Normal Mode Initialization with Elementary Surface Friction

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ABSTRACT

Various normal-mode initialization techniques are applied to a simple 12-level linear model with boundary layer friction, and results are compared to exact solutions of the model. It is found that Machenhauer's initialization scheme gives an approximate solution to the initialization of ageostrophic circulations due to friction; however, all or almost all vertical modes must be initialized and a moderate number of iterations are required. Second-order Baer–Tribbia initialization is found to be less effective than several iterations of the Machenhauer procedure. An iterative initialization based on bounded derivative theory and requiring the second time derivatives of the gravity modes to vanish gives excellent results, but a simple iterative scheme to achieve this diverges with moderate friction. The successful application of these procedures to the initialization of ageostrophic circulations due to friction in numerical weather prediction models will require careful utilization of, and possibly improved, iterative methods to achieve convergence and stability.

1. Introduction

The importance of ageostrophic circulations due to surface friction is well known (see, for example, Holton, 1972, Ch. 6). Reasonable initial values of such circulations for numerical weather prediction models would be desirable. However, most atmospheric analysis schemes are based in part on geostrophic constraints and therefore are not well suited to the analysis of ageostrophic circulations due to friction. In addition, the desired balanced ageostrophic circulations of a model will not be identical to those of the atmosphere and thus cannot be derived solely from atmospheric data. Some form of normal mode initialization (NMI) might be expected to produce those ageostrophic circulations, but in practice most NMI schemes are used only for deep gravity modes and have little effect on initializing friction-induced circulations. [The literature is replete with detailed discussions of NMI theory and applications, and we will not develop this topic herein. The interested reader is referred to Daley (1981) and Leith (1980) for comprehensive reviews.] Furthermore, there has been little examination of the potential performance of NMI for initializing frictionally-induced ageostrophic circulations.

Williamson and Temperton (1981) found with a nine-level model including boundary-layer friction that Machenhauer's (1977) NMI for five vertical modes produced little of the boundary-layer cross-isobaric flow associated with friction. However, when one iteration of the initialization with five vertical modes was followed by one iteration with all nine vertical modes, reasonable boundary-layer cross-isobaric flow appeared. Despite these apparently successful results in the boundary layer, Williamson and Temperton noted that the NMI procedure was on the verge of instability and that its performance was questionable. In addition, they had no numerical test for the accuracy of the boundary-layer divergence, nor did they examine the quality of the boundary-layer vorticity or the divergence outside the boundary layer. Using a higher-resolution fifteen-layer model with additional physics, Williamson and Temperton found no version of NMI that gave reasonable initialization of physics-induced ageostrophic circulations.

The National Meteorology Center (NMC) spectral model (Sela, 1982) also produces problems if NMI is applied for all vertical modes. One difficulty is that boundary-layer friction may be so strong that the frictional force at some locations in midlatitudes may exceed the Coriolis force; in such areas, NMI can give poor boundary-layer winds. Such poor performance with strong friction is not surprising, however, since in these select areas the Rossby number is not small.

To identify the problems associated with initialization of frictional circulations, we examine a model with boundary-layer friction. Because the model is simple and linear, we can find exact solutions to its equations of motion. One solution is of low frequency and includes the desired ageostrophic circulations due to friction. This exact solution can be used in tests of various NMI schemes.

2. Normal modes with friction

Let us examine the linearized equations representing a multilevel primitive equation model with friction. We simplify the equations by considering motion on a rotating f-plane. The equations are expressed in terms of vorticity, divergence and geopotential for 12 layers, and some of the vertical finite differencing of the NMC spectral model (Sela, 1982) is retained. At each level I, the vorticity and divergence equations are

$$\frac{\partial D_I}{\partial t} = f\xi_I - \nabla^2\phi_I - K\delta_{I1}D_I, \tag{1}$$

$$\frac{\partial \xi_I}{\partial t} = -fD_I - K\delta_{I1}\xi_I, \tag{2}$$

where D_I is the divergence at level I, ξ the vorticity, ϕ the geopotential, K is a boundary-layer friction constant, and δ_{I1} is zero for $I = 2$, 12 but unity for $I = 1$. The geopotential tendency is

$$\frac{\partial \hat{\phi}}{\partial t} = \mathbf{G}D, \tag{3}$$

where $\hat{\phi} = (\phi_1\phi_2, \cdots \phi_{12})^T$, a vector containing the 12 levels of geopotential, and $\mathbf{G}$ is a constant 12×12 matrix described by Sela (1982).

In Eqs. (1)–(3) we have taken a constant Coriolis acceleration; consequently, these equations have no east–west or north–south coupling except for the term involving $\nabla^2\phi$ in (1). Let us now expand the dependent variables in eigenfunctions of the Laplacian operator. Equations (1)–(3) will show no coupling of these eigenfunctions. Thus we may examine the system of equations for each eigenfunction separately, noting that the eigenvalues of $\nabla^2 = -C$, a measure of the horizontal length scale. Recall that Eqs. (1)–(3) also contain boundary-layer friction, not generally considered in normal-mode analyses.

For given values of f, K and C, we calculate the eigenvalues and vectors of system (1)–(3). Since the friction varies with vertical level, we cannot employ the usual expansion in vertical modes to find the exact solution. However, since the system is rather small we can calculate normal modes without expanding in vertical modes. We rewrite (1)–(3) as

$$\frac{\partial \bar{\psi}}{\partial t} = \mathbf{A}\bar{\psi}, \tag{4}$$

where

$$\bar{\psi} = (D, \xi, \phi)^T; \tag{5}$$

$\mathbf{A}$ is a real 36×36 matrix, and we determine its eigenvalues and eigenvectors. Note that 24 of the 36 eigenvectors or modes of $\mathbf{A}$ represent inertial gravity waves with relatively high frequency and some damping due to friction. Of the remaining 12 modes, 11 are steady-state, nondivergent, geostrophic, have a totally zero eigenvalue, and have no amplitude in the boundary layer. The remaining mode has a negative real eigenvalue (no frequency) indicating damped time behavior due to friction. For any values of the parameters examined here, this mode has no oscillations but only exponential time decay with a time scale longer than that of inertial oscillations; clearly it is a slow mode. This damped slow mode has significant boundary-layer divergence, with divergence of predominantly opposite sign outside the boundary layer. Its boundary-layer vorticity is significantly less than geostrophic, while its vorticity outside the boundary layer is approximately but not exactly geostrophic. This damped mode describing an Ekman-type circulation is the exact slow mode solution to our simple model with friction; henceforth we will refer to this exact damped slow mode as EDSM.

A low frequency solution to (1)–(3), using any linear combination of the 12 exact slow modes as initial values of D, ξ and ϕ, can be developed. Since the 11 exact slow modes with no damped behavior are nondivergent, geostrophic and have no amplitude in the boundary layer, they present no difficulty in conventional normal-mode initialization wherein the modes are calculated without friction. However, the EDSM is nongeostrophic, has boundary-layer flow, and is a critical test for conventional normal-mode initialization. Although the EDSM is a low frequency solution to (1)–(3), it is partially gravitational in terms of conventional modes without friction. Therefore, we will apply various types of conventional normal-mode initialization to the EDSM, treating friction as a nonlinear term. The results of these initialization experiments will be compared to the EDSM, and also tested to see if they give low frequency solutions.

For the specific case with $f = 10^{-4}$ s^{-1}, $K = \frac{1}{2}f$, and $C = 2.29 \times 10^{-11}$ m^{-2}, corresponding to a spherical harmonic with $n = 30$, the EDSM has an eigenvalue of -1.30×10^{-5} s^{-1}. Figure 1 shows the divergence and vorticity of the mode as a function of the vertical coordinate σ. The mode has been normalized so that the boundary-layer vorticity is 10^{-4} s^{-1}. Note that this damped slow mode has an Ekman-like vertical dependence in divergence, and a vorticity structure that decreases gradually toward zero at high model levels.

3. Model normal-mode initialization experiments

In this section, various NMI experiments will be described. The conventional normal modes of (1)–(3) without friction will be used in the NMI schemes, and friction will be treated as a nonlinear term. The selected NMI schemes may also be applied directly to the EDSM, and we will examine a few such cases.

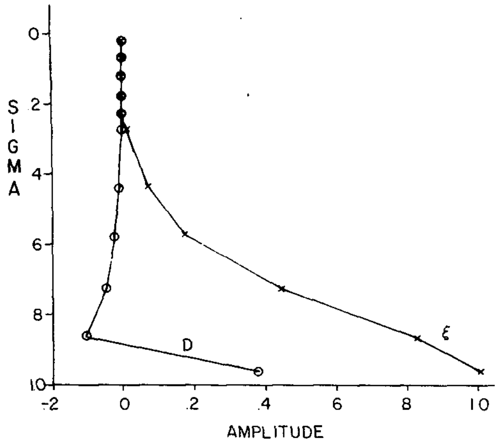

FIG. 1. Damped slow-mode vorticity and divergence.

However, we will apply most NMI experiments to the conventional slow-mode projection of the EDSM. Thereby we can examine how well each NMI regenerates the EDSM and whether it yields a low frequency solution.

a. The Machenhauer NMI

We first examine the performance of the Machenhauer (1977) initialization as a function of the number of vertical modes initialized. Let us consider the case with $f = 10^{-4}$ s^{-1}, $K = \frac{1}{4}f$, and $C = 2.29 \times 10^{-11}$

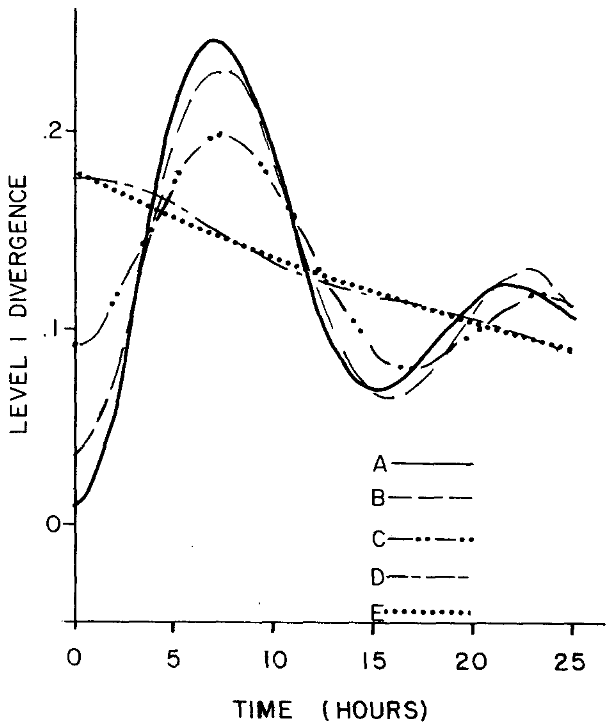

FIG. 2. Level 1 divergence versus time for: A, L12 + 4M5; B, L12 + 4M7; C, L12 + 4M9; D, L12 + 4M11; E, exact solution.

TABLE 1. Boundary-layer divergence as a function of number of vertical modes initialized (percent difference from exact values).

Number of vertical modes initialized				
5	7	9	11	12
94.8	80.0	49.1	2.8	1.9

m^{-2}. Each experiment begins with the slow-mode projection of the EDSM denoted by L12 and implying linear initialization for all 12 vertical modes. To this linearly initialized result, we apply four iterations of the Machenhauer NMI for various numbers of vertical modes. As an example, the notation L12 + 4M7 indicates linear initialization for 12 vertical modes followed by four iterations of the Machenhauer procedure for the first seven vertical modes. Figure 2 describes the model-generated surface divergence for the initialized experiments L12 + 4M5, L12 + 4M7, L12 + 4M9 and L12 + 4M11, as well as the nonadjusted EDSM. When only five vertical modes are initialized using the Machenhauer procedure, the initial boundary-layer divergence is barely 5.2% of the exact solution and oscillates substantially during the forecast. There is only minor improvement over the L12 + 4M5 result when seven and then nine vertical modes are included. When the L12 + 4M11 NMI is applied, however, the boundary-layer divergence differs by only 2.8% from the exact value and the integration with this initial condition shows only small-amplitude oscillations of the surface divergence in time. The L12 + 4M12 experiment (not illustrated) is still closer to the exact solution, with an initial boundary-layer divergence only 1.9% different from the exact value. The surface divergence values are presented in Table 1.

These results appear favorable if only the L12 + 4M11 boundary-layer divergence is examined. However, when we examine the divergence at level 5, which is near 500 mb, we find that the divergence oscillates substantially, (Fig. 3). A significant improvement at this level is noted for the case L12 + 4M12, indicating that all 12 modes must be included in the initialization to properly account for the friction circulation. Consequently, we will include all 12 vertical modes in the remaining initialization experiments.

We now examine the question of how many iterations the Machenhauer procedure requires. With only one nonlinear iteration following the linear step (experiment L12 + 1M12), the initial boundary-layer divergence is within 1.3% of the exact solution but shows some small-amplitude oscillations during the forecast. However, as noted from Fig. 3 the divergence of this experiment at level 5 shows a substantial high frequency oscillation. When an additional iteration is applied, the boundary-layer divergence does not

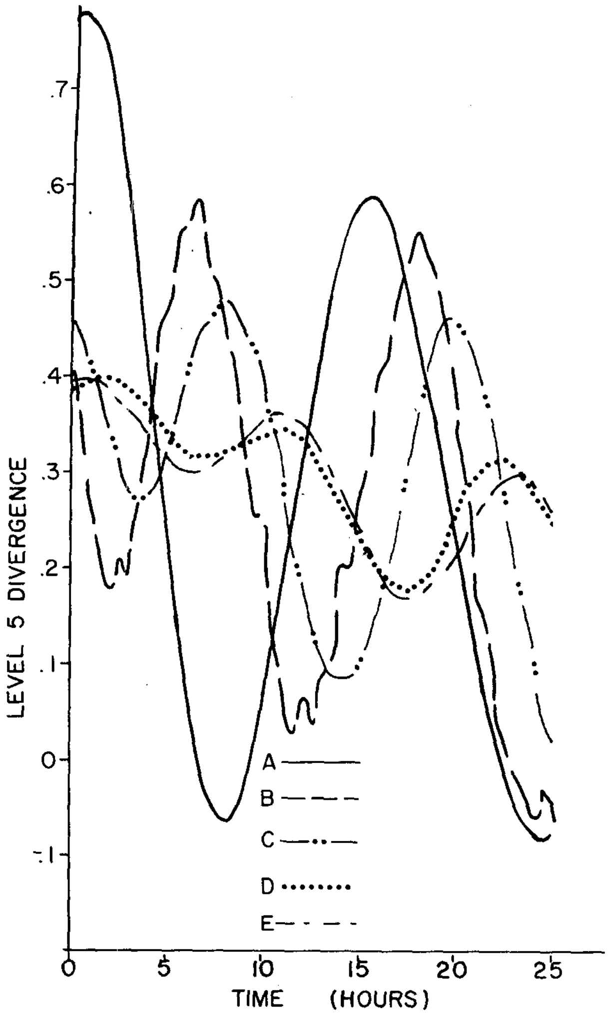

FIG. 3. Level 5 divergence versus time for: A, L12 + 4M11; B, L12 + 1M12; C, L12 + 2M12; D, L12 + 4M12; E, exact + 2M12.

oscillate during the forecast but the level 5 divergence is only somewhat less noisy than the L12 + 1M12 experiment (see Fig. 3). Figure 3 shows that four iterations of the Machenhauer procedure result in more suppression of noise than do two iterations; additional iterations yield little change, however.

The friction coefficient chosen for these experiments, $K = 0.25f$, although hardly small still satisfies the rationale for NMI based on an expansion in small Rossby number. However, that coefficient could become larger in some physical problems, and thus we investigate this possibility. We first note that if $K = 0.5f$, the results of the Machenhauer NMI are similar to those just described, but numerically the performance is worse. Previously, the L12 + 4M12 experiment resulted in a 1.9% difference between the boundary-layer divergence and the EDSM, whereas with $K = 0.5f$ that difference becomes 7.9%. When the friction is increased to $K = f$, the initialization still converges, but more slowly. With such large values of friction, four iterations of the Machenhauer procedure may not give a good approximation of vanishing gravity-mode tendencies; however, since vanishing gravity-mode tendencies are not the correct physical solution here and because such initialization might diverge with stronger friction, one could question whether more iterations are safe and/or worthwhile. The L12 + 12M12 experiment results in a 16.7% difference in boundary-layer divergence, and the amplitude oscillations in the level 5 divergence are almost as large as the divergence itself. When K is increased to $1.5f$, the Machenhauer procedure diverges. This is exemplified by noting that the L12 + 2M12 experiment results in a boundary-layer divergence error of 35.6% while the L12 + 6M12 experiment results in a 112% error plus a boundary-layer vorticity of the wrong sign.

The experiments described so far incorporate a value of C representing a relatively small length scale. If we choose C corresponding to a larger length scale, specifically $C = 0.76 \times 10^{-13}\ \mathrm{m}^{-2}$, representing a spherical harmonic of $n = 5$, we find that one iteration of the Machenhauer procedure yields reasonably good results. For example, with $K = 0.25f$, experiment L12 + 1M12 produces level 1 and 5 divergences that have the proper sign and only small-amplitude oscillations of order 10% in time. Further iterations yield better but not perfect results. Despite the imperfections of such an initialization, it is probably satisfactory for practical purposes. Since the Machenhauer procedure seems adequate for initializing friction circulations on the larger scales, we will examine other NMI experiments only for the case where $C = 2.29 \times 10^{-11}\ \mathrm{m}^{-2}$; i.e., the shorter scales.

In the above experiments using Machenhauer initialization, we began with the slow-mode projection of the EDSM and examined whether initialization could regenerate the correct ageostrophic circulations. If instead we apply a small number of iterations of the Machenhauer initialization directly to the EDSM, we find that the results are similar to those obtained using several iterations of the initialization after performing linear initialization. Since the EDSM has an exponential time decay, its gravitational tendency is small but nonzero; therefore, the Machenhauer scheme produces changes in attempting to make the gravitational tendencies zero.

b. *Baer–Tribbia NMI*

We now present a limited number of experiments utilizing the NMI technique of Baer and Tribbia (1977). Whereas the Machenhauer initialization uses an iterative approach to make the gravity-mode tendencies zero, the Baer–Tribbia initialization uses an expansion technique designed to give initial values of gravity modes with small but nonzero tendencies so that the modes evolve with no fast time behavior.

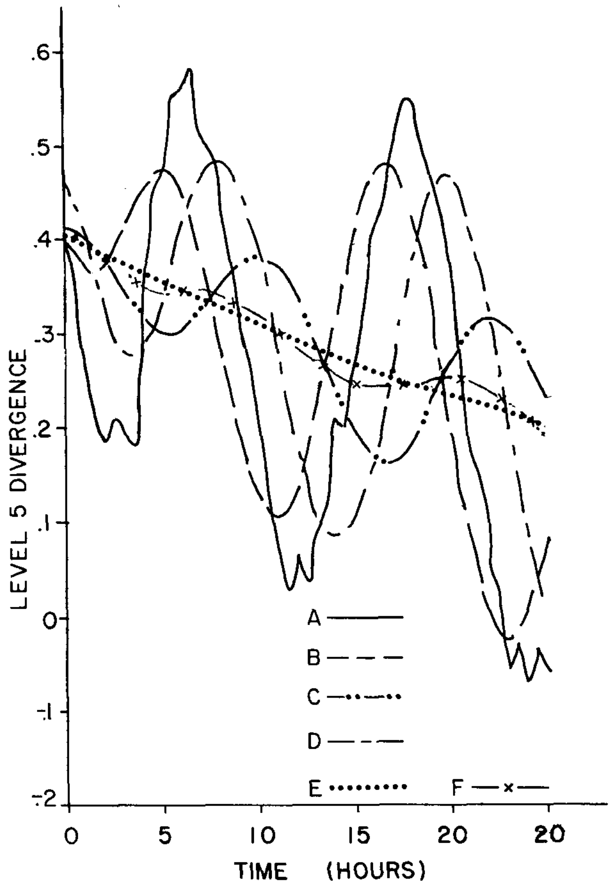

FIG. 4. Level 5 divergence versus time for: A, 1BT12; B, 2BT12; C, L12 + 2M12; D, L12 + $2\ddot{Y}12$; E, L12 + $6\ddot{Y}12$; F, 2BT'12.

Since the exact damped slow mode has a nonzero tendency, it may be that the Baer–Tribbia scheme will yield better results than a scheme that forces the gravity-wave tendencies to zero. We define a notation of experiments such that 1BT12 denotes first-order Baer–Tribbia initialization for 12 vertical modes, 2BT6 represents second-order initialization for six vertical modes, and so on.

Consider now the 1BT12 experiment with $K = 0.25f$ applied to the EDSM. Since the 1BT12 procedure is identical to L12 + 1M12 discussed previously, the forecast for the 1BT12 initialization results in boundary-layer divergence with small oscillations in time, but with divergence at level 5 oscillating sufficiently to change sign (Fig. 4).

The second-order initialization experiment (2BT12) provides results improved over the first-order experiment, but still not satisfactory. This may be seen by comparing the boundary-layer differences listed in Table 2. Note that although the difference $\delta\phi$ is smaller with second-order initialization, $\delta\xi$ is larger. Additionally, the level 5 divergence (Fig. 4) is less noisy with second-order initialization, but oscillates more than the results using L12 + 2M12 initialization.

The only difference between the 2BT12 and L12 + 2M12 initializations as applied herein is that the 2BT12 procedure includes the term $1/(i\omega)\partial Y^{(1)}/\partial \mathrm{T}$, where $Y^{(1)}$ represents the first-order gravity-mode correction, ω is the mode's frequency, and T describes the slow time variable. For our simple model, the above term can be expressed as $-(K/i\omega)Y^{(1)}$. This term allows the initialized gravity modes to have a small but natural nonzero tendency, whereas the Machenhauer constraint forces the gravity-mode tendencies to zero. Since the exact solution, the EDSM, has a nonzero tendency due to damping, we expect the Machenhauer procedure to have some error. Letting Z represent the EDSM, we find for the values of f and C used here that $\partial Z/\partial t \approx -\frac{1}{4}KZ$; i.e., the EDSM decays with a damping factor of roughly $\frac{1}{4}K$.

The 2BT12 initialization results in $\partial Y/\partial t \sim -KY$; this appears to be too large. To test this hypothesis, we modify the 2BT12 initialization such that the second-order term, $1/(i\omega)\partial Y^{(1)}/\partial \mathrm{T}$, is replaced by one fourth of its value. Although this test has no theoretical justification, the EDSM does tend to decay approximately as $\exp(-Kt/4)$. Letting the notation 2BT'12 describe this modified initialization, we see from Fig. 4, with $K = 0.25f$, that the 2BT'12 initialization results in a very smooth solution. Furthermore, the boundary-layer differences are small (Table 2). Evidently a higher order must be used with the BT12 initialization in order to obtain results superior to the iterated Machenhauer initialization.

Increasing the friction coefficient with the Baer–Tribbia initialization destabilizes the results in a way similar to that noted using the Machenhauer scheme. With $K = 0.5f$, experiment 1BT12 shows increased surface differences from the exact solution (Table 2) when compared to the same experiment with $K = 0.25f$. In addition, small oscillations show up in the time behavior of the boundary-layer divergence, and the level 5 divergence oscillates with changing sign. Not much improvement is noted in Table 2 when the second-order procedure (2BT12), is added,

TABLE 2. Differences (percent) from the exact values of the surface divergence ΔD, vorticity $\delta\xi$ and geopotential $\delta\phi$ for various initialization experiments and two values of the friction coefficient K/F.

Experiment	K/F	δD	$\delta\xi$	$\delta\phi$
1BT12	0.25	1.3	2.2	1.0
2BT12	0.25	1.3	2.8	0.44
2BT'12	0.25	1.3	0.14	0.03
L12 + $6\ddot{Y}12$	0.25	0.59	0.01	0.002
1BT12	0.5	5.6	9.1	4.0
2BT12	0.5	5.6	11.9	1.6
2BT'12	0.5	5.6	0.67	0.25
L12 + 6M12 + $6\ddot{Y}12$	0.5	0.37	0.17	0.07

and the oscillation in the level 5 divergence also is not substantially diminished. However, as previously noted from the $K = 0.25f$ case, experiment 2BT'12 does show some improvement in the boundary layer, and the level 5 divergence remains positive with small oscillations in time. Thus we may conclude that the results with the Baer–Tribbia scheme are consistent but less successful as the friction coefficient is increased.

c. Bounded derivative initialization

Although it would be interesting to perform a higher-order Baer–Tribbia initialization, we do not attempt that here because of the complexity of the calculations involved. Instead we will investigate an iterative procedure that is similar to second-order Baer–Tribbia initialization. If the technique of Browning *et al.* (1980) were applied to the model equations in normal-mode form, the results would indicate that requiring higher derivatives (and in particular the second time derivatives) of the gravity modes to vanish is superior to having the first time derivatives vanish. Leith (1980) supports this contention by pointing out that such initialization is equivalent through second order to Baer–Tribbia initialization. We explore this procedure with a tendency equation written as $\dot{Y} = i\omega Y + N$, where Y represents a gravity mode with frequency ω, and N represents all tendency terms not included in $i\omega Y$. The second-order equation in time becomes $\ddot{Y} = -\omega^2 Y + i\omega N + \partial N/\partial t$. Following a method of iteration similar to that used in the Machenhauer initialization procedure, we attempt to converge on the condition $\ddot{Y} = 0$. This is done by calculating $\ddot{Y}$ for a given value of the modes Y and then adjusting Y incrementally by δY, where $\delta Y = \ddot{Y}/\omega^2$. As with the Machenhauer scheme (which meets the condition that $\dot{Y} = 0$), this method converges for reasonable values of the variables. To test the scheme, we establish experiments with a notation similar to that used for our previously described experiments; i.e., for an experiment denoted $4\ddot{Y}12$, we apply four iterations to all 12 vertical modes.

Consider now experiments with $K = 0.25f$. Application of L2 + $2\ddot{Y}12$ to the EDSM produces results with substantial oscillations in the level 5 divergence, whereas an increase in the number of iterations to six (experiment L12 + $6\ddot{Y}12$) produces a very smooth level 5 divergence field in time, as may be seen from Fig. 4. Furthermore, the boundary-layer differences from the exact solutions of the dependent variables for this latter experiment are quite acceptable, being smaller than those from 2BT12 (as noted in Table 2). Additional iterations yield small changes in initial states and produce smooth time integrations. It appears that the $\ddot{Y}12$ procedure requires a number of iterations to converge, but also requires a reasonably balanced state initially. This was confirmed by applying $\ddot{Y}12$ directly to the EDSM and finding smooth solutions for any number of iterations. Additionally, a carefully prepared experiment (L12 + 2M12 + $2\ddot{Y}12$) for initial balance also produces a very smooth and accurate solution.

Application of the $\ddot{Y}12$ initialization with stronger friction ($K = 0.5f$) produces results that are barely convergent. The L12 + $6\ddot{Y}12$ experiment results in improved balance over the experiment with linear initialization, but the level 5 divergence oscillates with change of sign during the forecast. By combining methods and initializing with L12 + 6M12 + $6\ddot{Y}12$, the results are much improved, with the level 5 divergence showing only one small departure from exponential decay, and only small boundary-layer differences (see Table 2). Note that these differences are significantly smaller than those produced by the Machenhauer and second-order Baer–Tribbia initializations. It appears that L12 + 6M12 is a better starting point for the $\ddot{Y}12$ initialization than simply using L12. We note that application of the $\ddot{Y}12$ initialization directly to the EDSM without any linear step results in very accurate conditions, indicating that the $\ddot{Y}12$ procedure does not seriously disturb the exact solution. Furthermore, this suggests that the condition $\ddot{Y} = 0$ is superior to $\dot{Y} = 0$, although probably more difficult to achieve.

When we increase K to $0.75f$, we find that the $\ddot{Y}12$ procedure diverges quickly and gives poor results within a few iterations after applying the L12 initialization to the EDSM. By contrast, we recall that the Machenhauer procedure can converge with K somewhat larger than f, yielding a reasonable although not exact solution.

4. Discussion

Experiments with our simple friction model indicate that normal-mode initialization with the Machenhauer procedure requires the use of most if not all of the vertical modes to adequately initialize the friction circulation. Furthermore, several iterations are required, of which four seem to be the optimum number; i.e., additional iterations beyond four yield only marginal improvement. Tests appropriate to midlatitudes indicate that this procedure is convergent for all magnitudes of the friction coefficient not significantly exceeding the Coriolis parameter. In general, the results show a reasonable but not exact balance of the initial fields. This disparity can be explained by the constraint of vanishing gravity-mode tendency imposed by the Machenhauer scheme, whereas the true solution for the gravitational modes has a small nonvanishing tendency due to exponential time decay.

Application of the Baer–Tribbia method to second order with our friction model does not produce results that are as accurate as those obtained using the Machenhauer scheme with several iterations. This result suggests that the condition of vanishing gravity-mode tendencies gives a closer approximation to the exact solution than does utilizing those tendencies inherent in the second-order B–T scheme.

Another initialization scheme based on the bounded derivative method was also tested with our friction model. This scheme adjusts gravity modes initially with the constraint that each mode's second time derivative vanish. Interestingly, it can be shown that this scheme and the B–T procedure are equivalent to second order, although the former uses an iterative approach. Experiments with the scheme yield results that are better than those from the other two techniques tested, and are limited only by a greater sensitivity of the iterative procedure to the magnitude of the friction coefficient. It seems evident that the constraint of a vanishing second time-derivative for the gravity modes when forced by friction approximates the exact solution more closely than a requirement that the tendency vanish.

Both the second-order B–T initialization and the Machenhauer initialization with one or two iterations yield results that when applied to a friction model are less satisfactory than expected. The following discussion may give some insight into this failure. When the Machenhauer procedure is applied to our simple friction model with a geostrophic and nondivergent initial state, the first iteration adjusts only the divergence in response to the nonzero friction-induced vorticity tendency, with the other tendencies being zero. This divergence change results in a friction-induced nonzero divergence tendency. The next iteration responds to this tendency by adjustments to the vorticity and height fields. Additional iterations create further adjustments of the variable fields in this alternating pattern. Indeed, the same pattern appears as higher orders of the B–T scheme are applied. Thus we note that two iterations of the Machenhauer procedure or second-order B–T initialization involve only one adjustment in the divergence field followed by one adjustment in the vorticity and height. The correct balanced solution with friction cannot be achieved with such limited adjustment.

The achievement of adequate initialization of ageostrophic circulations by normal-mode initialization is likely to be more difficult with comprehensive numerical weather prediction models than with the simple model we have studied. Our model did not include the tropics, nonlinear friction or dynamics, and other physical processes. Although several iterations of the Machenhauer procedure with all vertical modes gave reasonable results, the technique could likely diverge when applied to more complex weather prediction models. Convergence of the iterative process in the initialization of these models when the bounded derivative procedure with vanishing second time-derivatives of the gravity modes is applied, might also prove difficult. It appears that generation of a satisfactory ageostrophic initial state arising from surface friction will require better high-order initialization schemes for all vertical modes. However, it may be difficult to successfully apply such advanced schemes in the presence of nonlinear dynamics. Considerable research will be needed to solve this problem. Moreover, the benefits of a successful solution will far outweigh the difficulties and the effort is strongly encouraged.

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