NOTES AND CORRESPONDENCE

The Seasonal Excursion of the Intertropical Convergence Zone

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ABSTRACT

A simple analytical model relates the seasonal excursion of the Intertropical Convergence Zone to seasonal changes in hemispheric forcing. The maximum value of the excursion increases with the seasonal heating change; it decreases with the equator-pole heating difference and with the heat storage capacity of the system as indicated by the lag of the response after the forcing.

The data analysis, published in an accompanying paper in this issue (Kraus, 1977), suggests a reduced cross-equatorial flow of energy toward the winter hemisphere during subtropical drought years when the tropical rainfall belt fails to penetrate into the monsoonal fringe areas of the summer hemisphere. As a rationale one can argue that any unusually large demand for heat in the winter hemisphere must be compensated by the collection of heat from an increased area in the summer hemisphere. This can be achieved by the displacement of the Intertropical Convergence Zone (ITCZ) into higher latitudes. The opposite happens during periods of reduced energy demand. The argument essentially involves the concept (or the assumption) of an ITCZ which is defined as being a divide of the meridional heat transport.

Let s denote the vertically integrated and zonally averaged total kinetic, potential, internal and latent energy per unit horizontal area of the atmosphere. The change of s is a function of the latitude and the season and can be written as

$$\frac{\partial s}{\partial t}=\frac{K}{r^2\cos\varphi}\frac{\partial}{\partial\varphi}\cos\varphi\frac{\partial s}{\partial\varphi}+F, \qquad (1)$$

where K is a generalized turbulent transfer coefficient which will be evaluated later, r the radius of the earth and F the zonally averaged, specific turbulent and radiative flux of energy through the bottom and the top of an atmospheric column with unit cross-sectional area. Eq. (1) can be solved if the forcing F is expressed in terms of Legendre polynomials $P_n(y)$, where $y=\sin\varphi$.

Values of F as a function of latitude and time have been tabulated recently by Oort and Vonder Haar (1976). They show that the temporal variation can be represented very adequately by annually and semi-annually variable, harmonic components. Unfortunately, their data refer to the Northern Hemisphere and no corresponding data have been published for southern latitudes. This makes some additional assumptions necessary, as the orthogonality condition for the Legendre polynomials requires integration from pole to pole.

The average energy supply to the atmosphere decreases from the equator toward both poles. In a first-order approximation the physiographic differences between the hemispheres and the inequalities caused by the excentricity of the earth's orbit can be neglected. This assumption makes the time-averaged energy supply to southern latitudes a mirror image of that in the north, and allows representation of the temporal mean values of F by a series of *even* Legendre polynomials with constant coefficients.

The annually variable component of the forcing is largest in high latitudes. The preceding assumption is equivalent to making this component zero on the equator. It is therefore represented by the *odd* polynomials. The case of the relatively small but not insignificant semiannual oscillation is slightly more complicated. Its amplitude has a maximum on the equator. This inevitably involves representation by at least some even polynomials. The Oort and Vonder Haar tables show that the semiannual oscillation of the boundary fluxes also reaches a second maximum at the time of both the solstices in the arctic. The reasonable assumption that the same happens in the antarctic suggests representation throughout of the semiannual oscillation by even polynomials. It will be seen below that this may not be entirely realistic; however, in the absence of southern data there is little one can do to improve it.

On the basis of these considerations, F is now represented in the form

$$F=\sum A_{1\nu}P_\nu \cos\omega t+\sum (A_{0\eta}+A_{2\eta}\cos 2\omega t)P_\eta, \quad (2)$$

where ν and η are odd and even integers; ω is the annual frequency.

It is instructive to consider first the very simple case of the highly truncated series

$$F\approx A_1P_1\cos\omega t+A_2P_2. \quad (2')$$

The forcing is now represented by an invariant parabola which has its apex on the equator plus an annual variation which has zero amplitude on the equator and which reaches its greatest values on the poles.

Introduction of (2′) into Eq. (1) and integration yields

$$s=\frac{\epsilon}{\omega}\left[\frac{\cos(\omega t-\gamma)}{(1+\epsilon^2)^{\frac{1}{2}}}yA_1+\tfrac{1}{6}(3y^2-1)A_2\right]+\text{constant}. \quad (3)$$

It may be recalled that $y = \sin\varphi$. The parameter

$$\epsilon\equiv\frac{\omega r^2}{2K} \quad (4)$$

represents the ratio of the frequency to the damping factor; it also determines the phase angle $\gamma = \text{arc}\tan\epsilon$.

We are concerned here with the latitude φ_0 of the ITCZ or with the corresponding value of y_0. Being specified as a divide of the heat flux, it follows that $\partial s/\partial\varphi = \partial s/\partial y = 0$ at $y = y_0$. Differentiation and rearrangement of (3) yields

$$y_0=-\frac{A_1}{A_2}\frac{\cos(\omega t-\gamma)}{(1+\epsilon^2)^{\frac{1}{2}}}.$$

The highest latitude reached by the ITCZ is therefore

$$(\varphi_0)_{\max}=\text{arc}\sin\frac{A_1}{A_2(1+\epsilon^2)^{\frac{1}{2}}}. \quad (5)$$

The last expression indicates how the maximum ITCZ excursion increases with the amplitude A_1 of the global, seasonal heating change and also with the value of K. It is reduced by a large amplitude A_2 of the mean meridional heating gradient.

The mean seasonal heating amplitude $A_1 \approx 14$ W m^{-2} and the meridional amplitude $A_2 \approx 66$ W m^{-2}. If one assumes further that extreme summer and winter temperatures lag 45 days after the solstices, one gets $\epsilon \approx 1$ from Eq. (4) and therefore

$$(\varphi_0)_{\max}\approx 9°.$$

An expression for the heating function F which is more realistic than (2′) can be based on the Oort and Vonder Haar tables. If all amplitudes are expressed in watts per square meter and if ωt is an integer multiple of 2π at the time of the (northern) winter solstice, one can approximate the downward flux of energy through the top of the atmosphere by the value of $2\pi r^2$ multiplied by the function

$$F_T \approx -(106P_2 + 3P_4) - (127P_1 - 34P_3)\cos\omega t + 13P_4\cos 2\omega t + (11P_0 - 8P_2)\sin 2\omega t + \cdots.$$

The terms which were neglected in this expansion all have amplitudes which are less than 5 W m^{-2}.

The upward flux of energy through the bottom of the atmosphere is approximated in a similar way by

$$F_B \approx (39P_2-60P_4)+(98P_1-60P_3)\cos\omega t+9P_1\sin\omega t -13P_4\cos 2\omega t-9P_0\sin 2\omega t+\cdots.$$

Some discussion of the last two expressions may help in an understanding of the following argument. The time-averaged flux through the top of the atmosphere decreases with latitude. The amplitude of the annual variation of this flux also reaches its maximum at relatively high latitudes. As the polynomials P_2 and P_1 are the only ones which change monotomically with latitude, they make the largest contribution to the expression for F_T. By contrast, the upward flux F_B from the surface reaches its maximum at mid-latitudes. It is negative in the tropics where the large downward flux of solar energy heats the oceans below and it is again negative in high latitudes where the atmosphere heats the underlying surface by conduction and infrared radiation. The P_3 and P_4 polynomials which reach extreme values in mid-latitudes at 27° and 41° therefore play a relatively large role in the representation of F_B.

The sum F of F_T and F_B represents the net heat supply to the atmosphere:

$$F \approx -67P_2 - 57P_4 - (29P_1 + 26P_3)\cos\omega t + 9P_1\sin\omega t - 9P_2\sin 2\omega t + \cdots. \quad (6)$$

It is interesting to note that the semiannual component of the net heating has maxima at the equinoxes corresponding to conditions at low latitudes. The $P_4\cos 2\omega t$ terms, which are associated with the polar maxima of both the top and bottom fluxes at the time of the solstices, cancel each other out. In low latitudes the meridional gradients of P_2 and P_4 have opposite signs. The mean meridional heating gradient, as represented by the differential of the first two terms in Eq. (6), therefore increases very slowly between the equator and 30° latitude. The differentials of the two polynomials have the same sign at high latitudes and that can be associated with a correspondingly large gradient of F.

The solution of Eq. (1) with the forcing expressed in Eq. (6) has the form

$$s\approx\frac{\epsilon}{\omega}[a_{02}P_2+a_{04}P_4+(a_{11}P_1+a_{13}P_3)\cos\omega t +(b_{11}P_1+b_{13}P_3)\sin\omega t+a_{22}P_2\cos 2\omega t +b_{22}P_2\sin\omega t]+\text{constant}. \quad (7)$$

The coefficients, as computed from the amplitudes of the input function (6), are specified by

$$\left.\begin{aligned} a_{02}&=-22.3 & a_{04}&=-5.7 \\ a_{11}&=-\frac{9\epsilon+29}{\epsilon^2+1} & b_{11}&=-\frac{29\epsilon-9}{\epsilon^2+1} \\ a_{13}&=-\frac{156}{\epsilon^2+36} & b_{13}&=-\frac{26\epsilon}{\epsilon^2+36} \\ a_{22}&=-\frac{18\epsilon}{4\epsilon^2+9} & b_{22}&=-\frac{27}{4\epsilon^2+9} \end{aligned}\right\}. \quad (8)$$

By definition, the integrals of the even Legendre polynomials from the equator to the pole are all zero. For the odd polynomials

$$\int_0^1 P_1 dy=\tfrac{1}{2}, \quad \int_0^1 P_3 dy=-\tfrac{1}{8}.$$

The area integral of s is therefore equal to $2\pi r^2$ multiplied by

$$S=\frac{\epsilon}{\omega}(a_* \cos\omega t+b_* \sin\omega t+\text{const}), \quad (9)$$

where

$$a_*=\tfrac{1}{2}a_{11}P_1-\tfrac{1}{8}a_{13}P_3, \quad b_*=\tfrac{1}{2}b_{11}P_1-\tfrac{1}{8}b_{13}P_3. \quad (10)$$

The tangent of the phase lag γ_* of S after the solstices

$$\tan\gamma_*=\frac{b_*}{a_*} \quad (11)$$

is again a function of ϵ and therefore by Eq. (4) a function of K. It is known that the atmospheric energy storage in the Northern Hemisphere reaches its maximum toward the end of July, about 40 days after the solstice. If one stipulates $\epsilon = 1$, Eqs. (10) and (11) would indicate a phase lag of 29 days. For $\epsilon = 2$, one gets a phase lag of 49 days. The phase lag increases slowly with increasing ϵ. Reasonably realistic lag values, however, can be associated only with values of K which are of order unity. This being the case, it follows from the defining relation (4) that K—the generalized horizontal transfer coefficient of a general circulation—must be of order ωr^2:

$$K=\frac{\omega r^2}{2\epsilon}\sim 4\times10^6 \text{ m}^2\text{ s}^{-1}.$$

The area-averaged rate of energy storage change

$$\frac{dS}{dt}=\epsilon(-a_* \sin\omega t+b_* \cos\omega t). \quad (12)$$

In Table 1 the values of dS/dt computed from Eq. (12) with $\epsilon = 1.6$ are compared with the values derived directly from observational records.

The agreement between (I) and (II) is fair. A value $\epsilon = 2$ would have given an almost exact fit to the amplitude and $\epsilon = 1$ would have provided a closer agreement between the phase angles. The main differences are due, however, to my assumption of an entirely symmetric semiannual oscillation. This inevitably eliminated the semiannual component from (9) and (12). An analysis of the actual data as represented by (II) in Table 1, however, indicates that the variations of the mean heat storage rate do contain in fact a semiannual component, which is small but not altogether negligible. This suggests that the semiannual oscillation in the south cannot be approximated entirely as a mirror image of that in the north. In the absence of southern data, the asymmetric component cannot be computed by the methods used in this paper. It might be possible to estimate it, by working backward from the values in (II), but there is not much to be gained from such a procedure. For the present purpose, the agreement between (I) and (II) is sufficient to show that the preceding very simple analysis with a constant K can produce results which are of the right order.

To compute the value of y_0 or the latitude φ_0 which corresponds to the forcing represented by Eq. (6) one differentiates Eq. (7) and sets the result equal to zero. With the use of expressions (8) and the explicit forms of the polynomials this now leads to a third-order algebraic equation in y_0 with time variable coefficients. An equation of this type can be solved numerically. There is some advantage, however, in seeking an approximate analytical solution which can provide some additional insight. Such a solution can be obtained when one considers that the ITCZ will always be at relatively low latitudes and that it is therefore permissible to neglect higher powers of $y_0 = \sin\varphi_0$ in the present context. With this assumption one gets for $y = y_0$

$$\frac{\partial}{\partial y}P_1=1, \quad \frac{\partial}{\partial y}P_2=3y_0, \quad \frac{\partial}{\partial y}P_3\approx-1.5, \quad \frac{\partial}{\partial y}P_4\approx 7.5y_0. \quad (13)$$

Introduction of these expressions into the differentials of (7) yields

$$0=\left(\frac{\partial s}{\partial y}\right)_{y=y_0}=[3a_{02}-7.5a_{04}+3(a_{22}\cos\omega t+b_{22}\sin 2\omega t)]y_0$$
$$+(a_{11}-1.5a_{13})\cos\omega t+(b_{11}-1.5b_{13})\sin\omega t. \quad (14)$$

The numerical value of the coefficients can be computed from the expressions (8) with a suitably chosen value of ϵ. For $\epsilon = 1.6$, after rearrangement and conversion to phase angle expressions and with t specified in days, we have

$$y_0=\frac{-10.8\cos\omega(t-56)}{24.3+6.2\sin 2\omega(t-23)}. \quad (15)$$

According to this formula, the highest northern

TABLE 1. Rate of atmospheric energy storage change (W m^{-2}) averaged over the Northern Hemisphere: (I) as computed from Eq. (12): (II) as tabulated by Oort and Vonder Haar (1976)

	Jan.	Feb.	Mar.	Apr.	May	June	July	Aug.	Sept.	Oct.	Nov.	Dec.
(I)	−3.7	1.5	8.2	11.6	12.0	9.0	3.7	−1.5	−8.2	−11.6	−12.0	−9.0
(II)	−2.1	2.1	7.2	11.5	12.2	9.8	3.7	−4.7	−11.0	−12.6	−10.0	−6.1

latitude reached by the ITCZ is

$$(\varphi_0)_{\max} = \arc\sin(y_0)_{\max} \approx 21°. \quad (16)$$

This value is reached not quite two months after the solstice, that is about mid-August. A corresponding numerical solution of the exact equation, in which higher powers of y_0 were not neglected, yields

$$(\varphi_0)_{\max} = 20.8°. \quad (16')$$

This value is reached 32 days after the solstice. The difference between (16) and (16′) is sufficiently small to warrant the additional truncation involved in the approximate equation [(14)].

The difference between the amplitude of 21° obtained from (15) and the amplitude of 9° obtained from (5) is rather large. So, for that matter, is the contrast between 21° and the amplitude of the real ITCZ excursion. In the model, the size difference between the solutions of (5) and (15) is due almost entirely to the small value of the denominator in the second case. The time-invariant component of the forcing which was used in Eq. (5) decreased monotomically from the equator to the pole. On the other hand, the mean forcing, specified by the formulation (6) which was used in (15), reaches a secondary maximum at about 30° latitude. Though this maximum is no longer evident in the meridional distribution of s, it causes the meridional gradient of s to be small in low latitudes. There one has

$$\frac{\partial \bar{s}}{\partial y} = \frac{\partial}{\partial y}(a_{02}P_2 + a_{04}P_4) \approx (3a_{02} - 7.5a_{04})y \approx 24.3y. \quad (17)$$

The size of this quantity decreases with a_{04} which is proportional to the mid-latitude heating from the surface. A relatively large value of a_{04} causes the denominator in Eq. (15) to be relatively small and that in turn can cause a rather excessive value of $(\varphi_0)_{\max}$. In physical terms, strong heating in mid-latitudes and relatively weak heating near the equator must cause the ITCZ to penetrate deeper into the summer hemisphere to allow for the collection of all the energy which may be demanded by cooling in the opposite winter hemisphere.

It is interesting to speculate that ice ages were probably characterized by a smaller mid-latitude upward heat flux from the much colder oceans known to have existed there. In terms of the present analysis this would decrease the amplitude of the P_4 polynomial, increase the denominator of Eq. (15) and therefore reduce the amplitude of the ITCZ excursion.

The Oort and Vonder Haar data are characterized by a very large spike in the meridional distribution of the heat supply to the atmosphere. The vertical convergence of the heat flux into an atmospheric column of unit cross section is indicated by their data tables as being more than five times larger in the 20–30° latitude belt than between 30° and 40°. It is shown to be more than 20 times larger between 20° and 30° than between 10° and 20°. One cannot help wondering whether the very large amplitude of this heating spike between 20° and 30° in the tables is in fact a true representation of actual condition. If it is, it would seem to require some explanation. In any case, it is very doubtful whether this feature is truly reflected in the Southern Hemisphere. However, the formulation (6) for the heating, being based on the data tables, makes allowance for the existence of this heating maximum in both hemispheres and this inevitably contributes to the unrealistically large value of $(\varphi_0)_{\max}$ in the expression (16).

Perhaps one should recall here the limitations of the present model. It deals with a hypothetical ITCZ which oscillates in a symmetric manner about a mean position at 0° latitude. The assumption of a constant K is crude and the value of $\epsilon = 1.6$, which was chosen for the numerical computations, is arbitrary. However, although the magnitude of ϵ has to be of order unity, the model is not very sensitive to its actual value; $\epsilon = 1$ or $\epsilon = 2$ would have yielded very similar results.

Realistic numbers should not be expected from the present simple model. It serves its purpose if it shows that the position of the ITCZ is probably affected by the global heat balance distribution and not simply by local conditions. It also indicates that the amplitude of the ITCZ seasonal excursion should increase with the heating difference between summer and winter. It is likely to decrease with an increasing mean heating difference between the equator and the poles, and with a reduced surface heating in mid-latitudes.

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