General Circulation Statistics on Short Time Scales

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ABSTRACT

The sensitivity of various zonal mean general circulation statistics to the choice of the averaging period used to define them is tested with upper-air data for the Northern Hemisphere taken from the NMC global analysis for the winter of 1976–77. We find that averaging periods of less than about 10 days do not permit a clear separation of total eddy momentum and heat fluxes into their transient and standing eddy components. Between 10 and 30 days, the definition of these components is less sensitive to the specific averaging period chosen during the winter. We illustrate one use of monitoring general circulation statistics on short time scales by studying the evolution of 10-day mean eddy fluxes of sensible heat and their relation to changes in the meridional temperature gradient during this winter. It appears, for this one season at least, that the standing waves regulated the temperature structure of midlatitudes, whereas the transient waves merely responded to the temperature gradient that was imposed.

1. Introduction

Until recently, the traditional approach to generating large-scale atmospheric circulation statistics has been to accumulate data for certain periods at individual rawinsonde stations and then subject these to some form of spatial analysis. An example of this approach is the compilation of zonal mean fields produced by Oort and Rasmusson (1971). Partly because of the substantial computer resources required by the spatial analysis step, it has generally been performed on station data that have been accumulated only for monthly or longer periods. Thus, the evolution of the general circulation on shorter time scales has not been regularly diagnosed using this approach.

In September 1974, another source of global circulation statistics became available when the National Meteorological Center (NMC) introduced a daily global data analysis scheme. In Rosen and Salstein (1980), we computed circulation statistics for the winter of 1976–77 with grid point data from the version of the NMC final analysis then in use. The eddy transports of momentum and heat appeared quite realistic despite the fact that mean meridional circulations were absent because the analyzed fields in this version were partly based on the rotational modes of Hough functions. In our earlier study, we had limited our calculations to the winter period as a whole. However, because the NMC analysis is performed on a regular array of grid points, the need for further map analysis is obviated, and it becomes straightforward to generate circulation statistics for a virtually unlimited variety of averaging periods. The implications of this new flexibility is the subject of our current study.

It is not obvious a priori how short a sequence of daily maps can be used to produce statistics that meaningfully describe the “general circulation”. One attribute usually desired of general circulation statistics is that they account separately for transient and standing eddy effects, and ideally the minimum averaging period selected would result in an unambiguous separation between the two. Because these quantities are nonlinear, however, a degree of ambiguity is always present; i.e., some component of the standing eddy flux computed on the basis of one choice of averaging period will almost certainly appear as a transient eddy flux in the context of a longer averaging period. We refer to this component as the “inter-period eddy” flux (a more precise definition is given in the next section). Mulokwa and Mak (1980) studied the interperiod eddy flux defined by the difference between using seasonal and annual averaging periods, i.e., the inter-seasonal fluctuation. Here we examine the fluxes due to interperiod eddies between sets of much smaller averaging periods, on the order of a few days to a month. We compare the size of these to the magnitude of the transient and standing eddy fluxes within each of the periods in order to draw conclusions about the suitability of each period for general circulation studies. Unfortunately, our conclusions must be considered tentative, since our study encompasses only a single winter. Also, for simplicity we deal entirely with
circulation statistics that have been zonally averaged and so ignore any local differences that may exist in the sensitivity to the time scale.

In addition to computing interperiod eddy fluxes, we also present results for various elements of the atmospheric energy cycle over the Northern Hemisphere calculated for each choice of averaging period. The differences that result among these integrated statistics provide further insight into the implications of using various averaging periods.

For the most part, our study deals simply with the effect of the averaging period on general circulation statistics. However, our mode of analysis also permits us to view the evolution of the general circulation during the 1976–77 winter. We find that the standing and transient eddy fluxes of heat in midlatitudes developed very differently during the course of the winter. This difference appears related to their opposite behavior with respect to temporal changes in the meridional gradient of temperature. To help quantify these results, we compute correlation coefficients between the eddy fluxes of heat and the temperature gradient in a manner reminiscent of the recent work by Lorenz (1979), van Loon (1979) and Stone and Miller (1980). Important differences in both temporal and spatial resolutions exist between our data and those used by these other authors, however.

2. Interperiod eddy fluxes

Given the nature of atmospheric processes, we can expect the mean value of a quantity to be different in (two) successive periods of equal length. Here we are concerned with the fluxes of momentum and heat associated with such interperiod deviations in the wind and temperature fields. In what follows, we relate these interperiod eddy fluxes to the transient and standing eddy fluxes within the averaging periods.

Suppose we have once-daily observations of a quantity \( x \) during a period \( c \). Then its time mean for the period is

\[
\bar{x}^c = \frac{1}{N(c)} \int x dt,
\]

where \( t \) is time and \( N(c) \) is the number of days in the period, and

\[
x^c = x - \bar{x}^c,
\]

is the departure of its value on a particular day during \( c \) from its time mean. For concreteness, let us deal with the eastward and northward components of wind, \( u \) and \( v \). Then for a point in space, we may use (1) and (2) to derive the following familiar relationship for the temporal covariance between \( u \) and \( v \) during period \( c \) (or, in physical terms, the net transient eddy northward flux of westerly momentum):

\[
\bar{u} v^c = \bar{u} \bar{v}^c - \bar{u}^c \bar{v}^c.
\]

Next, let us imagine that period \( c \) has been chosen so that it can be divided into two subperiods \( a \) and \( b \) which are equally long. We may, therefore, expand the right-hand side of (3) in terms of time averages over these shorter subperiods, i.e.,

\[
\bar{u} \bar{v}^c = \frac{1}{2}(\bar{u} \bar{v}^a + \bar{u} \bar{v}^b) - \frac{1}{4}(\bar{u}^a + \bar{u}^b)(\bar{v}^a + \bar{v}^b). \tag{4}
\]

Upon noting that \( \bar{u} \bar{v}^a = \bar{u} \bar{v}^a + \bar{u}^a \bar{v}^a \) and \( \bar{u} \bar{v}^b = \bar{u} \bar{v}^b + \bar{u}^b \bar{v}^b \) as in (3), and performing some algebraic manipulations, we may rewrite the right-hand side of (4) so that

\[
\bar{u} \bar{v}^c = \frac{1}{2}(\bar{u} \bar{v}^a + \bar{u} \bar{v}^b) + \frac{1}{4}(\bar{u}^a - \bar{u}^b)(\bar{v}^a - \bar{v}^b). \tag{5}
\]

According to (5), the net transient eddy momentum flux at a location during a period equals the average of the fluxes computed independently for the first and second halves of the period plus a contribution due to the change in the means of \( u \) and \( v \) from one half to the other. This latter contribution, which we refer to as an interperiod eddy flux, may be thought of as measuring the transience in "mean" winds when they are viewed in the context of a longer averaging period. Viewed in another sense, the first term on the right-hand side of (5) measures the temporal covariance on periods of less than \( N(a) = N(b) \) days, whereas the second term measures the covariance of \( N(a) = N(b) \) day averages within the \( N(c) = 2N(a) = 2N(b) \) day period. What we wish to know is the relative sizes of the first and second terms on the right-hand side of (5) as a function of \( N(c) \).

Actually, we wish to go beyond the relationship expressed in (5) for behavior at an individual point and consider conditions in the zonal mean. To do so, we introduce the standard definitions

\[
[x] = \frac{1}{2\pi} \oint x d\lambda,
\]

where \( \lambda \) is longitude. Using (1), (2) and (6), we may decompose the total flux of momentum across a latitude circle at a given level into three parts in the "mixed space-time domain", as it is referred to by Oort (1964):

\[
[\bar{u} v^c] = [\bar{u} \bar{v}^c] + [u' v'^c] + [\bar{u}^e \bar{v}^e], \tag{7}
\]

where the terms on the right-hand side represent the fluxes accomplished by the mean meridional circulation, the transient eddies and the standing eddies, respectively, all during period \( c \).

What we seek are expressions for the transient and standing eddy fluxes during period \( c \) in terms of these fluxes within its subperiods \( a \) and \( b \) plus a contribution made by the interperiod flux in the mixed space-time domain. Details of the derivation are given in the Appendix. Because our data set does not
contain any mean meridional circulation, our final results simply become

$$\overline{u'v'} = \frac{1}{2}(\overline{u'v'^a} + [\overline{u'v'}]) + \frac{1}{4}([\overline{u''} - \overline{u''}] (\overline{v''} - \overline{v''})),$$  \hspace{1cm} (8)

$$[\overline{u''v''}] = \frac{1}{2}([\overline{u''}v''^b] + [\overline{u''}v'']) - \frac{1}{4}([\overline{u''} - \overline{u''}] (\overline{v''} - \overline{v''})).$$  \hspace{1cm} (9)

Thus, in analogy with (5), the zonal mean transient eddy flux during a period is the average of the transient fluxes within its two halves plus a contribution associated with movement between the halves in the standing wave pattern around the zone. This contribution, given by the second of the two terms on the right-hand side of (8), is what we refer to in the remainder of this paper as the “interperiod eddy” flux of momentum (or sensible heat, when we replace \(u\) by temperature \(T\)). Conversely, the zonal mean standing eddy flux during period \(c\) is, according to (9), the average of the standing fluxes within the subperiods minus this interperiod eddy flux.

Our goal is to study the relative magnitudes of the various terms in (8) and (9) as functions of the length of period \(c\). If the interperiod eddy is consistently large for a particular choice of \(c\), then our premise is that a desirable separation between transient and standing eddy effects has not been achieved. In a sense, we are accomplishing a form of spectral decomposition in time, much in the manner of the “poor man’s spectral analysis” described by Lorenz (1979).

Our presentation, however, follows the traditional approach of viewing general circulation statistics so that we may assess the effect of different averaging times on such statistics directly.

3. Data and analysis approach

Our wind and temperature data were drawn from the NMC final global analysis for 0000 GMT for the period 1 December 1976–28 February 1977. Values were available at a regular array of grid points, spaced every 2½° in latitude and longitude, and at 12 pressure levels in the vertical (1000, 850, 700, 500, 400, 300, 250, 200, 150, 100, 70 and 50 mb). The global analysis scheme in operation at NMC during this winter incorporated a spectral data assimilation technique due to Flattery (1971) that utilized Hough functions in the spectral representation. Earlier, we calculated general circulation statistics for the winter as a whole with these data (Rosen and Salstein, 1980). The eddy fluxes of momentum and heat appeared to us to be quite realistic, by and large. We did find, though, that \([v]\) = 0 in this data set, so that meridional circulation cells and their associated transports are absent. This limitation is not of much concern to us here, however.

In the current study, we are interested in circulation statistics for periods shorter than the full winter season. We felt a priori that a period of at least 5 days is needed to provide a meaningful separation of transient from standing eddy effects. Accordingly, we divided the 90-day winter into 18 consecutive 5-day periods and used these as the bases for creating nine consecutive 10-day periods, six consecutive 15-day periods and four consecutive 20-day periods (see Fig. 1). For each 5-day period, we summed the daily values of the horizontal components of the wind and the temperature, along with their products and cross-products, at each grid point. Data for two days during the winter, 15 January and 16 February 1977, were missing; the statistics for the 5-day periods containing them were computed from averages of the other four days in each period. Grid point statistics for periods that are multiples of 5 days were formed by adding the appropriate shorter period statistics together. Fields of the eddy momentum and heat fluxes in the mean meridional plane were easily obtained for every period by zonally averaging the grid point values according to (6). (We have assumed a smooth earth, as did NMC in generating their daily analyses.)

Historically, the calendar month has served as a convenient period over which to accumulate circulation statistics. Therefore, in addition to the periods noted in Fig. 1, we will also present results based on data organized for each of the three calendar months

FIG. 1. Schematic depicting the organization of the 1976–77 winter into subperiods of 5, 10, 15 and 20 days duration.

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Fig. 2a. Cross sections of the mean winter angular momentum flux (\(\pm 2\pi a^2\), where \(a\) is the radius of the earth) by the transient and standing eddies determined on the basis of averaging periods of 5, 10 and 15 days, and calendar month (M). The last column presents the mean winter flux associated with the interperiod eddy between the 5- and 10-day, and also between the 15-day and monthly, subperiods. In each cross section, the ordinate is pressure (mb) and the abscissa is latitude. Units are \(\text{m}^2 \text{s}^{-2}\), and negative values are shaded and indicate a southward flux.

4. Results of interperiod eddy flux calculations

We computed the fluxes of both angular momentum and heat associated with the interperiod eddies that exist between each pair of 5-day periods that forms one of the nine 10-day periods indicated in Fig. 1 by setting $N(e) = 10$ and $N(a) = N(b) = 5$ in (8). We first calculated the zonal mean transient eddy fluxes for the 10-day period and its pair of 5-day subperiods separately and then obtained the interperiod eddy flux as a residual. A similar procedure was used for the interperiod eddies that exist between each pair of 10-day periods that forms one of our four 20-day periods. Finally, we adopted the same simple residual approach with $e = \text{calendar month}$ and $N(a) = N(b) = 15$, even though the three calendar months are not exact multiples of our 15-day periods. The error introduced by this approximation is small, however.

We will not present the fields of transient, standing or interperiod eddy fluxes for individual periods here. Instead, we offer in Fig. 2 the mean for the entire winter of the eddy fluxes of angular momentum determined on the basis of 5-, 10-, 15- and 20-day and monthly (M) averaging subperiods. The column labeled “Interperiod Eddy” is, in the case of the row marked “10” for example, the average of the nine individual fields of interperiod eddy flux arising between the nine pairs of 5-day periods. Because only four successive 20-day periods fit into our 90-day study period, we have isolated in Fig. 2b the mean flux derived from these, along with results for the corresponding eight 10-day periods and the average interperiod eddy between them.

According to Fig. 2, the maxima in both the transient and standing eddy momentum flux fields occur near 30°N and 200–250 mb for all averaging periods. The maximum mean transient eddy flux of angular momentum for the winter is 19.5 units when determined on the basis of 5-day averaging periods. When the averaging period is increased to 10 days, the transient component increases to 26.5 units, while the maximum in the standing eddy flux decreases from 40.6 to 34.9. The transition between the two averaging periods is, of course, measured by the interperiod eddy flux, whose maximum value of 7.1 is ~25% of the total transient eddy flux determined from the 10-day averages. In other words, in the vicinity of its maximum, about one-fourth of the transient eddy flux of momentum that is computed on a 10-day basis is due to the covariance of 5-day means within the 10-day periods. While this still leaves 75% of the flux to the covariance due to transient eddies on periods of less than 5 days, the 5-day averaging period seems too short to separate tran-
Fig. 3a. As in Fig. 2a, but for the mean winter sensible heat flux (−2ra cosθ). Units are 10^3 J kg⁻¹ m⁻³ s⁻¹.
sient from standing phenomena properly. On the other hand, from 10-day periods onward there is less change in the mean winter transient and standing eddy fluxes, although there is a general tendency toward a larger transient eddy flux as the averaging period progressively lengthens. This tendency does result in a cumulative difference between the 10-day result for \( [u'v'] \cos^2 \phi \) and that for the season as a whole (given by Rosen and Salstein, 1980) that is as large as 4.4 units, but this is still smaller than the maximum in the 5–10 day interperiod eddy alone.

In Fig. 3 we present comparable results for the wintertime mean of the eddy fluxes of sensible heat. Focusing on behavior in the region of the midlatitude tropospheric maximum in the fields, we again see that the largest change occurs in the transition between 5- and 10-day averaging periods, with relatively small changes occurring thereafter. Because we have not multiplied the fields in Fig. 3 by a factor of \( \cos^2 \phi \), as we did in connection with the angular momentum fluxes, behavior at high latitudes is more prominent than before. Two interesting results are visible in this region. The first relates to the sudden disappearance between the 10- and 15-day results of the relative maximum in standing eddy heat flux at about 80°N and 850 mb. This behavior is mirrored, of course, by the appearance of increased transient eddy fluxes in this region. After 15 days, less change is evident, so that in fact the largest values of the interperiod eddy flux occur here in the 10–20 day transition (Fig. 3b). In their study of winters during 1963–72, Blackmon et al. (1977) also found that a sizeable contribution to the transient eddy heat flux at 850 mb in high latitudes was made by waves with periods beyond 10 days (their Fig. 11a). The second point worth noting about high latitudes is that in the stratosphere the largest change in the transient and standing eddy fluxes occurs in going from 15-day to monthly averaging periods (accounting for the 4.4 value in the interperiod eddy flux for the 15-M transition).

The indication, therefore, is that waves at high latitudes this winter had relatively large transient eddy heat flux components at the lower frequencies studied here, whereas the transience in their counterparts at midlatitudes occurred more at higher frequencies. In any case, after including the proper geometric factors, the meridional flux of heat across complete latitude circles becomes negligibly small as the pole is approached, and the importance of our results for high latitudes diminishes in the context of most general circulation studies. On the basis of our midlatitude results for both eddy heat and momentum fluxes, therefore, we conclude that averaging periods of less than ~10 days are not suitable for computing statistics meant to define large-scale transient and standing waves. Beyond this 10-day limit, less information about the character of the
eddies seems to be lost by the specific choice of averaging period within the winter. It is interesting to note that this 10-day limit is fairly close to the characteristic time scale that separates effectively independent states of the atmosphere, according to Lorenz (1973) and Leith (1973).

Our results should not be construed to mean that transient waves with periods greater than 10 days are unimportant in transporting momentum or heat in midlatitudes. Thus, Blackmon et al. (1977) found that fairly sizeable wintertime transient eddy momentum fluxes over portions of Northern Hemisphere midlatitudes are associated with waves of 10–90 day periods (their Fig. 7a for “low-pass” filtered data). Unfortunately, a direct comparison between their results and ours is made difficult by several factors, including the fact that our study deals only with zonal mean fields. Also the results of Blackmon et al. pertain to winters other than ours, and as they note the interannual variability of their low-pass filtered data is large. A more fundamental difficulty in comparing our results, though, is related to our different sets of filter response functions. As we noted earlier, our approach closely resembles that of Lorenz’s (1979) poor man’s spectral analysis. Trenberth (1981) has recently demonstrated that the response functions implied by such an analysis consist of rather broad overlapping frequency bands, whereas the filters applied by Blackmon et al. (1977) have much sharper cut-offs. In any case, our purpose here has not been to perform a spectral decomposition per se, but rather to determine how much of the temporal covariance among the wind and temperature fields during the winter is captured by different averaging periods, irrespective of the precise frequencies of the transient waves that are involved.

One final note concerning the results in Figs. 2 and 3 is to recall that they are average, or composite, fields for the winter of 1976–77. In many cases, there is actually a fairly large deviation about these means. For example, although the average maximum value for \( [\bar{u} \bar{v}] \cos^2 \phi \) computed on a 5-day basis is, from Fig. 2, 19.5 m² s⁻², the maxima in the 18 independent fields range from 5.8 to 53.3 m² s⁻² and fluctuate greatly from one period to the next. On a 10-day basis, the range of the nine maxima is from 12.5 to 60.3 m² s⁻², whereas the winter mean field in Fig. 2 contains a maximum of 26.5. Similarly, a wide variation is found in the percentage of the 10-day transient eddy flux that is attributable to the 5–10 day interperiod eddy. Although the winter average of this percentage (cited earlier) is ~25%, it actually ranges from a low of 10% to a high of ~50% (during the period from 20–29 January). However, a smaller range is found in the percentages of the fluxes for the 20-day periods that are attributable to their respective 10–20 day interperiod eddies, and likewise for the relationship between the monthly statistics and the 15-day to monthly interperiod eddies. This result supports our conclusion that a lower limit of ~10 days is generally acceptable when choosing an averaging period over which to accumulate circulation statistics.

5. Energetics as a function of averaging period

Among the more likely candidates for routine monitoring with global gridded analyses are the various components of the atmosphere’s energy cycle. In this section, we study the effect of the choice of averaging period on those parts of the cycle that could be easily evaluated with the NMC analysis for the 1976–77 winter. On the basis of the results presented in Section 4, we expect to find that the energy terms depend on the averaging period, particularly those terms that involve transient and standing eddy fields.

Adopting conventional notation, the energy terms we have calculated may be written as:

\[
K_M = \frac{1}{2} \int [\bar{u}]^2 dm, \quad \text{where} \quad dm = 2\pi a^2 g^{-1} \cos \phi d\phi dp
\]

\[
K_{TE} = \frac{1}{2} \int ([\bar{u}^2] + [\bar{v}^2]) dm, \quad K_{SE} = \frac{1}{2} \int ([\bar{u}^*]^2 + [\bar{v}^*]^2) dm, \quad K_E = K_{TE} + K_{SE}
\]

\[
P_M = \frac{1}{2} c_p \int \gamma(T)^2 dm, \quad \gamma(T)^2 = [T] - \int_0^{\pi/2} [T] \cos \phi d\phi
\]

\[
P_{TE} = \frac{1}{2} c_p \int \gamma(T^2) dm, \quad P_{SE} = \frac{1}{2} c_p \int \gamma(T^*^2) dm, \quad P_E = P_{TE} + P_{SE}
\]

\[
C(K_E, K_M) = \int ([\bar{u} \bar{v}^*] \cos \phi + [\bar{u}^* \bar{v}* \cos \phi]) \frac{\partial}{\partial \phi} \left( \frac{[\bar{u}]}{a \cos \phi} \right) dm = C(K_{TE}, K_M) + C(K_{SE}, K_M)
\]

\[
C(P_M, P_E) = -\int \left( \gamma c_p \bar{\nu} T \bar{T}^* + \gamma c_p \bar{\nu}^* \bar{T} T^* \right) \frac{\partial}{\partial \phi} \left( \int [T] \right) dm = C(P_M, P_{TE}) + C(P_M, P_{SE}). \quad (10)
\]
In the above, $K$ refers to kinetic energy, in either its zonal mean (subscript $M$), transient eddy (TE) or standing eddy (SE) forms. Similarly, $P$ denotes available potential energy in one of its forms. The conversions from energy form $A$ to form $B$ are symbolized by $C(A, B)$. In this study, the volume integration extends in latitude ($\phi$) from equator to north pole and in pressure ($p$) from 1000 to 100 mb, i.e., we are just considering the energy cycle for the tropospheric portion of the Northern Hemisphere. Also in (10), $a$ is the radius of the earth, $g$ the acceleration due to gravity, $c_p$ the specific heat of air at constant pressure, and $\gamma$ the static stability factor. Although Stone and Miller (1980) indicate that important seasonal changes in static stability may occur, we have taken the expedient in our calculations of using the mean January profile for $\gamma$ presented by Peixoto and Oort (1974), since we are dealing here with conditions within a single season. We do not believe this simplification detracts from our current study. Finally, we note that we have simplified the standard formula for $K_M$ since $[\bar{v}] = 0$ in our data set.

In Fig. 4, we present the time series of the zonal kinetic energy $K_M$ during the 1976–77 winter computed for our different averaging subperiods. In this case, the different time series appear fairly similar, attesting to the relative stability of the $[\bar{u}]$ field from one subperiod to the next. There is, however, a small trend toward lower $K_M$ values as the averaging period is lengthened (Table 1). This decrease of kinetic energy in the zonal mean state reappears, of course, as a portion (albeit a small one) of the increase in $K_{TE}$ with increasing averaging period (Fig. 5). Most of this increase in $K_{TE}$, though, is related to the decrease in $K_{SE}$ with averaging period, also visible in Fig. 5. This behavior is in accord with our previous discussion of interperiod eddies, except here we are dealing with a hemispherically integrated measure of an interperiod eddy, whereas our earlier focus was on behavior at individual points in the mean meridional plane. As we found in Section 4, Table 1 shows that on average the largest incremental change in $K_{TE}$ and $K_{SE}$ occurs in going from 5- to 10-day subperiods, with smaller differences thereafter. Indeed, the single change in these energies made in going from 5–10 day averages is comparable to the cumulative difference between the 10-day and monthly averaged results.

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<td></td>
<td>(0.20)</td>
<td>(0.14)</td>
<td>(0.11)</td>
<td>(0.08)</td>
<td>(0.14)</td>
<td>(0.11)</td>
</tr>
<tr>
<td>$C(K_{TE}, K_M)$</td>
<td>0.44</td>
<td>0.61</td>
<td>0.66</td>
<td>0.70</td>
<td>0.46</td>
<td>0.51</td>
</tr>
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<td>(0.39)</td>
<td>(0.51)</td>
<td>(0.38)</td>
<td>(0.26)</td>
<td>(0.31)</td>
<td>(0.23)</td>
</tr>
<tr>
<td>$C(K_{SE}, K_M)$</td>
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<td>1.11</td>
<td>0.99</td>
<td>0.91</td>
<td>1.10</td>
<td>1.01</td>
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<td>(0.50)</td>
<td>(0.26)</td>
<td>(0.27)</td>
<td>(0.19)</td>
<td>(0.28)</td>
<td>(0.14)</td>
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<tr>
<td>$C(K_E, K_M)$</td>
<td>1.69</td>
<td>1.71</td>
<td>1.65</td>
<td>1.60</td>
<td>1.56</td>
<td>1.52</td>
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<tr>
<td></td>
<td>(0.77)</td>
<td>(0.61)</td>
<td>(0.51)</td>
<td>(0.44)</td>
<td>(0.45)</td>
<td>(0.36)</td>
</tr>
<tr>
<td>$C(P_M, P_{TE})$</td>
<td>2.51</td>
<td>3.10</td>
<td>3.32</td>
<td>3.48</td>
<td>2.81</td>
<td>3.01</td>
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<td>(1.02)</td>
<td>(1.04)</td>
<td>(0.99)</td>
<td>(0.60)</td>
<td>(0.67)</td>
<td>(0.22)</td>
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<tr>
<td>$C(P_M, P_{SE})$</td>
<td>4.54</td>
<td>3.98</td>
<td>3.80</td>
<td>3.64</td>
<td>4.17</td>
<td>4.00</td>
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<tr>
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<td>(1.38)</td>
<td>(1.09)</td>
<td>(0.73)</td>
<td>(0.46)</td>
<td>(1.01)</td>
<td>(0.97)</td>
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<td>$C(P_M, P_E)$</td>
<td>7.05</td>
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<td>7.11</td>
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<tr>
<td></td>
<td>(1.64)</td>
<td>(1.13)</td>
<td>(1.07)</td>
<td>(0.85)</td>
<td>(1.16)</td>
<td>(1.08)</td>
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Fig. 4. Time series of the zonal kinetic energy of the atmosphere between 1000 and 100 mb above the Northern Hemisphere for the 5-day (thin solid line), 10-day (heavy solid line), 15-day (thin dashed line), 20-day (heavy dashed line) and monthly (dashed-dotted line) averaging periods. The abscissa is the time scale, in units of days from 1 December 1976. Each value for $K_M$ is plotted as a dot at the midpoint of the appropriate period.
Fig. 5a. As in Fig. 4, but for the transient and standing eddy kinetic energies.

Total Eddy Kinetic Energy

Fig. 5b. As in Fig. 4, but for the total eddy kinetic energy.

Fig. 6. As in Fig. 4, but for the zonal available potential energy.

Fig. 7a. As in Fig. 4, but for the transient and standing eddy available potential energies.
Close parallels to the behavior cited above for kinetic energy exist among the time series for the potential energies plotted in Figs. 6 and 7. An additional point about $P_M$ worth noting concerns the "midwinter dip" evident in some of our time series for it. This rather prominent feature, discovered by Krueger et al. (1965), has been linked by McGuirk and Reiter (1977) to the occurrence of stratospheric disturbances. We see from Fig. 6 that the dip is resolved nearly equally well by the 5-, 10- and 15-day averages, but poorly by the 20-day and monthly periods.

We present results for the energy conversion terms listed in (10) in Figs. 8 and 9. To some extent, they all are marked by the presence of large oscillations in the 5-day results. In the case of $C(K_{SE}, K_M)$, for example, this phenomenon is very striking beginning around the end of January. We will not explore the basis for these oscillations here, but merely note that their presence raises the general problem of using fixed calendar dates to set averaging periods. If, for example, we had defined our 5-day periods starting from 3 December and not 1 December, then the curve depicting $C(K_{SE}, K_M)$ may have appeared quite different. Moreover, this fixed sampling problem is not necessarily confined to just the 5-day periods, since some oscillatory behavior is also evident in the 10-day results for $C(K_{SE}, K_M)$. In fact, such oscillations may also exist in results for longer averaging periods but are not visible here because of the shortness of our sample [McGuirk and Reiter (1976) have noted the presence of an apparent 24–27 day oscillation in certain general circulation statistics, which could impact results obtained even for the traditional calendar month]. A further study of this issue is beyond the scope of our current effort, however.


Motivated in part by the desire to improve techniques for parameterizing the northward eddy fluxes of sensible heat in climate models, van Loon (1979) and Stone and Miller (1980) each undertook empir-
Fig. 9a. As in Fig. 4, but for the transient eddy conversion of available potential energy.

Fig. 9b. As in Fig. 4, but for the standing eddy conversion of available potential energy.

ical studies that related changes in these fluxes to changes in the meridional temperature gradient. These studies examined the correlation between the two on seasonal or longer time scales. Higher frequency variations in the eddy heat flux and temperature gradient were included by Lorenz (1979) in his study of “forced” and “free” variations of weather and climate, although he did not separate transient from standing components.

We decided to supplement these works by using our data for the 1976–77 winter to examine the evolution of both transient and standing eddy heat fluxes on short time scales. To simplify our discussion in this section, we present results based just on our 10-day averaging periods, which we have already shown are appropriate for separating transient from standing eddy effects. (In fact, though, the results for the 10-day periods are similar in character to those obtained for our 5- and 15-day periods as well.)

Our approach was to calculate the vertically integrated (from 1000 to 100 mb) eddy fluxes of heat across each 2½° of latitude between 15°N and 70°N for each 10-day period of the winter, and then to correlate these nine numbers with time series of different measures of the meridional temperature gradient across each latitude. For each 2½° of latitude, the temperature gradients were derived from differences between the vertically averaged values of zonal mean temperature at latitudes to the south and north. [Stone and Miller (1980) noted that their results were largely independent of using the gradient at 1000 mb or a vertically averaged one.] If we use an overbar to refer to a 10-day mean and a caret to denote the vertical average from 1000 to 100 mb, then these differences can be written as

$$\Delta[\hat{T}](\phi) = [\hat{T}](\phi - \frac{1}{2} \Delta \phi) - [\hat{T}](\phi + \frac{1}{2} \Delta \phi).$$

We correlated $\Delta[\hat{T}]$ with the eddy heat fluxes for separate choices of $\Delta \phi$ from 5 to 30°, but for the most part the results were insensitive to the actual choice made. Therefore, we present only the case in which $\Delta \phi = 10°$.

Fig. 10 presents the profile of the correlation coefficient of contemporary values of the meridional temperature gradient across the 10° belt centered on a latitude with the eddy flux of sensible heat across that latitude, using data based on successive 10-day averages during the 1976–77 winter. Separate results are given for correlations with the transient and the standing eddy components of the heat flux, as well as for their sum. A negative correlation is indicative of a free variation in the flux, wherein its strength changes because of internal dynamics and the temperature gradient then responds to these heat flux changes. A positive correlation between the heat flux and the temperature gradient, on the other hand, implies the variation in the flux is forced; i.e., changes in external conditions force changes in the temperature gradient which in turn force similar changes in the flux (for example, the annual cycle in the eddy fluxes of heat is a forced phenomenon, as Stone and Miller amply demonstrate).

Because Lorenz (1979) performed his analysis in spectral form over the Northern Hemisphere, he ob-
tained results not at individual latitudes, but rather in wavenumber space for the hemisphere as a whole. On the scales we are studying here, Lorenz found a negative correlation between the total eddy heat flux and the temperature gradient for the Northern Hemisphere. Fig. 10 reveals, however, that this result is not true everywhere over the hemisphere, at least for this particular winter season. The sharpness of the transition around 40°N in the sign of the correlation coefficient for the total eddy flux is noteworthy.

Finally, Fig. 10 clearly depicts the tendency for the transient and standing waves to behave in opposite ways, a result also found by both van Loon (1979) and Stone and Miller (1980) in connection with longer time scales. Also, in our earlier study of winter mean conditions (Rosen and Salstein, 1980), we noted that the standing eddy heat fluxes were largest farther north from the region of maximum temperature gradient than were the transient eddy fluxes. This alone suggested to us that the two types of waves interacted with the temperature field differently, and in particular that the standing eddies were being forced by something other than just meridional temperature differences.

To examine the results contained in Fig. 10 more closely, we plot in Fig. 11 the time series of the total eddy heat flux and the temperature gradient at the latitudes where their correlation was most positive (37.5°N) and most negative (47.5°N). Focusing on conditions at 47.5°N first, we see that the standing eddy heat fluxes (which are strongest near this latitude) appear to be varying freely and, in turn, regulating the temperature gradient. Thus, when the heat flux increases, the gradient is diminished, and when the flux weakens, the gradient rebuilds. On the
other hand, the intensity of the transient eddy heat flux appears to be forced by the changes in the temperature gradient induced by the standing waves, i.e., when the gradient lessens (strengthens), the transient eddy heat flux lessens (strengthens).\(^2\)

The reason for the reversal in the signs of the (zero lag) correlation coefficients between 47.5 and 37.5°N is, according to Fig. 11, not due to changes in the behavior of the heat fluxes. Indeed, the time series of each component of the heat flux behaves much the same at the two latitudes, indicating the transient and standing waves maintain a sense of spatial cohesiveness across these latitudes. Rather, it is the time series of the temperature gradients at 47.5 and 37.5°N that are negatively correlated, and it is this circumstance that results in the differences between the two latitudes depicted in Fig. 10. [This contrast-

\[^2\] The reasonableness of this scenario is supported further by calculating the correlations among the various fields at different lags. The details are not presented here, but basically they demonstrate a tendency near this latitude for changes in the temperature gradient to lead changes in the transient eddy flux but to be led by changes in the standing eddy flux. In concert with this result, we also find that changes in the standing eddy flux tend to lead changes in the transient eddy flux.\]
tant weather such special winters bring and the need, therefore, to diagnose them separately. Obviously, though, it is important to extend our study to other periods before attempting to generalize the results of this section any further.

7. Final remarks

Our approach to investigating the effect of the averaging period on circulation statistics has been grounded in the traditional framework for general circulation studies. Thus, we have simply calculated the familiar zonally averaged transient and standing eddy flux fields for independent averaging periods and compared the results. Although we found that these fields did differ for all choices of averaging period, the single largest and most important changes generally occurred in going from 5- to 10-day periods. On this basis, we conclude that to separate properly zonal mean transient from standing eddy processes on less than seasonal time scales, averaging periods of at least 10 days or so are required.

Our conclusion is tempered, of course, by the realization that our study period may not be entirely representative. It is possible, for example, that the size of our interperiod eddy between 5- and 10-day averaging periods is affected by the blocking pattern this winter. On the other hand, the large fluctuations in many of our statistics from one part of the winter to the next tend to belie the argument that our results are all overly affected by persistence. We would recommend, however, that other years and seasons be studied along the lines followed here before more definitive conclusions are reached.

The best choice of averaging period for a particular application depends, of course, on the phenomenon being studied. Thus, 5-day periods may be suitable if linear quantities are involved, whereas 15- or 20-day periods may be too lengthy to resolve properly some behavior, such as the midwinter dip in $P_M$.

Finally, our analysis has allowed us to study the evolution of eddy heat fluxes in the general circulation, albeit for just a single season. With meteorological centers now providing global analyses on a routine basis, time series of such circulation statistics for longer periods can now be conveniently calculated, stored and studied. The challenge for the future is to turn these new data sets into keys for understanding fluctuations in our weather and climate.

Acknowledgments. We are grateful to A. J. Miller of NMC for his helpful suggestions and his support throughout the course of this study. Discussions with Profs. E. N. Lorenz and P. H. Stone clarified important aspects of our research. We also thank the reviewers for their worthwhile comments. Programming support was ably provided by Nancy Tripp. The National Meteorological Center/NOAA provided funds for this work under Contract NA-80-SAC-00730.

APPENDIX

The Interperiod Eddy Flux in the Mixed Space-Time Domain

To derive an expression for this flux, we begin by rewriting (7) in terms of averages over the subperiods
a and b. For convenience, we denote $\bar{\mathbf{u}' \mathbf{v}}$ + $\bar{\mathbf{u}^c \mathbf{v}^c}$ = $E^c$. Now,
\[
\bar{\mathbf{u}^c \mathbf{v}^c} = \frac{1}{2}([\bar{\mathbf{u}^c}] + [\bar{\mathbf{v}^c}])
= \frac{1}{2}([\bar{\mathbf{u}^a}][\bar{\mathbf{v}^a}] + E^c) + ([\bar{\mathbf{u}^b}][\bar{\mathbf{v}^b}] + E^c),
\]
but also,
\[
\bar{\mathbf{u}^c \mathbf{v}^c} = [\bar{\mathbf{u}^c}][\bar{\mathbf{v}^c}] + E^c
= \frac{1}{4}([\bar{\mathbf{u}^a}] + [\bar{\mathbf{u}^b})([\bar{\mathbf{v}^a}] + [\bar{\mathbf{v}^b}] + E^c.\]
Combining (A1) and (A2), we find that
\[
E^c = \frac{1}{2}(E^a + E^b)
+ \frac{1}{4}((\bar{\mathbf{u}^a} - \bar{\mathbf{u}^b})(\bar{\mathbf{v}^a} - \bar{\mathbf{v}^b}). (A3)
\]
According to (A3), the total (i.e., transient plus standing) eddy flux during a period is the average of the total eddy fluxes during its two halves plus a contribution due to the change in the zonal means of $\bar{\mathbf{u}}$ and $\bar{\mathbf{v}}$ during the halfs [cf. (5)]. We demonstrate next that this latter contribution is actually made entirely to the transient eddy part of $E^c$, and while doing so, we derive the expression for the interperiod eddy flux in the mixed space-time domain we seek.

Returning to (5) and taking its zonal mean yields
\[
\bar{\mathbf{u}^c \mathbf{v}^c} = \frac{1}{2}([\bar{\mathbf{u}^c}] + [\bar{\mathbf{v}^c}])
+ \frac{1}{4}((\bar{\mathbf{u}^a} - \bar{\mathbf{u}^b})(\bar{\mathbf{v}^a} - \bar{\mathbf{v}^b}). (A4)
\]
Noting from (6) that $[\mathbf{xy}] = [\mathbf{x}][\mathbf{y}] + [\mathbf{x}^\ast \mathbf{y}^\ast]$, we may rewrite the second term on the right-hand side of (A4) so that the transient eddy flux during period c is given by
\[
\bar{\mathbf{u}^c \mathbf{v}^c} = \frac{1}{2}([\bar{\mathbf{u}^c}] + [\bar{\mathbf{v}^c}])
+ \frac{1}{4}((\bar{\mathbf{u}^a} - \bar{\mathbf{u}^b})(\bar{\mathbf{v}^a} - \bar{\mathbf{v}^b}) + \bar{\mathbf{u}^c \mathbf{v}^c} + \frac{1}{4}((\bar{\mathbf{u}^a} - \bar{\mathbf{u}^b})(\bar{\mathbf{v}^a} - \bar{\mathbf{v}^b}). (A5)
\]
It therefore follows from (A3) that the standing eddy flux during period c is
\[
\bar{\mathbf{u}^c \mathbf{v}^c} = \frac{1}{2}([\bar{\mathbf{u}^a \mathbf{v}^a}] + [\bar{\mathbf{u}^b \mathbf{v}^b}])
- \frac{1}{4}((\bar{\mathbf{u}^a} - \bar{\mathbf{u}^b})(\bar{\mathbf{v}^a} - \bar{\mathbf{v}^b}). (A6)
\]
[Eq. (A6) is the general case of Eq. (5) of Mulokwa and Mak (1980) for the momentum flux due to interseasonal fluctuations.) From (A5) and (A6), it is clear that movement in standing wave patterns between periods, as measured by the last term in these equations, will affect both the transient and standing eddy fluxes in the longer period, but by opposite amounts. Moreover, the difference in the zonal

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