Modification of a Successive Corrections Objective Analysis for Improved Derivative Calculations

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ABSTRACT

The use of objectively analyzed fields of meteorological data for complex diagnostic studies and for the initialization of numerical prediction models places the requirements upon the objective method that derivatives of the gridded fields be accurate and free from interpolation error. A modification of an objective analysis developed by Barnes provides improvements in analyses of both the field and its derivatives. Theoretical comparisons, comparisons between analyses of analytical monochromatic waves, and comparisons between analyses of actual weather data are used to show the potential of the new method. The new method restores more of the amplitudes of desired wavelengths while simultaneously filtering more of the amplitudes of undesired wavelengths. These results also hold for the first and second derivatives calculated from the gridded fields. Greatest improvements were for the Laplacians of the height field; the new method reduced the variance of undesirable very short wavelengths by 72 percent. Other improvements were found in the divergence of the gridded wind field and near the boundaries of the field of data.

1. Introduction

In the analysis of weather data, it is often necessary to transform the data from nonuniformly distributed observation sites to a rectangular grid suitable for numerical calculations. The transformation is accomplished by some form of interpolation and is referred to as "objective analysis." An important class of objective analysis techniques known as "successive corrections methods (SC)" was first described by Bergthorsson and Doos (1955) and was introduced into operational meteorology by Cressman (1959). Though replaced by multivariate statistical techniques as the operational interpolation method in many numerical forecasting models, the successive corrections methods are still widely used for gridding meteorological data.

Because the SC methods have been widely used for gridding meteorological data, it is appropriate that they should be benchmarks for comparisons with newer objective analysis techniques (Seaman 1983; Schlatter et al. 1976; Davidson 1982). Further, since SC methods have been shown to be interchangeable with statistical interpolation methods (Bratseth 1986; Lorenc 1986), and because gridded fields can be produced more economically by the SC methods, the SC methods may remain important for future comparisons. It is only fair that the analysis parameters be set to yield the "best analysis" possible with the SC methods. Because the equations for numerical models and diagnostic models require derivatives and derivative products, it is crucial that the definition of "best analysis" include accurate derivatives calculated from objectively gridded fields of meteorological data.

This study presents a modification of the Barnes (1964, 1973) SC method to improve upon the "best analysis" obtainable by the technique. Here, "best analysis" is defined as the analysis that best restores the amplitudes of the wavelengths resolvable by the data and, conversely, best removes those wavelengths not resolvable by the data and also maintains or improves upon the accuracy of the derivatives calculated from the gridded fields. Some of the reasons for using the Barnes method for this study are: 1) it has been widely used for gridding meteorological data and has been made part of data assimilation and analysis packages (Koch et al. 1983; Smith and Leslie 1984), 2) its Gaussian weighting scheme has been incorporated into statistical methods (Seaman 1983), 3) the weight function is not arbitrarily truncated (a practice that introduces high frequency noise into the gridded fields), 4) the response characteristics of the method have been well-documented, and 5) the influence of data can be extended without changing the response characteristics of the weight function.

Achtemeier's (1987) results suggested that the desired improvements were possible if the traditionally used 2-pass version developed by Barnes (1973) were modified to include a third pass. Since the new method incorporates much of the original methodology devel-

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oped by Barnes, we adopt the terminology, *B*arnes *OB*jective *AN*alysis Three Pass (BOBAN3) for the 3-pass version. In the next section, we derive the response equations for BOBAN3 and demonstrate the theoretical increase in accuracy over the 2-pass method. In addition, comparative results from analyses of monochromatic sine waves are presented. In section 3, analyses of a jet streak embedded within an intense synoptic scale cyclone on 1200 UTC 11 April 1979 are presented to show how BOBAN3 improves over the 2-pass method. The significance of these results is discussed in section 4.

2. Theoretical analysis

Only portions of the theory developed by Barnes are reproduced here to provide the reader with the necessary background to understand this study. A more complete explanation is available in Barnes (1973) and is also contained in Koch et al. (1983) and Achtemeier (1987).

The horizontal distribution of an atmospheric variable can be described mathematically by a linear combination of an infinite number of sinusoidal waves, a Fourier Integral representation. If analysis requirements are to represent only components in the actual field larger than an arbitrarily chosen wavelength, one can apply a judiciously selected filtering function to the distribution. The effect upon the resulting depiction of the field can be demonstrated for a single sinusoidal wave of the form $f(x, y) = A \sin(ax)$, where $a = 2\pi/L$ and L is the wavelength. The filtered (or objectively analyzed) field, $g(x, y)$, is represented by

$$g(x, y) = \int_0^{2\pi} \int_0^{\infty} f(x + r\cos\Theta, y + r\sin\Theta) w(r, k) r\,dr\,d\Theta, \quad (1)$$

where the filter is a function of the distance, r, between the arbitrary point (x, y) and all other points in the field. Barnes (1964) chose a Gaussian function for $w(r, k)$

$$w(r, k) = (4\pi k)^{-1} \exp(-r^2/4k). \quad (2)$$

If the data concerning the atmospheric variable are distributed continuously over an infinite plane surrounding (x, y), as they are in this analytical example, then (1) becomes

$$g(x, y) = D(a, k) f(x, y), \quad (3)$$

where the amplitude response, $D(a, k)$, is given by

$$D(a, k) = \exp(-a^2 k). \quad (4)$$

Here k is a selectable parameter that determines the shape of the weighting curve for the filter.

Equation (4) gives the response of the Gaussian filter after one application. Barnes (1964) showed that more of the details inherent in the data could be recovered if the filtered residual differences between $f(x, y)$ and $g(x, y)$ were added to the original filtered field. However, several additions of smoothed corrections to a smoothed initial analysis were required to retrieve the important details inherent in the data. In 1973, he modified his method so that most of the important details are recovered in only two passes through the data. If $k = k_0$ and $D(a, k_0) = D_0$ for the initial pass, and $k_1 = \gamma k_0$ for the correction pass, then the final response D' is given by

$$D' = D_0(1 + D_0^{\gamma-1} - D_0^{\gamma}). \quad (5)$$

Here γ is an arbitrary weighting parameter that decreases the number of iterations required to achieve desired response.

Achtemeier (1987) found from theoretical studies that equivalent or better accuracy could be obtained over the 2-pass 1973 (traditional) method by a 2-pass method with the same filter parameters on the initial and correction passes (fixed method). These studies required the final responses for the traditional and fixed methods to be equal at an arbitrary "reference wavelength" (usually the $2S$ wavelength where S is the average separation between observations). Accuracy was defined by how well the method retains desired wavelengths and removes undesired wavelengths. Applications revealed that the more sparsely and irregularly distributed are the data, the less the results are in accord with the predictions of continuous theory. No single set of filter parameters for the 2-pass method produce the most accurate objective analysis for all wavelengths. The traditional method developed the most accurate objective analysis of the longer wavelengths. The fixed method added information present in the shorter wavelengths without increasing the contribution by undesired very short wavelengths.

This information about how different filter parameters modify the performance of the 2-pass objective analysis was used to develop the 3-pass method that includes the desirable properties of both the traditional and fixed methods. This modification is but a minor extension of the existing technique. A very smooth first pass captures the long wavelengths and an additional two passes with the fixed method retains the desired shorter wavelengths. Consider a 3-pass method for which the first-pass response is D_0 and the second- and third-pass responses are D_1 and D_2, respectively. The final response after three passes is

$$D' = D_0 + (1 - D_0) D_1 (1 + D_1^{\tau-1} - D_1^{\tau}), \quad (6)$$

where the third-pass response is related to the second-pass response by $D_2 = \tau D_1$. If $D_0 = 0$, the first pass only restores the mean of the data field and has zero response for all wavelengths. Then the final response reduces to the response for the traditional 2-pass method given by (5). The final response for BOBAN3 is obtained upon setting $\tau = 1$,

$$D' = 1 - (1 - D_0)(1 - D_1)^2. \quad (7)$$

Choosing a small D_0 (say $D_0 = 2.5 \times 10^{-4}$) allows the restoration of some of the longer wavelengths but has near zero response at the reference wavelength. Then the filter parameters for the fixed method give the desired final response at the reference wavelength after two correction passes. Figure 1 shows the theoretical final response curves for BOBAN3 and the traditional method. The responses are identical (0.25) at the reference wavelength ($2S$). BOBAN3 offers an additional 7–9 percent restoration of the $4S$–$5S$ wavelengths.

Though D_0 has near zero response at the reference wavelength, and therefore contributes negligibly to the final response there, its contribution to the final response at the longer wavelengths is not negligible. The solid line in Fig. 2 shows the differences between the final responses for the fixed method and BOBAN3 expressed as percent of the amplitude of the original wave. The standard of comparison is the fixed method. The negative percentages mean that inclusion of the third pass causes restoration of more of the original wave. There is no difference between the two methods at the reference wavelength ($2S$ wave). BOBAN3 restores 2 percent more of the $4S$ wave. It restores lesser percentages at the longer wavelengths because the 2-pass fixed method has almost perfectly restored these wavelengths already and there remains almost no additional information to add to the fixed analysis.

Following a procedure for the analysis of analytical data (Achtemeier 1987), we performed objective analyses by BOBAN3, the fixed method, and the traditional method. These analytical fields take the form,

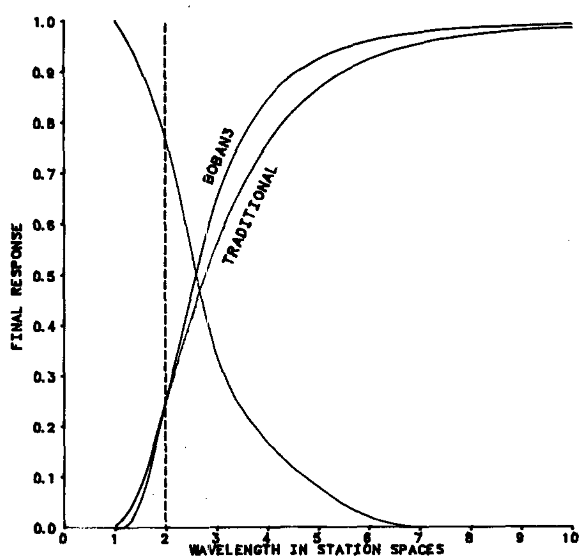

FIG. 1. Theoretical final response curves for BOBAN3 and the traditional Barnes method. The responses are identical (0.25) at the reference wavelength ($2S$). Third line: response curve for high-pass filter used for analysis of Laplacians of the 300 mb height field.

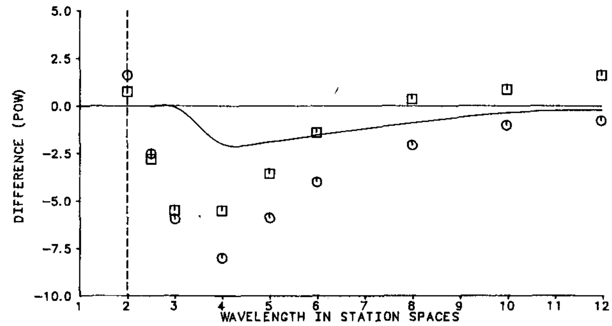

FIG. 2. (Solid line) Differences between the final responses for the fixed method and BOBAN3 for reference wavelength equal to $2S$. Relative analysis accuracies for the traditional method and the fixed method (squares) and traditional method and BOBAN3 (circles) for analyses of irregularly spaced data. The relative accuracy is the difference between the rms errors for each analysis pair normalized by the amplitude of the analytical wave and converted to percent.

$$f(x, y) = A \sin(2\pi x/L), \quad 2S \leq L \leq 12S. \quad (8)$$

RMS differences between the true field and each analysis were calculated for the grid interior defined as all points at least three grid points removed from the grid boundary. The statistic of relative performance is the difference between the rms differences for BOBAN3 and the traditional method and for the fixed method and the traditional method normalized by the amplitude of the analytical wave. This statistic approximates the percent of the amplitude of the original wave used for the comparisons of theoretical responses.

Figure 2 compares the relative performances of the BOBAN3 and fixed methods with the traditional method. The rms errors are taken from within the interior of a field of data with data distributed randomly at only one of the four corner points of each grid square. The standard of comparison is the traditional method. The results for the fixed method (squares) are superior in the $3S$ to $8S$ wavelength range and are inferior in the wavelengths greater than $8S$ relative to the traditional method. There is no relative increase in the accuracy of the BOBAN3 analysis (circles) for the short wavelengths because the first pass response for these waves is approximately zero. The analyses of the longer wavelengths by BOBAN3 were superior to the results from either the fixed or traditional methods. Greatest increases over the traditional method were approximately 8 percent for the $4S$ wavelength.

Comparison of the results from BOBAN3 and the fixed method (Fig. 2) reveals the following. Both methods yield similar improvements over the traditional method for wavelengths in the range $2S$–$3S$. For $4S$ or greater wavelengths, BOBAN3 outperformed the fixed method by 2–3 percent. These percentages are for the whole domain and, locally, the improvements can be much larger. Now the only difference between BOBAN3 and the fixed method is the very smooth first pass of BOBAN3 and the differences between the theoretical final responses of the two methods are given by the solid line. It is apparent that relative improvements by BOBAN3 in the $2S$–$5S$ wavelength range can be explained by the first pass of BOBAN3. Simply add the differences in the theoretical response curve (solid line) to the values for the fixed method. However, for wavelengths greater than $6S$, the relative improvements by BOBAN3 cannot be explained by the increase in the final response gained by including the third pass because the theoretical percentages increases are too small.

To determine if there also exists improvements of BOBAN3 over the traditional method not predicted by continuum theory, we repeated the comparative objective analyses for the one-dimensional monochromatic waves on a 21 by 21 grid and set the analysis parameters so that analyses by the two methods would be equally accurate at the $4S$ wavelength when applied to a grid of uniformly spaced data. It was necessary to require the equivalence with analyses of uniformly spaced data when it was found that the traditional method does not filter the short waves to the extent predicted by theory. We restored the $4S$ wave to 75 percent of its original amplitude. The solid line in Fig. 3 gives differences between the analyses in percent of the amplitudes of the original waves. The traditional method is the standard of comparison. Figure 3 shows that BOBAN3 has the desirable property of restoring more of the longer wavelengths (negative percentages) and filtering more of the short wavelengths (positive percentages) relative to the traditional method. However, BOBAN3 has restored about 2.5 percent more of the $4S$ wavelength.

Since the percentage differences at the $4S$ wavelength exist only when the two methods are applied to irregularly spaced data, the underlying cause for these differences cannot be determined from theory for a data continuum. We therefore seek a measure of the relative sensitivities of the two methods when applied to irregularly spaced data. One test is to compare the horizontal derivatives calculated along the y-axis of the analyzed fields. The analytical fields have no gradient in the y-direction. The method that is most sensitive to the data distribution should generate more high frequency noise in the fitted surface thus giving a "sawtooth" appearance to the analyzed wave.

The gradients along the y-direction of the fitted surfaces were calculated for each grid-point pair within the grid interior. Since the gradients of the true analytic surfaces are zero in this direction, any nonzero values are analysis errors. The rms errors for BOBAN3 were compared with the rms errors for the traditional method (the traditional method was used as the standard of comparison) and the results expressed as percent of the amplitude of the original wave per grid space. As shown by the dashed line in Fig. 3, these percentages are negative for the whole range of wavelengths studied, meaning that the BOBAN3 gradients

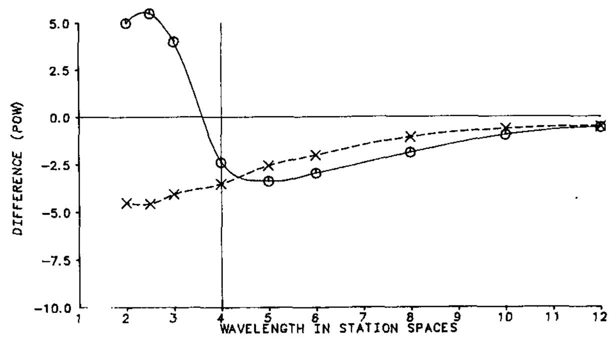

FIG. 3. (Solid line) Relative analysis accuracies for the traditional method and BOBAN3 for reference wavelength equal to $4S$. (Dashed line) Relative analysis accuracies for derivatives normal to the wave fronts for the traditional method and BOBAN3 expressed as percent of the amplitude of the original wave per grid space.

are smaller than the gradients calculated from the surfaces fitted by the traditional method.

To provide more detail into the behavior of these two analysis methods, we produce three pairs of cross sections in Fig. 4 showing first derivatives along the y-axes of the analyzed waves. The first cross section is taken along the crest of a wave, the second taken part way between an inflection point and a wave crest, and the third taken along an inflection point. The crosses (circles) identify the BOBAN3 (traditional) method. There are several ways to interpret these cross sections. First, the most accurate method gives the smallest departures from zero, the true slope. BOBAN3 almost always gave the smallest departures. Second, changes in the sign of the slope identify erroneous extrema in the analyzed surface. BOBAN3 produced the fewest short wavelength peaks (see point 11 Cross Section a). Third, the curvature is a measure of how well an analysis method produces smooth surfaces. The greater the "saw tooth" appearance of the lines, such as Fig. 4b, the larger is the "noise" contained in the second derivative.

3. Comparisons between BOBAN3 and traditional method analyses of real data

It was shown in the last section that BOBAN3 analyses of irregularly distributed data are superior to comparative analyses by the traditional method in the following ways. 1) BOBAN3 restores more of the amplitudes of the desirable long and short waves, 2) BOBAN3 filters more of the amplitudes of the undesirable short waves, and 3) BOBAN3 generates smaller noise waves upon analyses with irregularly spaced data. The above results are based upon analyses of one dimensional monochromatic waves and there remains some question whether the improvements are meteorologically significant and justify the added expense of a third pass. We therefore perform comparative anal-

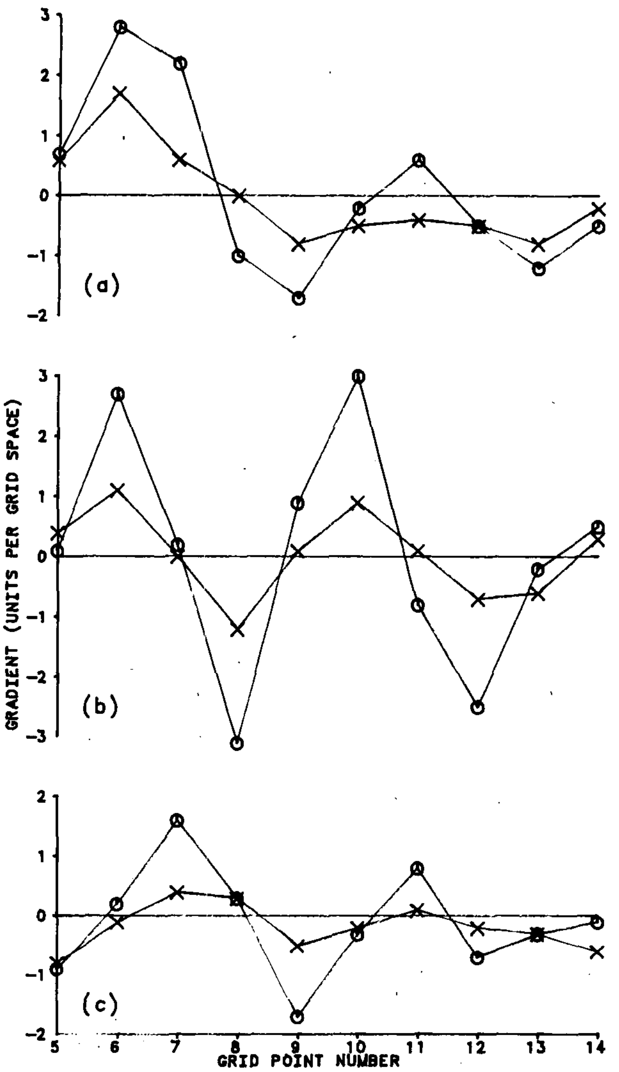

FIG. 4. Cross sections normal to the wave front showing derivatives of 4S wave along (a) wave crest, (b) midway between wave crest and inflection point and (c) inflection point. Circles (traditional method) and crosses (BOBAN3).

yses of meteorological data and also compare the first and second derivatives calculated from the analyzed fields.

An intense cyclone was located over the western United States at 1200 UTC, 11 April 1979 (SESAME I). It was a synoptic system that consisted mostly of long wavelengths (relative to the data distribution) for which the continuum theory predicts only small differences between the two methods. However, the event also included several jet streaks and associated gradients in the height and temperature fields. The data were gridded onto a 40 × 25 mesh with 100 km grid spacing and analyses done for wind components, temperature, and height at 100 mb intervals from the surface to 100 mb. Derivative fields such as divergence, vorticity, and Laplacian of the heights were calculated also. Analysis parameters were selected so that the final responses for the 2S wavelength were 0.25.

Differences that could be considered as meteorologically significant were found mostly in areas of strong height and temperature gradients and also within important small scale features, such as jet streak maxima and the depth of the major cyclone. Comparative results for the 300 mb heights are presented in Fig. 5. Gradients were largest at this level and the relative impacts of the two methods upon the final analyses were most apparent. Heights analyzed by the traditional method and BOBAN3 are shown in Fig. 5a, b. The most notable difference is that the low over New Mexico (shaded area) has been deepened 20 m by BOBAN3. Table 1 shows the observed 300 mb heights at four stations surrounding the low and also shows heights bilinearly interpolated to the station locations from the traditional and BOBAN3 analyses. The four stations are identified by the circled dots in Fig. 5a, b. The average interpolation error for the four stations was 30 m for the traditional method and 19 m for BOBAN3; thus BOBAN3 gave a better fit to the data. Please note that other choices for the analysis parameters would improve the fit, however, it would be necessary to increase the response of the 2S wavelength. The relative improvement of the fit by BOBAN3 is about 2–4 percent of the approximate 500 m amplitude of the 300 mb height pattern.

The Laplacian of the height field is proportional to the geostrophic vorticity and therefore is an important meteorological variable. The Laplacians of the height fields analyzed by the traditional method and BOBAN3 (Fig. 5c and 5d) reveal the advantages of BOBAN3 in three ways. First, both Laplacian fields show evidence for cyclonic curvature and steep height gradients from New Mexico through Nebraska and from Washington through Nevada. The amplitude of the pattern over New Mexico is 8–10 percent larger revealing that the BOBAN3 analysis has restored more of the desired short wavelengths. Second, the patterns calculated from the BOBAN3 Laplacians are smoother and show less apparently nonmeteorological short wavelength features. And third, the BOBAN3 analyses appear to be less sensitive to boundaries in the data field (Achtemeier 1986). This advantage of BOBAN3 is more tenuous because there exist other methods for reducing the impacts of data boundaries (Koch et al. 1983).

In order to identify the short wavelengths in the Laplacian fields, we removed most of the longer wavelengths with a high pass filter (Fig. 1) that consists of a 2-pass Barnes analysis (fixed method). This filter, applied to the gridded data, passes 75 percent of the 2S wave and up to 100 percent of the shorter wavelengths. Figures 5e and 5f show the short wavelengths remaining after the application of the filter. The patterns consist of some small fraction of meteorologically significant wavelengths and mostly undesirable high frequency "noise" waves introduced through the grid-

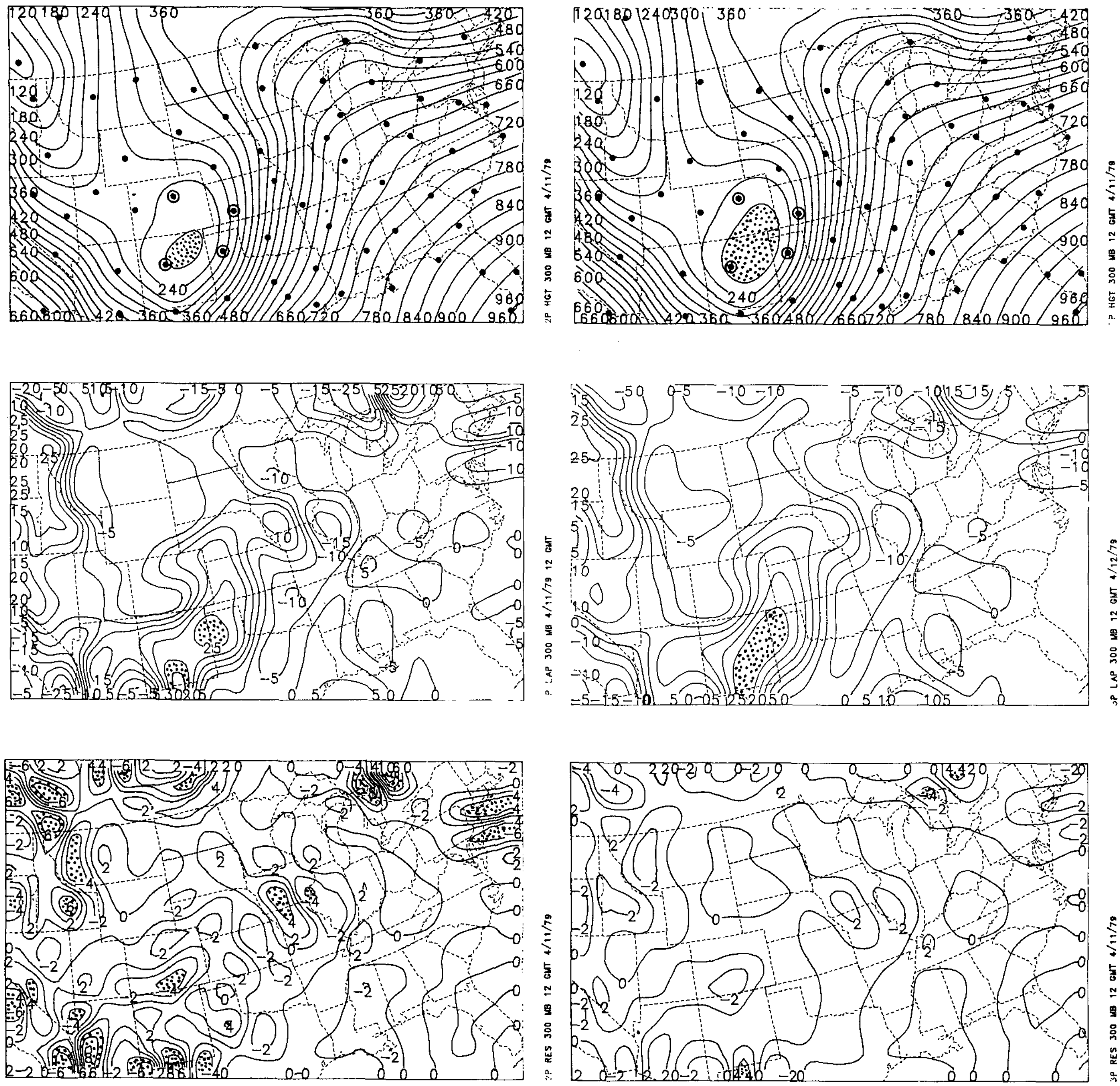

FIG. 5. Comparative results for analyses of the 300 mb heights at 1200 UTC, 11 April 1979 for the traditional method (left panels) and BOBAN3 (right panels). Height contours (a and b), Laplacian of heights (c and d), and Laplacian residuals after filtering (e and f). Units converted to geostrophic vorticity are 10^{-5} s^{-1} for panels c, d, e and f.

ding of irregularly spaced data. They are mostly $2S$ waves. An amplitude of 2 units corresponds to an error in geostrophic vorticity of approximately 2×10^{-5} s^{-1}. Largest amplitudes are found within and around the deep trough. By contrast, Fig. 5f shows these waves much suppressed by BOBAN3. Furthermore, the almost imperceptible extension of the ridge in the height field over Illinois and a weak trough over Nebraska (Fig. 5a) into a large data void located over Missouri has introduced a couplet, a positive and negative center, in the Laplacian field (Fig. 5c) over the same area. Consisting mostly of analysis error, it has been reduced by approximately 50 percent in the BOBAN3 analysis. Excluding three rows and columns of grid points along the boundary from the calculations, the variance of the high frequency wavelengths in Figs. 5e and 5f was reduced 72 percent by BOBAN3.

Streamlines calculated by an accurate method proposed by Achtemeier (1979) and isotachs of the 300 mb wind (Fig. 6a, b) reveal only subtle differences.

TABLE 1. Observed, traditional, and BOBAN3 300 mb heights and winds at four rawinsonde sites. Calculated winds are geostrophic winds.

	Heights (Z–8700)			Winds (deg/m s^{-1})		
Station	Observed	Traditional	BOBAN3	Observed	Traditional	BOBAN3
Amarillo, TX	214	268	255	200/37	206/64	204/68
Albuquerque, NM	195	218	207	10/17	307/23	313/29
Dodge City, KA	246	283	270	195/47	183/42	179/47
Denver, CO	224	231	223	45/17	45/05	50/13

BOBAN3 has increased the jet-streak maximum over Kansas by about 2 m s^{-1}. Overall, BOBAN3 reduced the rms differences between the observations and vector wind speed interpolated to the locations of the observations from the gridded fields by approximately 6 percent. The field of differences between the two analyses (Fig. 6c) reveals that BOBAN3 added about 2 m s^{-1} along the full length of the jet streak to bring the wind speeds into better agreement with the observations. Elsewhere, the rather large differences in wind speed over Idaho and parts of Montana are the result of a shift by BOBAN3 of the deformation field southwestward toward the jet streak located over Nevada. Since the deformation field was located within an area devoid of observations, it was not possible to determine whether the southwestward shift by BOBAN3 was an improvement to the analysis.

Comparisons between fields of derivatives calculated from the components of the wind gridded by the two methods (Fig. 7) reveal that BOBAN3 increased the magnitudes of the cyclonic shear near the jet streak and also the anticyclonic vorticity over Illinois by approximately one unit (Fig. 7a, b). These increases were accomplished without the addition of analysis noise. The improvements by BOBAN3 in the divergence (Fig. 7c, d) appear mostly in the divergence patterns. Patterns of divergence calculated from components of the vector wind gridded by the traditional method reveal a sensitivity to the data distribution in that the patterns tend to be broken into small centers located in data voids midway between observation sites. For example, the zone of divergence that extends from Texas into the Great Lakes appears as four centers, one each over north Texas, Arkansas, southeastern Illinois, and central Wisconsin. Several of these centers are present in the BOBAN3 analysis but much reduced magnitudes suggest that BOBAN3 is less sensitive to the data distribution. Smoother, more continuous fields of divergence are also found in the analysis of the convergence over Iowa and Kansas, and the area of divergence that extends from Canada southward through the High Plains to near the Colorado border.

4. Discussion

Achtemeier's (1987) analysis of the Barnes (1964, 1973) method for gridding irregularly spaced weather data found that improvements, particularly in the retrieval of short but resolvable wavelengths, were possible if the traditional 2-pass method were replaced with

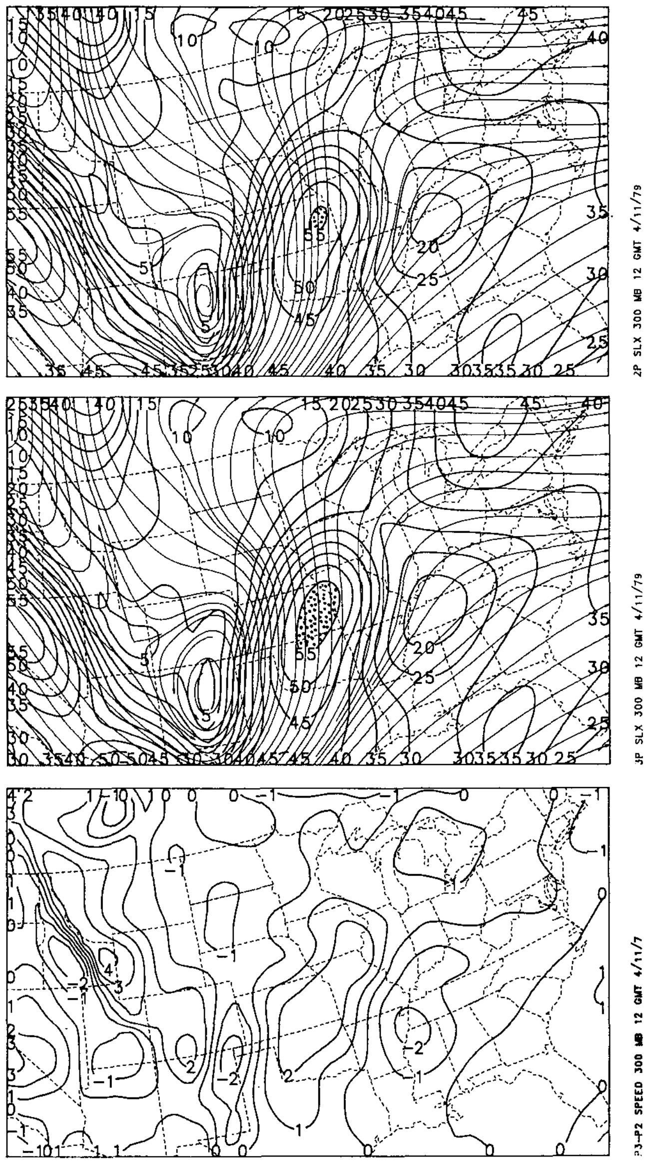

FIG. 6. Streamlines and isotachs by (a) the traditional method and (b) BOBAN3 of the 300 mb wind. (c) Differences in wind speed (m s^{-1}) between BOBAN3 and the traditional method.

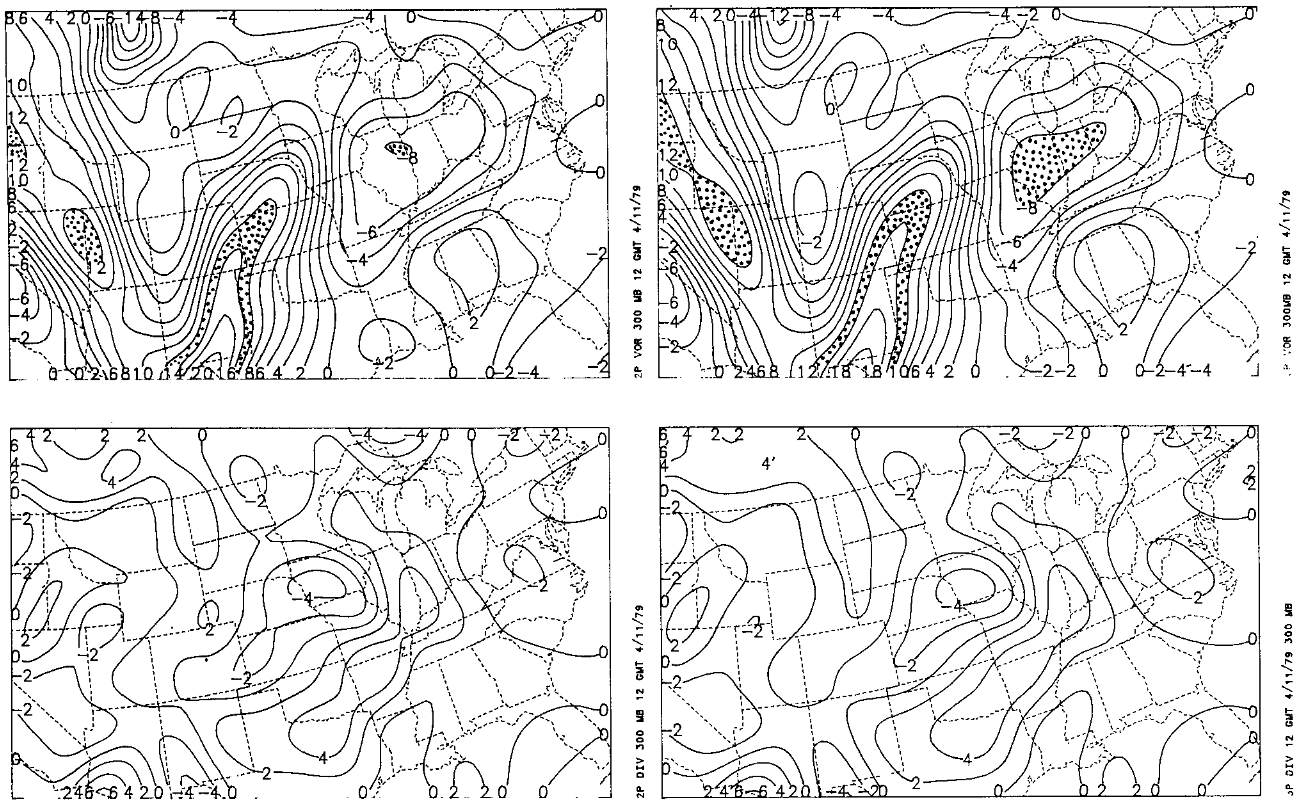

FIG. 7. Comparison of vorticity and divergence for the traditional method (left panels) and BOBAN3 (right panels). Vorticity (a and b) and divergence (c and d). Units are 10^{-5} s^{-1}.

a 3-pass method. In this paper, the theory for the 3-pass method (BOBAN3) was advanced and tested with analyses of analytical and real data. Comparisons between the traditional method and BOBAN3 verified the theoretical prediction that BOBAN3 would, upon specification of analysis parameters, restore some of the desired longer wavelengths, restore more of the desired shorter wavelengths, and filter more of the undesired short wavelengths. The more specific results of this study are as follows.

1) Both theoretical results and comparisons between analyses of one-dimensional analytical waves showed that BOBAN3 would restore 7–9 percent more of the desired short wavelengths (3–5S). BOBAN3, upon analysis of the 300 mb 1200 UTC 11 April 1979 height and wind fields, restored approximately six percent more of the magnitudes of short wavelength features such as jet-streak maxima (2–4 m s^{-1}) and the depth of the major cyclone (20 m). This result is in accord with theoretical predictions.

2) Comparisons between analyses of one-dimensional analytical waves by BOBAN3 and the traditional method revealed that the magnitudes of errors in the first derivatives of fields gridded by BOBAN3 were reduced. In the study of the behavior of derivatives of the gridded wind field, it was found that the divergences calculated from the BOBAN3 wind field were not dismembered into small centers located midway between observations to the same extent as were the divergences calculated from wind fields gridded by the traditional method.

3) The analysis of the second derivatives was done with the Laplacians of the heights, an important meteorological variable proportional to the geostrophic vorticity. The magnitude of the cyclonic center calculated from the height field gridded by BOBAN3 was approximately 6 percent greater than that calculated from heights gridded by the traditional method. In addition, BOBAN3 produced a simultaneous reduction of the variance of the undesired 2S wavelengths by 72 percent. Given the locations of these waves with respect to the locations of the observations, it is apparent that they originated mostly through the interpolation of irregularly spaced data. The ability of BOBAN3 to reduce short wavelength noise in the gridded fields is in accord with the results of the analytical studies.

All derivatives were calculated from gridded fields by finite differences. Both vorticity and divergence were calculated from centered differences on a staggered grid $\{\zeta = [v(i, j+1/2) - v(i, j-1/2) - u(i+1/2, j) + u(i-1/2, j)]/dx; D = [u(i+1/2, j) - u(i-1/2, j) + v(i, j+1/2) - v(i, j-1/2)]/dx\}$. Peppler and

Smith (1984) found that finite differences smooth the magnitude of vorticity by approximately 2 percent for wavelengths and grid intervals similar to those used in this study. However, since finite differences were used for derivative calculations from fields gridded by both methods, we have isolated just the relative improvements in analysis accuracy of BOBAN3 over the traditional method. Caracena (1987) has developed a modification of successive corrections objective analysis that allows for the mapping of derivatives directly from the observations. Multipass schemes such as BOBAN3 can be easily adapted to it. Thus, as tests with real data verify the method, further improvements in the resolution of derivatives appear possible.

Finally, what changes in analysis parameters are required to convert existing programs of the traditional method into BOBAN3? Since the parameter $\tau = 1$ in BOBAN3, it is necessary to find appropriate values for $4k_0$ and $4k_1$ ($4k_2 = 4k_1$).

The two values for $4k$ that BOBAN3 needs are found through (4) and (7) and are

$$4k_0 = -(\lambda^*/\pi)^2 \ln(D_0^*) \qquad (9)$$

$$4k_1 = -(\lambda^*/\pi)^2 \ln[1 - (1 - D'^*)^{1/2}]. \qquad (10)$$

The following may be said about the variables on the right-hand sides of (9) and (10): (i) The most useful value for the reference wavelength is the minimum resolvable wave, $\lambda^* = 2S$, where S is the average separation between observations when the observation density is relatively uniform. If the observation density is highly irregular, some other choice for S may be more ap propriate. (ii) The first-pass response is $D_0^* = 2.5 \times 10^{-4}$ so that its contribution to the final response at the reference wavelength in (7) is negligibly small. (iii) D'^* is selected according to the accuracy of the data and the analysis needs.

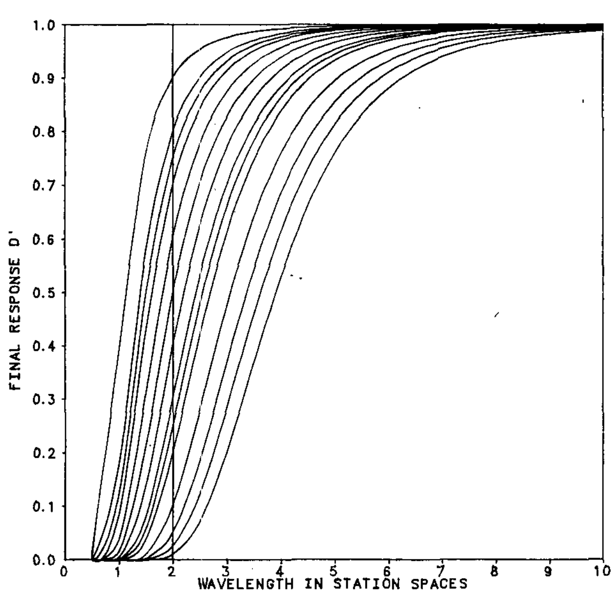

FIG. 8. Plot of final response D' as a function of wavelength.

Figure 8 shows curves of D' as a function of wavelength. The analyst may find D'^* in several ways. He may choose to limit the amplitude of the $2S$ wave in the final analysis by picking D'^* directly from the intersection of the response curve with the vertical $2S$ line or he may desire to restore a specified fraction of the amplitude of some other wavelength. In this latter case, it is necessary to find the value of the response at the desired wavelength from Fig. 8 and follow the final response curve to its intersection with the $2S$ line to find D'^*.

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