A Further Study of Spectral Energetics in the Winter Atmosphere

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ABSTRACT

The contributions of standing (time-mean) and transient (time-departure) waves to the atmospheric spectral energetics are analyzed using the NMC (National Meteorological Center) data of winter 1976–1977. It is found that the standing long waves are responsible for the major horizontal sensible heat transport and also for the significant horizontal momentum transport. Furthermore, the major contents of $A_K$ (eddy available energy) and $K_e$ (eddy kinetic energy) of standing waves are in the long-wave regime. However, the spectral energetics analysis indicates that the standing long waves are energetically less efficient than the transient long and short waves. It is suggested that the lower efficiency of the standing long waves in the atmospheric energetics may be one of the physical factors causing the underforecast of the standing long waves in the numerical weather prediction models.

1. Introduction

Atmospheric energetics analysis has been employed as a diagnostic tool in general circulation studies since Lorenz (1955) proposed his schematic energy cycle. As is well known, the dynamics and energetics of large and small scales of atmospheric disturbances are different. Saltzman (1957) introduced spectral analysis along latitude circles into the Lorenz energy cycle, to explore the role played by various scales of motion in the atmospheric circulation. Energetics analysis in physical space can be found in Peixoto and Oort (1974) and Oort and Peixoto (1974), while analysis in the spectral domain can be found in Wiin-Nielsen et al. (1963, 1964) and Steinberg et al. (1971). Saltzman (1970) and Tsumatsu (1979) also made thorough reviews of the spectral energetics of the atmosphere.

It is observed that the atmospheric flow consists of standing (time-mean) and transient (time-departure) wave motions. The former may be induced by the major mountain ranges (e.g., Charney and Eliasen, 1949; Bolin, 1950) and by the differential heating (e.g., Smagorinsky, 1953; Döös, 1962), while the latter may be stimulated by atmospheric instability (Charney, 1947; Eady, 1949; Green, 1960; Burger, 1962). Therefore, it would be instructive to further examine the separate contributions to the atmospheric energetics of standing (time-mean) and transient modes in the different wave regimes. Of course, the spectral energetics of the atmosphere have been investigated in numerous studies. However, the effort was made in this study to explore the role played by standing and transient waves in the atmospheric spectral energetics. It is hoped that documentation of this type of spectral energetics provides further verification of the numerical simulation of atmospheric circulation. To arrive at this goal, we display the contributions of standing and transient modes to the spectral energetics in the long- and short-wave regimes with the latitudinal pressure distribution of various energy variables.

The most serious error observed in numerical weather predictions occurs in the long waves (Baumhefner and Downey, 1978; Hollingsworth et al., 1980; Daley et al., 1981). The spectral analysis of medium-range forecast experiments at the ECMWF (European Centre for Medium-Range Weather Forecasts) by Hollingsworth et al. indicates that the energy content of the standing mode in the long-wave regime is too small in the forecast model. The efficiency of the spectral energetics in both the long- and short-wave regimes will be used to discuss the possible causes of the forecast error occurring in the long-wave regime.

The NMC data of winter 1976–77 are used for this study. This winter is selected because the standing mode is more intense as indicated by some recent studies (e.g., Edmon, 1980). Since we are particularly interested in the long-wave regime which contains most of the standing waves, it would be helpful to analyze the winter in which the standing eddies are strongest. A brief description of the spectral energetics scheme and data is presented in Section 2. The spectral analysis of momentum and sensible heat transport is discussed in Section 3. The spectral analysis of available potential and kinetic energies is shown in Section 4. The spectral analysis of energy conversions is depicted in Section 5. An energy diagram for the long- and short-wave regimes and the
efficiency of spectral energetics in different wave regimes will be discussed in Section 6. Section 7 will be devoted to summary of this study.

2. Spectral energetics scheme and computation

Following Saltzman's formulation and Steinberg et al.'s (1971) expression, the spectral form of the energy equations can be written as

\[
d\frac{K_Z}{dt} = \sum_{n=1}^{N} C(K_n, K_Z) + C(A_Z, K_Z) - D(K_Z),
\]

\[
d\frac{K_n}{dt} = -C(K_n, K_Z) + CK(n|m, l) + C(A_n, K_n) - D(K_n),
\]

\[
d\frac{A_Z}{dt} = -\sum_{n=1}^{N} C(A_Z, A_n) - C(A_Z, K_Z) + G(A_Z),
\]

\[
d\frac{A_n}{dt} = C(A_Z, A_n) - C(A_n, K_n) + CA(n|m, l) + G(A_n),
\]

where \( K \) and \( A \) represent kinetic and available potential energy, respectively. The subscripts \( Z \) and \( n \) denote zonal and wave components. \( C(P, Q) \) represents the energy conversion from \( P \) to \( Q \). Note that

\[
\sum_{n=1}^{N} C(K_n, K_Z) = C(K_{EL}, K_Z),
\]

\[
\sum_{n=1}^{N} C(A_Z, A_n) = C(A_{EL}, A_Z).
\]

Energy equations of \( K_{EL}, K_{ES}, A_{EL} \) and \( A_{ES} \) are obtained by summing Eqs. (2) and (4) from \( n = 1 \) to \( 4 \) and from \( n = 5 \) to \( 18 \)

\[
d\frac{K_{EL}}{dt} = -C(K_{EL}, K_Z) + C(K_{ES}, K_{EL}) + C(A_{EL}, K_{EL}) - D(K_{EL})
\]

\[
d\frac{K_{ES}}{dt} = C(K_{ES}, K_Z) - C(K_{ES}, K_{EL}) + C(A_{ES}, K_{ES}) - D(K_{ES})
\]

\[
d\frac{A_{EL}}{dt} = C(A_{EL}, A_Z) - C(A_{EL}, K_{EL}) - C(A_{EL}, A_{ES}) + G(A_{EL})
\]

\[
d\frac{A_{ES}}{dt} = C(A_{ES}, A_Z) - C(A_{ES}, K_{ES}) + C(A_{EL}, A_{ES}) + G(A_{ES})
\]

where

\[
C(K_{EL}, K_Z) = \sum_{n=1}^{4} C(K_n, K_Z)
\]

\[
C(K_{ES}, K_Z) = \sum_{n=5}^{18} C(K_n, K_Z)
\]

\[
C(A_{EL}, A_Z) = \sum_{n=1}^{4} C(A_Z, A_n)
\]

\[
C(A_{ES}, A_Z) = \sum_{n=5}^{18} C(A_Z, A_n)
\]

\[
C(A_{EL}, K_{EL}) = \sum_{n=1}^{4} C(A_n, K_n)
\]

\[
C(A_{ES}, K_{EL}) = \sum_{n=5}^{18} C(A_n, K_n)
\]

\[
C(A_{EL}, A_{ES}) = \sum_{n=1}^{4} CA(n|m, l) = -\sum_{n=5}^{18} CA(n|m, l)
\]

\[
C(K_{ES}, K_{EL}) = \sum_{n=5}^{18} CK(n|m, l) = -\sum_{n=1}^{4} CK(n|m, l).
\]

A schematic energy diagram to describe Eqs. (5)–(8) is shown in Fig. 15.

The data used in this study, 12-hourly observations
of $u$, $v$, and $T$ at the mandatory levels (1000, 850, 700, 500, 400, 300, 250, 200, 150 and 100 mb) for winter 1976–77 (Dec–Jan–Feb), were obtained from the NMC global analyses archived at the National Center for Atmospheric Research. The $\omega$ field was evaluated using the quasi-geostrophic $\omega$ equation, which was solved by a scheme designed by Chen et al. (1981). The integration of energy variables was performed between 15 and 85°N, and from 1000 to 100 mb.

The sum of wavenumbers 1–18 will be regarded as the total wave disturbances. In order to facilitate the discussion of the spectral energetics statistics, the definition of standing (time-mean) and transient (time-departure) waves proposed by Lorenz (1967) is adopted in this study. The contribution of standing waves is obtained by using only time-mean variables in the spectral energetics. However, the contribution of transient waves containing the interactions of three variables, for example transient $C(A_2, A_{EL})$, involves two parts: one is the interaction of only purely transient modes and the other is the interaction between two transient and one standing mode.

3. Sensible heat and momentum transport

The main objective of this study is to examine the contributions to the atmospheric spectral energetics by the standing and transient waves in different wave regimes. The distribution of various quantities as a function of latitude and pressure will be used to illustrate the computational results. Each figure contains eight diagrams. Total, long and short waves will be displayed from left to right. Then, total, standing and transient waves will be displayed from top to bottom of each figure.

a. Sensible heat transport

The total eddy sensible heat transport (Fig. 1a) is characterized by double maxima. One is located
in the lower troposphere around 50°N and 700 mb and the other in the lower stratosphere around 55°N and above 200 mb. The eddy sensible heat transport is northward through the entire atmosphere, except for small portions in the lower stratosphere in the tropics and near the pole. This transport distribution is similar to that seen in previous studies (e.g., Wiin-Nielsen et al., 1963, 1964; Oort and Rasmusson, 1971). The long-wave regime (Fig. 1b) accounts for the major part of the total eddy sensible heat transport. The maximum transport in the short-wave regime (Fig. 1c) is much weaker and somewhat south of that in the long-wave regime. This situation seems consistent with the relative locations of long- and synoptic-scale waves.

The sensible heat transports by standing and transient waves are depicted in Figs. 1d and 1f. Rosen and Salstein (1980) also computed these transports for the same winter (1976–77). It is interesting to note that this transport by standing and transient waves bears a similarity to that by the long and short waves. This situation shows that the standing long waves (Fig. 1e) are the major agent of sensible heat transport. Lau (1979) did a similar computation using the data of 11 winters. His result shows that standing and transient waves have comparable contributions to the sensible heat transport. The transient waves have a minor maximum around 65°N due to long waves. The contrast between Figs. 1g and 1h indicates that the transient long waves are active in the high latitudes and the transient short waves in the middle latitudes. Furthermore, Figs. 1c and 1h reveal that the sensible heat transport of transient waves is controlled by the short-wave regime.

b. Momentum transport

The total eddy momentum transport (Fig. 2a) occurs mainly in the upper troposphere. The northward transport is the most intense around 30°N and 250 mb, while the maximum southward transport occurs around 60°N and 300 mb. The boundary which separates the northward and southward transports is located around 50°N. The northward transport is more intense than the southward. This distribution is similar to that seen in the studies of Wiin-Nielsen et al. (1963, 1964) and Oort and Rasmusson (1971). The northward transport in the short-wave regime
is somewhat weaker than that in the long-wave regime south of 50°N. The southward transport north of 50°N is dominated by the long-wave regime.

The momentum transports by the standing and transient waves (Figs. 2b and 2c) are very comparable in both northward and southward transports. Our result agrees with Mak's (1978) finding that the meridional transport of momentum by the standing waves is comparable to that by the transient waves. However, Lau (1979) shows that the northward transport by transient waves is much more significant than that by standing waves. The momentum transport by the standing and transient waves in different wave regimes shows that the transport by standing waves is mainly performed by the standing long waves. The transport transient waves is mainly contributed by the long waves north of 50°N and by the short waves south of 50°N. This situation confirms again that the transient long waves are active in high latitudes and short waves in lower latitudes. Furthermore, the comparisons between Figs. 2c and 2b demonstrates that the momentum transport by transient waves is attributed to the short-wave regime.

4. Available potential and kinetic energies

a. Available potential energy

The distribution of eddy available potential energy ($A_E$) as a function of latitude and pressure (Fig. 3a) indicates that the whole troposphere contributes to $A_E$. The maximum $A_E$ appears at 500 mb and 45°N. This $A_E$ distribution is similar to the result of Newell et al. (1974). Interestingly, Peixoto and Oort (1974) found another $A_E$ maximum near the surface in high latitudes. This $A_E$ maximum is regarded by them as due to the mechanism of baroclinic energy release associated with the polar front. However, this $A_E$ maximum does not appear in our computation.

The significant contrast between long wave ($A_{EL}$) and short wave ($A_{ES}$) available potential energy (Figs. 3b and 3c) indicates that $A_{EL}$ is dominant. Note that the contour interval of $A_{ES}$ is half of that of $A_{EL}$. The contributions of standing and transient waves to the $A_E$ distribution are somewhat different from those of long and short waves. The standing waves have a maximum value at 500 mb and 50°N, while the transient waves have double maxima south
and north of the standing wave maximum. In addition, the standing and transient waves contribute very comparably to $A_E$.

Let us examine further the $A_E$ content of standing and transient waves in different wavenumber regimes. It is revealed by the comparison between Figs. 3d and 3e that the standing $A_E$ is essentially dominated by the standing long waves. The resemblance between Figs. 3c and 3b shows that $A_{E5}$ is contributed by the transient short waves. It is also important to point out that the transient long waves also make a significant contribution to the transient $A_E$.

The energy spectrum has long been an interesting subject in atmospheric energetics. The $A_n$ spectrum is shown in Fig. 4. The upper curve is total $A_E$, while the lower is standing $A_n$. It is observed that standing $A_n$ decreases more rapidly than total $A_n$ as the wavenumber increases. This situation indicates that no significant standing $A_n$ exists in the short-wave regime. A $-3$ power law also appears in the high wavenumber range in observational $A_n$ spectra (e.g., Wien-Nielsen, 1967). The total $A_n$ spectrum in Fig. 4 displays a $-3$ power law between wavenumbers 8 and 18.

b. Kinetic energy

The contribution to the eddy kinetic energy ($K_E$) from standing and transient waves in different wave regimes (Fig. 5) appears mainly in the upper troposphere. The $K_E$ distribution as a function of latitude and pressure (Fig. 5a) is similar to those distributions seen in the studies of Oort and Rasmusson (1971) and Peixoto and Oort (1974). $K_E$ has a maximum value existing at 250 mb and 35°N, located somewhat below and to the north of the maximum zonal kinetic energy (not shown) or maximum west-erly (Edmon, 1980; Rosen and Salstein, 1980). The major contributor of $K_E$ is $K_{E5}$, revealed from Figs. 5b and 5c. It is interesting to note that $K_{E5}$ has a minimum in high latitudes.

The standing $K_E$ (Fig. 5d) is larger than the transient $K_E$ (Fig. 5f), which contrasts with the situation for $A_E$. The transient $K_E$ also has a minor maximum in high latitudes. The observation of standing and transient $K_E$ in different wave regimes reveals that the standing $K_E$ is attributed to the standing long waves. $K_{E5}$ is mainly contributed by the transient short waves. $K_E$ of transient long waves has double maxima: one in midlatitudes and the other in high latitudes. The latter may well be due to the $K_{E5}$ maximum in high latitudes.

A $K_n$ spectrum is displayed in Fig. 6. The high wavenumber regime ($n = 8-18$) follows a $-3$ power law like $A_n$ and $K_n$ spectra in observational studies (e.g., Wien-Nielsen, 1967). Standing $K_n$ decreases more rapidly than the total $K_n$ as wavenumber increases. This situation is similar for standing $A_n$ and also indicates that the short-wave regime has less $K_n$ content.

5. Energy conversions

a. Conversion between zonal ($A_Z$) and eddy ($A_E$) available potential energy

The conversion between $A_Z$ and $A_E$, $C(A_Z, A_E)$ is determined by the eddy transport of sensible heat and the zonal temperature gradient. It was shown in Fig. 1 that the maximum eddy transport of sensible heat occurs in midlatitudes where the maximum zonal temperature gradient occurs. Fig. 7a reveals that the contribution to $C(A_Z, A_E)$ comes from the whole troposphere in midlatitudes with a maximum value at 500 mb and 40°N. The negative values appear above 200 mb in midlatitudes where the negative zonal temperature gradient occurs in the lower stratosphere because of the cold tropics and warm pole. Wien-Nielsen (1967) found that the maximum $C(A_Z, A_E)$ occurs at 600 mb in winter. Oort and Peixoto (1974) showed that the maximum values occur at the jet stream level and at lower levels where cyclone activities are significant. In fact, Newell et al. (1970) found significant $C(A_Z, A_E)$ near the surface in midlatitudes.

Let us recall that the sensible heat transport shown in Fig. 1 was mainly carried out by long waves. Therefore, it may be expected that $C(A_Z, A_{E5})$ should be much larger than $C(A_Z, A_{E2})$. However, it turns out that $C(A_Z, A_{E5})$ (Fig. 7c) is only somewhat smaller than $C(A_Z, A_{E2})$ (Fig. 7b). The same situation occurs in the standing and transient $C(A_Z, A_E)$ (Figs. 7d and 7f). In other words, the sensible heat transport by transient eddies is much weaker than that by standing eddies. Nevertheless, we still find that standing $C(A_Z, A_E)$ is only slightly larger than transient $C(A_Z, A_E)$. We might tentatively conclude
that the short waves and transient waves more effectively convert \( A_z \) to \( A_E \). One interesting point regarding the distribution of \( C(A_Z, A_E) \) is that the maximum values of \( C(A_Z, A_{E5}) \) and transient \( C(A_Z, A_E) \) appear south of the maximum values of \( C(A_Z, A_{E1}) \) and standing \( C(A_Z, A_E) \). The locations of maximum values in various modes of this energy conversion are similar to those in the sensible heat transport.

The spectral distribution of \( C(A_Z, A_a) \) as a function of wavenumber is depicted in Fig. 8. The bimodal distribution stands out very clearly with maxima at wavenumbers 2 and 6. This bimodal spectral distribution of \( C(A_Z, A_a) \) also can be seen in Saltzman (1970) and Steinberg et al. (1971). The shaded area represents the contribution of standing waves which is only significant in the low-wavenumber regime.

b. **Conversion between eddy available potential \( (A_E) \) and kinetic \( (K_E) \) energy**

The conversion between \( A_E \) and \( K_E \), \( C(A_E, K_E) \), is evaluated in terms of the covariance between the temperature and vertical velocity departures from their area means. Therefore, \( C(A_{E1}, K_E) \) can also be regarded as the upward transport of sensible heat. The evaluation of \( C(A_E, K_E) \) is hindered by the com-

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**Fig. 5.** As in Fig. 3, except for kinetic energy \((K)\).

**Fig. 6.** As in Fig. 4, except for \( K_E \).
Fig. 7. Latitudinal-pressure distribution of the conversion between $A_2$ and $A_4$: (a) $C(A_2, A_2)$, (b) $C(A_2, A_{EL})$, (c) $C(A_2, A_{ES})$, (d) standing $C(A_2, A_{EL})$, (f) transient $C(A_2, A_{EL})$, (g) transient $C(A_2, A_{ES})$, (h) transient $C(A_2, A_{ES})$. Units: $10^{-4}$ W m$^{-2}$ mb$^{-1}$.

Computation of the $\omega$ field. The $\omega$ field used in this study is based upon the scheme shown in Chen et al. (1981).

Chen et al. (1981) has provided a detailed discussion of $C(A_2, K_E)$. In order to make this paper self-contained, we also would like to present results of our computation. The contribution to $C(A_2, K_E)$ (Fig. 9a) comes almost from the whole troposphere with a maximum value at 500 mb and 40°N. Tomatsu (1979) also presented the latitudinal-pressure distribution of $C(A_2, K_E)$ which is similar to Fig. 9a. Oort and Peixoto (1974) calculated this energy conversion in terms of the work done by the pressure gradient force, i.e., generation of kinetic energy. It is observed that their latitudinal-pressure distribution of $C(A_2, K_E)$ is very different from the gradient in Fig. 9a. The long- and short-wave regimes (Figs. 9b and 9c) provide a comparable contribution to $C(A_2, K_E)$. In other words, $C(A_{EL}, K_{EL})$ and $C(A_{ES}, K_{ES})$ are somewhat comparable. The comparison between $C(A_{EL}, K_{EL})$ and $C(A_{ES}, K_{ES})$ is very different from that between the horizontal transport of sensible heat by the long- and short-wave regimes.

The pronounced contrast between the standing and transient $C(A_2, K_E)$, (Figs. 9d and 9f) is opposite to that between the sensible heat transports by the standing and transient waves. In other words, the transient $C(A_2, K_E)$ is dominant. Oerlemans (1980) found that the latitudinal distributions of horizontal and vertical sensible heat transport by transient

1 The relationship between the generation of kinetic energy and $C(A_2, K_E)$ can be illustrated by

$$-\nabla \cdot \nabla \phi' = -\nabla \cdot (\phi \nabla) - \frac{\partial}{\partial \rho} (\rho \omega') - \frac{R}{p} \omega' T.'$$

The zonal averages of $-\nabla \cdot (\phi \nabla)$ and ($\partial / \partial \rho$)($\phi \omega'$) are not zero. Therefore, the latitude-pressure distributions of zonally-averaged $-\nabla \cdot \nabla \phi'$ and $-(R/p)\omega' T'$ are not the same. Suppose $\omega = 0$ at the surface and top, the integration of the above equation over a closed domain is

$$\frac{1}{A_g} \int_{\phi_0}^{\phi_f} -\nabla \cdot \nabla \phi' d\rho = \frac{1}{A_g} \int_{\phi_0}^{\phi_f} \left( \frac{R}{p} \right) \omega' T' d\rho.$$

In other words, the area-mean generation of kinetic energy and $C(A_2, K_E)$ are equal.
waves have good similarity. Therefore, a relationship between these two sensible heat transports proposed by Saltzman and Vernekar (1971) based upon a baroclinic instability theory can be fitted in Oerle-
mans result. If we compare Figs. 9f and 1f carefully, Oerle-
mans' finding can still be seen in our results. However, we should note that the horizontal sensible heat transport is dominated by the standing waves and the vertical sensible heat transport is mainly controlled by the transient waves.

Some statements should be made on the contribution to $C(A_E, K_E)$ from various modes of atmospheric motion. Standing $C(A_E, K_E)$ is contributed to standing $C(A_{EL}, K_{EL})$. Transient $C(A_E, K_E)$ is contributed by both transient $C(A_{EL}, K_{EL})$ and $C(A_{ES}, K_{ES})$, which are comparable. Transient $C(A_{ES}, K_{ES})$ is significant in midlatitudes, while transient $C(A_{EL}, K_{EL})$ is significant at higher latitudes. We are thus provided more evidence that the transient long waves are active in high latitudes. We should also point out that the transient short waves account for most of $C(A_{ES}, K_{ES})$.

An interesting point can be observed by comparing the horizontal and vertical sensible heat transports (Figs. 1 and 9); the maximum value of various modes of the latter is always south of the former.

The spectral distribution of $C(A_m, K_m)$ is shown in Fig. 10. A bimodal distribution is still visible with maxima at wavenumbers 2 and 7. The contribution from standing waves mainly exists only in the low wavenumber regime. This spectral distribution of $C(A_m, K_m)$ is similar to that shown in Wiin-Nielsen (1959) and observed in the winter conditions of Saltzman (1970). In order to complete the Lorenz energy cycle, we should also examine $C(A_Z, K_Z)$, which is displayed in Fig. 11a. As many observational studies have shown, $C(A_Z, K_Z)$ is usually small. This can be understood because $C(A_Z, K_Z)$ is evaluated by the latitudinal integration of positive and negative values alternating in the latitudinal-pressure cross section. The three-cell structure can be seen clearly in Fig. 11. The large negative value between 50 and 70°N is due to the Ferrel cell. Fig. 11b depicts the distribution of standing $C(A_Z, K_Z)$. It is obvious that this conversion is due to the standing meridional circulation.

c. Conversion between eddy ($K_E$) and zonal ($K_Z$) kinetic energy

The conversion between $K_E$ and $K_Z$, $C(K_E, K_Z)$, is evaluated by the correlation between the eddy transport of momentum and the gradient of zonal flow. It was shown in Section 3 that the northward eddy momentum transport occurs south of 50°N and the southward transport north of 50°N. The north–south gradient of zonal flow (not shown) is only sig-}

Fig. 8. As in Fig. 4, except for $C(A_s, A_s)$. Standing component is shaded. Units: 10^{-2} W m^{-2}.
of $C(K_{E}, K_{Z})$ may not guarantee the magnitude of the area integration of $C(K_{E}, K_{Z})$.

Fig. 12 also shows that the momentum transport by transient waves south of 50°N is slightly larger than that by standing waves. It can be observed that the magnitude of transient $C(K_{E}, K_{Z})$ (Fig. 12f) is also larger than standing $C(K_{E}, K_{Z})$ (Fig. 12d). The standing $C(K_{E}, K_{Z})$ is explained by standing $C(K_{EL}, K_{Z})$. Transient $C(K_{E}, K_{Z})$ is contributed comparably by both transient $C(K_{EL}, K_{Z})$ and $C(K_{ES}, K_{Z})$.

Recall that the area integration of $C(K_{n}, K_{Z})$ is the difference between an area of positive large-valued $C(K_{n}, K_{Z})$ and negative large-valued $C(K_{n}, K_{Z})$. The difference between two large values has the possibility of uncertainty. This may be the reason that the spectral distribution of $C(K_{n}, K_{Z})$ varies greatly from study to study. Wiin-Nielsen et al. (1963, 1964) presented $C(K_{n}, K_{Z})$ of three Januaries for 1959, 1962 and 1963. There is no similarity of the spectral distribution of $C(K_{n}, K_{Z})$ in these three months. Some other studies, e.g., Saltzman (1970), show that all waves transfer the kinetic energy to zonal flow except wave 3. However, Steinberg et al. (1971) found that the long waves receive kinetic energy from the zonal flow, but the short waves feed energy into zonal flow. Our result (Fig. 13) shows that most of $K_{n}$ is fed into $K_{Z}$, except at waves 3 and 5. In addition, standing wave 2 has a significant contribution to $C(K_{n}, K_{Z})$. It may be because a persistant blocking ridge rode over the west coast of the North America during this winter. It was found that wave two was highly amplified in this winter (Chen and Shukla, 1982).

6. Energy diagram and discussion

In order to complete the analysis of spectral energetics, we also compute the nonlinear exchange of the available potential and kinetic energy as Steinberg et al. (1971). They found that the available potential energy cascades from long waves to smaller-scale waves and the kinetic energy cascades from
synoptic-scale waves to long waves and short waves. These characteristics of nonlinear exchange in the atmospheric spectral energetics are shown in Figs. 14a and 14b. The standing mode does not contribute significantly to the cascading of $A_n$. On the contrary, the standing mode of long waves plays a significant role in the cascading of $K_n$.

The energetics of long- and short-wave regimes is displayed schematically in Fig. 15. The conventional energetics study (Oort, 1964; Oort and Peixoto, 1974) expressed in terms of the Lorenz energy cycle shows that the zonal available potential energy ($A_Z$) is transformed to the eddy available potential energy ($A_E$) due to the sensible heat transport by baroclinic eddies. The vertical motion is induced by this transport so that $A_E$ is converted to eddy kinetic energy ($K_E$). The eddy motion transports momentum against the gradient of zonal velocity to speed up the zonal flow. Therefore, $K_E$ is converted to maintain the zonal kinetic energy ($K_Z$).

The eddies energies $A_E$ and $K_E$, from the conventional energy diagram of atmospheric large-scale motion are split here into long- and short-wave energies $A_{EL}$, $A_{ES}$, $K_{EL}$ and $K_{ES}$, shown in Fig. 15. However, two extra energy conversions are added. Both are nonlinear transfers, one between $A_{EL}$ and $A_{ES}$, i.e., $C(A_{EL}, A_{ES})$, and the other between $K_{EL}$ and $K_{ES}$, i.e., $C(K_{EL}, K_{ES})$. The directions of these two conversions in our result are $A_{EL} \rightarrow A_{ES}$ and $K_{ES} \rightarrow K_{EL}$. In addition to these two extra energy conversions, the energy cycles of both long- and short-wave regimes are consistent with the conventional one.

The generations of available potential energies $G(A_Z)$, $G(A_{EL})$, $G(A_{ES})$, and dissipations of kinetic energies $D(K_Z)$, $D(K_{EL})$, and $D(K_{SE})$, are evaluated in terms of residual method and displayed in Table 1. It is interesting, to note that $G(A_Z) \gg G(A_{EL}) \gg G(A_{SE})$ and $D(K_{EL}) \approx D(K_{SE}) > D(K_Z)$. In Fig. 15, the numerical values in parentheses represent the contributions from the standing mode.

The classic baroclinic instability theory (Charney, 1947; Eady, 1949) shows that the cyclone waves are most energetic in midlatitudes and are most likely to release the available potential energy. The long waves are much less unstable compared to cyclone waves (Green, 1960; Burger, 1962). Therefore, it might be expected that the short-wave regime is more efficient than the long-wave regime in atmospheric energetics.

As shown in Table 2, $A_{ES}$ is only 20% of $A_E$ and the maximum value of sensible heat transport by short waves is about a factor of 2.5 smaller than that by long waves. However, $C(A_Z, A_{ES})$ contributes
Fig. 12. Latitudinal-pressure distribution of the conversion between $K_E$ and $K_Z$: (a) $C(K_E, K_Z)$, (b) $C(K_{EL}, K_Z)$, (c) $C(K_{ES}, K_Z)$, (d) standing $C(K_E, K_Z)$, (e) standing $C(K_{EL}, K_Z)$, (f) transient $C(K_E, K_Z)$, (g) transient $C(K_{EL}, K_Z)$, (h) transient $C(K_{ES}, K_Z)$. Units: $10^{-4}$ W m$^{-2}$ mb$^{-1}$.

39% to $C(A_E, A_Z)$. $K_{ES}$ contains 39% of $K_E$, but $C(A_{ES}, K_{ES})$ provides 52% of $C(A_E, K_E)$. $C(K_{ES}, K_Z)$ contributes 55% of $C(K_E, K_Z)$, while the maximum value of the northward momentum transport by short waves is about 82% of that by long waves. It may be difficult to define the efficiency of the atmosphere energetics. Nevertheless, the argument shown above clearly demonstrates that the short waves are energetically more efficient.

The forecast experiments analyzed by Baumhefner and Downey (1978), Hollingsworth et al. (1980) and Daley et al. (1981) indicate that the long waves are predicted less accurately than the synoptic-scale waves. The discussion of the energetics efficiency of the short waves based upon Table 3 may provide some clue to this issue. Since the short waves are energetically more efficient, it may be expected that the short waves of the numerical prediction model can respond to the initial input data more rapidly and develop faster than long waves. Based upon the spectral energy analysis of the forecast experiments performed with the ECMWF model, Hollingsworth et al. (1980) found that the errors in the forecasts of the long waves occurred in the forecast of the standing mode of these waves. Hollingsworth et al. did not further examine the
model energetics to determine what may have caused these errors.

In order to explore the possible causes of this problem by means of spectral energetics, we prepared Table 3 to show the contributions from both standing and transient waves in the long- and short-wave regimes. It is obvious that the short-wave regime is dominated by the transient mode. However, the standing and transient $A_{EL}$ and $K_{EL}$ are, in general, comparable. Let us go back to compare the maximum value of sensible heat and momentum transport by the standing and transient long waves. The maximum momentum transport by transient long waves is about one-third of that by the standing long waves south of $50^\circ$N. The maximum sensible heat transport by transient long waves in midlatitudes is less than one-fifth of that by standing long waves. Table 3 shows that transient $C(A_2, A_{EL})$ is one-third of the standing value, transient $C(A_{EL}, K_{EL})$ is larger than standing, and transient $C(K_{EL}, K_2)$ is about 41% of standing. It seems that the transient long waves are also energetically more efficient than the standing long waves. The errors occurring in the forecasts of standing long waves may be attributed to the energy input of long waves in the initial field which develops the transient mode of the long waves much faster than the standing waves (Tribbia, 1981).

Table 1. Generations of available potential energies ($G$) and dissipations of kinetic energies ($D$) evaluated by the residual method. Units: W m$^{-2}$.

<table>
<thead>
<tr>
<th></th>
<th>$G(A_2)$</th>
<th>$G(A_{EL})$</th>
<th>$G(A_{ES})$</th>
<th>$D(K_2)$</th>
<th>$D(K_{EL})$</th>
<th>$D(K_{ES})$</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>3.13</td>
<td>-0.53</td>
<td>-0.34</td>
<td>-0.61</td>
<td>-1.0</td>
<td>-0.92</td>
</tr>
</tbody>
</table>
Table 2. \( K_E, A_E, C(A_{22}, A_E), C(A_{22}, K_E) \) and \( C(K_E, K_2) \) of the total, long- and short-wave regimes. The numbers in parentheses are the ratios of energy variables of either long- or short-wave regimes and that of the total waves. Units for \( K_E \) and \( A_E \) are \( 10^7 \) J m\(^{-2}\), and for \( C(A_{22}, A_E), C(A_{22}, K_E) \) and \( C(K_E, K_2) \) W m\(^{-2}\).  

<table>
<thead>
<tr>
<th></th>
<th>( K_E )</th>
<th>( A_E )</th>
<th>( C(A_{22}, A_E) )</th>
<th>( C(A_{22}, K_E) )</th>
<th>( C(K_E, K_2) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>Total</td>
<td>9.5</td>
<td>8.3</td>
<td>3.09</td>
<td>2.24</td>
<td>0.31</td>
</tr>
<tr>
<td>Long waves</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(1–4)</td>
<td>5.8 (61)</td>
<td>6.6 (80)</td>
<td>1.90 (61)</td>
<td>1.06 (47)</td>
<td>0.14 (45)</td>
</tr>
<tr>
<td>Short waves</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>(3–18)</td>
<td>3.7 (39)</td>
<td>1.7 (20)</td>
<td>1.19 (39)</td>
<td>1.17 (52)</td>
<td>0.17 (55)</td>
</tr>
</tbody>
</table>

7. Concluding remarks

The spectral energetics proposed by Saltzman (1957) has been employed to verify the numerical simulation of some general circulation models (e.g., Tenenbaum, 1976; Baker et al., 1977). Since the atmospheric flow consists of standing and transient waves, the examination of the contribution of these two types of atmospheric motions to the spectral energetics would be a further step to verify the numerical simulation of the sophisticated atmospheric models. An attempt was made in this study to achieve this goal.

Recently, Holopainen (1978) and Holopainen and Oort (1981) have diagnostically shown that the atmospheric time-mean flow may be forced by the transient waves. Murakami (1981) examines the nonlinear exchange of kinetic energy between transient and standing waves. Since the standing and transient waves co-exist in the atmosphere, the interaction between these two kinds of wave motion is indispensable and should be examined. However, these investigations are beyond the scope of the present study.

It is found that the energy content of atmospheric standing waves is mainly in the long-wave regime. Furthermore, the standing long waves are responsible for the major horizontal sensible heat transport and the significant momentum transport. However, detailed analysis of the contributions to the atmospheric energetics, from the standing and transient waves in the long- and short-wave regimes, indicates that the standing long waves are energetically less efficient than the transient long and short waves.

The importance of the long waves to numerical weather prediction was stressed in many studies (e.g., Wiin-Nielsen, 1976). It was also pointed out by Hollingsworth et al. (1980) that the major deficiency of numerical weather forecasts is due to the underforecast of the standing long waves. The physical cause of this problem in numerical weather forecasts may not be easy to answer. A recent attempt was made by Daley et al. (1981). However, the comparison between the energetics efficiency of standing and transient waves in various wave regimes seems to suggest that the underforecast of the standing long waves may be due to their lower energetics efficiency. Our spectral energetics of winter 1978–79 also show the same conclusion, although the energy content of the standing long waves is much smaller than the transient waves. In any event, the suggestion that the low energetics efficiency of standing long waves may have caused the underforecast of standing long waves needs further investigation.

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References


