Synoptic Scale Disturbances with Circular Symmetry

ALAN J. THORPE

Department of Meteorology, University of Reading, Reading, RG6 2AU, U.K.

(Manuscript received 28 August 1985, in final form 3 February 1986)

ABSTRACT

The balanced flow structure of various classic synoptic-scale disturbances is reviewed using the invertibility principle for isentropic potential vorticity (IPV) distributions. Complete solutions are shown for cold and warm core structures of various types. The basic model imagines the tropopause to be the interface between the lower potential vorticity of the troposphere and the approximately six-fold larger value typical of the lower stratosphere. The sensitivity of the structure to the potential temperature variation along the tropopause and at the surface is described. Results are presented in diagrammatic form to allow easy diagnosis of the vortex structure from synoptic data available at perhaps only a few levels. The point is made that upper air IPV and surface potential temperature distributions are often the most crucial in accounting for the balanced flow structure.

1. Introduction

On occasion, the atmosphere exhibits structures that are nearly axisymmetric at all levels. Examples are cut-off lows, blocking highs, and tropical cyclones. Furthermore, in a large number of cases the flow can be understood in terms of a more or less linear superposition of such a symmetric structure and a simple mean flow. A concise way of describing these disturbances in terms of simple dynamical concepts has recently been reviewed by Hoskins et al. (1985). They concentrate on the isentropic potential vorticity (IPV) as the fundamental dynamical entity in synoptic scale and other balanced flows. In particular they describe the invertibility principle, which states that if the spatial distribution of IPV is known and the flow satisfies a balance condition, such as geostrophic or gradient wind balance, then it is possible, in principle, to determine the details of that balanced state. This principle was adumbrated in earlier papers by Eliassen and Kleinschmidt (1957).

In certain respects this approach is the inverse of that traditionally used by synopticians for the determination of the (quasi-geostrophic) vorticity from the horizontal Laplacian of the geopotential height contours. This inverse problem of specifying the higher order variable, IPV, and determining the primary variables $\mathbf{v}$, θ, and ϕ is particularly illuminating because the IPV is often a conserved quantity following the motion of an air parcel. (Nonconservation occurs in the presence of diabatic and frictional sources and sinks. Even in such circumstances the volume integral of the mass-weighted IPV can only be changed by boundary fluxes. Furthermore, in saturated flow the moist PV based on the equivalent potential temperature is conserved.) Also, it is known that the troposphere on the synoptic scale can be considered to have only a small horizontal variation of IPV with the largest contrast in IPV being across the tropopause. It is therefore often realistic to describe synoptic scale disturbances as being characterized mainly by two regions of air—the low IPV of the troposphere and the high IPV of the stratosphere. This simplifying view of the structure of such disturbances is the main power of the invertibility approach.

In this paper, which is in part a summary of aspects of other papers, a small catalog of axisymmetric structures will be shown in the form of figures to illustrate the insight to be derived from these ideas. The limitation to symmetric flows is *not essential* to the invertibility concept. Nor is geostrophic balance: the flows illustrated here satisfy the more accurate gradient wind balance condition and so cover a wider class of phenomena than those describable in terms of geostrophic balance. The mathematical details will not be described in this paper but can be found in Shutts and Thorpe (1978), and Thorpe (1985a), as well as in Hoskins et al. (1985). The particular approach to be followed here will be the same as that described in Thorpe (1985a). That paper considered the application to tropical cyclones, whereas here we will describe structures more applicable to midlatitudes.

2. Invertibility principle

We imagine an axisymmetric flow in gradient wind and hydrostatic balance. It is then possible to derive the following inversion equation expressing the invertibility principle for the balanced structure of such a vortex:

$$\frac{1}{f^2}\frac{\partial^2\Phi}{\partial R^2}+\left(1+\frac{2}{Rf^2}\frac{\partial\Phi}{\partial R}\right)^2\frac{1}{\rho q}\frac{\partial^2\Phi}{\partial Z^2}-\frac{3}{Rf^2}\frac{\partial\Phi}{\partial R}=1$$

where

$\Phi = \phi + \frac{1}{2}v^2$	potential function
$q = \frac{g}{f\theta_0}P,\ P = \frac{\zeta \cdot \nabla\theta}{\rho}$	potential vorticity
$\theta = \frac{\theta_0}{g}\phi_z$	potential temperature
θ_0	a constant reference value of θ
$R = r\left(1 + \frac{2v}{fr}\right)^{1/2}$	transformed radius
$Z = z$	transformed height
$v = \frac{r}{Rf}\frac{\partial\Phi}{\partial R}$	gradient wind
ϕ	geopotential
$\zeta = \nabla \times \mathbf{v} + f\mathbf{k}$	absolute vorticity.

This equation results from a coordinate transformation from (r, z) to (R, Z). The mathematical details of the transformation can be found in Schubert and Hack (1983), following earlier work of Shutts and Thorpe (1978). Note that the equality between z and Z does not imply equality between the derivatives $(\partial/\partial Z)_R$ and $(\partial/\partial z)_r$. This inversion equation is one of a set of equations describing the evolution of the flow in terms of PV, which can be found in Thorpe (1985a). The solutions to be presented here are shown in physical coordinates, the inverse transformation having already been made. A less concise form for the inversion equation exists in isentropic coordinates, see Hoskins et al. (1985). It should be stressed that the inversion equation allows diagnosis of the balanced structure of any (time-dependent) vortex just as the omega equation allows diagnosis of the ageostrophic components.

In this paper we shall assume the two fluid models alluded to earlier in which the troposphere is characterized by a constant value of $\rho q = 10^{-4}$ s^{-2} and the stratosphere is characterized by a constant value of $\rho q = 6 \times 10^{-4}$ s^{-2}. In the terminology of Hoskins et al. (1985), these values correspond to $P = 0.3$ units at the surface and $P = 7.5$ units just above the tropopause (1 unit = 10^{-6} m^2 s^{-1} K kg^{-1}). Note that a constant value of ρq implies that P varies with height as the inverse of the density. Given these positive values of q the inversion equation is elliptic and can be solved numerically given suitable boundary conditions. Hence we obtain $\Phi(R, Z)$, from which all other balanced fields in physical space can be evaluated using the above defining equations and by performing the inverse transformation to physical coordinates. In these circumstances the solution to the above inversion equation is determined by specifying the potential temperature distribution on the tropopause and on the horizontal and vertical boundaries of the domain containing the disturbance.

For simplicity we take the boundaries of the domain to be $z = 0$ km and 16.7 km and $r = 6000$ km. At the upper boundary, θ is a constant, and at the lateral boundary the tropopause is assumed to be at $z = 10$ km with the appropriate values of $\partial\theta/\partial z$ there consistent with $\zeta = f\mathbf{k}$ at this radius. The various structures to be shown will therefore depend only on the assumed θ variations on the tropopause and at the ground. Note that the position of the tropopause is found as a result of the inversion and is not specified a priori. This approach directly prescribes variations in PV *along isentropes*. The alternative is to prescribe a geographical distribution of PV without reference to isentropes. In that case the tropopause position would be specified and the isentropic gradients along the tropopause would be deduced as a result of the inversion. These variations will be taken to be

$$\theta_s' = \Delta\theta_s \cos^2(\pi R/2R_0), \quad R < R_0$$

$$\theta_s' = 0, \quad R \geqslant R_0$$

$$\theta_t' = \Delta\theta_t/[1 + (2R/R_0)^2],$$

where suffices s and t refer to the surface and tropopause respectively, and where a prime indicates deviation from the values when r is large. A value of $R_0 = 1667$ km has been used. Thus the structures will depend only on the amplitudes $\Delta\theta_t$ and $\Delta\theta_s$.

3. Disturbance structure

The results will be shown as cross sections of θ' and v, and of ζ_r/f and h'. Here ζ_r is the vertical component of the relative vorticity vector, $\theta' = \theta - \theta_0$, and $h' = (\phi - \phi(r_1, z))/g$ where $r_1 = 6000$ km. Thus θ' is the potential temperature deviation from a constant reference value and h' is the geopotential height deviation from that at the lateral boundary of the domain. Therefore it can be shown that

$$\phi(r_1, z) = gz + \frac{g}{\theta_0}\int_0^z \theta'(r_1, z)\, dz,$$

where $\theta'(r_1, z)$ is a simple function of height due to the constant, but different, gradients of θ' in the troposphere and stratosphere at the lateral boundary.

The disturbances to be shown can be considered in three categories: those with $(\Delta\theta_s = 0, \Delta\theta_t \neq 0)$, $(\Delta\theta_s \neq 0, \Delta\theta_t = 0)$, and $(\Delta\theta_s \neq 0, \Delta\theta \neq 0)$. In particular the following nomenclature can be used for vortices with

$\Delta\theta_s = 0,$	$\Delta\theta_t < 0$	cold core, upper cyclone
$\Delta\theta_s = 0,$	$\Delta\theta_t > 0$	warm core, upper anticyclone
$\Delta\theta_s < 0,$	$\Delta\theta_t = 0$	cold core, lower anticyclone
$\Delta\theta_s > 0,$	$\Delta\theta_t = 0$	warm core, lower cyclone
$\Delta\theta_s < 0,$	$\Delta\theta_t < 0$	cold core: upper cyclone, lower anticyclone
$\Delta\theta_s > 0,$	$\Delta\theta_t > 0$	warm core: upper anticyclone, lower cyclone
$\Delta\theta_s < 0,$	$\Delta\theta_t > 0$	anticyclone: lower cold, upper warm core
$\Delta\theta_s > 0,$	$\Delta\theta_t < 0$	cyclone: lower warm, upper cold core.

A few salient features apparent from the figures are as follows.

(i) Disturbances with cold or warm tropopauses (i.e., $\Delta\theta_t \neq 0$ as in Figs. 1, 2, 5, 6, 7 and 8) have core thermal anomalies which change sign at the level of the jet core. The anomaly signs described previously refer to heights below this level.

(ii) The characteristic shape of isentropes which pass through an elevated or depressed tropopause should be noted. For example, in the case of Fig. 1, an upper troposphere isentrope slopes up to meet the tropopause, then dips down in the core of the stratospheric "tongue." Such an isentrope defines the cutoff nature of the structure, in that there is a PV anomaly along an isentrope which in adiabatic frictionless flow cannot be reunited with its stratospheric reservoir. The same comments apply to the blocking high cases, for example, Fig. 2. This characteristic isentrope shape also implies that at a constant height or pressure level one finds an annulus of low/high θ at the tropopause.

(iii) Classic warm and cold core structures in which the relative vorticity changes sign with height require *both* tropopause and surface thermal anomalies to coexist. If only one of these anomalies is nonzero, then, although the relative vorticity changes with height according to the thermal structure, it cannot change sign.

(iv) As in Hoskins et al. (1985), these flows can be discussed in terms of the PV anomalies on isentropes with their associated vorticity and static stability anomalies. For example, one sees much reduced tropospheric static stability in the case described in Fig. 6, which in the presence of a suitable moisture source might lead to increased convective activity.

(v) It should be noted that these structures are representative of synoptic scale disturbances for the assumed values of PV. Diabatic and frictional processes will change the PV locally and so, for example, after the onset of deep convection one would expect the stratospheric protrusion of Fig. 1 to be eroded. The importance of these sources/sinks of PV is brought out in Thorpe (1985a), where the role of tropospheric variations of PV is discussed in terms of the intensity of tropical cyclone circulations.

(vi) Although the systems are referred to as synoptic scale, the dynamics would suggest that this may be misleading. The local Rossby number can be of order 0.4, and there are significant variations in static stability consistent with the fact that the structure of the disturbances would not be well represented by the quasi-geostrophic approximation. The analysis leading to the inversion equation is a generalization of the two-dimensional semigeostrophic approximation to curved flows. Some of these disturbances are therefore perhaps better thought of as mesoscale from a dynamical viewpoint.

A consequence of these statements is that the cross sections derived from observations, with which to compare the structures described in this paper, are few in number. In particular, nearly all observational studies have considered cyclonic vortices, e.g., Palmen (1949), Peltonen (1963) and Shapiro (1978). It is clear from the results presented here that most of the observed features can be accounted for using the invertibility principle. There is a need for observational cross sections through anticyclonic vortices, as well as for the more complicated cyclonic structures, such as those described by Figs. 5, 6, 7 and 8.

(vii) As shown in Hoskins et al. (1985), cutoff regions of PV typically arise after a period of advection of a wave along the tropopause (see Fig. 5 in that paper). Before the cutoff occurs, a thin filament of PV connecting the disturbance with the stratospheric reservoir is evident. It is possible to use a two-dimensional inversion equation to understand the structure of this filament. This equation can be obtained from that quoted here by taking the limit of R large and $r/R \rightarrow 1$ for small isobar curvature.

(viii) Intercomparison of these structures allows one to assess the extent to which the contributions to the structure from upper IPV and surface temperature anomalies can be linearly, or otherwise, superposed. Comparing the cold core, upper cyclone case (Fig. 1) with the warm core, upper anticyclone case (Fig. 2) shows that the surface pressure anomalies associated with blocking ridges are significantly less than those for cutoff lows. The sum of their surface pressure deviations with those for the opposite sign surface thermal anomalies (Fig. 4 and Fig. 3 respectively) almost exactly produces the surface pressure deviation of the composites (Fig. 6 and Fig. 8 respectively). However, this linear superposition is not so good for tropopause and surface thermal anomalies of the same sign. Such attempts to superpose the contributions from the "building blocks" of the potential vorticity distribution will help to give insight into the quantitative differences between structures, such as blocking highs and cutoff lows, which appear, from these arguments, to be closely related from a dynamical viewpoint.

4. Discussion

A set of representative structures have been presented using the simplifying notion that the atmosphere can be thought of as having constant ρP in the troposphere and a larger constant ρP in the stratosphere. No attempt has been made to choose parameters to compare with particular observed cases; rather, the parameters have been chosen to give insight into the sensitivity of the structures to the variations of temperature at the surface and at the tropopause. It is clear, for example, that the representation of the tropopause in any numerical weather prediction model is a factor of crucial importance if the synoptic scale flow is to be described accurately. An improved representation might result from increased vertical resolution near the tropopause in current models or from the use of isentropic coordinates.

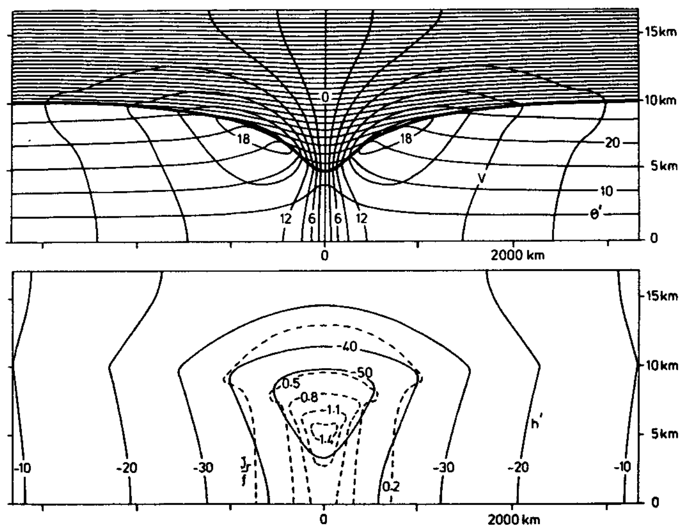

FIG. 1. Cold core, upper cyclone: $\Delta\theta_s = 0$. $\Delta\theta_t = -24$ K. In this and other figures v is in m s^{-1}, θ' in K, h' in dam, the tropopause position is shown by the bold solid line, and the label 0 on the horizontal axis indicates the core (and axis of symmetry) of the disturbance. The equivalent pressure deviation at the surface in the center of the vortex is $p_0' = -47$ mb.

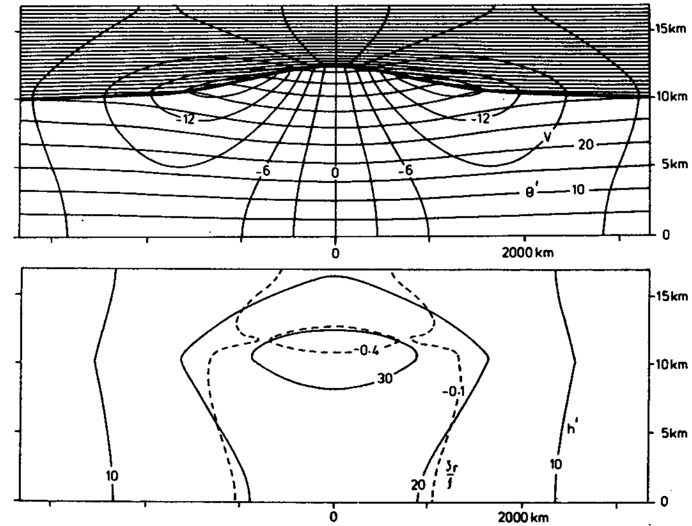

FIG. 2. Warm core, upper anticyclone: $\Delta\theta_s = 0$, $\Delta\theta_t = 24$ K. Here p_0' = +22 mb.

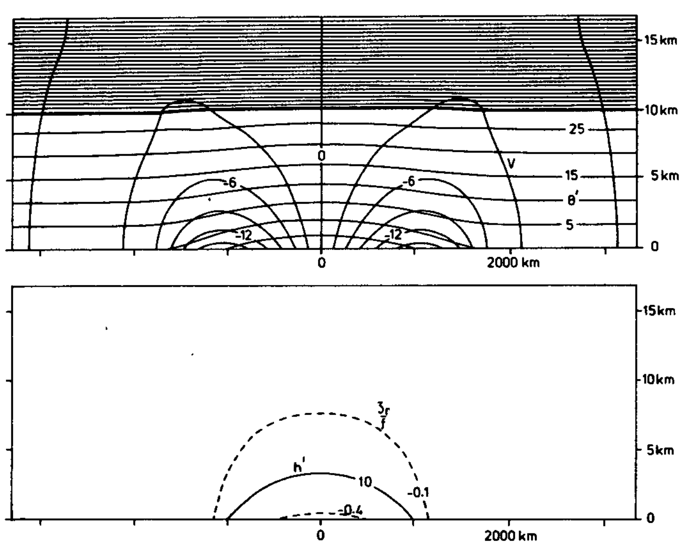

FIG. 3. Cold core, lower anticyclone: $\Delta\theta_s = -10$ K, $\Delta\theta_t = 0$ K. Here p_0' = 18 mb.

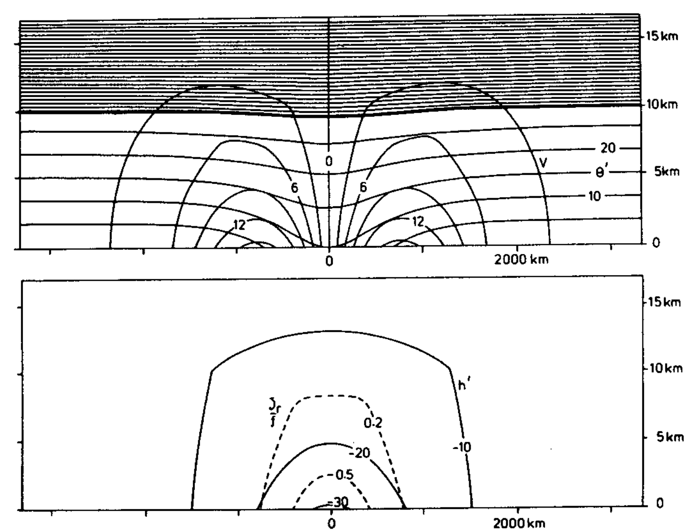

FIG. 4. Warm core, lower cyclone: $\Delta\theta_s = +10$ K, $\Delta\theta_t = 0$ K. Here p_0' = −31 mb.

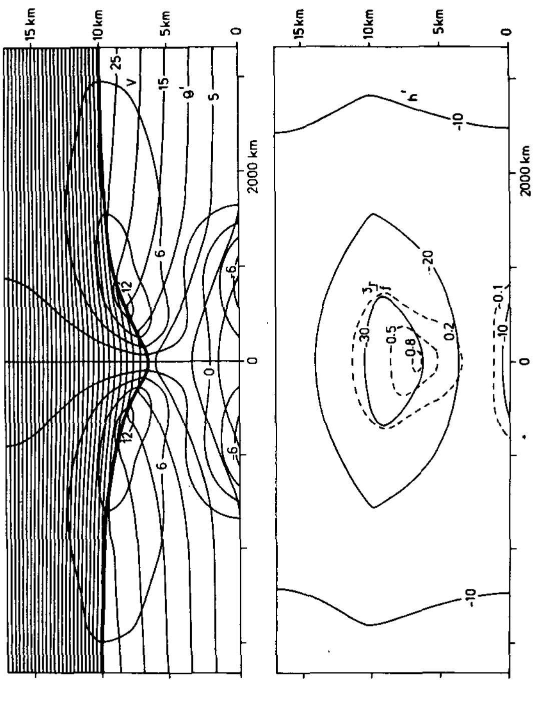

FIG. 5. Cold core, upper cyclone, lower anticyclone: $\Delta\theta_s = -10$ K, $\Delta\theta_t = -24$ K. Here $p_0' = -8$ mb.

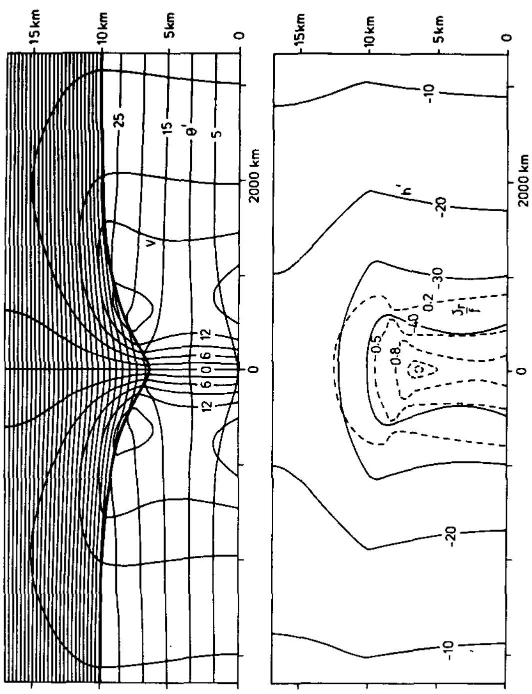

FIG. 6. Cyclone, lower warm, upper cold core: $\Delta\theta_s = +5$ K, $\Delta\theta_t = -16$ K. Here $p_0' = -46$ mb.

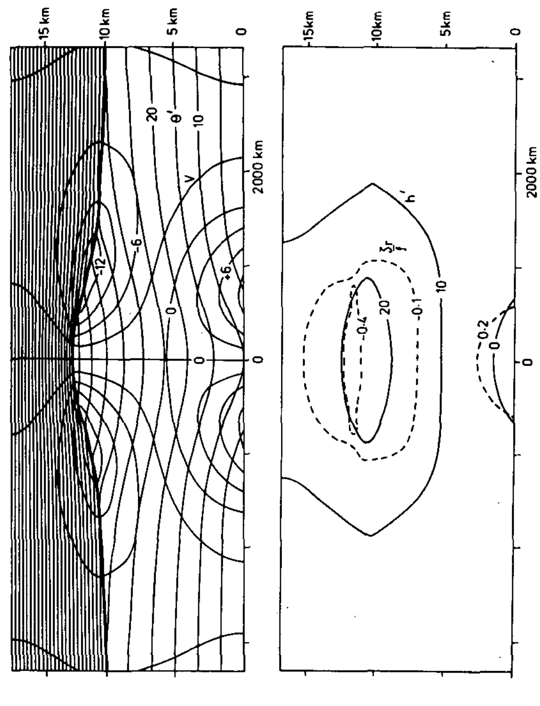

FIG. 7. Warm core, upper anticyclone, lower cyclone: $\Delta\theta_s = +10$ K, $\Delta\theta_t = +24$ K. Here $p_0' = -5$ mb.

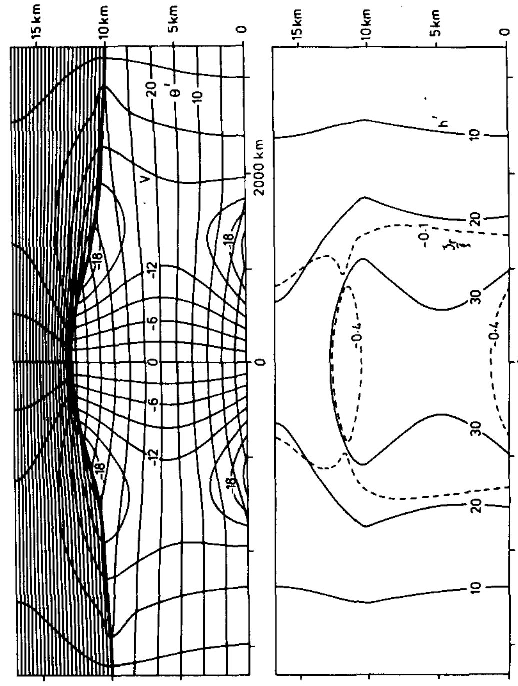

FIG. 8. Anticyclone, lower cold, upper warm core: $\Delta\theta_s = -10$ K, $\Delta\theta_t = +24$ K. Here $p_0' = +39$ mb.

Using this formulation, it is also possible to develop equations describing the ageostrophic (or rather agradient) flow and the changes in the PV following the motion (see Thorpe, 1985a, and Hoskins et al., 1985). Even from the balanced flow structures themselves one can deduce some of the preference for enhanced or suppressed convective activity. Furthermore, if, for example, an upper cutoff feature is moving relative to the lower level flow, then the invertibility principle predicts ascent upstream and descent downstream of the feature: that is, ascent occurs in regions of positive PV advection that increase with height.

The changes in PV due to diabatic and frictional process will erode these structures over representative timescales. Despite the complicated smaller-scale structure and dynamics of these processes, it is possible to make some simple deductions of their role in modifying the synoptic scale structure. For example, if convection occurs, it can be shown that a decrease (increase) in PV will take place above (below) the maximum latent heating, subject, however, to the constraint that the volume integral of ρP has to remain a constant. Therefore, convection will tend to erode the upper tropospheric anomaly in PV while generating a lower tropospheric anomaly in PV. The latter anomaly might plausibly be expected to be eroded itself by frictional effects within the planetary boundary layer. An example of the changes in PV produced by moist slantwise convection at a front can be found in Thorpe and Emanuel (1985).

In principle, other boundary conditions at the surface can be incorporated into the solution of the inversion equation. For example, an isothermal surface can easily be included and this, it is interesting to note, determines *both* the potential temperature and pressure deviations at the surface (see Thorpe, 1985b). In this case the scale of the vortex is solely determined by the tropopause temperature variations.

Acknowledgments. I would like to thank Dr. M. E. McIntyre and Mr. W. A. Norton for comments on the submitted version of this paper, which resulted in a more precise exposition of the invertibility principle in this revised version.

REFERENCES

Eliassen, A., and E. Kleinschmidt, Jr., 1957: Dynamic meteorology. *Handbuch der Physik.* Vol. 48, 1–154.

Hoskins, B. J., M. E. McIntyre and A. W. Robertson, 1985: On the use and significance of isentropic potential vorticity maps. *Quart. J. Roy. Meteor. Soc.,* **111,** 877–946.

Palmen, E., 1949: Origin and structure of high-level cyclones south of the maximum westerlies. *Tellus,* **1,** 22–31.

Peltonen, T., 1963: A case study of an intense upper cyclone over eastern and northern Europe in November 1959. *Geophysica,* **8,** 225–251.

Schubert, W. H., and J. J. Hack, 1983: Transformed Eliassen balanced vortex model. *J. Atmos. Sci.,* **40,** 1571–1583.

Shapiro, M. A., 1978: Further evidence of the mesoscale and turbulent structure of upper-level jet stream-frontal zone systems. *Mon. Wea. Rev.,* **106,** 1100–1111.

Shutts, G. J., and A. J. Thorpe, 1978: Some aspects of vortices in rotating stratified fluids. *Pure Appl. Geophys.,* **111,** 993–1006.

Thorpe, A. J., 1985a: Diagnosis of balanced vortex structure using potential vorticity. *J. Atmos. Sci.,* **42,** 397–406.

——, 1985b: Potential vorticity and the structure of tropical cyclones. *Preprints 16th Conf. on Hurricanes and Tropical Meteorology,* Houston, *Amer. Meteor. Soc.,* 113.

——, and Emanuel, K. A., 1985: Frontogenesis in the presence of small stability to slantwise convection. *J. Atmos. Sci.,* **42,** 1809–1824.