# Computations of Transmittance and Radiance in Infrared Water Vapor Sounding Channels

MING-DAH CHOU

*Laboratory for Atmospheric Sciences, Goddard Space Flight Center, Greenbelt, MD 20771*

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ABSTRACT

The method originally developed by Chou and Arking (1981) for computing the absorption of solar radiation by water vapor has been extended to the computations of transmittance and radiance in infrared water vapor sounding channels. It utilizes the wing-scaling approximation and the $k$-distribution approach. The effects of the instrument response function, zenith angle and the variation of the Planck radiance with wavenumber can be easily and accurately included in computing transmittance and radiance. The method can be effectively applied to any tropospheric water vapor sounding channels in the infrared. Compared to line-by-line calculations, which can be considered as the most accurate but too time consuming to be operational, the rms error in transmittance of the method at any pressure level is less than 0.009 for the three operational HIRS/2 water vapor sounding channels. The rms error in brightness temperature is less than 0.2°C which is about a factor of 2–5 smaller than the instrument noise.

## 1. Introduction

The contribution of a given layer in the atmosphere to the satellite-measured radiation is a function of the energy emitted by that layer and the transmission between the satellite and that layer. Assuming the temperature profile is known, the satellite-measured radiation in water vapor channels is related to the vertical profile of transmittance which in turn is a function of atmospheric water vapor profile. It is therefore necessary to compute transmission functions for water vapor in order to retrieve the water vapor profile from a set of satellite-measured radiances. Since the atmospheric temperatures and constituents retrieved from satellite-measured radiation are sensitive to the noise in radiance, a high degree of accuracy in the calculation of transmission functions is required. Line-by-line calculations of transmission functions can be considered as the most accurate method but is too time consuming to be practical. Although the calculation of transmission functions can be greatly simplified by using band models such as the statistical model (Goody, 1952), errors in the transmission function could be as large as 0.2 when compared to line-by-line calculations. In addition, the effect of the instrument response function is difficult to include using band models.

Weinreb and Neuendorffer (1973) introduced a method for computing transmittance in water vapor channels which utilizes Smith's (1969)[1] transmittance model for homogeneous paths. The coefficients in Smith's model were obtained by a least square fit of transmittances to line-by-line calculations. The homogeneous model introduces a standard devia-

[1] Smith, W. L., 1969: A polynomial representation of carbon dioxide and water vapor transmission. ESSA Tech. Rep. NESC 47, National Environmental Satellite Center [NTIS 69N23573].

tion of ~0.01 in transmittance. For inhomogeneous paths, errors are expected to be larger.

Recently, a functional relationship between water vapor transmittance and atmospheric conditions has been derived by McMillin *et al.* (1979). The coefficients in the function were also obtained by a least-square fit to line-by-line calculations. The method is computationally fast and accurate with a possible exception for sounding channels with strong absorption. In deriving the functional relationship between transmittance and atmospheric conditions, it was assumed that the absorption coefficient varies only slowly within a channel's bandpass. For sounding channels with strong absorption, the important region to the upwelling radiances is in the higher atmosphere where molecular lines are very narrow compared to the mean line spacing. The absorption coefficient within a channel's bandpass varies several orders of magnitude (see Fig. 2), which is contrary to the assumption that the absorption coefficient does not change much within a channel's bandpass. As can be seen from their Table 1, the computed transmittance in water vapor channels is less accurate when absorption is stronger.

By properly choosing a reference pressure and scaling the absorption coefficient according to the behavior in the wings of absorption lines, Chou and Arking (1981) developed a fast and accurate method for computing the absorption of solar radiation by water vapor between the top of the atmosphere and any pressure level in the atmosphere. Since the satellite-measured radiation also is related to the transmittance between the top of the atmosphere and pressure levels in the atmosphere, the same method is applied in this study to the computation of transmittance in infrared water vapor channels, specifically for the HIRS/2 instrument on TIROS-N satellite (Schwalb, 1978).[2] The effects of the instrument response function and zenith angle can be correctly taken into account. In addition to the transmittance computations, we also extend the method to the computation of radiance so that the variation of the Planck function with wavenumber is accurately taken into account. It will be seen in the results of this study that the method for computing upwelling radiances is very efficient, especially when the Planck radiance varies significantly within a channel's bandpass.

## 2. Methods

### *a. Computation of transmittance*

Theoretically, the transmission (or absorption) function for a nonscattering inhomogeneous path in the atmosphere can be computed if we know the atmospheric conditions and the absorption line parameters such as the position $\nu_0$, strength $S$, half-width $\alpha$ and shape. The size of spectral intervals for the HIRS/2 water vapor channels is on the order of 100 wavenumbers (see Fig. 1) and involves hundreds of absorption lines. Since the mean half-width is smaller than the mean line spacing, the absorption coefficient is a rapidly varying function of wavenumber. For accurate transmittance computations using line-by-line methods, the number of points at which transmittances must be computed is of the order of $10^4$. Each point, in turn, involves tens of absorption lines and, therefore, line-by-line methods are not practical when transmittances are to be computed repeatedly. Nevertheless, the transmittance model introduced in this study is based on detailed line-by-line structures.

The monochromatic transmittance between the satellite and any pressure level $p$ in the direction $\cos^{-1}\mu$ from the zenith is given by

$$\tau_\nu(p) = \exp[-u_\nu(p)], \tag{1}$$

where

$$u_\nu(p) = \frac{1}{\mu g}\int_0^p k_\nu[p',T(p')]q(p')dp', \tag{2}$$

$k_\nu$ is the absorption coefficient at wavenumber $\nu$, $T$ the temperature, $q$ the specific humidity and $g$ the gravitational acceleration.

Following Chou and Arking (1980, 1981), the absorption coefficient can be computed from the wing-scaling approximation given by

$$k_\nu(p,T) = k_\nu(p_r,T_r)\left(\frac{p}{p_r}\right)^m \bar{R}(T,T_r), \tag{3}$$

where $p_r$ and $T_r$ are the reference pressure and temperature, respectively, and $m$ the scaling parameter. For transmittance averaged over spectral intervals, the scaling parameter $m$ should be close to zero for weak absorptions and close to one for strong absorptions. For the sounding channels investigated in this study, the scaling parameter $m$ is empirically chosen to be 0.9. The function $\bar{R}(T,T_r)$ is the mean effect of temperature on the absorption coefficient given by

$$\bar{R}(T,T_r) = \int_{\nu_1}^{\nu_2} R_\nu(T,T_r)d\nu/(\nu_2 - \nu_1), \tag{4}$$

with

$$R_\nu(T,T_r) = \left(\frac{T_r}{T}\right)^{1/2} \frac{\sum_i \left[\frac{S_i(T)\alpha_i(p_r,T_r)}{(\nu - \nu_{0i})^2}\right]}{\sum_i \left[\frac{S_i(T_r)\alpha_i(p_r,T_r)}{(\nu - \nu_{0i})^2}\right]}. \tag{5}$$

Here $\nu_1$ and $\nu_2$ define the bandpass of a channel, and $i$ the index for molecular lines. The temperature

[2] Schwalb, A., 1978: The TIROS-N/NOAA A-G satellite series. NOAA Tech. Memo. NESS 95 [NTIS 79N12135].

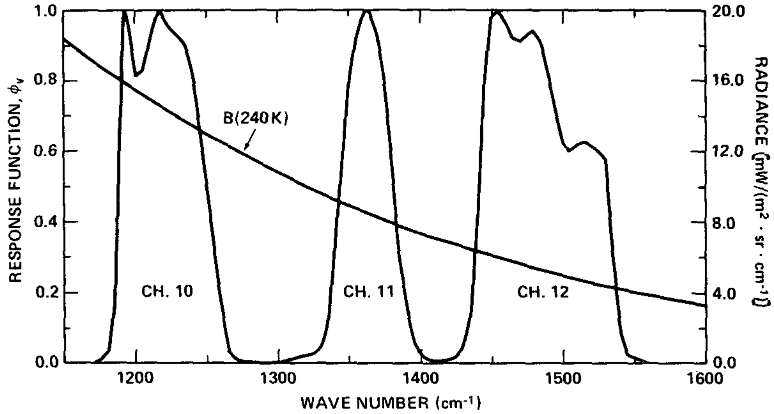


FIG. 1. Instrument response functions (maximum normalized to 1) for the HIRS/2 water vapor channels and Planck radiance at 240 K.

scaling factor $R_\nu(T,T_r)$ is derived from a Lorentz line function and the assumption that radiative transfer is dominated by the far wing portions of molecular lines where $(\nu - \nu_0) \gg \alpha$. The value of $R_\nu$ is not defined at $\nu = \nu_{0i}$. In practice, we computed $R_\nu$ at discrete points with a 0.01 cm$^{-1}$ interval. In computing the mean temperature scaling term $\bar{R}$, we simply skip those points where $|\nu - \nu_{0i}| < 0.01$ cm$^{-1}$. In this study the reference temperature is chosen to be 240 K, and the reference pressure is chosen according to the opacity of sounding channels. As the opacity of a sounding channel decreases, the peak of the radiative weighting function shifts downward, and the reference pressure should be chosen at a lower level. The reference pressures used for the three HIRS/2 water vapor channels are listed in Table 1.

With the substitution of (3), Eq. (1) becomes

$$\tau_\nu(p) = \exp[-k_\nu(p_r, T_r)w(p)], \quad (6)$$

where the scaled water vapor amount $w(p)$ is defined by

$$w(p) = \frac{1}{\mu g}\int_0^p q(p')\left(\frac{p'}{p_r}\right)^m \bar{R}[T(p'),T_r]dp'. \quad (7)$$

The advantage of using the wing-scaling approximation [Eqs. (3) and (7)] is that the wavenumber dependence of the absorption coefficient is separated from temperature and pressure. Within a narrow spectral interval where the Planck radiance and the instrument response function can be practically considered as constant, wavenumbers with the same value of $k$ at the reference condition $(p_r,T_r)$ will have a common value of $k$ at any other conditions when (3) is used. These wavenumbers are therefore radiatively identical and can be treated as one identity. The transmittance averaged over a spectral interval can then be computed by using the $k$-distribution method which replaces the integration over wavenumber by the integration over the absorption coefficient,

$$\frac{1}{\Delta\nu}\int_{\Delta\nu} \exp(-k_\nu w)d\nu = \int_{\log k=-\infty}^{\log k=\infty} f(k)\exp(-kw)d\log k, \quad (8)$$

where the function $f(k)$ is the distribution of the magnitude of $k$ in the spectral interval such that the fraction of $\log k$ contained in the interval $\log k \pm \frac{1}{2} d\log k$ is $f(k)d\log k$. Compared to the absorption coefficient $k$ which varies very rapidly with $\nu$, the transmittance $\exp(-kw)$ is a smooth function of $k$, and the time required for computing the trans-

TABLE 1. Parameters for HIRS/2 water vapor sounding channels.

| Channel | Central wavenumber (cm$^{-1}$) | Half-power bandwidth (cm$^{-1}$) | Reference pressure (mb) | Reference temperature (K) | Scaling parameter | $\bar{R}$ (200 K, $T_r$) | $\bar{R}$ (280 K, $T_r$) |
|---|---|---|---|---|---|---|---|
| 10 | 1225 ± 4 | $60^{+10}_{-3}$ | 375 | 240 | 0.9 | 0.615 | 1.673 |
| 11 | 1365 ± 5 | 40 ± 5 | 275 | 240 | 0.9 | 0.672 | 1.471 |
| 12 | 1488 ± 4.7 | $80^{+15}_{-4}$ | 188 | 240 | 0.9 | 0.887 | 1.262 |

mittance can be greatly reduced by using the $k$-distribution method. It requires about 15 points in the $k$ domain for computing the mean transmittance as compared to thousands of points in the $\nu$ domain.

If we divide the spectral bandpass of a sounding channel into $n$ small intervals, the mean transmittance can be expressed as

$$\bar{\tau}(p) = \sum_{i=1}^{n} \bar{\phi}(i) \int_{\Delta\nu(i)} \tau_\nu(p) d\nu, \quad (9)$$

where $\bar{\phi}(i)$ is the mean normalized response function of the instrument in the interval $\Delta\nu(i)$. Substituting (6) into (9) and utilizing the $k$-distribution method [Eq. (8)], the mean transmittance is reduced to

$$\bar{\tau}(p) = \int h(k) \exp[-kw(p)] d \log k, \quad (10)$$

where the weighted $k$-distribution function is defined by

$$h(k) = \sum_{i=1}^{n} \bar{\phi}(i) f(i,k) \Delta\nu_i, \quad (11)$$

where $f(i,k)$ is the distribution function of the magnitude of $k$ at the reference conditions $T_r$ and $p_r$ in the spectral interval $\Delta\nu(i)$. Since $h(k)$ is independent of atmospheric conditions, it can be precomputed using detailed line-by-line calculations.

### b. *Computation of outgoing radiance*

In a clear atmosphere, the satellite-measured radiance $I$ can be expressed as

$$I = \int \phi_\nu d\nu \left[ B_\nu(T_s)\tau_\nu(p_s) - \int_0^{p_s} B_\nu[T(p)] \frac{\partial \tau_\nu(p)}{\partial p} dp \right], \quad (12)$$

where the subscript $s$ denotes the surface, $B_\nu(T)$ the Planck radiance.

The difficulty in computing radiance from (12) is the integration over wavenumbers. Usually, Eq. (12) was reduced to a much simpler form by suitably representing the radiance within the spectral bandpass by its mean values $\bar{B}(T)$

$$I = \bar{B}(T_s)\bar{\tau}(p_s) - \int_0^{p_s} \bar{B}[T(p)] \frac{\partial \bar{\tau}}{\partial p} dp. \quad (13)$$

Strickly, there is no simple way to compute $\bar{B}(T)$ since it depends on atmospheric conditions. It has been found that if we properly choose a reference wavenumber $\nu'$ and approximate $\bar{B}(T)$ by $B_{\nu'}(T)$, the error in the computed brightness temperature[3] for the three HIRS/2 water vapor channels is $<0.4$ K (D. Chesters, private communication). However, computations can be made faster by integrating (12) by parts and defining

$$G(w,T) = \int \phi_\nu \tau_\nu(w) B_\nu(T) d\nu, \quad (14)$$

which reduces (12) to

$$I = G[0,T(0)] + G[w(p_s),T_s] - G[w(p_s),T(p_s)] + \int_{T(0)}^{T(p_s)} \frac{\partial}{\partial T} G[w(p),T(p)] dT(p), \quad (15)$$

where $T_s$ is the surface temperature, and $T(p_s)$ the air temperature immediately above the surface. Outgoing radiances can be easily computed from (15) by given precomputed tables of $G(w,T)$.

The advantages of using (15) over (13) to compute outgoing radiance are as follows:

1) The effect of the variation of Planck radiance with wavenumber is exactly taken into account. As will be seen later, the error arising solely from the use of $B_{\nu'}(T)$ to represent the mean radiance within the spectral bandpass is equivalent to the overall error introduced by using the present method.

2) There is no need to compute $\bar{\tau}$. The computation is faster by using (15) than (13) since the function $G(T,w)$ is precomputed.

## 3. Application to TIROS-N water vapor channels

There are three channels in the HIRS/2 instrument on TIROS-N satellite for retrieving tropospheric humidities. The instrument response function (with maximum normalized to 1) of these channels are shown in Fig. 1. Channel 12 is located near the center of the 6.3 $\mu$m band that can be used to determine upper tropospheric water vapor content. Channels 10 and 11 are less opaque than channel 12 and are designed for retrieving, respectively, lower and mid-tropospheric humidities. Central wavenumbers and half-power bandwidths of the three channels are listed in Table 1. To show how the Planck radiance varies within the three spectral bandpasses, the Planck radiance at 240 K from 1150–1600 $cm^{-1}$ is also plotted in Fig. 1. It can be seen that the change in Planck radiance is large, ~40% for channel 12 and 25% for channels 10 and 11.

Assuming a Voigt line function[4] and using the

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[3] Brightness temperature $T_B$ is defined as a blackbody temperature such that $I = \int \phi_\nu B_\nu(T_B) d\nu$. Given values of $I$ the brightness temperature is computed using Newton's method.

[4] The broadening of an absorption line is dominated by molecular collision at high pressure and Doppler effect at low pressure. For the sounding channels shown in Fig. 1, the region of equal importance of molecular collision and Doppler effect is at 40 mb which is far above the altitudes of importance in this study. However, the transmittance between the top of the atmosphere and any level in the troposphere is related to the optical thickness above that level, which includes the stratosphere. Although the Doppler effect can be neglected locally in the troposphere, the Voigt profile is used in this study in order to achieve maximum accuracy.

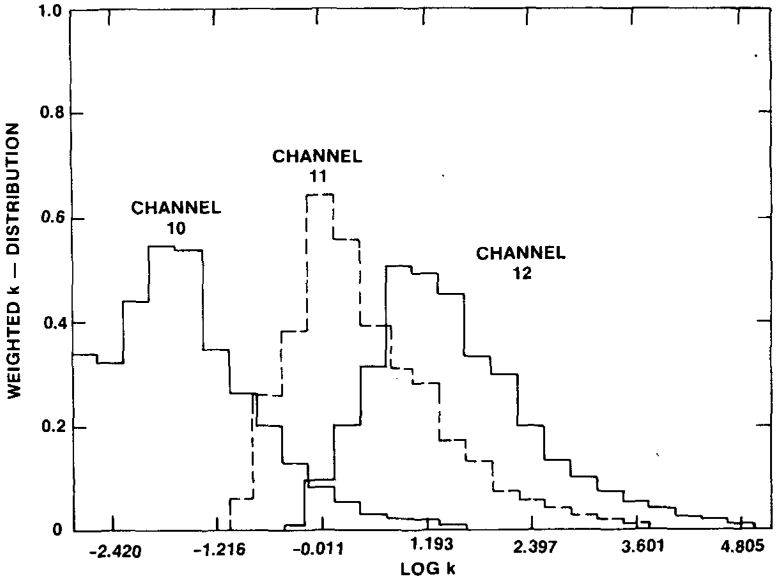


FIG. 2. Weighted $k$ distribution function ($g^{-1}$ $cm^2$) for the HIRS/2 water vapor sounding channels. Reference pressure and temperature used are listed in Table 1.

molecular line parameters compiled by McClatchey *et al.* (1973),[5] the functions $h(k)$ and $G(w,T)$ are precomputed from (11) and (14) using line-by-line methods. In generating $h(k)$ and $G(w,T)$, the band-passes of the sounding channels are first divided into numerous intervals with 1.0 $cm^{-1}$ wide [i.e., $\Delta\nu(i) = 1.0$ $cm^{-1}$]. Each interval is further divided into 100 points with 0.01 $cm^{-1}$ apart. The absorption coefficient $k$ and transmittance $\tau_\nu$ at $(p_r,T_r)$ are then computed at these points, and a histogram of the absorption coefficient is created for each 1.0 $cm^{-1}$ spectral interval, which is $f(i,k)$. Fig. 2 shows the histograms of $h(k)$ for the three water vapor channels. The reference pressure $p_r$, temperature $T_r$, scaling parameter, $m$ and the temperature correction factor $\bar{R}$ are listed in Table 1. In Table 1 values of $\bar{R}$ are given only at two temperatures, other values of $\bar{R}$ are interpolated using a quadratic fit with $\bar{R}$ (240 K, $T_r$) = 1.

In practice, the mean transmittance of a sounding channel is computed by replacing (10) with

$$\bar{\tau}(p) = \sum_{j=1}^{J} h(k_j)\Delta \log k_j \exp[-k_j w(p)], \qquad (16)$$

where $J$ is the total number of the absorption coefficient. If it is only for the reason of accuracy, one would like to choose $\Delta \log k$ as small as possible. It has been found that the computation can be made very fast yet accurate by choosing $\Delta \log k = \log 2 = 0.30103$ which gives

$$\exp[-k_j w(p)] = \exp[-2k_{j-1}w(p)], \qquad (17)$$

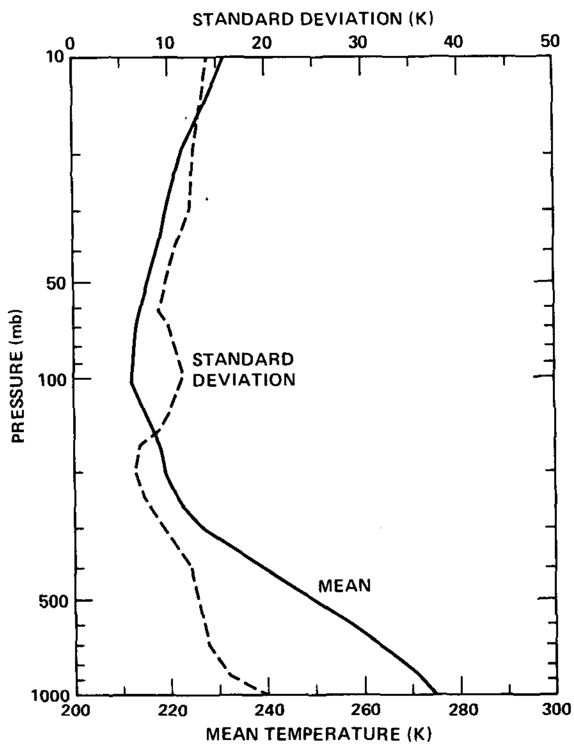


FIG. 3. Mean and standard deviation of the 11 temperature profiles used to test the methods for computing transmittance and radiance.

[5] McClatchey, R. A., W. S. Benedict, S. A. Clough, D. E. Burch, R. F. Calfee, K. Fox, L. S. Rothman and J. S. Garing, 1973: AFCRL atmospheric absorption line parameters compilation. Environ. Res. Pap., No. 434, AFCRL-TR-73-0096 [NTIS AD 762904].

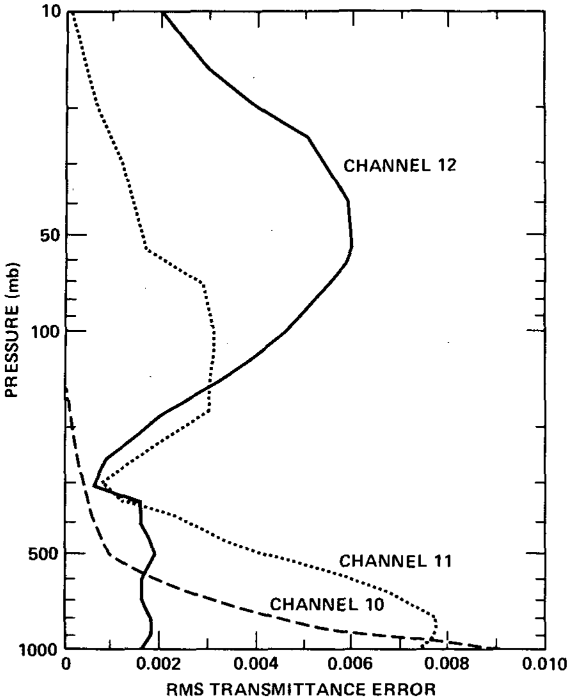


FIG. 4. rms errors in the computed transmittance for the HIRS/2 water vapor channels, for 11 profiles viewed at two angles.

For this particular choice of the absorption coefficients, we need only to compute the transmittance for the first value of $k$, other values of transmittance can be generated by taking the square of the transmittance for the previous value of $k$. As shown in Fig. 2, the number of the absorption coefficient used are 15, 16 and 18 for channels 10, 11 and 12, respectively. At the first glance, the increment in $k$ seems to be very large by choosing $k_j = 2k_{j-1}$, the change in transmittance from one value of $k$ to the next is not as large as it seems to be.

Using line-by-line methods and the methods introduced in this study, we have computed transmittances and outgoing radiances for 11 widely separated atmospheres. They include wet-hot tropical atmospheres and dry-cold subarctic-winter atmospheres. The mean and standard deviation of the 11 temperature profiles are plotted in Fig. 3. It can be seen that the standard deviation is rather large with a maximum of 20°C near the surface. The vertically integrated water vapor content ranges from 0.112 to 5.367 g cm$^{-2}$ with a mean of 1.62 g cm$^{-2}$ and a standard deviation of 1.82 g cm$^{-2}$. In the computations, the atmospheres are divided into 110 levels between 1 mb and the surface, and two zenith angles (0 and 45°) are used to simulate satellite scanning. Thus, for each sounding channel, we have 22 sets of transmittance and 22 radiances to test the validity of the method.

Using the line-by-line calculation as a standard, the rms errors in transmittance versus pressure for the three water vapor channels are shown in Fig. 4. The maximum rms error is $<0.009$ for all channels. For the lower and mid-tropospheric channels (10 and 11), the maximum rms error occurs at lower troposphere. For channel 12 which is relatively opaque, the maximum occurs in the stratosphere. The local minima in the upper troposphere for channels 11 and 12 are due to the fact that the reference pressure is chosen, respectively, at 275 and 188 mb where the absorption coefficients are computed exactly. The maximum error in transmittance is 0.017, 0.013 and 0.006 for channels 10, 11, and 12, respectively.

With the small rms errors in transmittance as shown in Fig. 4, it is expected that errors in the computed outgoing radiance will also be small. Rms errors (against line-by-line calculations) in the computed outgoing radiance for the three water vapor channels are listed in Table 2. Compared to the instrument noise equivalent radiance (NEΔR), the present method introduces much smaller errors (about a factor of 2–5 smaller). The rms error in brightness temperature is $<0.2$ K for all channels. Also, shown in Table 2 are the mean and maximum errors in brightness temperature. The maximum error is $<0.4$ K for all cases.

The important region of the atmosphere to the outgoing radiance shifts downward for channels which are less opaque, and the reference pressure should be chosen at a larger value. The particular choice of the reference pressures listed in Table 1 is according to this principle. However, the accuracy of the computed transmittance and radiance does not depend critically on the exact position of the reference pressure level. For example, the rms error in brightness temperature increases only from 0.12 to 0.13 K for channel 10, when $p_r$ is shifted from 375 to 450 mb and from 0.19 to 0.21 K for channel 11 when $p_r$ is shifted from 275 to 325 mb. The maximum rms errors in transmittance have been found to be essentially unchanged for both cases. Similar situations also have been found for channel 12 when $p_r$ is changed from 188 to 225 mb.

TABLE 2. Noise equivalent radiance (NEΔR) of the instrument and rms, mean and maximum errors in the computed brightness temperature of the HIRS/2 water vapor channels. The unit of radiance is mW(cm·sr)$^{-1}$.

| Channel | NEΔR | rms error Radiance | rms error $T_B$ (K) | Mean error (K) | Maximum error (K) |
|---|---|---|---|---|---|
| 10 | 0.16 | 0.10 | 0.12 | 0.00 | −0.23 |
| 11 | 0.22 | 0.09 | 0.19 | 0.08 | 0.40 |
| 12 | 0.11 | 0.02 | 0.10 | 0.07 | 0.14 |