produced by the intermediate-scale waves of the middle latitudes. Unfortunately, this cannot be done. There are other factors that can also strongly affect the symmetry of a wave, in much the same way that the non-quasigeostrophic motions did in this study.

Perhaps the strongest of these additional effects would be horizontal asymmetries in the zonal wind profile (which are always there) and the spherical geometry of the earth. Ample evidence of this is presented in Figs. 18 and 19 of Gall (1976a). These figures display the various eddy fluxes of waves 5 and 7 in a linear model, using a zonal flow taken from a GCM. The main difference between these two waves, other than their wavelength, is that each developed at different latitudes where the horizontal asymmetries of the mean zonal wind were quite different. For both waves, the primitive equations were used to compute the fluxes; therefore, non-quasigeostrophic effects are included. Yet the fluxes of $\overline{u'v'}$, $\overline{v'\phi'}$ and $\overline{u'\omega'}$ for wavenumber 7 had the opposite sign of the corresponding fluxes of wavenumber 5.[4]

Clearly, these differences cannot be attributed entirely to non-quasigeostrophic effects, and they are as large as the differences between the quasigeostrophic and primitive equation models outlined in this study. These differences must be related to the spherical geometry of the earth and to horizontal asymmetries of the zonal wind profiles. Some discussions of the effects of spherical geometry on the fluxes of zonal momentum and heat by linear baroclinic waves are presented by Simmons and Hoskins (1976).

The only reliable way to calculate the eddy fluxes for linear baroclinic waves in an arbitrary observed zonal flow is to use a primitive equation model that is applied to the sphere. However, it has been noted by Gall (1976b) that even the fluxes calculated in this way can differ considerably from the fluxes as observed in the GCM.

7. Conclusions

Non-quasigeostrophic effects, defined here as differences between solutions in quasigeostrophic and primitive equation models due to terms that are dropped from the quasigeostrophic equations, introduce significant differences in the structure of unstable baroclinic waves of intermediate-scale wavelengths in middle latitudes. This was found to be true for a wave where the usual quasigeostrophic approximations are good (the Rossby number is small). These structural differences, in turn, result in pronounced changes in some of the eddy fluxes of the wave in the primitive-equation model as compared to a quasigeostrophic model, to the extent that even the sign is different in certain regions. This was found to be especially true of the northward fluxes of zonal momentum, $\overline{u'v'}$, and of geopotential, $\overline{v'\phi'}$. The distributions of the horizontal and vertical fluxes of heat, $\overline{v'T'}$ and $\overline{\omega'\alpha'}$, in the primitive equation model were, on the other hand, similar to those predicted by quasigeostrophic theory.

The apparent sensitivity of the various fluxes to factors such as the spherical geometry of the globe and to horizontal asymmetries in the zonal mean flow, in addition to non-quasigeostrophic effects, does not mean that the results presented here are of only academic interest. The results of this study indicate that non-quasigeostrophic effects must be included if accurate estimates of the eddy fluxes by the intermediate-scale waves of middle latitudes are to be made. The results presented here are only for linear waves; however, it is reasonable to assume that if the differences between the primitive and quasigeostrophic models are significant in the linear wave, then they will likely be large in the nonlinear wave.

Acknowledgments. Most of the computer calculations reported here were performed while the author was a postdoctoral research associate at the Geophysical Fluid Dynamics Laboratory, Princeton, N. J. Support for the latter stages of this research was supplied by National Science Foundation Grant DES75-15889.

The author wishes to thank D. Golder of GFDL for his aid in preparing the necessary computer programs. The author is also indebted to Margaret Sanderson Rae of the University of Arizona for editing the manuscript.

APPENDIX

List of Symbols

t	time
T	temperature
x	horizontal coordinate in the eastward direction
y	horizontal coordinate in the northward direction
p	pressure
p_*	surface pressure
u	eastward wind component
v	northward wind component
ω	dp/dt
ζ	vorticity
α	specific volume
θ	potential temperature

[4] Note that a number of previous studies have indicated that the vertical flux of zonal momentum is downward in *linear* baroclinic waves (Kuo, 1952; Saltzman and Tang, 1975; Stone, 1972). The results of Gall (1976a, Fig. 19) indicate that this flux could have either sign.

ϕ geopotential
σ stability parameter
f Coriolis parameter
$(\bar{\ })$ zonal average
$(')$ departure from the zonal average
$(_s)$ symmetric part
$(_{as})$ antisymmetric part
γ an arbitrary variable
R gas constant for dry air
c_p specific heat of air at constant pressure
κ R/c_p

REFERENCES

Brown, J. A., 1969: A numerical investigation of hydrodynamic instability and energy conversions in the quasi-geostrophic atmosphere: Part I. *J. Atmos. Sci.*, **26**, 353–365.

Gall, R. L., 1976a: A comparison of linear baroclinic instability theory with the eddy statistics of a general circulation model. *J. Atmos. Sci.*, **33**, 349–373.

——, 1976b: Structural changes of growing baroclinic waves. *J. Atmos. Sci.*, **33**, 374–390.

Hollingsworth A., 1975: Baroclinic instability of a simple flow on the sphere. *Quart. J. Roy. Meteor. Soc.*, **101**, 495–528.

Holton, J. R., 1972: An introduction to dynamic meteorology. *International Geophysics Series*, Vol. 16, Academic Press, 76–78.

Hoskins, B. J., 1975: The geostrophic momentum approximation and the semigeostrophic equations. *J. Atmos. Sci.*, **32**, 233–242.

Kuo, H. L., 1952: Three-dimensional disturbances in a baroclinic zonal current. *J. Meteor.*, **9**, 260–277.

McIntyre, M. E., 1970: On the non-separable parallel flow instability problem. *J. Fluid Mech.*, **40**, 273–306.

Moura, A. D., and P. Stone, 1976: The effects of spherical geometry on baroclinic instability. *J. Atmos. Sci.*, **33**, 602–616.

Saltzman, B., and C. Tang, 1972: Analytical study of the evolution of an amplifying baroclinic wave. *J. Atmos. Sci.*, **29**, 427–444.

——, and ——, 1975: Analytical study of an amplifying baroclinic wave: Part II. Vertical motions and transports. *J. Atmos. Sci.*, **32**, 243–259.

Simmons, A. J., and B. J. Hoskins, 1976: Baroclinic instability on the spheres: Normal modes of the primitive and quasi-geostrophic equations. *J. Atmos. Sci.*, **33**, 1454–1477.

Song, R. T., 1971: A numerical study of the three-dimensional structure and energetics of unstable disturbances in zonal currents: Part II. *J. Atmos. Sci.*, **28**, 565–586.

Stone, P., 1972: On non-geostrophic baroclinic stability: Part III. The momentum and heat transports. *J. Atmos. Sci.*, **29**, 419–426.