Scale Analysis of Marine Winds in Straits and along Mountainous Coasts[1]

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ABSTRACT

The complicated wind regimes in straits which develop in response to different large-scale pressure fields are investigated by scale analysis of the equations of motion. Adjustment of the mass and motion fields in straits O(10s km) in width is governed by four nondimensional numbers: separate along- and cross-strait Rossby numbers, a strait drag coefficient, and a stratification parameter, which relates the internal Rossby radius of deformation to the width of the strait. The wind field is in approximate geostrophic balance with an imposed cross-channel pressure gradient. An along-channel pressure gradient is primarily balanced by ageostrophic acceleration of the wind field down the axis of the strait (the gap wind). Vertical motion and the accompanying horizontal divergence in the near-surface wind field can be large even for moderately stable stratification; as a consequence, there may be particularly abrupt transitions of the surface wind field at the exits of straits, where there is a rapid change of the scaling parameters to match coastal conditions.

The scale analysis also applies to open coasts with the Rossby radius of deformation replacing the width of the strait as the offshore length scale. For the mountainous coasts along Alaska, Canada and Norway, a typical Rossby radius is O(80 km); within this distance an alongshore pressure gradient will be principally balanced by the ageostrophic terms in the momentum equation. Since the coastal Rossby radius is smaller than the grid size of present numerical weather prediction models, geostrophic adjustment is not correctly modeled for landfalling storms along mountainous coasts.

1. Introduction

A variety of wind regimes exists in straits as a function of the orientation of the large-scale pressure field to the channel axis (Reed, 1931; Yoshino, 1975; Mass, 1981). A central issue is the degree of geostrophic adjustment between the mass and motion fields in straits as a function of channel width. The intent of this note is to present unifying concepts for winds in straits, based upon scale analysis of the governing equations, to guide future data analysis and modeling studies. A key element of the analysis is the boundary condition of no transport through the side walls. As a consequence, the ageostrophic response of marine winds near mountainous coastlines is similar to that of a wide strait with the Rossby radius of deformation replacing the strait width as the offshore length scale.

One limitation of the analysis is that the sea level pressure field must primarily be externally imposed upon the strait. The regional response of sea level pressure in the lee of coastal mountains is governed by the magnitude of an internal Froude number, $\mathrm{Fr} = V_a(DN)^{-1}$, where V_a is the upstream velocity, D the height of the ridge, and N the Brunt–Väisälä frequency of the incident air mass (Walter and Overland, 1982). For onshore flow with Fr O(1), regional perturbations in the sea level pressure field can be induced in the lee of mountains (Smith, 1980). Such a case was the 13 February 1979 Hood Canal storm (Reed, 1980) in which a 4 mb low was induced by the presence of the Olympic Mountains in western Washington State. The addition of the pressure gradient from the induced low to the pressure gradient from the large-scale background flow resulted in surface winds of 50 m s^{-1} in the confined channel to the south of the low center. In the years since the Hood Canal Storm no feature of similar magnitude has been produced in western Washington, although there is an indication of pressure troughing in the lee of the Olympic Mountains on the one or two major winter storms each year. The more common situation is the low Froude number regime (Walter and Overland, 1982) in which the motion field behind the ridge responds principally to the overlying large-scale pressure field modified by secondary circulations in the strait itself. This note discusses the low Froude number regime.

A second limitation of the analysis is that it considers the regional response of marine winds for width scales of 5–100 km and length scales of 25–500 km. It excludes strong local cross-channel flows on scales O(km) due to valleys or gaps which drain into the main strait. Downslope acceleration plays a role in these local cross-channel flows, whether they

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are valley breeze in origin (Mass, 1982), or bora-like (Reed, 1981).

This study is concerned with marine channels which are at sea level and have relatively low surface friction coefficients. While the theory in principle can apply to gaps on land, these often have some influence of slope. Here gravitational acceleration, even for slight slopes, can have a major impact on the regional surface wind field (Ehrlich, 1953). The next section presents the scale analysis; the third section compares the results of the analysis to observational studies of marine winds in straits.

2. Scale analysis of ageostrophic motion in straits

The purpose of this section is to determine the relative importance of various terms in the equations of motion for geophysical parameters relevant to flow in straits. The analysis orients the x_* axis in the cross-channel direction and the y_* direction along the main axis of the channel. We define L as the length scale in the y_* direction, l the width scale in the x_* direction O(10 km), and D the height of the surrounding topography, where by hypothesis:

$$\frac{l}{L} \ll 1, \quad \text{and} \quad \delta \equiv \frac{D}{l} \ll 1. \qquad (1)$$

For motion on these length scales, the Boussinesq hydrostatic system of equations is appropriate:

$$\frac{du_*}{dt_*} - fv_* = -\frac{1}{\rho_s}\frac{\partial p_*}{\partial x_*} + \frac{1}{\rho_s}\frac{\partial \tau_{x_*}}{\partial z_*}, \qquad (2)$$

$$\frac{dv_*}{dt_*} + fu_* = -\frac{1}{\rho_s}\frac{\partial p_*}{\partial y_*} + \frac{1}{\rho_s}\frac{\partial \tau_{y_*}}{\partial z_*}, \qquad (3)$$

$$\frac{\partial p_*}{\partial z_*} = -\rho_* g, \qquad (4)$$

$$\frac{\partial u_*}{\partial x_*} + \frac{\partial v_*}{\partial y_*} + \frac{\partial w_*}{\partial z_*} = 0, \qquad (5)$$

$$\ln\theta_* = \frac{1}{\gamma}\ln p_* - \ln\rho_* + \text{constant}, \qquad (6)$$

$$\frac{d\theta_*}{dt} = H_*, \qquad (7)$$

with

$$\frac{d}{dt_*} \equiv \frac{\partial}{\partial t_*} + u_*\frac{\partial}{\partial x_*} + v_*\frac{\partial}{\partial y_*} + w\frac{\partial}{\partial z_*}, \qquad (8)$$

where (u_*, v_*, w_*) are the dimensional velocity components in the (x_*, y_*, z_*) directions, f the Coriolis parameter (assumed constant), g the acceleration of gravity, (τ_{x_*}, τ_{y_*}) are the horizontal shear stress components, p_* pressure, ρ_* density, ρ_s a reference density, θ_* potential temperature; $\gamma = R/C_p$ with R the gas constant for dry air and C_p the specific heat at constant pressure, and H_* the heating rate divided by the specific heat of air.

We define U as the velocity scale in the cross-channel or x_* direction and V as the velocity scale in the along-channel or y_* direction. We introduce nondimensional variables (Pedlosky, 1979, p. 541) as follows:

$$u_* = Uu, \quad x_* = lx,$$
$$v_* = Vv, \quad y_* = Ly,$$
$$w_* = Ww, \quad z_* = Dz, \quad t_* = Tt. \qquad (9)$$

All "asterisk" variables are physical quantities, capital letters are physical scales, and all lower case letters are nondimensional variables O(1). The time scale is selected so that it is of the same order as the along-channel advective time scale,

$$T \sim \frac{L}{V}. \qquad (10)$$

This time scale is considered short compared to the rate of change of external forcing and the heating rate H_* in (7).

To allow for the possibility that the mass flux can be balanced either in the horizontal or vertical plane, each term in the continuity equation (5) is set to equal order:

$$\frac{V}{L}\frac{\partial v}{\partial y} + \frac{U}{l}\frac{\partial u}{\partial x} + \frac{W}{\delta l}\frac{\partial w}{\partial z} = 0. \qquad (11)$$

Note that it is the variation of velocity over a given length scale, not the magnitude of the velocity, that is important to the scaling. Equation (11) leads to geometric constraints on the relative velocity amplitudes,

$$U = \frac{l}{L}V, \quad W = \delta U. \qquad (12)$$

Scaling of the pressure and density variables is more subtle. Since a purpose of the study is to make comparisons of the ageostrophic components of the momentum balance to the Coriolis acceleration, an appropriate scale for the pressure gradient force is the geostrophic balance. The scaling for pressure is then

$$p_* = p_s(z) + \rho_s flV(p_0 + p), \qquad (13)$$

where p_s is the hydrostatic variation of pressure with height in the absence of motion, p_0 the nondimensional, externally prescribed large-scale pressure field, and p the nondimensional perturbation pressure caused by mass adjustment within the strait. To scale density, one can anticipate, based upon the hydrostatic equation (4), that the buoyancy force per unit mass due to density perturbations will be on the same order as the vertical pressure gradient. Thus, we scale density from pressure using (4) and (13):

$$\rho_* = O[\rho_s flV/(gD)]$$
$$= \rho_s(z)(1 + R_l F\rho), \qquad (14)$$

where

$$R_l = \frac{V}{fl}, \quad F = \frac{f^2 l^2}{gD}, \tag{15}$$

and ρ is the nondimensional perturbation density. The equation of state (6) provides a relation of potential temperature to density and suggests that potential temperature should be scaled in a manner similar to density by a rest state value θ_s and a nondimensional deviation θ scaled as in (14):

$$\theta_* = \theta_s(z)(1 + R_l F \theta). \tag{16}$$

In nondimensional form, the system of equations become the along-strait (y) and cross-strait (x) equations of motion, the continuity equation and the thermodynamic energy equation:

$$R_l\left[\frac{dv}{dt} + C_D'(u^2 + v^2)^{1/2}v\right] + u = -\frac{\partial p_0}{\partial y} - \frac{\partial p}{\partial y}, \tag{17}$$

$$R_l \frac{l^2}{L^2}\left[\frac{du}{dt} + C_D'(u^2 + v^2)^{1/2}u\right] - v = -\frac{\partial p_0}{\partial x} - \frac{\partial p}{\partial x}, \tag{18}$$

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = 0, \tag{19}$$

$$R_l S^{-1}\frac{d\theta}{dt} + w(1 + R_l F\theta) = 0, \tag{20}$$

with

$$\frac{d}{dt} \equiv \frac{\partial}{\partial t} + u\frac{\partial}{\partial x} + v\frac{\partial}{\partial y} + w\frac{\partial}{\partial z}. \tag{21}$$

The stress gradient terms have been scaled by the surface stress divided by D, so that the relative importance of surface friction to the field acceleration terms is given by a "strait" drag coefficient,

$$C_D' = \frac{C_{D*}L}{D}, \tag{22}$$

and C_{D*} is a drag coefficient $O(10^{-3})$; if the surface frictional layer is shallower than D, this change can be incorporated into C_D'. It will be shown later that $R_l F \ll 1$ in (20).

There are two separate measures of nonlinearity for the momentum equations. In the down-strait equation of motion (17) the Rossby number is

$$R_l = \frac{V}{fl}, \tag{23a}$$

while in the cross-strait direction (18) the corresponding Rossby number is

$$R_L = \frac{l^2}{L^2}R_l, \tag{23b}$$

which is much smaller than (23a).

The stratification parameter S is

$$S = (F\theta_s)^{-1}\frac{\partial \theta_s}{\partial z} = \frac{N^2 D^2}{f^2 l^2} = \frac{l_R^2}{l^2}, \tag{24}$$

where N is the Brunt–Väisälä frequency of the rest state atmosphere which is assumed to be constant with height, i.e.,

$$N^2 = \frac{g}{\theta_s}\frac{d\theta_s}{dz_*} = \frac{g}{D\theta_s}\frac{d\theta_s}{dz}. \tag{25}$$

Note that S^{-1} is the square of the ratio of the width of the channel to the internal Rossby radius of deformation,

$$l_R = \frac{ND}{f}. \tag{26}$$

3. Discussion

To assign numerical values to the geophysical scaling, reference is made to parameters of Shelikof Strait (Table 1), which is between Kodiak Island and the Alaskan mainland and the Strait of Juan de Fuca, which is between Washington State and Vancouver Island. The major geophysical characteristics of Shelikof Strait are its large width and high latitude, both of which favor geostrophic adjustment; the Strait of Juan de Fuca is narrower.

In the down- and cross-strait momentum equations [(17) and (18)], the bracketed terms (local acceleration, advection, and friction) are scaled by down- and cross-strait Rossby numbers, indicators of the importance of the inertial acceleration to the Coriolis force. Since $l \ll L$, the cross-strait Rossby number given by (23b) is much smaller than the down-strait Rossby number. Given a cross-strait pressure gradient, $\partial p_0/\partial x$, and $R_L \ll 1$, (18) predicts a near

TABLE 1. Geophysical scales for Shelikof and Juan de Fuca Straits. Data are from Overland and Walter (1981), Walter and Overland (1982) and Macklin *et al.* (1984).

Parameter	Shelikof Strait	Strait of Juan de Fuca
l	50×10^3 m	20×10^3 m
L	200×10^3 m	140×10^3 m
V	15 m s^{-1}	12.5 m s^{-1}
D	1300 m	1000 m
N	0.6×10^{-2} s^{-1}	0.7×10^{-2} s^{-1}
f	1.24×10^{-4} s^{-1}	1.08×10^{-4} s^{-1}
C_D	1.5×10^{-3}	1.5×10^{-3}
l_R	63×10^3 m	65×10^3 m
l/L	0.25	0.14
R_l	2.4	5.8
R_L	0.15	0.11
C_D'	0.23	0.21
F	3.0×10^{-3}	4.8×10^{-4}
l/l_R	0.79	0.31
S	1.60	10.5

geostrophic balance between the pressure gradient and the down-strait wind. This condition was observed in the outer Strait of Juan de Fuca by Mass (1981) and Schoenberg (1983).

If the large-scale pressure gradient is oriented along the axis of the strait, $\partial p_0/\partial y \neq 0$, then by (17) it will be balanced primarily ($R_l > 1$) by the ageostrophic acceleration of the flow down the axis of the channel, and gap winds will result (Reed, 1931; Overland and Walter, 1981). This seems reasonable because at both shores the cross-channel velocity component must vanish, due to the requirement for zero transport through the boundary, leaving only the acceleration and friction terms to balance the pressure gradient. Also, (18) must be satisfied for $\partial p_0/\partial x = 0$. This condition implies that the local mass field will adjust to the along-strait velocity, giving greater sea level pressure on the right side of the channel when looking downwind, and indeed this condition was observed in Shelikof Strait by Macklin, Overland and Walker (1984). For both Shelikof Strait and Juan de Fuca Strait, the strait drag coefficient C_D' is less than unity, implying that friction is not as important as acceleration in the momentum balance.

We are now in a position to discuss the magnitude of vertical motion within straits by considering the thermodynamic energy equation (20). One feature of straits is the frequent occurrence of strong convergence zones in the horizontal winds at the surface. Large vertical velocities can be inferred, and examples of potential temperature soundings in straits obtained from aircraft dropsondes often indicate continuous, weak stable stratification (Overland and Walter, 1981; Walter and Overland, 1982; Macklin, Overland and Walker, 1984). The seaward extent of gap winds tends to end with abrupt transitions, rather than as horizontal spreading of a jet. The vertical velocity is scaled by the product of the along-strait Rossby number R_l and the inverse of the stratification parameter S^{-1}, the square of the ratio of the width scale to the Rossby radius of deformation. When the product is O(1), as it will be in most straits, the local mass field will adjust to the horizontal velocity field, forcing three-dimensional circulations and large surface wind divergence. For the gap wind case this implies a cross-channel secondary circulation within the strait. At the exit of straits there will be an abrupt change in the scaling parameters, and one can expect a rapid transition toward the open coast problem when one coastline continues or quasi-geostrophic flow when both coastlines terminate, which is the appropriate geophysical scaling of synoptic weather systems.

At the point where the width of the channel exceeds the Rossby radius of deformation l_R the appropriate length scale for the distance away from the shore becomes fixed at the Rossby radius (see discussions in Pedlosky, 1979, and Csanady, 1982). Under this condition, the circulation near the two coastlines of the strait will approximate that of an open coastline with topography of order D. The alongshore length scale L is now associated with the scale of variation of the large-scale pressure field. The interpretation of the thermodynamic energy equation (20) with $l = l_R$ implies significant vertical motion for the open coastline case and the alongshore momentum equation (17) implies an ageostrophic momentum balance to an imposed alongshore pressure gradient.

So far in the analysis we have treated the external large-scale pressure field as given. However, as a cyclone approaches a mountainous coastline, the quasi-geostrophic balance is disrupted by the no transport boundary condition for the lower part of the atmosphere and a new ageostrophic momentum balance will be established either by accelerating winds or by the formation of Kelvin waves (Gill, 1977; Bannon, 1981). Since present numerical prediction models do not resolve the coastal Rossby radius O(80 km) for typical mountain heights (1500 m) and onshore air masses [$N = 0.7 \times 10^{-2}$ s^{-1}; Walter and Overland, 1982] of Alaska, Canada and Norway, the geostrophic adjustment problem will not be correctly treated in the models for landfalling storms (Hsieh and Gill, 1984).

4. Conclusions

Ageostrophic flow through sea level channels or straits and along mountainous coasts is an important regional meteorological phenomenon. The length and width of the channel, its latitude, air mass stability and large-scale sea level pressure gradient determine the magnitudes of the four fundamental scaling parameters: the down- and cross-strait Rossby numbers, the strait drag coefficient, and the stratification parameter. The magnitudes of these parameters control the adjustment of the mass and velocity fields in the channel. Most straits, including Shelikof Strait, which is very wide, are characterized by $R_l > 1$, $R_L \ll 1$, $C_D' < 1$, and $S > 1$.

Scale analysis indicates near-geostrophic surface flow for a pressure gradient orthogonal to the strait axis, and gap winds for a pressure gradient parallel to the strait axis. These conditions hold even for Shelikof Strait where the width of the strait is of the same order as the Rossby radius of deformation, the appropriate offshore length scale for an open coast. The analysis indicates large vertical velocities when the product of the along-strait Rossby number and the inverse of the stratification parameter is O(1); this is true for most straits and along mountainous coasts. There should be abrupt transitions in the horizontal velocity fields at the exits to straits as the change in geophysical parameters rapidly modify the scaling parameters.

Within a Rossby radius of mountainous coasts, the boundary condition of no transport through the coastline prohibits a geostrophic balance to an alongshore pressure gradient, which implies a separate meteorological regime than the quasi-geostrophic regime further offshore.

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