On the Use of Lower Saturation Criteria for Release of Latent Heat in NWP Models

MUKUT B. MATHUR
National Meteorological Center, Washington, DC 20233

30 December 1982 and 12 April 1983

ABSTRACT

In several numerical models the large-scale release of latent heat is evaluated when the mixing ratio q exceeds a certain fraction (SATRH < 1) of its saturation value q_s. The predicted mixing ratio at the end of a time step in the above case is always less than q_s. It is shown that when a fractional value of SATRH is used in the model, the net heating of the atmosphere is significantly weaker than when the release of latent heat in the model is evaluated at supersaturated grid points only. Results of integrations with a symmetric hurricane model show that the intensity of the simulated storm is therefore weaker when a lower value of SATRH is used.

1. Introduction

The large-scale release of latent heat (isobaric condensation of water vapor) in most numerical models is evaluated when the mixing ratio $q(T, p)$ exceeds $q_{sm}(T, p)$, where

$$q_{sm}(T, p) = \text{SATRH}\, q_s(T, p). \qquad (1)$$

Here SATRH ≤ 1, and $q_s(T, p)$ is the saturated mixing ratio at the grid point with temperature T and pressure p. We shall refer to SATRH = 1 (below) as the unmodified case (simulates condensation at a supersaturated point in the real atmosphere); and to SATRH < 1 as the modified case. Note that in the modified case, use of a fixed value of SATRH, say 0.8, implies that at the end of any time step $q(T, p) \leq 0.8\, q_s(T, p)$. The relative humidity (RH) in such a model cannot attain values greater than 80%. The National Meteorological Center (NMC) uses SATRH values ranging from 0.8 to 0.96 in their models.

The vertical profiles of T and q, for which cooling due to the vertical advective term $-\omega\partial\theta/\partial p$ is balanced by the large-scale release of latent heat (supersaturation arising from the upward advection of moisture $-\omega\partial q/\partial p$) for modified and unmodified cases are presented in Section 2. Here ω is the vertical p velocity, and θ is the potential temperature. The profiles for the unmodified cases are given by pseudo-adiabats. The profile in the modified case is found to be significantly colder than the corresponding profile in the unmodified case.

In order to show the impact of using a fractional value of SATRH in numerical models, the results from the time integrations of the NMC two-dimensional hurricane model in the modified and unmodified cases are compared in Section 3. The initial disturbance intensifies into a severe hurricane in the unmodified case and a nearly pseudo-adiabatic profile is attained near the center. The intensity of the simulated disturbance is much weaker in the modified case (SATRH = 0.8) and the vertical potential temperature profile attained near the center is nearly the same as derived for the modified case in Section 2.

2. Condensation due to large-scale upward motion in the model's saturated environment

a. Unmodified case

We consider the case when the lapse rate at a grid point A is such that $T_e(p) = T_c(p)$, and $q_e(p) = q_c(p)$. Here, subscript e denotes the value at the grid point and c the value along a pseudo-adiabat through the (T, p) at the lowest information level. Because this atmospheric state is moist-adiabatically neutral, a Kuo-type convective parameterization scheme [or convective adjustment scheme similar to one used in the NMC operational Limited-area Fine Mesh model (LFM)] would not change this profile.

We now consider the changes in (q_e, T_e) at the grid point A, due to a large-scale upward motion, i.e., changes associated with the vertical advective term in the (x, y, p) coordinate system and any isobaric condensation resulting therefrom. The change in q_e is given by

$$\frac{\partial q_e}{\partial t} = -\omega\frac{\partial q_e}{\partial p} = -\omega\frac{\partial q_c}{\partial p}.$$

This increase in moisture (q_c for a pseudo-adiabatic lapse rate decreases with p) would give rise to supersaturation. If all the excess moisture is condensed, then

$$\frac{\partial q_e}{\partial t} = 0, \qquad (2)$$

$$\frac{\partial\theta_e}{\partial t} = -\omega\left(\frac{\partial\theta_c}{\partial p} + \frac{\theta_c}{c_pT_c}L\frac{\partial q_c}{\partial p}\right). \quad (3)$$

Here, L and c_p are respectively the latent heat of condensation and the specific heat of air at constant pressure.

The sum on the right side of (3) is vanishingly small. To show the above, first note that the equivalent potential temperature θ_E [Eq. (4)] is constant along a pseudo-adiabat, i.e.,

$$\theta_E = \theta_c \exp(Lq_c/c_pT_c), \quad (4)$$

where

$$\theta_c = T_c\left(\frac{p_0}{p}\right)^{0.286},$$

p is pressure (kPa) and p_0 = 100 kPa.

Logarithmic differentiation of (4) gives

$$\frac{1}{\theta_c}\frac{\partial\theta_c}{\partial p} + \frac{L}{c_pT_c^2}\left(T_c\frac{\partial q_c}{\partial p} - q_c\frac{\partial T_c}{\partial p}\right) = 0. \quad (5)$$

The second term inside the parentheses is at least an order of magnitude smaller than the first term. Neglecting the second-order term yields

$$\frac{1}{\theta_c}\frac{\partial\theta_c}{\partial p} + \frac{L}{c_pT_c}\frac{\partial q_c}{\partial p} = 0. \quad (6)$$

We now summarize the above results. Consider the case when a pseudo-adiabatic lapse rate develops at grid point (A) at time t in a numerical model. The temperature and mixing ratio at A are the same as along a pseudo-adiabat through the lowest level's temperature. No change in this lapse curve at A would occur in the model's next time step due to moist convection, if a Kuo-type convective parameterization procedure (or the NMC LFM type convective adjustment procedure) is used in the model. Furthermore, if the large-scale release of latent heat is evaluated when q_e exceeds the saturation value q_s (SATRH = 1, unmodified case), then according to Eqs. (3) and (6),the pseudo-adiabatic lapse curve at A would not change because of the large-scale upward motion. The cooling due to the vertical advection term $-\omega\partial\theta/\partial p$ in the model is balanced by the large-scale release of latent heat (supersaturation associated with the upward advection of moisture $-\omega\partial q/\partial p$). Therefore the lapse curve (4) is unchanged with respect to the convective as well as large scale upward motion.

b. Modified case

We consider the case when the (vertical) lapse rate at a grid point B satisfies

$$\theta'_E = \theta'_c \exp[Lq'_c/c_pT'_c] = \text{constant}, \quad (7)$$

where

$$q'_c = q_{sm}(T'_c, p) = \text{SATRH}\,q_s(T'_c, p). \quad (8)$$

Here θ' and T' are the potential temperature and temperature, respectively, along the curve θ'_E = constant.

The change in (q_e) due to the large-scale upward motion at the grid point B would be

$$\frac{\partial q_e}{\partial t} = -\omega\frac{\partial q_e}{\partial p} = -\omega\frac{\partial q'_c}{\partial p}.$$

For all values of SATRH < 1, $T'_c(p) < T_c(p)$. Furthermore, q'_c decreases with p. Therefore the increase in moisture due to the vertical advective term would give rise to $q_e > q_{sm}(T'_c, p)$. If the excess moisture is condensed, so that $\partial q_e/\partial t = 0$, then

$$\frac{\partial\theta_e}{\partial t} = -\omega\left(\frac{\partial\theta'_c}{\partial p} + \frac{\theta'_c}{c_pT'_c}L\frac{\partial q'_c}{\partial p}\right).$$

From considerations similar to those presented in the unmodified case, it can be shown that the sum on the right-hand side of the above equation is vanishingly small. Therefore the lapse curve (7) is unchanged with respect to the large-scale upward motion in the modified case.

c. Comparison between unmodified and modified cases

The lapse rate curves, given by Eqs. (4) and (7) respectively, are shown in Fig. 1 for temperatures of 293.16 K and 301.6 K at 100 kPa. Note that the temperatures above 50 kPa along curves B [modified case, Eq. (7) with SATRH = 0.8] are much colder than those along the corresponding curves A [unmodified case, Eq. (4)]. The above differences decrease for larger values of SATRH.

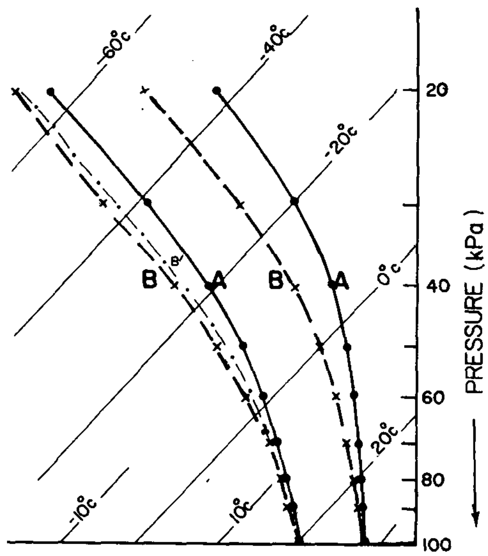

FIG. 1. Vertical lapse rate profiles through temperatures of 20 and 28°C, at 1000 mb for 1) unmodified case curves A, $\theta_E = \theta \exp(Lq_s/c_pT)$ = constant; and 2) modified case curves B, $\theta'_E = \theta \exp(0.8Lq_s/c_pT)$ = constant. B' is a lapse rate between B and A.

We now consider the case when a column above a grid point is predicted to be saturated with convective clouds during the time integration of a numerical model. The lapse curve in this vertical column is given by (4) for the unmodified case and (7) for the modified case. The vertical column will be much warmer in the unmodified case compared to that in the modified case. *Since the lapse rate in a column saturated with the convective clouds in the real atmosphere is nearly given by (4), the above discussion implies that, in general, the net heating in a conditionally unstable column is underpredicted in a numerical model that uses values of* SATRH *less than unity.*

3. Numerical results

The vertical temperature profile in wall clouds near the center of hurricanes is observed to nearly coincide with the pseudo-adiabatic lapse curve through the temperature and pressure point at the base of these clouds. This structure of the wall cloud is well simulated in the axially symmetric hurricane models in the unmodified case. Results of numerical integrations show that temperatures near the center in the simulated storm are colder when a fractional value of SATRH is used. The intensity of the simulated storm is therefore weaker in the modified case.

The NMC two-dimensional hurricane model developed by Hovermale (personal communication, 1980) is used in this study. This model uses σ (Phillips, 1957) as the vertical coordinate and has ten vertical layers. The horizontal grid spacing is 60 km with 22 points in the radial direction. In the operational cycle, the two-dimensional model is integrated on an f plane (Coriolis parameter corresponding to the latitude of the observed storm) to a nearly steady-state solution. A value of SATRH = 0.8 is used and the sea surface temperature is constant (usually 301.2 K) in time and space. Air–sea exchange of sensible and latent heat and the Kuo-type convective parameterization procedure are incorporated in the model. The initial state consists of a symmetric vortex (maximum tangential winds of ~30 m s^{-1}). The vortex simulated by this two-dimensional model is superimposed as a perturbation on the operationally (Hough) analyzed fields. The above procedure prescribes the initial data for the time integration of the NMC three-dimensional hurricane model [Movable Fine-mesh model (MFM)].

The results of integration of the two-dimensional hurricane model for the unmodified (SATRH = 1) and modified (SATRH = 0.8) cases are now presented. The value of f at 25°N is used in these experiments. A quasi-steady state is attained in both experiments after 50 h. The variation of the maximum tangential velocity ($V_{\theta\max}$) with time in the two cases is shown in Fig. 2. The maximum winds are located in the lower troposphere close to the center. The maximum winds near the center at 60 h are less than 36 m s^{-1} in the modified case, whereas they attain values close to 60 m s^{-1} in the unmodified case. The lapse rate at the grid point next to the center closely approximates the pseudo-adiabat through the lowest level temperature in the unmodified case (curve A in Fig. 3) and the vertical profile (7) in the modified case (curve B in Fig. 3).

A Kuo-type convective parameterization scheme is used in the two-dimensional model. This parameterization scheme tends to adjust the environmental temperature and moisture profiles toward that of the model's convective cloud which is given by the pseudo-adiabat [Eq. (4)] through the lowest layer temperature in both modified and unmodified cases. It is noted above that when the quasi-steady state is attained, the lapse rate near the center lies close to that given by a pseudo-adiabatic profile [Eq. (4)] in the unmodified case, but lies close to the profile given by (7) in the modified case. The above lapse rate profile is therefore determined by the saturation criteria used for the large-scale release of latent heat and not by the convective cloud (pseudo-adiabatic) profile used in the model.

The above behavior of the model is explicable by the considerations presented in Section 2. Consider the case when the profile B (Fig. 1) satisfying Eq. (7) is simulated in a column in the modified case at time t. This profile is unchanged with respect to the large-scale upward motions. We now assume that the convective release of latent heat is invoked in this column. Let B′ be the lapse curve (Fig. 1) in this column after the convective release of latent heat and the large-scale condensation [supersaturation ($q > q_{sm}$) may occur because of the moistening of the column by convection] have been evaluated. Because the adjusted $q(T, p) \leq q_{sm}(T, p)$, this column will be cooled

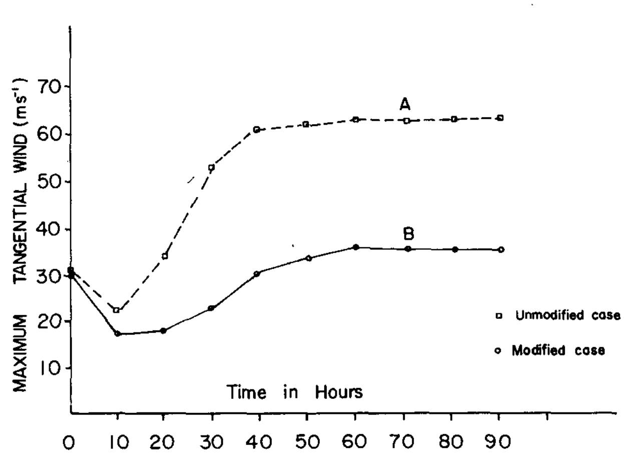

FIG. 2. Maximum tangential winds ($V_{\theta\max}$) during the time integration of the two-dimensional hurricane model: 1) unmodified case curve A, SATRH = 1; and 2) modified case curve B, SATRH = 0.8. Note that maximum winds are ~60 m s^{-1} at 60 h in the unmodified case while they are less than 36 m s^{-1} in the modified case.

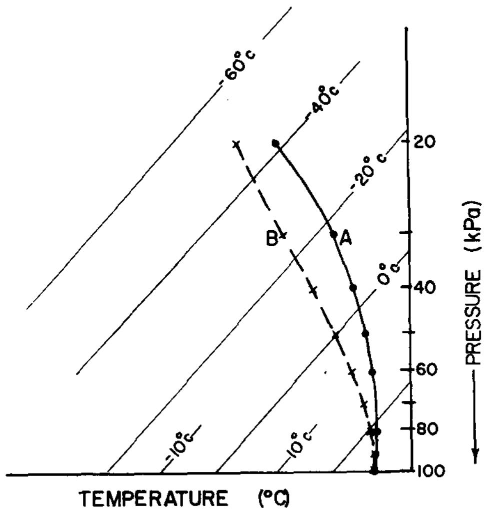

FIG. 3. Vertical temperature profiles near the center of simulated symmetric hurricanes at 60 h: 1) unmodified case curve A, SATRH = 1; and 2) modified case curve B, SATRH = 0.8. Note that profile A nearly coincides with a pseudo-adiabat and profile B is much colder than profile A.

as a result of large-scale motion in the next time step. The cooling caused by the vertical advection term is larger than the heating caused by the large-scale release of latent heat (supersaturation associated with upward advection of moisture). The lapse rate would be adjusted toward the modified curve B. The net result would be that the lapse rate will oscillate around modified curve B in the above column. This oscillation can be removed by specifying that the model convective cloud is given by Eq. (7) [and not by Eq. (4)] in the modified case.

4. Concluding remarks

Many parameterization schemes used to realistically simulate the convective release of latent heat in the numerical models tend to adjust the conditionally unstable regions in the model toward the moist adiabatically neutral state given by the pseudo-adiabat through the base of the convective cloud. This neutral lapse rate may only be simulated in the model provided the isobaric condensation of water vapor takes place when the mixing ratio exceeds the saturation value at the grid point (unmodified case, SATRH = 1). This is because only under these conditions, given the moist adiabatic lapse rate at the grid point, is the cooling due to the vertical advective term $-\omega\partial\theta/\partial p$ balanced by the isobaric condensational release of latent heat; supersaturation occurs due to the upward vertical advection of moisture ($-\omega\partial q/\partial p$ term). *It is worth noting that in order to make a realistic simulation of the adjustment of the conditionally unstable atmosphere toward the moist neutral state, the large-scale release of latent heat in the conditionally unstable regions must be included in the model.* The large-scale release of latent heat at conditionally unstable grid points has been omitted in some previous works (e.g., Mathur, 1975; Krishnamurti *et al.*, 1973).

Thermodynamic considerations presented in Section 2 and numerical results presented in Section 3 show that when the large-scale release of latent heat is evaluated in the model at less than 100% saturation (modified case, SATRH < 1), the conditionally unstable regions are adjusted toward a modified vertical lapse curve (curves B in Figs. 1 and 3). The temperatures along these modified curves are much colder than those along the corresponding pseudo-adiabats through the base of the convective clouds. *Therefore, the net heating of the atmosphere is much weaker in the modified case than in the unmodified case. The warm core of hurricanes can apparently only be simulated if* SATRH *is assigned a value close to unity.*

The use of SATRH < 1 in the models is based on the premise that the value of a variable at the grid point represents an average value of the variable in a three-dimensional box centered at the grid point. The large-scale condensational release of latent heat simulates the formation of layered clouds in the atmosphere. The layered clouds in the real atmosphere would, in general, occupy only a fractional surface area of the grid box at any given time. Therefore, although the relative humidity RH in the clouds is 100%, the average value of RH over the grid box in the real atmosphere (with only a fraction of the grid box area occupied with layered cloud) would be less than 100%. The large-scale release of latent heat in a numerical model may therefore be invoked at a value of RH less than 100%. A simple scheme which allows the RH at the model grid point to attain a value of 100% (situations in which the grid box is completely covered with clouds) and also evaluates the large-scale release of latent heat when the grid box is partially covered with clouds (RH > 80%) has been formulated. The fractional area (β) covered by the layered clouds is evaluated from the value of RH at the grid point and its mean value over the surrounding grid points. The large-scale release of latent heat is calculated over the fractional area β of the grid box. The structure of the simulated hurricane using the NMC two-dimensional hurricane model and the above scheme is similar to that obtained in the unmodified case. The formulation of this scheme and the numerical results with the two-dimensional hurricane model will be presented in a subsequent paper.

REFERENCES

Krishnamurti, T. N., M. Kanamatsu, B. F. Ceselski and M. B. Mathur, 1973: Florida State University tropical prediction model. *Tellus,* **25,** 523–535.

Mathur, M. B., 1975: Development of banded structure in a numerically simulated hurricane. *J. Atmos. Sci.,* **32,** 512–522.

Phillips, N. A., 1957: A coordinate system having some special advantages for numerical forecasting. *J. Meteor.,* **14,** 184–185.