Persistent Anomalies of the Extratropical Northern Hemisphere Wintertime Circulation: Geographical Distribution and Regional Persistence Characteristics

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(Manuscript received 1 October 1982, in final form 19 May 1983)

ABSTRACT

We have studied the geographical and regional persistence characteristics of wintertime Northern Hemisphere 500 mb height anomalies, focusing particular attention on the behavior of strong anomalies that persist beyond the durations associated with synoptic-scale variability ("persistent anomalies"). We have also examined the persistence characteristics of certain dominant regional patterns of low-frequency variability.

There are three major regions for the occurrence of persistent anomalies: the North Pacific to the south of the Aleutians, the North Atlantic to the southeast of Greenland, and from the northern Soviet Union northeastward to over the Arctic Ocean. These regions have relatively high numbers of both persistent positive anomaly and persistent negative anomaly cases. For moderate magnitudes and durations, the numbers of positive and negative cases in each region are about the same; however, for larger magnitudes and longer durations, the number of positive cases exceeds the corresponding number of negative cases. Analyses with data that have been low-pass filtered (removing periods of less than 6 days) reveal that part (but not all) of the discrepancy between positive and negative cases results from the relatively greater likelihood that negative anomalies will experience brief interruptions by transient disturbances.

For durations beyond about 5 days, the probability that an anomaly which has lasted \( n \) days will last at least one more day is nearly constant. This nearly constant probability of continuation resembles the behavior obtained for a linear first-order autoregressive process (red noise). Nevertheless, there are significant differences in persistence between the positive and negative anomalies and red noise, particularly at large magnitudes, with the positive anomalies typically more persistent than either the negative anomalies or red noise.

A simple nonlinear autoregressive model is described that simulates many of the observed deviations from red noise, and possible physical sources for the nonlinearities are discussed.

Relationships between the initial anomaly value and its subsequent 12 h change are then studied. The height changes are decomposed into two parts: a mean change and a deviation from the mean change. Mean change variations are examined for evidence of multiple "quasi-equilibria" (multiple anomaly values having mean changes of zero). Mean change variations are also determined for the temporal coefficients of certain dominant regional patterns of low-frequency variability. Although the temporal fluctuations of the patterns exhibit considerably more persistence than found for the corresponding local height anomalies, neither the patterns nor the local anomalies display convincing evidence of multiple quasi-equilibria.

1. Introduction

A characteristic of weather evident to even a casual observer is the tendency toward persistence. Periods marked by the unusual persistence of highly anomalous weather conditions are of particular practical importance, often attracting intense interest in the community-at-large. The problem of forecasting these extreme events is among the most fundamental and challenging in weather prediction.

Empirical schemes for extended-range weather prediction (see Nicholls, 1980 for a recent review) have often used subjective or quasi-objective techniques in attempting to identify recurrent flow patterns associated with persistent abnormal weather conditions. Perhaps the most frequently cited example, blocking, is generally defined by certain anomalous flow patterns that typically persist beyond the periods associated with synoptic-scale variability (Elliott and Smith, 1949; Rex, 1950a, b; Sumner, 1954). Teleconnection studies (Wallace and Gutzler, 1981) also provide evidence for the existence of coherent forms of low-frequency behavior. Nevertheless, our current understanding of the nature and characteristics of persistent flow anomalies remains quite limited.

Theoretical studies of predictability (Leith, 1978; Moritz and Sutera, 1981) also often draw a close connection between predictability and persistence. Such studies frequently model the statistics of atmospheric

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fluctuations by a first-order linear autoregressive (red noise) process (Leith, 1973), although recent research suggests that higher order models may sometimes be more appropriate (Katz, 1982). Alternatively, frequency spectra are often presented, with relatively greater power at low frequencies generally being considered as indicative of relatively greater long-range predictability. These conventional statistics, while highly useful, do not provide a basis for addressing questions such as:

1) What are the favored regions for the occurrence of persistent positive (or negative) anomalies? Do the locations of the favored regions vary depending on the sign, magnitude or duration of the anomalies?

2) Are positive or negative anomalies typically more persistent?

3) Does the probability that an anomaly will persist depend on its magnitude or duration?

In the present study, we will adopt an approach similar to, but more general than, that usually adopted in blocking studies in order to ascertain the geographical and regional persistence characteristics of anomalies. We will also briefly examine the persistence characteristics of certain dominant patterns of low-frequency variability. A major objective throughout will be to provide detailed comparisons between the persistence characteristics of positive anomalies and negative anomalies; we will also compare the observed behaviors with the behaviors anticipated for certain simple statistical and dynamical models of atmospheric persistence.

2. Data

We will focus here on anomalies of the extratropical Northern Hemisphere wintertime 500 mb geopotential heights. The basic data set consists of twice-daily (0000 GMT and 1200 GMT) National Meteorological Center (NMC) final analyses of the Northern Hemisphere 500 mb geopotential heights for the 14 winter seasons from 1963–64 through 1976–77. The NMC analyses are discussed in Appendix E. The winter season is defined as the 90-day period from 1 December through 28 February. Prior to calculations, data were spatially interpolated by a 16-point Bessel scheme (Dahlquist and Bjorck, 1974, p. 280) from the NMC octagonal grid to a 5 degree latitude by 5 degree longitude grid over the region from 20 to 90°N. Missing or obviously incorrect analyses were replaced by linear interpolations in time. Less than 1% of the 500 mb height analyses required replacement.

3. Procedure for obtaining geographical distributions

a. Anomaly normalization

Raw height anomalies are defined as the departures of the analyzed heights from the corresponding long-term seasonal trend values. The seasonal trend time series at a point is determined by a least-squares quadratic fit to the 14–winter mean time series for that point (i.e., the first value of the winter mean time series is the average of the 14 different 0000 GMT 01 December values, the second value is the average of the 1200 GMT 01 December values, etc.). The raw height anomalies $h'$ have been normalized by a scale factor which is inversely proportional to the sine of latitude:

$$z = \frac{\sin 45}{\sin \theta} h'.$$

(1)

The scaling factor is motivated by a recent study on atmospheric energy dispersion (Hoskins et al., 1977) showing that height-field analyses provide a poor indication of the meridional component of energy propagation. This shortcoming is due to the latitudinal variation of the Coriolis parameter, which biases height-field responses toward high latitudes. Hoskins et al. suggest that quantities like streamfunction or vorticity provide better indicators of horizontal energy propagation. Note that this normalization is similar to that used in obtaining a geostrophic streamfunction from height data.

Although there is no a priori basis for expecting that large numbers of positive (or negative) latitude-normalized anomalies will be associated with climatological-mean ridges (or troughs), we might anticipate that regions characterized by relatively high temporal variance will be more likely to have frequent strong anomalies (or, from perhaps a more phenomenological viewpoint, that regions where strong anomalies frequently occur will tend to have relatively high variance). Elsewhere (Dole, 1982), we have considered the effect of regional differences in the height variance on the geographical distributions by instead normalizing the local height anomalies by their respective standard deviations (i.e., standardizing the height anomalies). Although both the latitude-normalized anomalies (LNA’s) and standard deviation–normalized anomalies (SDN’s) provide some distinct information on the behavior of the height fields, for two reasons we will focus primarily on the LNA’s. First, consistent with the attitude adopted in studies of blocking, we are interested in anomalies that are strong and long-lived relative to anomalies in other regions; normalizing by local standard deviations, however, tends to mask geographical variations in the intensity of the anomalies. Second, through the geostrophic relation, fields of LNA’s provide a much better indication of associated wind and vorticity anomalies and therefore are more amenable to direct physical interpretation. In the remainder of this section and in the following section, then, the term “anomalies” refers to the latitude-normalized anomalies.
b. Method for defining cases

We will define a "persistent anomaly" at a point if an anomaly at that point persists beyond some threshold value for a specified duration. The method, illustrated in Fig. 1, is as follows:

1) Specify a magnitude criterion $M$ and a duration criterion $T$, where for positive anomaly cases $M \geq 0$ and for negative anomaly cases $M \leq 0$.

2) Define the occurrence of a persistent positive (negative) anomaly case at a particular grid point satisfying selection criteria $(M, T)$ if the anomaly at that point remains equal to or greater (less) than $M$ for at least $T$ days.

3) Define the duration $D$ for a positive (negative) case as the time from which the anomaly first becomes greater (less) than $M$ to the time when the anomaly next becomes less (greater) than $M$ at that point.

Note that these criteria act as lower bounds, so that all events which meet or exceed the threshold values are counted as persistent anomaly cases satisfying the specified criteria.

The numbers of persistent anomaly cases occurring over the 14 winter seasons were determined for each point for the following values of selection criteria:

$$M = \pm 0 \ m, \pm 50 \ m, \ldots, \pm 250 \ m$$
$$T = 5 \ days, 10 \ days, \ldots, 25 \ days$$

where the $0 \ m$ threshold identifies runs of non-negative values for the positive cases and non-positive values for the negative cases.

In the following section, we will present results for a few values that illustrate a number of the most interesting characteristics of the geographical distributions. For display purposes, the fields of the numbers of cases have been lightly smoothed by applying a two-dimensional nine-point spatial filter (Shapiro, 1970). This filter effectively removes fluctuations having wavelengths $\leq 1500 \ km$ but does not otherwise affect the general character of the spatial variability.

4. Geographical distributions

Fig. 2a displays the geographical distribution of the number of positive cases satisfying the criteria $(+150 \ m, 5 \ days)$. These values include contributions from rather strong but short-lived events. Three major regions of maximum frequency of occurrence are evident: 1) over the central North Pacific to the south of the Aleutians (PAC); 2) over the eastern North Atlantic to the southeast of Greenland (ATL); and 3) over the northern Soviet Union extending north-eastward to over the Arctic Ocean (NSU). There is considerable latitudinal variability in the number of cases, despite the latitude-dependent normalization, with maxima occurring near 50°N for the ATU and PAC regions and near 60°N over NSU. Cases satisfying this set of criteria are rare to the south of 30°N, over southern and eastern Asia and over central North America.

The corresponding distribution for negative anomaly cases satisfying $(−150 \ m, 5 \ days)$ is shown in Fig. 2b. There are several striking similarities between the positive and negative distributions. The greatest numbers of negative cases also occur over the PAC, ATL and NSU regions, although for this set of selection criteria the latter maximum is displaced somewhat westward of the corresponding positive center. There is additionally a fourth region of high negative occurrences over the extreme eastern North Pacific. For PAC and ATL, the maxima in the number of positive cases exceed the corresponding maxima for the negative cases; for NSU the maxima are comparable.

![Fig. 1. Method for defining cases. A persistent positive (negative) anomaly case of duration $D$ satisfying criteria $(M, T)$ is defined at a point if the anomaly at that point exceeds (is less than) the threshold value $M$ for at least $T$ days. Examples are given for both positive anomaly cases $(M > 0)$ and negative cases $(M < 0)$.](image-url)
Fig. 2c shows the sum of the two previous distributions. The range in the number of cases is substantial: the four major regions have in excess of 20 cases over the 14 winter seasons, while central North America and large areas of Asia and the subtropics have fewer than four events satisfying these criteria over the period.

Fig. 3 displays similar positive, negative and sum distributions for selection criteria (+100 m, 10 days) and (−100 m, 10 days). These maps show a substantial reduction in the number of cases from the previous values. The three regions PAC, ATL and NSU continue to have the greatest number of cases, with the positive maxima exceeding the negative maxima for each region. There is only a weak indication, however, of the fourth region of high negative occurrences seen in the earlier analyses to the west of British Columbia. Examination of distributions with other criteria (not shown) shows that this maximum is present for all values of the magnitude criterion at 5 days but is not strongly evident for any values by 10 days. This suggests that although strong negative anomalies are frequently present in this region, they are typically relatively short-lived.

For values of the duration criterion beyond 10 days the PAC, ATL and NSU regions continue to have the greatest number of persistent anomaly cases. There are few cases at these durations except for low values of the magnitude criterion. Aside from an overall decrease in the number of cases, the greatest qualitative change evident at low magnitudes and long durations is the relative increase in the total number of cases in NSU compared with either ATL or PAC. This can be seen by comparing Fig. 3c with Fig. 4, which shows the sum of the cases satisfying (+50 m, 15 days) or (−50 m, 15 days). For relatively large magnitudes and short durations, then, the ATL and PAC maxima exceed the NSU maximum, while for relatively small magnitudes and long durations the NSU maximum is slightly larger. Also evident in Fig. 4 are relative increases in the numbers of cases to the west of Greenland and over the subtropics of the western North Pacific and central North Atlantic. Examination of maps for other criteria (not shown) indicates that these features are most prominent for small values of the magnitude criterion. This suggests that although 500 mb height anomalies are typically relatively weak in these regions, they may also be rather persistent.

Inspection of time series for individual points reveals that a number of persistent events occur which may not satisfy certain selection criteria due to brief (∼1 day) interruptions by mobile transient distur-

Fig. 2. The number of cases in 14 winter seasons satisfying the (a) positive anomaly criteria (+150 m, 5 days); the (b) negative anomaly criteria (−150 m, 5 days); and (c) the sum of the cases in (a) and (b). Contour intervals are 2 in (a) and (b), and 4 in (c).
bances. In order to assess this effect, a low-pass filter that removes periods $\leq 6$ days was applied to the data and the analysis described above was repeated. Fig. 5 displays the filter response function. Fig. 6 shows distributions obtained from the filtered data for selection criteria of ($+100$ m, 10 days) and ($-100$ m, 10 days). Comparing these distributions with the corresponding distributions for the unfiltered data (Fig. 3), we see that the three key regions (PAC, ATL and NSU) continue to have the highest number of cases. The most striking change is the relatively large increase in the number of negative cases occurring in the key regions. Whereas for the unfiltered data the number of positive cases exceeds the number of negative cases, for the low-pass filtered data the numbers of positive and negative cases are comparable. The total numbers of cases satisfying these criteria are increased by 50–100% over the corresponding unfiltered values, primarily as a result of the increase in the negative cases. The regional distributions, however, are otherwise almost unchanged from the unfiltered data. Low-pass distributions for other values of the selection criteria (not shown) also reveal a relatively greater increase in the number of negative cases, although at large thresholds (magnitudes $\geq 150$ m), there are still more positive than negative cases.

To estimate possible influences of interannual variability on the numbers and locations of persistent anomalies, geographical distributions were also calculated for anomalies that were defined with respect to the mean for each winter season (after removing the long-term seasonal cycle by the method described in Section 3). The results (not shown) are quite similar to those obtained for the earlier distributions, although the values of the maxima are slightly reduced (5–25%). This indicates that interannual variations in the seasonal mean provide relatively small contributions toward determining the occurrence (or non-occurrence) of persistent anomalies. Indeed, we anticipate that some of the interannual variability in the means is at least partly attributable to the occurrence of anomalous events that may themselves last a relatively small fraction of a season, as suggested by studies of sampling fluctuations in long-term means (Leith, 1973; Madden, 1976).

The gross structure of the persistent anomaly distributions closely resembles that of the daily variance of the wintertime 500 mb heights described by Blackmon (1976). The PAC, ATL and NSU regions, identified as having high numbers of major persistent anomaly cases, correspond with the major centers of large variance. This relationship is not surprising, since persistent anomalies can be expected to provide

FIG. 3. The number of cases in 14 winter seasons satisfying the (a) positive anomaly criteria ($+100$ m, 10 days); the (b) negative anomaly criteria ($-100$ m, 10 days); and (c) the sum of the cases in (a) and (b). Contour intervals of 2.
major contributions to the low-frequency variance (periods beyond 10 days) and, as Blackmon demonstrates, the daily height variance is dominated by low-frequency contributions.

The relatively large increase in negative events for the filtered data suggests that negative anomalies are more likely than positive anomalies to experience brief transient interruptions (predominantly from disturbances having periods $\leq 6$ days). Further analyses of temporal behavior (Dole, 1982) indicate that this is related primarily to changes in storm activity accompanying the persistent anomalies, with relatively more frequent and vigorous eddies over the region typically associated with the negative cases.

5. Regional persistence characteristics

Our primary emphasis in this section will be on comparing the detailed persistence characteristics for regions characterized by relatively high and low numbers of persistent anomalies. For the former, we will focus on the eastern North Atlantic (ATL) and central North Pacific (PAC) regions, and for the latter, the central North America (AME) and eastern Asia (EAS) regions. For brevity, the results for the Northern Soviet Union (NSU) persistent anomaly region are not presented in detail, but are summarized at the end of Section 6. Fig. 7 shows the locations of the five regions.

For ease of display, the anomalies have been normalized by their standard deviations; average values of the standard deviations for the regions are 179.3 m (PAC), 174.2 m (ATL), 162.2 m (NSU), 113.5 m (AME) and 44.8 m (EAS). Calculations are first performed at individual grid points; distributions for a region are then determined by combining the distributions obtained at each point in a 3 by 3 grid with 5$^\circ$ latitude and 10$^\circ$ longitude spacings centered on a point within the region. The center points are 50$^\circ$N, 20$^\circ$W (ATL); 50$^\circ$N, 165$^\circ$W (PAC); 40$^\circ$N, 90$^\circ$W (AME); 30$^\circ$N, 90$^\circ$E (EAS); and 60$^\circ$N, 50$^\circ$E (NSU). Although the areas of the regions vary, the general statistical characteristics of the distributions at points within each region are nearly constant. The combined distributions for the regions have smaller sampling errors than the corresponding distributions at the individual points.

Fig. 8 presents, for each of the four regions, distributions of the numbers of runs of positive and negative anomalies that exceeded selected magnitude thresholds for at least a given duration. For comparison, estimates of the 5% and 95% confidence limits are also displayed for similar distributions generated by red noise processes appropriate for the regions; Appendix A describes the method used for estimating the confidence limits.

We see that after about 3–5 days the distributions for all regions form nearly straight lines, with shallowest slopes at low thresholds and largest slopes at high thresholds. On this semi-logarithmic plot, the local slope of the distribution is proportional to the probability of continuation of the run, with shallower slopes indicating a higher probability of continuation. The almost constant slopes at durations beyond a few days imply that the runs rather quickly assume a nearly constant probability of continuation. For many of the curves, a change from a steeper to a shallower slope occurs over the first few days, indi-
cating that the probability that an anomaly will continue is initially increasing with increasing duration. This initial transitional behavior is intuitively plausible: at the start of the run, the anomaly value will generally be relatively close to the threshold, since it has recently crossed this value, and is therefore rather likely to return across the threshold. As the run continues, the expected value of the anomaly moves away from the threshold, asymptotically approaching a limiting value.

Fig. 9 illustrates this behavior for the PAC and ATL regions. Note the tendency unique to the PAC positive anomalies for the average anomaly to peak after 5–7 days before asymptotically approaching a slightly lower value. This behavior is slightly suggestive of a preferred time scale for PAC positive anomalies. Charney et al. (1981) provide other observational evidence for a possible preferred time scale for persistent positive anomalies of about 7–10 days, although their result is obtained by combining distributions at all longitudes (for latitudes between 50 and 70°N), and therefore does not allow for possible regional differences. Both our results and those of Charney et al., however, suggest that any deviations are small from the primary asymptotic behaviors.

For EAS, the distributions of positive and negative runs displayed in Fig. 8 are almost identical; for the other regions, there are systematic differences between corresponding positive and negative curves. At nonzero thresholds, the positive anomalies in these regions are typically more persistent. In general terms, the distributions for the real data follow the shape of the distributions generated by the corresponding red noise process. In particular, the red noise curves also show a tendency for relatively steeper slopes at short durations and shallower, nearly constant slopes at long durations. Nevertheless, some of the differences in behavior between positive and negative distributions and the red noise distributions appear clearly significant.

A question of considerable theoretical as well as practical interest is whether the anomaly \( z' \) tends to particular (possibly multiple) “preferred” values. We have examined distributions of anomaly values for evidence of multiple modes, perhaps suggesting multiple quasi-equilibria. Fig. 10 presents distributions of anomaly values for the 14 winter seasons of twice-daily data. The PAC distribution displays a marked positive skewness. The ATL distribution is more rectangular than a corresponding normal distribution, with relatively few values near the mean and in the tails and relatively many values at moderate magnitudes; indeed, a few of the distributions for individual points within the ATL region (not shown) hint at bimodality. The EAS and AME distributions, in con-

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**FIG. 6.** As in Fig. 3, but for the low-pass filtered data.
contrast, do not differ markedly from normal distributions. The general characteristics of the regional distributions appear broadly typical of distributions at points throughout the key regions (cf. White, 1980). White's results suggest that for plausible choices of the number of degrees of freedom, anomaly distributions in the PAC and ATL regions may be significantly non-Gaussian.

We have looked further for evidence of multiple quasi-equilibria by examining the relationship of the height anomaly tendency to the height anomaly value, searching for those anomaly values where the mean anomaly tendency vanishes. Our approach is as follows: we first compute average values of the 12 h change $\Delta z'$ for initial values of $z(t)$ within intervals (bins) of width 0.1 (i.e., from $-3.05$ to $-2.95$, $-2.95$ to $-2.85$, etc.), and denote this average change curve by

$$\overline{\Delta z'} = f(z').$$

We then define a potential function $U$ by

$$U(z' = x) = -\int_{x_1}^{x} f(z')dz' + C, \quad x_1 < x < x_2,$$

Fig. 8. Number of events in 14 winter seasons when a run of anomalies above or below a set threshold lasts for at least a given duration, for regions (a) PAC, (b) ATL, (c) AME and (d) EAS. The solid lines are for positive events above standardized thresholds of 0.0 (top), +0.5 (middle) and +1.0 (bottom). The dotted lines are for negative events below 0.0 (top), −0.5 (middle) and −1.0 (bottom). The shaded areas contain the corresponding estimates of the 5 to 95 percentile ranges for red-noise processes appropriate to the regions (see Appendix A for further details).
where \( x_1 \) and \( x_2 \) represent, respectively, the lower and upper limits of the range of anomaly values having at least 10 observations per bin. The arbitrary constant \( C \) is chosen for graphical convenience.

By virtue of the integration, the potential function \( U \) displays less sensitivity to random sampling errors than the average change curve. Note that, since the slope of \( U \) is proportional to the average 12 h change, the maxima and minima in \( U \) correspond to anomaly values having mean tendencies equal to zero; these may be considered as quasi-equilibrium points. Furthermore, the maxima in \( U \) correspond to unstable points and the minima to stable points (in the sense that small departures will tend on average to grow or decay, respectively). Our approach is analogous to that of Benzi et al. (1982); although the climate models they study are sufficiently simple that the potential functions can be obtained directly from the model equations.

Fig. 11 displays the average 12 h changes and the potentials as functions of the initial anomaly value. We see that for each of the four regions there is only a single minimum in the potential corresponding to a single preferred “climatic” value. We will call the value of the anomaly corresponding to this minimum \( z_0' \). Although the precise values of the minima are rather poorly defined, examination of the change curves suggests that in some regions (particularly ATL) \( z_0' \) may be different from zero, indicating that the heights may on average be decaying toward a value other than the long-term mean.

The shapes of the potential functions also show considerable regional variability. For EAS, the potential function closely resembles a parabola, reflecting the almost exact proportionality of the 12 h change to the initial anomaly value. Potential functions in the other regions have much steeper slopes for negative than positive anomalies; correspondingly, the negative anomalies typically have larger average changes back to \( z_0' \). The overall character of the potential function and average change curves supports our earlier results in suggesting the generally greater persistence of positive than negative anomalies in these regions.

6. Comparison to simple statistical models of persistence

a. The relationship between observed and red noise persistence

We noted in the Introduction that the statistics of atmospheric fluctuations have frequently been modeled by linear autoregressive processes, perhaps most commonly by a first-order process (often called red noise). Our results, however, indicate that there are systematic differences in persistence between positive and negative anomalies. These differences are generally most evident at large anomaly values. These variations in persistence cannot be accounted for by first-order (or higher order) linear autoregressive models. Further, comparisons of the geographical distributions for unfiltered and low-pass filtered data suggest that there may also be differences in high-frequency variability between positive and negative anomalies. In this section, we will examine in more detail some observed variations in persistence that are not contained in linear autoregressive models. We will also introduce a very simple nonlinear autoregressive model that roughly simulates a number of the observed statistical characteristics in the persistent anomaly regions.
We will first investigate the relationship between observed and red noise persistence characteristics. The model for a discrete red noise process obeys the difference equation

\[ z(t + 1) - \bar{z} = a(z(t) - \bar{z}) + e(t + 1), \]

(4)

where \( \bar{z} \) is the mean value of the process, \( a \) is a constant having a value between 0 and 1, and \( e(t) \) is a white noise process having constant variance \( v^2 \). In the absence of the white noise, the time-series decays toward a single equilibrium value \( \bar{z} \) with a decay time given by \( (1 - a)^{-1} \).

A classical problem (see, for example, Chatfield, 1975, pp. 65–70) is to obtain optimal estimates of \( a \) and \( v \) from a given time series. As Chatfield notes, the least-squares estimate of \( a \) can be obtained by treating (4) as a linear regression problem with \( z(t + 1) - \bar{z} \) as the "independent" variable. An important assumption in this analysis is that the values of \( a \) and \( v \) do not depend on the value of \( z \). We will examine whether this assumption is supported by the observations. Our approach is as follows.

For standardized anomaly intervals (bins) of width 0.1, all anomaly values initially falling within the bin, \( z'(t) \), and the corresponding values 12 h later, \( z'(t + 1) \), are determined. For each bin having at least 10 values of \( z'(t) \) and \( z'(t + 1) \), a least-squares estimate of the parameter \( a \) is then obtained for the equation

\[ [z'(t + 1) - z'_o] = a[z'(t) - z'_o], \]

(5)

where \( z'_o \) is the appropriate regional quasi-equilibrium value determined in the previous section. An estimate of the corresponding value for \( v^2 \) is obtained from the unexplained variance in this regression calculation.

Our analysis differs in two basic respects from the classical approach: first, we have attempted to obtain local, rather than global, estimates for \( a \) and \( v^2 \) and, second, we have included a parameter \( z'_o \) since, as previously shown, \( z' \) may tend to a value not necessarily equal to the long-term mean value (equal to zero by definition of the anomalies). We may therefore recover the classical approach results by performing the analysis over a single bin of infinite bin width with \( z'_o \) set to zero (for the global analysis, this is the least-squares estimate of the equilibrium value for the time series). Later, we will discuss in some detail how variations in the choice of \( z'_o \) may affect the appearance of the results; however, results obtained by instead assuming that \( z'_o = 0 \) are qualita-
Fig. 11. The average 12 h changes in the standardized anomaly (top thick line) and the corresponding potential function $U$ (lower thick line) defined in (3) as functions of the initial anomaly value. The thin lines above and below the change curve give the 95% confidence interval for the average changes. The value of the arbitrary constant $C$ in (3) is chosen such that the minimum value of $U$ is $-1$. For regions (a) PAC, (b) ATL, (c) AME and (d) EAS.

Fig. 12 displays, as functions of $z'(t)$, the estimates of $a$ and $v$. Aside from values near $z'_0$, where there is considerable uncertainty in the least-squares estimates, systematic changes in $a$ are evident as a function of the initial anomaly value. The behavior of $a$ with respect to $z'$ is consistent with our earlier results in indicating that positive anomalies are typically more persistent than negative anomalies. This discrepancy tends to increase with increasing anomaly magnitudes. The trend is very slight over EAS, but quite pronounced in the other regions. For ATL, the distribution of $a$ appears almost bimodal, with values of around 0.85 for $z'(t) < -0.5$ and values around 0.95 for positive anomalies. Although such changes appear small, the consequences for the persistence characteristics may be large: for the ATL values, corresponding $e$-folding times for red noise decay are around 3 days for the negative anomalies and about 10 days for the positive anomalies.

For the PAC region, there is a small range of anomaly values near $z'_0$ where the estimated value of $a$ exceeds 1. Although this result may seem surprising (since for a red noise process $a$ must lie between 0 and 1), values that exceed 1 for a limited range of initial anomalies are not prohibited for a nonlinear process (and indeed must occur for a first-order process with multiple equilibria). Values of $a$ that locally exceed 1 may also occur if the value of $z'_0$ is incorrectly specified.

This can be seen by interpreting $a$ in (5) not only as a measure of persistence, but also as an indicator of whether the anomalies tend to return to $z'_0$: absolute values of $a$ less than (greater than) 1 indicate that departures from $z'_0$ tend to decrease (increase) with time. In a first-order system with multiple equilibria, the transition from $a < 1$ to $a > 1$ defines the region of attraction for $z'_0$. To see how an incorrect specification of $z'_0$ may be manifested in the estimate for $a$, consider a first-order nonlinear process with a single stable quasi-equilibrium value at $z'_0 > 0$, so that anomalies initially less than (greater than) $z'_0$ tend to be followed by larger (smaller) values. If we incorrectly assume that the quasi-equilibrium value is zero, then for initial anomaly values between 0 and $z'_0$, the
estimate of $a$ obtained from (5) would exceed 1 (since on average $z'(t + 1) > z'(t)$ over these values). The results in the PAC region appear to reflect our uncertainty in the precise quasi-equilibrium value, and suggest that the specified value of $z'_o$ may be slightly too high.

The estimated noise intensity $v$ displayed in Fig. 12 also shows interesting variations. Over EAS, the trend with $z'$ is very slight. Over the other regions, however, substantial trends are evident. In these regions, the smallest values occur for large positive anomalies; values vary by a factor of about 2–3 between maxima and minima. The anomaly values for the noise maxima show considerable regional variability; these are plausibly associated with shifts in the storm paths located adjacent to the regions. These relationships are described more fully elsewhere (Dole, 1982).

Analyses similar to those described in this and the previous section have also been conducted for the NSU region. The results (not shown) display trends in $a$ and $v$ that are qualitatively similar to, although slightly less pronounced than, the trends described for the ATL and PAC regions. Principal quantitative differences suggest that the NSU region is characterized by somewhat higher overall persistence (values of $a$ range from about 0.90 for strong negative anomalies to just below 1.00 for strong positive anomalies) and somewhat lower noise (values of $v$ range from about 0.35 for negative anomalies to around 0.20 for strong positive anomalies). Consistent with the high persistence, the NSU potential function appears relatively flat, with a single minimum at an anomaly value near zero.

b. A simple nonlinear model of the observed persistence characteristics

We observed earlier that distributions of anomaly values for the persistent anomaly regions (Fig. 10) displayed some apparent non-Gaussian characteristics. For a red noise model with a Gaussian white noise input, however, the resulting anomalies will also have a Gaussian distribution. As a rough test of how variations in $a$ and $v$ might affect the shapes of anomaly distributions, we have generated synthetic time
series from the following simple first-order nonlinear model for the anomaly time series:

\[ z'(t + 1) = a_0 + a_1 z'(t) + a_2 z'(t)^2 + e(t), \]

where \( e(t) \) has a Gaussian distribution with zero mean and a standard deviation \( v(t) \) that also depends on the initial anomaly value, i.e.,

\[ v(t) = v_0 + v_1 z'(t) + v_2 z'(t)^2. \]

Details of the procedure are described in Appendix B. Note that for a red noise model, \( a_2, v_1 \) and \( v_2 \) are identically zero. For now, we motivate this model by observing the roughly quadratic dependence on \( z' \) of our empirical results for the changes (Fig. 11) and for \( v \) (Fig. 12); later, we will discuss possible physical motivations.

Fig. 13 displays distributions of the height anomaly values obtained from the model time series for the PAC and ATL regions. Note that the quasi-equilibrium value \( z'_0 \) does not necessarily determine the mode for the distribution; although \( z'_0 \) is larger for PAC than ATL, the mode for the PAC distribution is smaller. Comparisons of the model distributions to the corresponding observed distributions (Fig. 10) reveal a number of striking similarities, particularly in the pronounced positive skewness (with modes at about \(-0.6\)) of both PAC distributions, and the marked flattening (low kurtosis) of both ATL distributions. Distributions of anomaly run durations for the model-generated series (not shown) also display asymmetries in persistence between positive and negative anomalies similar to those described earlier. Thus, the crude nonlinear model appears capable of replicating many of the most pronounced deviations from simple red noise behavior that are observed in anomaly time series obtained from the persistent anomaly regions.

7. Persistence characteristics of recurrent anomaly patterns

So far, we have examined the persistence of anomalies at fixed spatial points without regard to the concurrent flow pattern. We noted earlier, however, that in previous investigations blocking was associated with certain anomalous flow patterns. We will now investigate the persistence characteristics of a few anomaly patterns that are sometimes associated with extreme blocking events (Dole, 1982).

Dole (1982) used several different techniques in attempting to identify and represent recurrent persistent anomaly patterns. We will apply some of his results to define quantitatively spatial patterns; a more extensive description and discussion of these results will be reported in a subsequent paper. The methods employed in this section are described in more detail in Appendix C.

Dole first examined the composite mean structure of the anomaly fields for cases defined by long-duration, large anomalies [selection criteria of \((100 \text{ m}, 10 \text{ days})\) and \((-100 \text{ m}, 10 \text{ days})\)] at key points in the PAC, ATL and NSU regions. He found that the anomaly patterns associated with negative and positive cases were very similar, but of opposite sign. For each of the regions, he then performed an empirical orthogonal function (EOF) analysis (see e.g., Davis, 1976) of the case mean patterns, in order to extract the single spatial pattern that best describes the patterns of both the positive and negative anomaly cases. Fig. 14 displays for the PAC and ATL regions the dominant EOF patterns calculated from the case.
Fig. 14. Dominant EOF patterns of persistent anomaly cases occurring in regions (a) PAC and (b) ATL. The EOF's are normalized such that the maximum weight has a value of 100. Contour intervals of 20 units.

data. The patterns for the PAC and ATL regions resemble two of the major teleconnection patterns identified by Wallace and Gutzler (1981) from 15 winters of monthly mean 500 mb heights for the extratropical Northern Hemisphere. Wallace and Gutzler also note that the PAC pattern appears in other studies on teleconnections (e.g., Dickson and Namias, 1976).

The EOF analyses provide a basis for quantitatively defining the spatial structure of the patterns. We can gain some insight into the temporal variability and persistence characteristics of these patterns by analyzing the corresponding EOF time coefficients generated for the 14 winter seasons of (unfiltered) twice-daily height anomaly data; details of this approach are described in Appendix C. As in the previous section, the time series are normalized to a mean of zero and a variance of unity.

Fig. 15 shows the average 12 h changes and corresponding potential functions for the PAC and ATL EOF time coefficients. The average changes of the EOF anomalies are very small; in fact, at almost all values, we cannot reject the pointwise hypothesis that
the average change is zero. The potential functions, which have been multiplied by a factor of 10 to reveal details, do suggest a more interesting behavior. For both regions, there is an apparent double potential well, with the deepest well for small negative values (around $-0.5$) and a shallower well at moderate positive values (around 1.0). In these regions, negative values of the EOF time coefficient are associated with relatively strong zonal flows while large positive values are associated with blocking (Dole, 1982). The double potential well behavior is slightly suggestive of multiple quasi-equilibria (Charney and DeVore, 1979; Charney and Straus, 1980). Using the significance test described in Appendix D, however, it appears that such a structure has a probability of between about 5% and 50% of occurring by chance, depending on the red noise process chosen to model the behavior of the EOF time series and on the criterion chosen to define significant minima in the potential function. Therefore, a more extensive set of data would be required to ascertain whether the double potential well reflects a typical aspect of the temporal variability or, rather, is only a characteristic of our particular sample.

8. Discussion

Our results show several areas of broad agreement with the results described in previous blocking studies, but also important differences. As we might anticipate, the locations of the PAC and ATL maxima in the frequency of occurrence of persistent positive anomalies are in approximate conformity with the locations of frequent blocking described by Rex (1950b), Sumner (1954) and White and Clark (1975). Not described in these studies, however, are the substantial numbers of persistent negative anomalies that also occur in these regions, nor the third area of frequent persistent anomalies centered over the northern Soviet Union. Also, Rex (1950b) finds that Atlantic blocking exceeds Pacific blocking by a factor of 2, whereas our results suggest that persistent positive anomalies are about equally frequent in the two regions.

Although part of these differences may be attributable to sampling variations, differences in the methods used for selecting cases are also likely to account for some discrepancies. For example, in most blocking studies the long-term mean is not removed from the height fields before defining cases. We might therefore anticipate that strong ridges or highs will show a geographical bias toward the mean ridge positions located over the eastern Atlantic and eastern Pacific; the northern Soviet Union, however, would not be particularly favored. Indeed, the mean upper-level flow over the eastern Atlantic is also characterized by a weak split flow structure, with only relatively modest anomalies required to produce the marked split flow structures that Rex identifies with blocking. In contrast, the PAC persistent anomaly region, located just to the north of a more intense zonal wind maxima, requires relatively strong anomalies to produce pronounced split flows. Thus, although we find that many of the persistence characteristics of anomalies in the PAC, ATL and NSU regions are grossly similar, we anticipate that statistics derived from flow patterns occurring in these regions may vary considerably.

We find no evidence for a strongly preferred duration for persistent anomalies, nor any indication of pronounced periodicities. Rather, for sufficiently long durations, the number of events decays nearly exponentially with increasing duration. The similarity of the observed distributions to the distributions gen-
gerated by a red noise process suggests that many of the persistent anomalies may arise from fluctuations, sometimes called "climate noise" (Leith, 1973), that are generally assumed to be unpredictable on long time scales.

Nevertheless, the observed behaviors are in important respects unlike those obtained in red noise (or other linear autoregressive) models. In particular, we find that the rapidity of the decay depends on the anomaly value: generally, positive anomalies decay more slowly than negative anomalies, with the discrepancy increasing at increasing anomaly magnitudes. Also, the variance of the changes (the noise) is typically considerably smaller for strong positive than strong negative anomalies, decreasing the probability of a rapid jump back to a more normal value with reduced persistence. The variations in both the mean height changes and the noise indicate that, once established, strong positive anomalies are relatively more apt to persist. These variations are also reminiscent of the picture presented by synopticians that strong highs (frequently associated with blocking) are relatively likely to persist for extended durations, and that they are also often accompanied by a marked local reduction in the daily variability associated with mobile disturbances.

We have suggested that variations in the intensity of the noise that depend on the anomaly value are at least partially related to variations in the storm paths adjacent to the anomalies. Another possible source for the differences between positive and negative anomalies is suggested by considering the divergence term in the vorticity tendency equation (Holton, 1979, p. 94):

\[
\frac{\partial \zeta}{\partial t} = -v \cdot \nabla \zeta - v \frac{df}{dy} - (\zeta + f) \nabla \cdot v
\]

\[
- \left[ \omega \frac{\partial \zeta}{\partial p} - \hat{k} \cdot (\frac{\partial V}{\partial p} \times \nabla \omega) \right],
\]

where \( \zeta \) is the relative vorticity, \( f \) the Coriolis parameter and \( \omega \) the vertical motion in pressure coordinates; all other notation is standard.

Note that the magnitudes of the relative and planetary vorticity advection terms and the terms in the brackets do not depend on the sign of the relative vorticity. For quasi-geostrophic motions, the terms within the brackets are neglected and the divergence term in (8) is linearized by replacing the absolute vorticity \((\zeta + f)\) by a mean vorticity value \(f_0\). This latter approximation is probably most reasonable at small values of the height anomalies, where we anticipate that associated vorticity perturbations will generally be weak. Very large height anomalies, however, are typically accompanied by large values of relative vorticity, with positive (negative) height anomalies associated with negative (positive) relative vorticity. We therefore anticipate from (8) that, for a given divergence, vorticity (and associated height) changes will occur more rapidly for negative than positive height anomalies, with the differences generally increasing with increasing anomaly magnitudes.

Since there are other possible sources for differences in persistence characteristics between positive and negative anomalies (e.g., asymmetries in structure) that are beyond the scope of this study, a definitive identification of the major reasons for differences cannot presently be made. The vorticity argument, however, does also provide a possible explanation for some apparent regional variations that we observed in the average change curves (Fig. 8). Although these curves appeared to disclose more differences in persistence between positive and negative anomalies in PAC and ATL than in EAS, recall that the anomalies in Sections 5 and 6 were normalized to unit variance. A normalized deviation of 1 unit is equivalent to about 170–180 m in PAC and ATL, but only about 45 m in EAS; thus, the corresponding vorticity anomalies may be sufficiently small in EAS that asymmetries due to relative vorticity variations may scarcely be evident. If this is the case, then some of the differences we have observed between the regions may be due more to differences in the physical magnitudes of disturbances (which are masked by scaling by the standard deviations) than to qualitative changes in persistence characteristics. This suggests that special care must be taken in interpreting and comparing results between regions (both ours and those of others) that are derived from standardized height anomalies.

Finally, we found that the average changes always tend to reduce the local height anomalies to a single value near zero (reflecting a decay toward the long-term mean); however, fluctuations that are manifested in the variance about the mean changes (which we have termed noise) may either decrease or increase an anomaly. If these fluctuations are generally too weak, then the anomalies may never achieve values where the differences between positive and negative anomalies (including increased positive anomaly persistence) become important; but if for strong positive anomalies the fluctuations are relatively too vigorous, then the probability will be increased of a rapid jump back to a more normal value with reduced persistence. To obtain the observed characteristics of persistent anomalies, then, we may need to model correctly not only the physical processes that determine the average changes, but also those that determine the variance of the changes about the average values.

9. Conclusions

We have studied the geographical and regional persistence characteristics of wintertime Northern Hemisphere 500 mb heights. We have also examined the persistence characteristics of certain dominant patterns of low-frequency variability.
The main points to emerge from the geographical distribution analyses are as follows:

1) There are three major regions of frequent occurrence of persistent anomalies: the North Pacific to the south of the Aleutians, the North Atlantic to the southeast of Greenland and from the northern Soviet Union northeastward to over the Arctic Ocean.

2) For moderate magnitudes and durations, the numbers of positive and negative cases in each region are about the same. For very short durations (1–2 days) there are more negative than positive cases, while at longer durations and at large magnitudes, the number of positive cases exceeds the corresponding number of negative cases.

3) Similar analyses performed on data that have been weakly low-pass filtered have regional distributions that are almost unchanged from the unfiltered data, although the total numbers of cases are increased by 50–100% over the corresponding unfiltered values, primarily as a result of an increase in the negative cases. The relatively large increase in negative events for the filtered data indicates that negative anomalies are more likely than positive anomalies to experience brief transient interruptions (from disturbances having periods < 6 days). At large magnitude thresholds, however, there are still more positive than negative cases.

The regional analyses indicate that:

1) In persistent anomaly regions, the probability that an event which has lasted \( n \) days will last at least \( n + 1 \) days increases up to about \( n = 5 \) days and is thereafter nearly constant. The nearly constant probability of continuation is accompanied by a nearly constant average anomaly value.

2) The general shapes of the persistence curves resemble those generated by a red noise process. Nevertheless, there are significant differences in behavior between the positive and negative distributions and the red noise distributions, particularly at large magnitudes and extended durations. At large magnitudes, the positive anomalies are typically considerably more persistent than either the negative anomalies or red noise.

3) For each region, analyses of the relationship between initial anomaly values and their subsequent 12 h changes indicate that the height anomalies tend toward only one quasi-equilibrium value (mean change of zero). Mean changes are considerably less rapid for large positive anomalies than large negative anomalies. The variance of the changes about the mean change (noise) also depends on the anomaly value, with the smallest noise for large positive anomalies. Both the mean changes and noise variations favor the relatively greater persistence of strong positive anomalies compared to strong negative anomalies.

4) Many of the statistical characteristics of anomalies in the persistent anomaly regions that are not consistent with red noise (or other linear autoregressive models) are well-simulated by a simple nonlinear autoregressive model, where the nonlinearities reflect the variations in mean changes and noise accompanying changes in the anomaly values.

5) The temporal fluctuations of the dominant regional patterns of low-frequency variability exhibit considerably more persistence than found for the local height anomalies, but also do not convincingly demonstrate evidence of multiple quasi-equilibria.

Our results reveal a number of typical aspects of the behavior of persistent anomalies that will require theoretical explanation. At a minimum, a comprehensive theory of persistent anomalies should be able to account for the observed geographical variations, the simple decay characteristics and the asymmetries in persistence between positive and negative anomalies.

**Acknowledgments.** Some of this work formed a part of the first author’s thesis research at MIT, which was initiated under Prof. Jule Charney and completed under Prof. Frederick Sanders; their insights and infectious enthusiasm for understanding meteorological phenomena were always inspirational. We are also grateful to several people for their comments and constructive criticisms on preliminary versions of this paper, including Profs. Mark Cane, Fred Sanders, Peter Stone and Kevin Trenberth, and Drs. Brian Farrell, Claude Frankignoul, Rick Rosen and Ed Sarachik. We also thank Ms. Isabelle Kole for her help in drafting several of the figures.

Support for the first author’s work was provided by MIT by National Science Foundation Grant NSF-76-20070 ATM and at Harvard by NASA Grant NGL-22-007-228. Support for the second author’s work was provided by NASA Grant NSG-5113, during a visit to MIT on leave kindly granted by the New Zealand Meteorological Service. Some of the calculations were performed on the National Center for Atmospheric Research (NCAR) computer system. NCAR is supported by the National Science Foundation.

**APPENDIX A**

**Estimating Confidence Limits for the Number of Events Generated by a Red Noise Process**

Although some relatively weak (primarily asymptotic) results have been obtained for the level-crossing properties of autoregressive time series (see, e.g., Cramer and Leadbetter, 1967; or Blake and Lindsey, 1973), to the authors’ knowledge there is as yet no theory giving, for a discrete, linear, autoregressive process, and for an arbitrary magnitude threshold,
the analytical form of the distribution of the number of events lasting at least a given duration. Nor is there any theory giving the spread of values expected by chance. The estimated 5–95% confidence intervals shown shaded in Fig. 8 were instead obtained from synthetic time series generated using (4). The values of \( a \) used were 0.89 for PAC and ATL, 0.85 for EAS, and 0.80 for AME. In each case this is, to two decimal places, the square root of the 1-day lag (lag 2) autocorrelation coefficient based on all 9 points within the specified region. This estimate for \( a \), rather than the more conventional estimate of the half-day (lag 1) autocorrelation, was made in order to allow us to incorporate both 0000 and 1200 GMT data into our estimates while at the same time minimizing possible spurious influences of the diurnal cycle. With this estimate for \( a \), the variance of the added white noise component was then set such that the values in the time series had an expected variance of unity.

To match 14 winter seasons of real data at the 9 points of each region, 126 different time series each of 180 values were generated; each series started with a random value drawn from a normal distribution with zero mean and unit variance. Then, as for the real data, the distribution of the number of events lasting for at least a duration of \( D \) days was computed. This whole procedure was done a total of 99 times, using different “seeds” for the pseudo-random-number generator employed. For each duration, the 99 different answers for the numbers of events lasting at least that long were ranked, and the 5th and 95th were taken to be estimates of the 5% and 95% confidence intervals, respectively.

**APPENDIX B**

**Generating Histograms of Synthetic Data**

The histograms in Fig. 13 were obtained by using (6) and (7) to generate, for each region, the equivalent of 280 winter seasons of data. The counts of the number of anomalies in each bin of width 0.1 were divided by 20 before being plotted, to conform to the 14 winter seasons of real data.

To obtain the coefficients required for (6) and (7), polynomial regression was used to fit quadratics to \( z(t + 1) \) as a function of \( z(t) \), and to the residual noise [from the fit of Eq. (5)] as a function of \( z(t) \). The resulting coefficients are:

For PAC:  
\[
\begin{align*}
 a_0 &= -0.028595, \\
 v_0 &= 0.454694, \\
 a_1 &= 0.900297, \\
 v_1 &= 0.026972, \\
 a_2 &= 0.028944, \\
 v_2 &= -0.056280.
\end{align*}
\]

For ATL:  
\[
\begin{align*}
 a_0 &= -0.023604, \\
 v_0 &= 0.428632, \\
 a_1 &= 0.923137, \\
 v_1 &= -0.066647, \\
 a_2 &= 0.025474, \\
 v_2 &= -0.044802.
\end{align*}
\]

**APPENDIX C**

**Empirical Orthogonal Function Analysis**

The dominant empirical orthogonal function was obtained as the eigenvector associated with the largest eigenvalue of the matrix

\[
A = \sum_{i=1}^{N} h_{ij} h_{jk}, \quad j, k = 1, 2, \ldots, M.
\]

The elements \( h_{ij} \) are mean values of the latitude-normalized anomaly values [as defined by Eq. (1)] for case \( i \), where \( i \) varies from 1 to the number of cases \( N \) (14 for PAC and 15 for ATL), and for the \( j \)th gridpoint, where \( j \) varies from 1 to \( M \). Because of practical limitations the analyses were performed over limited regions (50° latitude by 160° longitude, with 5° spacing in latitude and longitude, so \( M = 363 \)) which had the strongest indication of pattern recurrences. The spatial structure of the dominant EOF was found to be insensitive to changes in the location of the boundaries.

The time coefficient \( q_i \) of the dominant EOF \( f_j(j = 1, \ldots, M) \) for time \( i \) was calculated using

\[
q_i = \sum_{j=1}^{M} h_{ij} f_j,
\]

where \( h_{ij} \) is the latitude-normalized anomaly value for time \( i \) and grid point \( j \).

**APPENDIX D**

**Significance Test for Multiple Minima in the Potential Function**

A Monte Carlo technique was devised to test for the significance of the double potential well structure appearing in Fig. 15. The arbitrary criteria defining the occurrence of a minimum were deliberately chosen to be simple and easily satisfied. Our objective is to avoid underestimating the probability of occurrence of multiple minima by chance, so as not to overestimate the significance of the observed double potential well.

First, synthetic time series were generated using a red noise process with a value of \( a \) of 0.97—this is, to two decimal places, the square-root of the 1-day lag autocorrelation coefficient for the time series of
the dominant EOF coefficient for both PAC and ATL. One hundred time series were generated, each being equivalent to 14 winter seasons of EOF coefficients. Each time series was then subjected to the same analysis performed for the EOF coefficients, and the potential functions calculated.

The potential functions were then examined using 10 different criteria to define a minimum: for criteria $i$, a minimum was said to exist at bin $j$ (of width 0.1) if the potential at $j$ was less than the potentials at bins $j + 1, j + 2, \ldots, j + i$ and $j - 1, j - 2, \ldots, j - i$. The following table gives the resulting percentage of the generated potential functions having more than one minimum:

Criteria: 1 2 3 4 5 6 7 8 9 10
Percent with multiple minima: 97 78 54 32 11 5 4 2 2 1

The PAC and ATL potentials have multiple minima for all criteria from 1–6 and 1–7, respectively. Thus, if we choose a posteriori a criterion of 6, then according to the above table there would be a 5% probability that a red noise process modeled on the behavior of the EOF time series for PAC and ATL would have a potential function with multiple minima at least as well-defined as those observed. However, a priori we might, instead, have considered a criterion of 4 to define a significant minimum—there would then be a 32% probability of significant multiple minima occurring by chance.

The significance of the multiple minima clearly depends strongly on the criterion chosen. It is also sensitive to the value chosen for $a$—if 0.98 is used instead of 0.97, then the probability of obtaining multiple minima satisfying criterion 6 by chance increases to 21%, and of satisfying criterion 4 to 47%. This sensitivity to the choice of criterion and value of $a$ suggests extreme caution in interpreting the significance of the double potential wells in Fig. 15.

**APPENDIX E**

**The NMC Data Set**

The data base for the present study consists of twice-daily NMC analyses of 500 mb geopotential heights. The analyses are products of the forecast–analysis cycle at NMC. The “first guess” for the analyses is the previous (12 h) model forecast. This first guess is then updated by observations obtained from the surface and radiosonde observational networks, aircraft reports, winds inferred from satellite imagery, etc. The transient data sources often compose a significant fraction of the total number of reports: Jenne (1975) presents a typical time in July 1968 in which 48% of the upper-air reports were from conventional sources (RAOB and RAWIN reports) and 52% were from other sources (primarily aircraft winds and winds inferred from satellite data). The latter sources are probably most valuable in filling gaps over otherwise data-sparse regions such as the North Atlantic and North Pacific oceans. Over these regions, the NMC analyses should have an advantage over other analysis schemes that incorporate only fixed base data.

Up through 1965, the first guesses for both the 0000 and 1200 GMT analyses were based on a 3–level baroclinic model. From 1966 through 1972, this model was usually operation at 1200 GMT; a 6-layer primitive equation model (Shuman and Hovemare, 1968) was operational at 0000 GMT. Between 1972 and September 1974 the 6-layer model was used for both analyses. Through the period, numerous lesser changes in models and procedures also occurred. Although these changes undoubtedly introduce changes in the first-guess fields and hence in the analyzed fields over data-sparse regions, results obtained by Lau (1978) and Dole (1982) suggest that systematic changes during this time are generally small. Throughout this period, the observed data were incorporated into the analyses through a successive correction scheme similar to that described by Cressman (1959).

In September 1974, NMC introduced a global spectral forecast model together with a Hough function analysis scheme. As noted by Rosen and Salstein (1980), the Hough scheme places a strong constraint on the analyzed winds such that they are essentially nondivergent. This eliminates features such as the mean meridional cells. Rosen and Salstein indicate, however, that for analyses of midlatitude waves and transports the Hough analyses appear quite acceptable.

Comparison of the NMC schemes with schemes using only fixed station data (Lau and Oort, 1981) suggests that areas with data-rich regions the analyses are highly similar, whereas in data-sparse regions the NMC analyses show a more detailed structure. For our purposes, the incorporation of the additional mobile data sources (primarily ship and aircraft observations) in the NMC analyses presents a considerable advantage in attempting to obtain geographical distributions of persistent anomalies. In addition to these sources, at least one radiosonde station was located within or near the boundary of each of the regions that we selected for detailed study. In particular, for the PAC region, stations were located at Cold Bay (55°N, 162°W), St. Paul Island (57°N, 170°W), King Salmon (58°N, 156°W), Kodiak (57°N, 152°W), Adak (52°N, 176°W) and, for much of the period, weather ship 4YP (50°N, 145°W). For the ATL region, there was a radiosonde station located at Valentia, Ireland (51°N, 10°W); in addition, for most of the period, at least one of the following weather ships was located at or near positions within the region: 4YJ (52°N, 20°W), 4YK (45°N, 15°W), 4YC
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