REVIEW

The Cumulus Parameterization Problem

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(Manuscript received 15 June 1982, in final form 25 May 1983)

ABSTRACT

In the past two decades there has been extensive research into the nature of atmospheric convection and scale interactions in cumulus regimes. A major goal of these efforts has been to advance the state of the art in cumulus parameterization. This paper briefly reviews the cumulus parameterization problem in terms of fundamental principles, goals and dynamic constraints as they apply to parameterization in mesoscale and larger scale numerical models. Several popular current schemes are discussed in terms of their relationships to these overall aspects of the problem.

1. Introduction

Since atmospheric prognostic models are constrained to discrete time and space intervals, it is necessary to parameterize any significant physical processes which occur on scales smaller than those resolved by the model. Processes which can be readily approximated in terms of local values and gradients at grid points, such as turbulent diffusion of momentum in a stably stratified fluid, have been incorporated into models for many years. However, in a conditionally unstable atmosphere where cumulus clouds are present, the problem becomes complex. The subgrid-scale vertical motions are integrally related to the release of latent heat. The eddies extend over large vertical distances and often have characteristics which are poorly related to the large-scale properties at a level. The problem of parameterizing convective processes in a conditionally unstable atmosphere is referred to as cumulus parameterization, although it includes phenomena other than cumulus clouds.

This paper presents a brief review of the problem and practice of cumulus parameterization. In the required interests of brevity, no attempt will be made to trace the complete history of research in this area or to summarize all of the current schemes. Cho's (1975) review paper discussed the development of cumulus parameterization, while Ooyama (1982) and Anthes (1982) described the evolution of parameterization in terms of its application to tropical cyclone modeling. Betts (1974) discussed scale interactions in convective regimes and their relevance to cumulus parameterization. In addition to the references cited in this text, Houze and Betts (1981) present a review of recent studies of convection with particular emphasis on the results of the Global Atmospheric Research Program's Atlantic Tropical Experiment (GATE).

Individual convective updrafts and downdrafts have horizontal dimensions on the order of 0.1–10 km, similar to the meso-γ scale defined by Orlanski (1975). Models with grid spacings on this order or less can explicitly resolve convective elements, bypassing the need for parameterization. Examples are convective cloud models [see review papers by Cotton (1975) and Schlesinger (1982) and the hurricane simulations of Rosenthal (1978)]. This paper addresses cumulus parameterization only as it pertains to grid scales greater than ~10 km (meso-β and larger circulations).

2. Non-precipitating convection

Cumulus clouds can be divided conveniently into two groups, non-precipitating and precipitating. The former are relatively shallow with cloud tops generally less than 3 km above the surface. Since they do not involve a net release of latent heat when averaged over their lifetimes, they may be modelled conceptually as ordinary turbulent eddies with extended vertical dimensions resulting from condensation. Small cumulus can exist without low-level convergence or deep conditional instability. While these clouds are important in maintaining the cloud layer (Betts, 1978), they are sufficiently different from precipitating cumulus that they are usually parameterized differently. Models of shallow cumulus layers based on mixed-layer type models show good agreement with observations (e.g., Albrecht et al., 1979).

3. Precipitating convection

Precipitating cumulus clouds form in conditionally unstable atmospheres. They occur in the presence of low-level convergence existing on a scale larger than that of an individual cumulonimbus (cb) updraft.
(~1 km) (e.g., Ogura and Chen, 1977; Frank, 1978). By definition they cause a net release of the latent heat of condensation and most often involve ice phase transitions as well. However, the difficulties in parameterizing cb convection involve more than the complexities of the cloud-scale physics and microphysics. Conditional instability typically exists over horizontal areas which are very much greater than the area of an individual cloud. The net heating produced by the release of latent heat appears to be capable of providing the energy to form and maintain circulations on scales larger than the cloud or convective scale. The following discussion of dynamic considerations affecting parameterization is similar in part to arguments presented by Ooyama (1982) in his review of tropical cyclone modelling. It is recognized that some of the points are idealized and somewhat over-simplified. However, it is felt that the arguments presented represent a useful framework for bringing some conceptual coherency to the various aspects of the parameterization problem.

a. Scale considerations

1) DEFINITIONS OF FLOW REGIMES

It is convenient to relate the horizontal scales of atmospheric circulations (L) to the Rossby radius of deformation [R' in Eq. (1), defined for a stably stratified fluid]:

\[ R' = \frac{NH}{(\gamma + f)^{1/2}(2VR^{-1} + f)^{1/2}}, \]

where N is the Brunt-Väisälä frequency, H the scale height of the circulation, \( \gamma \) the relative vorticity, f the Coriolis parameter, V the rotational component of the wind, and \( R \) the radius of curvature. The radius of deformation is derived from consideration of the relative resistances to vertical displacements resulting from static stability and to horizontal displacements resulting from inertial stability. \( R' \) is proportional to the vertical scale height (H). This is often assumed to be the depth of the troposphere in cb regimes, but the effective H (and hence \( R' \)) associated with convective feedback to larger scales may be substantially smaller if the relevant effects of the clouds and related circulations are largely confined to shallower layers. \( R' \) varies inversely with the Coriolis parameter (f) and hence usually becomes smaller at higher latitudes (Fig. 1). It also decreases with increasing relative vorticity.

Fig. 1, closely based on Ooyama’s (1982) Fig. 1, defines three regimes classified according to the horizontal scale and the Rossby radius of deformation. Region I includes small-scale turbulence and individual convective cells and clouds, which are probabilistic phenomena with respect to any model with resolution coarser than those used in individual cloud models. Region III is the nearly balanced flow regime where large-scale circulations evolve slowly, and secondary circulations (including vertical motions) are largely controlled by the primary circulations. Region II encompasses circulations with significantly unbalanced flow in which the divergent component is no longer a secondary circulation and may even be the primary mode. When strong internal energy sources are present, the dynamics of region II circulations are complex and not well understood. It is not obvious to what degree systems existing in this regime are deterministic. It does seem likely that parameterizations for models of region II systems will have to be more sophisticated than those used for region III systems, for reasons outlined below.

It is important to clarify the distinctions between the dynamic and spatial horizontal scales of a circulation. The former refers to the size of a circulation relative to \( R' \), while the latter refers to the physical dimensions of the circulation. In the ensuing discussions the term “dynamically large” will refer to circulations with \( L \gg R' \) (i.e., region III) and “dynamically small” will refer to circulations where \( L < R' \) (region II). Terms such as “mesoscale” and “synoptic-scale” will be used to denote spatial dimensions only. Mesoscale will refer to circulations with horizontal dimensions of approximately 20–2000 km [meso-\( \beta \) and meso-\( \alpha \) scale (Orlanski, 1975)]. Synoptic-scale will refer to circulations with dimensions of 2000–10,000 km.
2) SCALES OF CONVECTIVE PROCESSES

There are several ramifications to the relationship between $L$ and $R'$, but three have particular importance with respect to convective regimes and cumulus parameterization. The first concerns the characteristic lifetimes of circulations. In general, dynamically small rotational circulations tend to decay rapidly due to the cascade of energy to smaller scales. Dynamically large circulations tend to exhibit predominantly horizontal, nearly balanced flow and decay more slowly, with typical lifetimes on the order of several days or more. This principle has been used as justification for the neglect of dynamically small circulations in models of slowly-varying systems in spatially large domains. However, in convective regimes the above argument is insufficient. There are numerous types of convective systems which may exist up to several days, but which are small compared to $R'$ (e.g., Houze and Betts, 1981; Maddox, 1982).

Individual cb clouds (region I, Fig. 1) have typical scales of 1 to 10 km and are dynamically small everywhere except where $R'$ is unusually small, such as in the eyewall of a tropical cyclone (Shapiro and Willoughby, 1982) or possibly in some severe thunderstorms. However, conditionally unstable layers commonly extend over very large horizontal domains, e.g., thousands of kilometers on a side over the tropical oceans. Therefore, there is often a large potential energy source available to drive and sustain circulations within a range of scales which are small compared to $R'$ but quite large compared to individual cumulonimbus clouds. Observations suggest that strong mesoscale circulations frequently develop concurrently with or as a result of latent heating. This occurs in both tropical cloud clusters (Frank, 1978; Leary, 1979) and middle-latitude mesoscale convective complexes (Maddox, 1980). Mesoscale convective systems tend to have lifetimes on the order of 1 day unless they develop strong rotational flow and become dynamically large circulations, such as tropical cyclones. Their lifetimes appear to be limited primarily by the ability of the systems to resupply themselves with low-level conditionally unstable air through horizontal advection (Ooyama, 1982).

The second point is that dynamically small circulations have very complex and sensitive interactions between the convection and spatially larger scale divergence fields. When the atmosphere is heated on a dynamically small scale (such as by the release of latent heat in a mesoscale convective system), the divergence is not significantly constrained due to lack of stiffness of the rotational flow. (Stiffness refers to resistance to lateral displacement resulting from inertial stability.) The heating tends to excite gravity waves and causes perturbations of the divergence field. One net result is a lateral dispersal of most of the induced warming over a scale of approximately $R'$ (Shapiro and Willoughby, 1982). Therefore, while deep convection may be controlled by convective to mesoscale processes as well as by larger scale divergence, the convection itself can influence divergent flow over spatially large areas. These complex scale interactions make it difficult to separate convective scale and spatially larger scale processes in region II circulations.

The third related point is that interactions between heating on a dynamically small scale and the rotational flow on that scale are relatively small. Geostrophic adjustment experiments show that when initial temperature perturbations are assumed over dynamically small areas, the final balanced state temperature and height fields are much weaker than the initial perturbations (Schubert et al., 1980). The final state rotational flows are similarly weak.

It is worth noting that dynamically small scale perturbations of the rotational winds tend to be initially more stable than do perturbations of the temperature or mass fields. The reason is that during the geostrophic adjustment process for dynamically small systems, the mass fields tend to adjust more to the rotational wind fields than vice-versa. (The final balanced rotational flow is still likely to decay rapidly for such systems.). Thus, although the energy of the diabatic heating in cumulus clouds is generally greater than the energy associated with convective momentum rearrangements and related processes, the latter may have greater influence on the rotational component of the circulations (Schubert et al., 1980; Fulton and Schubert, 1980). Several diagnostic studies have shown that subgrid-scale motions have significant effects upon the momentum and vorticity fields of tropical mesoscale and synoptic-scale systems, although the physical mechanisms are more complex than can be explained by simple cloud transport models (Stevens et al., 1977; Shapiro, 1978; Cho et al., 1979; Reeves et al., 1979).

3) IMPLICATIONS FOR PARAMETERIZATION

From the above scale arguments it can be seen that the design of a parameterization scheme must depend heavily upon the nature of the circulation to be modelled. For example, often the problem is to model the slowly varying rotational flow of a dynamically large system (region III). The stiffness of the flow in such a system inhibits the ability of embedded convective or mesoscale systems to drive the large-scale divergence and thereby to enhance their own forcing. There is a tendency for the system to respond to latent heating by adjustment of the rotational component of the wind (Haltiner and Williams, 1980; Shapiro and Willoughby, 1982). The cumulus parameterization problem seems to be somewhat simpler for region III systems than for those of region II, since the former have stronger relationships between the large-scale flow and the convection. In addition, circulations of region III systems are generally less sensitive
to the exact rate or timing of the simulated latent heat release than are those of dynamically small circulations.

If the circulation to be modeled is a region II system, where the divergent wind field is relatively strong, the parameterization required must be more complex. Since latent heating can influence the divergent flow over very large spatial areas, and since convection is greatly influenced by both large-scale divergence and mesoscale circulations, the interactive complications are enormous. A parameterization scheme must simulate very accurately the rate at which convective systems release latent heat in a conditionally unstable atmosphere. In addition, the parameterization may be required to simulate the evolution of intense mesoscale systems which may be linked only weakly to the large-scale circulations. Finally, it may be necessary to parameterize the direct effects of convection and mesoscale circulations upon the momentum fields due to their potentially significant effects upon the rotational flow.

b. Mesoscale circulations

It is well documented that latent heat release in convective regimes is generally organized in a variety of mesoscale patterns. These range from individual lines of cumulus (10–100 km) to larger squall lines, cloud clusters, and mesoscale convective complexes (~100–500 km). A typically complex example is shown schematically in Fig. 2. The reasons why convection organizes in such patterns are not fully understood, although the tendency for deep cumulus to form in lines along boundaries such as frontal zones and gust fronts is consistent with the view that low-level convergence is required. The ubiquitous presence of mesoscale circulations has a number of complicating effects:

1) The scale of the mesoscale circulation may approach $R'$ in which case the latent heat release can have important feedback to the rotational flow. This is particularly likely to occur at high latitudes and in systems with high initial values of vorticity. It might also be a factor in mesoscale circulations with shallow vertical dimension, but this is hypothetical.

2) The rate of convective heating in a large-scale conditionally unstable region may well be determined more by the nature of mesoscale circulations than by the large-scale flow. Failure to resolve or parameterize these circulations will result in incorrect specification of the timing and magnitude of the latent heat release.

3) Mesoscale updrafts and downdrafts are now thought to be common features of convective systems. Evidence of these features has been observed in tropical (Zipser, 1977; Houze, 1977) and mid-latitude systems (Ogura and Chen, 1977; Sanders and Paine, 1975). They have been modeled in mesoscale numerical models (Kreitzberg and Perkey, 1977; Brown, 1979; Fritsch and Chappell, 1980a, b). Since mesoscale vertical motions differ substantially from convective-scale vertical motions, they would seem to require separate treatment in parameterization schemes. Johnson (1980) has explored the roles of mesoscale vertical motions in tropical convective systems.

4) Mesoscale circulations and their resultant cloud patterns may cause important subgrid-scale modifications of the radiational heating.

5) Unresolved mesoscale organization and circulations affect virtually all of the model assumptions concerning the effects of the large-scale fields upon clouds. For example, what are the initial properties of air entrained by updrafts? This problem raises questions as to the feasibility of parameterizing convection using individual cloud models in numerical models with coarse grids.

6) As previously discussed, diagnostic analyses of convective weather systems reveal large unexplained residuals in the momentum and vorticity budgets. These residuals cannot be explained fully by simple convective-scale transport models. It is likely that at least some of the apparent source/sink terms result from effects of mesoscale circulations, particularly in the upper troposphere.

7) The dynamics of convective mesoscale circulations are poorly understood. The above arguments concerning "large" and "small" circulations are simplistic and do not directly address possible stable modes of circulation at spatial scales intermediate between those of convective drafts and $R'$. This is likely to be a problem where strong available internal energy sources exist, such as in a conditionally unstable environment. Emanuel (1979, 1982) discussed some of the recent work in this area. An additional complication is that formerly small circulations which persist in time can eventually become large as the vorticity increases and $R'$ decreases. An example is tropical cyclogenesis.

c. Closure assumptions

Cumulus parameterization schemes must determine the vertically integrated net rate of condensation and must then distribute the convective heating and moistening in the vertical. Depending upon the model and application, momentum processes may have to be included as well. Since the convective processes are subgrid scale, all schemes require assumptions. In a sense all of these assumptions are closure assumptions of the parameterization. However, most schemes contain numbers of relatively minor assumptions which may be changed without altering the essence of the parameterization. This is particularly true of those used to distribute quantities vertically. The term “closure
Fig. 2. Schematic of the flow in a mesoscale tropical squall line (Zipser, 1977).
mass flux at cloud base \( (M_c) \) is estimated from the three-dimensional mass convergence in the sub-cloud layer [including subsidence in clear air (\( \bar{M} \)) and in convective downdrafts (\( M_d \))]:

\[
M_c = \bar{M} - M_d - \bar{M}.
\]

By allowing subsiding mass at cloud base to contribute to the forcing of updrafts, it is hoped that the scheme will provide a more realistic time evolution of the convection and reduce the dependence on scale.

A related closure hypothesis is the specification of the net latent heating as a function of the total column moisture convergence. Since surface fluxes of moisture fluctuate less than does horizontal moisture convergence, the total moisture convergence tends to be dominated by the latter process during convectively active periods. The obvious appeal of this approach is the direct physical link between moisture and cumulus convection. Net condensation estimates obtained as residuals in diagnostic moisture budget analyses generally have agreed well with independent radar estimates of rainfall. A significant drawback is the need to specify empirically the division of moisture between condensation and storage in the column.

2) Equilibrium Assumptions

The other major type of parameterization closure is based upon the assumption that precipitating convection acts to modify a conditionally unstable atmosphere toward some equilibrium state at a specified rate. If the approximate nature of such an end state can be specified, it provides a convenient mechanism for estimating the magnitude of the convection. One widely used assumption is that deep moist convection acts to restore the atmospheric lapse rate to neutral or stable conditions. This forms the basis for convective adjustment schemes, usually applied to models with grid meshes of 100 km or more (e.g., Manabe et al., 1965). It has also been used in mesoscale models (Fritsch and Chappell, 1980a, b). Another view, usually used in models with coarser grids, is that the grid-scale convective potential energy in a convective regime (analogous to the area between the sounding and the moist adiabat of a rising parcel) tends to evolve slowly relative to the convection. Thus, the clouds may be thought of as responding to changes in larger-scale circulations in a manner which maintains the existing level of conditional instability. This idea is the basis for the quasi-equilibrium hypothesis of the Arakawa and Schubert (1974) parameterization.

On the convective scale and toward the lower end of the mesoscale (<100 km) deep convection tends to stabilize the atmosphere. The primary mechanisms are convective downdrafts which replace unstable boundary layer air with cooler and drier air from the
cloud layer (Fritsch et al., 1976; Zipser, 1977; Barnes and Garstang, 1982; Fitzjarrald and Garstang, 1981; Gaynor and Ropelewski, 1979). A relatively mild example of convective modification of the boundary layer is shown in Fig. 3. Lowering boundary layer thermodynamic energy values has the effect of reducing maximum updraft temperatures over their entire vertical extent. Warming at middle and upper tropospheric levels is characteristically weaker than boundary layer cooling and plays a smaller role in vertical stabilization (Frank, 1980; Fritsch et al., 1976). The stabilization resulting from convective overturning is sufficient to inhibit further convective development until the boundary layer recovers, a time of at least several hours even over the tropical oceans (Fitzjarrald and Garstang, 1981). On scales larger than those of individual mesoscale cloud lines or clusters, only a fraction of the domain is directly affected by convective wakes (Johnson, 1981). The rest of the area remains relatively undisturbed, and the undisturbed portions remain unstable. Observationally, it appears that complete stabilization is best

**FIG. 3.** Before (solid) and after (dashed) composites of 14 convective disturbances during GATE (tropical maritime convection). The quantities shown are specific humidity ($q$), moist static energy ($h$), saturated moist static energy ($h^*$) and potential temperature ($\theta$).
used as a closure assumption only for grid scales no larger than individual mesoscale cloud lines. Stabilization of a fraction of a grid space, as in soft convective adjustment, is one method for extending this closure to larger grids.

The quasi-equilibrium type of closure is clearly most applicable toward the larger end of the range of grid scales. Lord and Arakawa (1980) have shown that a wide variety of convectively active environments have very similar cloud work functions [Arakawa and Schubert’s (1974) measure of convective available potential energy] implying that this is a nearly universal property of convective atmospheres. Most of the data sets they surveyed were averaged in time and had effective horizontal resolutions on the order of a few hundred kilometers, comparable to a general circulation model grid space. One problem is that even on this scale, the conditional instability often increases considerably on time scales of a few days or so without significant increases in convective activity (Fig. 4). Once triggered, the convection persists while the large scale stabilizes. Instantaneous relationships between precipitation and the magnitude of the conditional instability are generally weak. Thompson et al. (1979) found that the strongest convection in GATE easterly waves occurred when conditional instability was near minimum. If the goal is to model dynamically large slowly evolving flow fields, then an equilibrium closure of the quasi-equilibrium type may work well. If, however, the parameterization must simulate the convective activity on shorter time scales (e.g., 1 day) or on smaller space scales, then problems are to be expected.

In addition to the above stability considerations, there may be a physical constraint restricting use of the quasi-equilibrium assumption to dynamically large scales. On scales small compared to $R'$, the release of latent heat tends to increase the divergent portions of the flow field, as previously noted. In such situations the convection is partially driving the larger scale flow and should not be construed as adjusting totally to the latter.

4. Current approaches

a. Types of schemes

In line with the discussion of Section 2, it is best to divide the parameterizations into two groups:
1) Mesoscale model parameterizations
2) Coarse grid model parameterizations.

The difference, as defined here, is that the former are used in models which have grid meshes fine enough (<50 km) to allow explicit resolution of mesoscale circulations. Depending upon the objectives of the studies and the grid meshes used, these circulations may include mesoscale updrafts and downdrafts with scales of cumulonimbus anvils (10–100 km), or entire convective lines such as squall lines (scales of hundreds of kilometers). Mesoscale models must parameterize convective-scale processes. Coarse-grid models must somehow parameterize both convective and mesoscale processes including their many mutual interactions.

b. Mesoscale model parameterizations

In the past decade there have been numerous attempts to simulate mesoscale circulations associated with convection and to study interactions between these circulations and the convection. The following studies are representative of the state of the art, al-

![Fig. 4. Time series of the conditional instability ([CI]), defined as the mean temperature difference between a rising undilute surface parcel and its environment, between cloud base and 175 mb, averaged horizontally over the GATE A/B array—400 km radius] and radar-estimated rainfall [P(R), averaged over the GATE master array—204 km radius].
though all of them are constantly evolving. The characteristics described are as of the time of the cited publications unless otherwise noted. Each simulates the effects of convection using a model of a single cloud type at each grid point, consistent with the fine meshes of the models. The first two schemes below do not contain convective-scale downdrafts or momentum fluxes, but inclusion of these processes is clearly possible within the frameworks of the schemes.

Kreitzberg and Perkey (1976, 1977) employed sequential plume models to simulate the effects of convection. The plumes were activated whenever grid-point conditional instability exceeded a critical value determined by cloud depth and continued until the instability dropped below the threshold—an equilibrium closure. The total cloud base mass flux at cloud base was determined iteratively by requiring the hydrostatic pressures in the plume and in the subsiding environment to be equal. The authors simulated mesoscale rainbands similar to those found in extratropical cyclones.

Brown (1979) used a one-dimensional updraft plume model to simulate convection in his two-dimensional mesoscale model. His parameterization also included subsidence between plumes and detailed specification of the fluxes and transformations of condensation products. The closure assumption was that the total cloud mass flux at 900 mb ($M_0$) was directly proportional to the large-scale mass flux ($\bar{M}$) at that level:

$$M(900) = \beta \bar{M}(900).$$

The proportionality constant ($\beta$) was empirical and generally assumed to be greater than 1 in agreement with numerous diagnostic studies such as those of Yanai et al. (1973) and Gray (1973). This closure does not constrain the convection to produce an equilibrium state. Values of $\beta$ and the validity of Eq. (3) in general are both quite scale-dependent as discussed in Section 3c1. Brown modeled evaporation-driven mesoscale downdrafts as well as mesoscale anvil updrafts occurring in conjunction with cumulonimbus convection.

Fritsch and Chappell (1980a,b) developed a parameterization which included both convective updrafts and downdrafts. Both were one-dimensional entraining plumes. The magnitude of the convective mass flux was determined iteratively by requiring the scheme to stabilize a grid point within a specified advective time period. This closure hypothesis may be thought of as a rather sophisticated form of convective adjustment and is somewhat similar to that of Kreitzberg and Perkey (1977). The Fritsch and Chappell scheme includes such features as vertical motion in environmental air and the effects of vertical wind shear on the downdrafts. They also utilized a simple momentum transport and mixing scheme. Their parameterized model was designed to simulate convectively driven mesoscale systems in the middle latitudes, and the scheme is currently being incorporated into a number of other models.

Since all of the above parameterizations employ cloud models, they have considerable flexibility with respect to specification of vertical eddy heating and moistening profiles. Anthes (1977b), Anthes and Keyser (1979) and Gyakum (1981) have demonstrated the great sensitivity of numerical models to the assumed vertical profiles of convective heating, and Rosenthal (1979) has shown the sensitivity of tropical cyclone models to details of the cloud models. Since our understanding of convective processes is both incomplete and rapidly evolving, model flexibility is generally desirable. By adjusting cloud model assumptions it is possible to tune such a parameterization and to incorporate new features such as convective momentum and vorticity fluxes. However, great care must be taken to insure that the cloud models perform as intended over a wide range of larger scale conditions.

It should be noted that the closure hypotheses of the above schemes may limit their use to models with meso-$\beta$ scale or finer grid meshes. The Kreitzberg and Perkey and the Fritsch and Chappell schemes determine the total cloud mass flux by criteria which depend on stabilization of a grid point. Stabilization of the troposphere by deep convection typically occurs over areas with diameters of tens to hundreds of kilometers. Thus, it can be resolved explicitly by fine-mesh models but not by coarse-grid models. Brown's $\beta$ constant is highly scale-dependent, and relationships between low-level vertical motion and cloud mass flux weaken with increasing domain size. These scale considerations are not a problem in the fine-mesh mesoscale models for which the above schemes are intended.

c. Coarse-grid model parameterizations

The majority of the research concerning cumulus parameterization has focused on its application to models with coarse grids. Although such models are subject to the same physical laws as are the mesoscale models above, they have the added handicap that they are unable to resolve mesoscale circulations and must parameterize the significant effects of such circulations. All of these schemes assume that the convection and the circulations resolved by the model exist on such different time and space scales that they may be separated to some degree. Therefore the convection can be specified in terms of the grid-point circulation, and the effects of the convection can be determined separately and fed back to the large scale. The ability of parameterizations to accomplish these aims in light of the increasingly obvious complexity of the intermediate mesoscale circulations is a topic of current widespread concern.
Some of the most widely used schemes are discussed below. Several references are made to diagnostic studies and semi-prognostic tests. It must be emphasized that while such tests are extremely valuable, they fall short of validating a parameterization scheme. Relatively subtle imbalances between the convective processes and the grid-scale circulation can have profound effects on the subsequent evolution of the ensuing circulation. Unfortunately, observational evidence is seldom sufficient to evaluate all of the fields accurately enough for adequate verification of single-time-step simulations. Fully time-dependent simulations are ultimately required, since the primary goal of any cumulus parameterization scheme is to produce accurate prognostic evaluations.

1) MOIST CONVECTIVE ADJUSTMENT

Convective adjustment refers to an approach where the lapse rate of a saturated conditionally unstable layer is adjusted to neutrality in a specified time interval (usually less than one hour). Although models using this scheme have simulated some of the features of large-scale tropical circulations, the technique tends to give inaccurate rainfall rates in semi-prognostic tests and yields unrealistic vertical lapse rates of temperature and moisture (Krishnamurti et al., 1980). Better results have been obtained with a modified version known as soft convective adjustment proposed by Manabe et al. (1965), Miyakoda (1969) and Kurihara (1973). The latter scheme assumes that the adjustment occurs only within a fraction of the grid area. This fraction must be specified and is clearly scale-dependent. Soft convective adjustment tends to give improved time-averaged rainfall rates, although the semi-prognostic tests of Krishnamurti et al. (1980) suggest a time phase problem which would restrict its use to simulations of systems with time scales greater than those of synoptic-scale systems.

Convective adjustment is a simple and economical method of parameterizing convection. Since it bypasses most of the physical processes involved, it has limited flexibility and is not optimal for investigating interactions between the convection and larger scales. However, the concept of using changes in the vertical lapse rate of temperature at a grid point as a closure hypothesis is currently being used in mesoscale models as noted above. As our observations of large-scale changes in the presence of convection improve, it is quite likely that more sophisticated adjustment schemes will evolve.

2) MOIST CONVERGENCE WITH CLOUD MODELS

Charney and Eliassen (1964) used total column moisture convergence to estimate convective heating in their paper on conditional instability of the second kind. Kuo (1965, 1974) refined the concept by distributing the heating and moistening vertically with a simple cloud model, and it is often referred to as the Kuo scheme. When used with a flexible partitioning of the moisture convergence between precipitation and moistening of the column, it produces reasonable rainfall rates in semi-prognostic tests (e.g., Krishnamurti et al., 1980). This is consistent with numerous diagnostic studies which show that during periods of strong convection, changes in precipitable water are small compared to moisture convergence and rainfall. The empirical partitioning is spatially scale-dependent and may well vary in time due to evolution of mesoscale circulations. This is an area for further inquiry. The moisture convergence closure assumption is closely coupled to the model’s grid-scale divergence fields.

Kuo’s (1974) simple cloud model produces a vertical heating profile which is proportional to the temperature difference between a rising cloud parcel and its environment. This profile often agrees rather well with time-averaged observations of convective heating during periods of heavy rainfall, but it does not compare well with heating profiles obtained for individual observation times during GATE (Song, 1982). Anthes (1977a) used a more complex cloud model to show that the vertical heating profile depends significantly on cloud size. This leads to the question of how to specify the cloud size. As our observational base improves, it may become possible to specify typical cloud sizes or even cloud size distributions as functions of the grid-point circulation. Considerable progress has been made in documenting the statistics of cloud and radar echo populations and their characteristics (e.g., Lopez, 1977, 1978; Zipser and LeMone, 1980; Houze and Cheng, 1977). The problem of obtaining realistic vertical profiles of moisture and momentum processes using simple cloud models has been pursued in a number of recent studies, many of which are summarized in Houze and Betts (1981).

The Kuo scheme and its relatives are widely used. This technique frequently has been used to model the slowly evolving circulations of tropical cyclones (Rosenthal, 1970; Anthes, 1977b). The biggest problems are the need to specify the moisture partitioning (a poorly known quantity) and the inability to resolve mesoscale circulations (a problem common to all coarse-grid models). As grid spacings become larger, there may be an additional problem of specifying a suitable population of cloud types as a function of the resolvable circulation.

3) ARAKAWA AND SCHUBERT (1974)

Two important features of the Arakawa and Schubert scheme are the quasi-equilibrium closure hypothesis and the use of a spectrum of cloud sizes at
each grid point. Quasi-equilibrium assumes that the cloud ensemble reacts sufficiently rapidly to changes in the grid-scale flow so that changes in the cloud work function (analogous to the positive area between the cloud parcels and their environment on a tephiogram) are minimized. As previously discussed, this assumption is quite scale-dependent, becoming more reliable with increasing horizontal scale. The small-scale limit for application of this scheme is a matter of considerable debate, and the answer will have to be determined by the evaluation of many numerical simulations. The Arakawa and Schubert scheme generally produces good rainfall rates in semi-prognostic studies (Krishnamurti et al., 1980; Lord, 1982). This is consistent with observations that net tropospheric temperature changes associated with deep convection, averaged over regions with diameters on the order of hundreds of kilometers, are very small compared to the magnitude of the latent heat release (Frank, 1980; Fritsch et al., 1976). Whether this occurs predominantly due to response of the larger scale circulation to the release of latent heat, or to response of the convection to changes in the larger scale flow, is the subject of contention.

The spectral cloud ensemble constitutes a step forward in the flexibility of cloud-model-based parameterizations. Lord (1982) found good agreement between model-produced and observed vertical profiles of heating and moistening in semi-prognostic tests using GATE data. Payne (1982) has experimented with adding convective and mesoscale downdrafts and boundary-layer wake effects to the scheme. He reports improvements in the vertical heating profiles and increased mass fluxes in middle and low clouds. Inclusion of vertical momentum fluxes is quite possible. The use of a spectrum of clouds is applicable to other parameterization schemes as well, provided that the closure provides a realistic partitioning of cloud mass fluxes. Ooyama (1971) recognized that proper determination of such a "dispatcher function" would constitute a sufficient closure hypothesis.

The Arakawa and Schubert parameterization is probably the most complex existing scheme. It is currently being used in the UCLA general circulation model (e.g., Lord et al., 1982). It has also been used in simulations of tropical cyclones, which have small values of $R'$ by tropical standards (Hack and Schubert, 1980). This parameterization is subject to the same problems concerning resolution of mesoscale circulations as are other coarse grid schemes.

d. Resolvable cumulus

Rosenthal (1978) and Yamasaki (1977) developed tropical cyclone models with resolvable cumulus clouds with the idea of bypassing cumulus parameterization altogether. This is an attractive approach, although with existing computers the requirement for a grid size small enough to resolve the cumulus limits its applicability. It appears best suited for simulations of systems such as the tropical cyclone where nested grids are appropriate and where the large-scale flow stabilizes sufficiently to prevent the convective drafts from becoming unduly violent. The latter is a problem because relatively coarse grids, desirable for economy, can resolve only rather large updrafts.

5. Conclusions

Two decades ago cumulus parameterization was viewed as a far simpler problem than it is today. The atmosphere has not become more complex, but our understanding of its complexities has increased. This does not mean that hopes for solutions to the parameterization problem are fading, only that the problem must be viewed less naively than was once thought.

Deep convective fluxes are integrally related with mesoscale circulations, and the latter often have distinct lives of their own. As understanding of the various scales of circulations and their interactions has developed, it has become clear that the task of parameterizing the net effects of all subgrid-scale processes in a convective regime is difficult. It is equally clear that our understanding of the dynamics of convective weather systems is at best incomplete, although there has been continual progress. With almost every new observational or theoretical discovery, there is a new constraint upon parameterization. It seems ever less likely that a comprehensive scheme or even approach will emerge to satisfy all models.

Design of any cumulus parameterization scheme should begin with a review of first principles and the basic dynamics relevant to the systems to be modeled. It is assumed that fully turbulent processes, which must be treated statistically, can be handled adequately using existing approaches. Parameterization schemes suitable for simulations of large, quasi-balanced circulations are still in a state of evolution due largely to uncertainties regarding the natures and importance of mesoscale circulations. Nevertheless, development of such parameterizations seems to be a tractable problem.

Systems with significantly unbalanced and often divergent flow (region II of Fig. 1) remain something of an enigma. When the atmosphere is conditionally unstable, the dynamics of such circulations are complex and less well understood than those of the other flow regimes. Development of cumulus parameterization techniques for these circulations is less advanced than for the others due to the greater complexity of the interactions between convection and larger scales. It appears that parameterizations for region II simulations will have to be very sensitive to the rate of convective release of latent heat, since dynamically small-scale heating tends to affect divergence over spatially large areas. Progress in this area
will have to occur concurrently with progress in theoretical understanding of mesoscale circulations, and it is therefore encouraging that there is considerable current research addressing the latter.

Acknowledgments. The author is grateful to Dr. Katsuyuki Ooyama and Dr. Lloyd Shapiro for their helpful comments and advice. Thanks are also extended to Miss Mary Morris who assisted with the manuscript and figures.

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