NOTES AND CORRESPONDENCE

A Simple Model of Over-forecasting*

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ABSTRACT

Probabilistic forecasts of the occurrence of precipitation have been used routinely in the United States since 1965. Studies of the reliability of such forecasts often show a tendency towards over-forecasting (i.e., for forecast probabilities to exceed observed relative frequencies). A simple model is described that explains over-forecasting in terms of an asymmetric loss function. The model is applied to some results of a previous study.

1. Introduction

Probabilistic forecasts of the occurrence of precipitation have been used routinely in the United States since 1965. A subjective precipitation probability forecast reflects the forecaster's degree of belief that measurable precipitation will fall during a specified period at a specified location. The comprehensive review article by Murphy and Winkler (1984) discusses the history of probability forecasting in meteorology, as well as other issues, and contains many important references.

Murphy and Winkler (1977) discuss the reliability of subjective precipitation probability forecasts. Reliability refers to the correspondence between forecast probabilities and observed relative frequencies of precipitation. Clemen and Murphy (1986) present some results from an experiment involving 6411 subjective precipitation probability forecasts for the Southeast National Weather Service area during the warm season, with 0000 cycle time, and 12–24 hours lead time. These results, which are plotted in Fig. 1, indicate that the forecasts are very reliable, in the sense that there is close correspondence between forecast probabilities and observed relative frequencies. However, as Clemen and Murphy (1986) point out, there is a tendency to over-forecast (i.e., for the forecast probabilities to exceed the relative frequencies). Over-forecasting is commonly seen in similar studies of forecast reliability. The purpose of this note is to present a simple model for over-forecasting.

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In the next section, we present the model. We apply this model to the results of Clemen and Murphy (1986) in section 3. Section 4 contains some concluding remarks.

2. A model for over-forecasting

The basic idea underlying our model for over-forecasting is that the probability forecast announced by the forecaster may be different from the forecaster's true subjective probability assessment.

Let R be the random variable defined by

$$R = \begin{cases} 1, & \text{if precipitation occurs} \\ 0, & \text{otherwise.} \end{cases}$$

Also, let π be the forecaster's subjective probability that $R = 1$ and let p be the forecaster's announced probability that $R = 1$. Finally, let:

$$L(p) = \begin{cases} L_0(p), & \text{if } R = 0 \\ L_1(p), & \text{if } R = 1 \end{cases}$$

be the loss associated with announced probability p. Note that this loss depends on the outcome of R. The forecaster chooses the optimal announced probability, p^*, by minimizing expected loss which is given by:

$$E(L(p)) = \pi L_1(p) + (1 - \pi)L_0(p). \tag{1}$$

The optimal announced probability, p^*, is the value of p that minimizes (1) over the unit interval.

We briefly consider three examples.

First, consider symmetric quadratic loss:

$$L_1(p) = A(1 - p)^2$$
$$L_0(p) = Ap^2$$

for $A > 0$. This loss function is symmetric in the sense that

$$L_1(p) = L_0(1 - p)$$

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(e.g., the loss associated with probability forecast 0.80 when precipitation occurs is equal to the loss associated with probability forecast 0.20 when precipitation does not occur). In this case, $p^* = \pi$, and the forecaster's announced forecast is the same as the forecaster's true subjective probability assessment.

Second, consider general linear loss:

$$L_1(p) = A(1 - p)$$

$$L_0(p) = Bp$$

for $A, B > 0$. In this case

$$p^* = \begin{cases} 1, & \text{if } B/A < \pi/(1 - \pi) \\ 0, & \text{otherwise.} \end{cases}$$

That is, under linear loss the optimal forecast is either 0 or 1. Since actual announced probability forecasts typically take values between 0 and 1, it is clear that linear loss is not used in practice.

Third, consider general quadratic loss:

$$L_1(p) = A(1 - p)^2$$

$$L_0(p) = Bp^2$$

for $A, B > 0$. In this case, the optimal announced probability is given by

$$p^* = \pi/[\pi + (1 - \pi)b] \qquad (2)$$

where b is the ratio B/A. If $b < 1$, the loss function is conservative in the sense that $L_1(p) > L_0(1 - p)$ (e.g., the loss associated with probability forecast 0.80 when precipitation occurs is greater than the loss associated with probability forecast 0.20 when precipitation does not occur). The term "conservative" is used in the sense of "safe". This usage implies that, loosely speaking, rain is bad, and that we would rather err on the side of precaution. For conservative quadratic loss, $p^* > \pi$, and over-forecasting will occur even if subjective probability assessments are perfectly reliable.

3. Application

In this section, we apply the general quadratic loss model to the results shown in Fig. 1. Specifically, under certain simplifying assumptions, we estimate b for these data.

For the purposes of this exercise, we make the following assumptions:

(i) The forecasters operate under general quadratic loss with common b.

(ii) The forecasters' subjective probability assessments are perfectly reliable.

(iii) The precipitation events (occurrences and nonoccurrences) are independent.

The expression for p^* given in (2) can be inverted to give

$$\pi = bp^*/[1 + (b - 1)p^*]. \qquad (3)$$

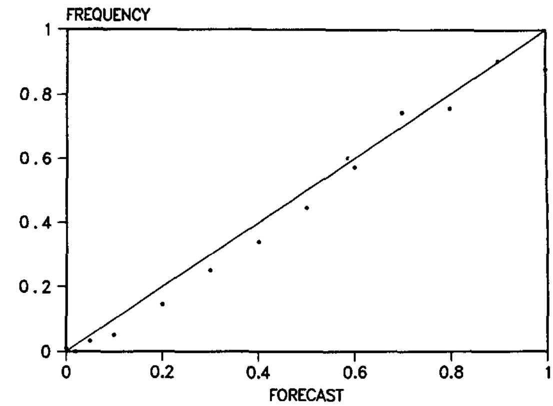

FIG. 1. Reliability diagram for the data given by Clemen and Murphy (1986).

Under the assumptions outlined above, the relative frequencies in Fig. 1, which we denote f_i, $i = 1, \cdots, N$, are approximately normally distributed with

$$E(f_i) = bp_i^*/[1 + (b - 1)p_i^*] \qquad (4)$$

$$\text{var}(f_i) = E(f_i)(1 - E(f_i))/n_i \qquad (5)$$

where p_i^* is the announced probability forecast, and n_i is the number of observations, associated with f_i. The approximate normality of f_i follows from the fact that the raw frequencies, $n_i f_i$, are binomially distributed, and from the well-known normal approximation to the binomial distribution.

Let $\mathbf{f}$ be the vector of f_i, $\hat{\mathbf{f}}$ be the vector of estimates of the f_i found by substituting a value $\hat{b}$ for b in the right-hand side of (4), and $\mathbf{W}$ be the diagonal matrix with entries given by (5). The maximum likelihood estimate of b is found by minimizing the weighted residual sum of squares:

$$(\mathbf{f} - \hat{\mathbf{f}})^T \mathbf{W}^{-1} (\mathbf{f} - \hat{\mathbf{f}})$$

over $\hat{b}$, where the superscript denotes transpose.

We make three remarks.

First, because (4) is nonlinear in b, the minimization must be carried out numerically.

Second, because $\mathbf{W}$ depends on b, and b is unknown, we will use an iterative procedure. Under this procedure, the current estimate of b will be used to form $\mathbf{W}$, and $\mathbf{W}$ will then be used to find a new estimate of b.

Third, under our simple model, $\text{var}(f_i) = 0$ for $p_i^* = 0, 1$. This follows from the fact that the announced probability forecasts are equal to the true subjective probability assessments (i.e., $p^* = \pi$) in these cases, for all values of b. Since we have assumed perfect reliability of the subjective probability assessments, this implies that f_i is necessarily 0 or 1 if p_i^* is 0 or 1, respectively. In fact, from Fig. 1, there is a slight tendency to under-forecast when $p^* = 0$ (over-forecasting is, of course, impossible in this case) and a rather substantial tendency to over-forecast when $p^* = 1$. In terms of our

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simple model, these discrepancies can be explained by inadequacies in the assumed loss function, by unreliability in the subjective probability assessments, or by both. In any case, to avoid problems arising from the singularity of **W**, we will remove these two data points from the analysis.

The minimization procedure outlined above converged after three iterations. The final estimate of b was 0.77. Further results are presented in Table 1 and plotted in Fig. 2. The estimated relative frequencies from the model are quite close to the observed relative frequencies particularly when p^* is 0.6 and less. The model fails to capture under-forecasting for $p^* = 0.7$ and over-forecasting for $p^* = 1$. Given the simplicity of the model and the fact that the specification was chosen from first principles (i.e., without reference to the data), these results are surprisingly good.

4. Discussion

The purpose of this note has been to present a model for over-forecasting. The elements of this model are the recognition that announced forecasts and true subjective probabilities may be different, and that systematic over-forecasting can arise from an asymmetric loss function that is conservative. It is important to realize that over-forecasting need not be a problem, provided that society (by which we mean users of the forecast) is aware of the forecaster's loss function and makes an appropriate correction. A problem will arise if society fails to distinguish between announced forecasts and true subjective probabilities. Even in this case, however, the problem may be short-lived, as society learns by experience to correct announced forecasts.

Under our model, over-forecasting is a systematic effect. An alternative explanation is that over-forecasting arises from errors in the forecasting process. Under this alternative explanation, the tendency to over-forecast might be expected to diminish over time, as forecasters make the kind of correction (implicitly or explicitly) that we have made in the previous section. It would be interesting to see whether the degree of over-forecasting (e.g., indexed by the parameter b) tends to diminish over time, or whether it persists.

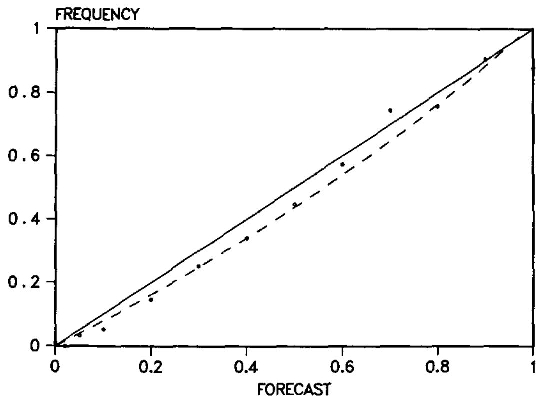

FIG. 2. Reliability diagram for the data given in Fig. 1 showing observed and predicted points.

TABLE 1. Some results from fitting the over-forecasting model to the data shown in Fig. 1.

p_i^*	f_i	$\hat{f}_i$	n_i
0.00	0.01	0.00	1521
0.02	0.00	0.015	13
0.05	0.032	0.039	94
0.10	0.052	0.079	990
0.20	0.146	0.161	937
0.30	0.251	0.248	923
0.40	0.340	0.339	564
0.50	0.446	0.435	482
0.60	0.572	0.536	386
0.70	0.744	0.642	203
0.80	0.756	0.755	135
0.90	0.904	0.874	73
1.00	0.878	1.00	90

One immediate extension of this work would be to generalize the loss function. One possibility is to make the parameter b depend on p^*. A second extension would be to study the relationship between over-forecasting, as summarized by the parameter b, and other variables such as the use to which the forecast is put (e.g., recreational, agricultural, etc.), or the severity of the event that is the object of the forecast. Although we have assumed that forecast conservatism is associated with over-forecasting, it is not difficult to imagine situations in which under-forecasting is conservative. An example of such a situation might be forecasting for agricultural purposes in an arid region.

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