NOTES AND CORRESPONDENCE

An Accuracy Goal for a Comprehensive Satellite Wind Measuring System

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ABSTRACT

Variational analysis with a geostrophic constraint is used to estimate a critical accuracy for a satellite lidar wind measuring system. This accuracy is such that the combination of satellite winds with satellite temperatures can produce analyses with an accuracy equal to that obtained from a rawinsonde network. An important assumption allowing this estimate to be made is that the satellite wind and temperature measurements are made with a spatial density equal to that of the rawinsonde network.

1. Introduction

The accuracy of meteorological analyses over the ocean is important not only for oceanic forecasts, but also because it quickly affects the accuracy of forecasts for downstream continental areas. For example, 48 h flow pattern forecasts by the National Meteorological Center are significantly less accurate over the western half of North America than over the eastern half. Halem *et al.* (1982) have presented evidence that atmospheric temperatures measured from polar orbiting satellites can improve the oceanic analysis and forecasts on the western coasts of continents. An analysis improvement could also result from a future satellite wind measuring program, if its accuracy and spatial coverage were sufficient. An active Doppler lidar system exploiting aerosol backscatter has been proposed as a possibility for this (Post, 1979; Huffaker, *et al.*, 1980). This would be an expensive system, however. The extent to which it would improve analyses and forecasts is therefore a very serious question.

2. Variational analysis

Observing system simulation experiments, such as were used in designing the Global Weather Experiment, can be used to estimate forecast improvements due to a new data source. These experiments are expensive, however, and are believed to exaggerate the improvement. This note applies principles of large-scale meteorological analysis to examine the satellite wind accuracy question in a simpler way. It can do this only because it assumes that the spatial coverage and density of satellite temperatures and winds over the ocean would be approximately the same as that of rawinsondes over land. This is already true for the satellite temperature system, since 12 h of data from a polar orbiting satellite produces about 3500 columnar retrievals. When divided into the area of the globe this produces a horizontal density of $1/(382\ \text{km})^2$, roughly equivalent to the density of rawinsondes in the United States.

This simplicity however allows only one question to be answered in a simple way: *If we assume that the density of satellite wind measurements is the same as that of the other systems, how accurate must the oceanic satellite winds be so that in combination with satellite temperatures they will lead to oceanic analyses whose accuracy is equal to continental analyses from the standard rawinsonde network?*

Modern large-scale analysis methods combine observational data with forecast "first guess" fields by statistical methods that weight all information according to its assumed error properties, and by enforcing kinematic constraints between wind and pressure fields (Lorenc, 1981). These constraints are defined by the "slow mode" components of the total field that are to be analysed (Phillips, 1981). (The "fast" modes are computed by nonlinear initialization.) In extratropical latitudes the slow mode constraint can be simplified to the geostrophic relation

$$\mathbf{v} = (g/f)\mathbf{k} \times \nabla z, \tag{1}$$

in which z is the isobaric height, $\mathbf{v} = (u, v)$ is the wind, g gravity, $\mathbf{k}$ the vertical unit vector, and f the Coriolis parameter ($10^{-4}\ \text{s}^{-1}$). The analysis must also interpolate information from observation locations to the regular analysis grid.

A simpler analysis method embodying (1) is the variational analysis proposed by Sasaki (1958). At any one level this consists of minimizing the horizontal integral of

$$(z - z_0)^2 + \epsilon|\mathbf{v} - \mathbf{v}_0|^2, \tag{2}$$

where z_0 and $\mathbf{v}_0$ are observations, z, $\mathbf{v}$ are the analysed fields to be determined subject to (1), and ϵ is the ratio of observation errors:

$$\epsilon = \frac{\overline{\delta z_0^2}}{\overline{\delta u_0^2}} = \frac{\overline{\delta z_0^2}}{\overline{\delta v_0^2}}. \tag{3}$$

This definition of ϵ is appropriate for uncorrelated errors. (It therefore is reasonably satisfactory for observational errors, but cannot be used for forecast errors.) However, because the variational analysis method I will apply is for a very idealized network, it will be necessary to use the following chain of arguments.

1) Assign typical observation errors for wind and for height to the combined satellite system over the ocean and separately so for the continental rawinsonde system.

2) Pretend that the two observation systems are not only equally dense, but are located on a regular lattice.

3) Apply Sasaki's method separately to each of the simplified systems, and derive for each the relation between its analysis accuracy and its data accuracy. Equate the two analysis accuracies to derive an expression for the satellite wind accuracy that, in combination with the satellite height (temperature) accuracy, will produce an idealized variational analysis over the ocean whose accuracy is equal to that from the continental system.

4) The accuracy results so obtained for either system will differ from that obtained in operational practice. It will overestimate the analysis accuracy because the real observations in either system are not on a regular lattice. It will underestimate the analysis accuracy because the variational approach in step 3 has ignored information from earlier data as carried forward in the "first guess" used in an optimal interpolation system. *Both of these effects will be quantitatively similar for the continental rawinsondes and the oceanic satellite system, however.*[1] We can therefore interpret the satellite wind accuracy value obtained in step 3 as also ensuring equal analysis accuracy in the real world of irregular observation nets and optimum interpolation analyses.

[1] It is reasonable to expect, in the future, that the asynoptic character of the satellite observations will be adequately treated by more frequent analysis updating than is employed in current optimum interpolation systems (6 h intervals). Some small improvement in satellite temperature retrievals can also be relied on to cancel the negative effect of horizontal correlation of error in the satellite temperatures that is ignored here.

By using the calculus of variations, Sasaki showed that minimization of (2) subject to (1) is achieved by the z field that satisfies the equation

$$\epsilon G^2\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right) - z = \epsilon G\left(\frac{\partial v_0}{\partial x} - \frac{\partial u_0}{\partial y}\right) - z_0, \tag{4}$$

where $G = g/f$. For a plane wave in which $(z, z_0, u_0, v_0) = \mathrm{Re}(Z, Z_0, U_0, V_0)\exp[i(\alpha x + \beta y)]$, the solution is

$$Z = [Z_0 - i\epsilon G(\alpha V_0 + \beta U_0)][1 + \epsilon G^2(\alpha^2 + \beta^2)]^{-1}. \tag{5}$$

If Z_t denotes the true field amplitude for wavenumbers α, β, we can define the error amplitudes

$$\left.\begin{aligned} \delta Z &= Z - Z_t \\ \delta Z_0 &= Z_0 - Z_t \\ \delta U_0 &= U_0 - U_t = U_0 + iG\beta Z_t \\ \delta V_0 &= V_0 - V_t = V_0 - iG\alpha Z_t \end{aligned}\right\}, \tag{6}$$

since the "true" field is also geostrophic (Phillips, 1981). Eq. (5) can now be rewritten as

$$\delta Z = [\delta Z_0 - i\epsilon G(\alpha\delta V_0 - \beta\delta U_0)] \times [1 + \epsilon G^2(\alpha^2 + \beta^2)]^{-1}. \tag{7}$$

We multiply this by its complex conjugate, and take the statistical expectancy under the assumption that δZ_0, δV_0 and δU_0 are uncorrelated:

$$\overline{\delta Z^2} = [\overline{\delta Z_0^2} + \epsilon^2 G^2(\alpha^2\overline{\delta V_0^2} + \beta^2\overline{\delta U_0^2})] \times [1 + \epsilon G^2(\alpha^2 + \beta^2)]^{-2}. \tag{8}$$

The coefficients δZ_0, δV_0, δU_0 come from Fourier representation of the individual observational error at grid points. Since the latter are taken to be uncorrelated, we can set $(\overline{\delta Z_0^2}, \overline{\delta V_0^2}, \overline{\delta U_0^2})$ equal to $N^{-1}(\overline{\delta z_0^2}, \overline{\delta v_0^2}, \overline{\delta u_0^2})$, where N is the number of observations. The analysis error $\delta z_{\alpha\beta}$ for this single wavenumber is of course correlated at neighboring grid points. The spatial average of its square, $\overline{\delta z^2}_{\alpha\beta}$, will be equal to $(1/2)\overline{|\delta Z|^2}$. If we now use $\overline{\delta u_0^2} = \overline{\delta v_0^2}$ and the definition (3) of ϵ, the result above can be written as

$$\frac{1}{2N}\frac{1}{\overline{\delta z^2}_{\alpha\beta}} = \frac{1}{\overline{\delta z_0^2}} + G^2\left(\frac{\alpha^2}{\overline{\delta u_0^2}} + \frac{\beta^2}{\overline{\delta v_0^2}}\right). \tag{9}$$

The formula for $\overline{(\delta v)^2}_{\alpha\beta}$ is readily derived from this and retains the relative weighing factor $G^2(\alpha^2 + \beta^2)$ between $\overline{\delta z_0^2}$ and $\overline{\delta u_0^2} = \overline{\delta v_0^2}$. It is well known from geostrophic adjustment theory (see also Daley, 1980) that at short wavelengths the adjusted fields reflect primarily the input vorticity distribution while at long wavelengths it is the input pressure (height) field that determines the adjusted motion. Eq. (9) expresses this relation in a slightly different way. It differs by showing explicitly how high accuracy in one type of data

TABLE 1. Observational root-mean-square errors. The satellite height error assumes no error at 1000 mb and is typical of oceanic retrievals.

	500 mb	250 mb
Rawinsonde		
δz	12 m	25 m
$\delta u_0 = \delta v_0$	3.8 m s^{-1}	5.9 m s^{-1}
Satellite		
δz_0	34 m	41 m

(e.g., satellite winds) can make up for poor accuracy in the other types of data (e.g., satellite temperatures).

Alternatively we can recognize that the true fields satisfy the relation

$$\overline{u^2}_{\alpha\beta} = G^2\beta^2\overline{z^2}_{\alpha\beta},\ \overline{v^2}_{\alpha\beta} = G^2\alpha^2\overline{z^2}_{\alpha\beta} \quad (10)$$

and rewrite (9) as a sum of signal-to-noise ratios for this wavenumber:

$$\frac{1}{2N}\frac{\overline{z^2}_{\alpha\beta}}{\overline{\delta z^2}_{\alpha\beta}} = \frac{\overline{z^2}_{\alpha\beta}}{\overline{\delta z_0^2}} + \frac{\overline{u^2}_{\alpha\beta}}{\overline{\delta u_0^2}} + \frac{\overline{v^2}_{\alpha\beta}}{\overline{\delta v_0^2}}.$$

Summation over all α, β reduces this to an integrated measure of signal-to-noise ratio of the analyzed z field:

$$\frac{1}{2N}\sum_{\alpha,\beta}\frac{\overline{z^2}_{\alpha\beta}}{\overline{\delta z^2}_{\alpha\beta}} = \frac{\overline{z^2}}{\overline{\delta z_0^2}} + \frac{\overline{u^2}}{\overline{\delta u_0^2}} + \frac{\overline{v^2}}{\overline{\delta v_0^2}}. \quad (11)$$

The annual mean values of transient kinetic energy and isobaric height variances that have been tabulated by Oort and Rasmussen (1971; pp. 90, 92 and 97) may be inserted in (10) to estimate a "dominant" value for $\alpha^2 + \beta^2$. If the values at 45°N are used, the results, when expressed as an effective horizontal wavelength $L = 2\pi(\alpha^2 + \beta^2)^{-1/2}$, are

$$\left.\begin{array}{l}\text{At 500 mb:}\quad L = 6600\ \text{km}\\ \text{At 250 mb:}\quad L = 8100\ \text{km}\end{array}\right\}. \quad (12)$$

These evidently reflect major contributions by planetary wavenumbers 5–6 to the transient motion, undoubtedly at periods longer than several days. Shorter wavelengths are important for 2–3 day forecasts, however, and must also be considered.

Table 1 lists some observational errors that are used in the analysis systems at the National Meteorological Center.[2] Critical values of δv_0^2 for a satellite wind program can then be defined by first equating δu_0^2 to δv_0^2 and then equating the right side of (9) for the continents (raob) to its value for the ocean (sat). The result is

$$\frac{1}{\overline{\delta v_0^2}_{\text{sat}}} = \frac{1}{\overline{\delta v_0^2}_{\text{raob}}} + \left(\frac{fL}{2\pi g}\right)^2\left(\frac{1}{\overline{\delta z_0^2}_{\text{raob}}} - \frac{1}{\overline{\delta z_0^2}_{\text{sat}}}\right). \quad (13)$$

The results for different wavelengths are shown in Table 2. The required errors are of course smaller than the rawinsonde wind error of Table 1 in order to make up for the relatively inaccurate satellite height values. (A reasonable inaccuracy of 10 m in the 1000 mb reference height values for the satellite heights will produce only minor reduction of these critical velocity errors.) Values in Table 2 for the L's defined in (12) can be interpreted as being based on requiring equal values for the integrated signal to noise ratio of (11).

Although these results are based on a variational analysis of an observing system with idealized geometry, the reasoning presented after Eq. (3) argues that the analysis accuracies over continents and oceans would also be equal to one another in a real system. I caution the reader, again, that *the above reasoning depends greatly on the assumption of an observing density for the satellite system that is equal to that of the rawinsonde system.* Reassessment of the observational errors listed in Table 1 would also require recalculation of the results in Table 2.

TABLE 2. Root-mean-square values (m s^{-1}) of satellite wind component error $\delta u_0 = \delta v_0$ that will, in combination with satellite temperature data, produce mid-latitude analyses over the ocean as accurate as the conventional rawinsonde system does over continents. The figures should be multiplied by $\sqrt{2}$ to give rms vector wind errors.

L (10^6 m)	500 mb	250 mb
2	2.7	5.0
4	1.8	3.7
6	1.2	2.8
8	1.0	2.2

[2] Aircraft winds over the ocean are useful in present practice, even though they are available only in limited volumes. They can be ignored here, however, against the large amount of wind data from the satellite that is assumed in this paper. The rawinsonde errors cited here are taken from information supplied by the European Centre for Medium Range Weather Prediction.

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