

NOTES AND CORRESPONDENCE

An Ice-Water Saturation Adjustment

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ABSTRACT

A reasonably accurate and noniterative saturation adjustment scheme is proposed to calculate the amount of condensation (and/or deposition) necessary to remove any supersaturated vapor, or amount of evaporation (and/or sublimation) necessary to remove any subsaturation in the presence of cloud droplets (and/or cloud ice). This proposed scheme can be implemented for a nonhydrostatic cloud model. The derivation of the scheme, evaluation of its performance and tests for sensitivity to variations in a few key parameters will be presented in this note.

1. Introduction

A saturation adjustment scheme that calculates the amount of condensation (and/or deposition) necessary to remove any supersaturated vapor, or the amount of evaporation (and/or sublimation) necessary to remove any subsaturation in the presence of cloud droplets (cloud ice) is needed for a nonhydrostatic cloud model (e.g., Ogura 1963; Soong and Ogura 1973; Klemp and Wilhelmson 1978; Cotton and Tripoli 1978; Clark 1979; Schlesinger 1980; Lin et al. 1983; Lord et al. 1984; Tao and Soong 1986; and many others). One approach frequently used is a relaxation technique (e.g., Newton-Raphson method) to iteratively balance the heat exchange and change of phase of water substance. A water-phase only saturation adjustment without a need for iterative computation was first proposed by Soong and Ogura (1973). A modified version of their scheme with inclusion of an ice-phase is the subject of this note. The performance, limitations and assumptions used in this scheme are also discussed.

A two-dimensional version of our cloud model (Soong and Ogura 1980; Tao and Soong 1986) is used to evaluate the performance of the proposed adjustment scheme. In addition to the Kessler-type of two-category liquid-water (cloud water, q_c ; and rain, q_r) parameterized microphysics, a sophisticated three-category ice-phase scheme (cloud ice, q_i ; snow, q_s ; and hail, q_g) is included (Lin et al. 1983). The particles comprising the cloud water and cloud ice fields are each assumed to be monodisperse and to advect with

the airflow, having no appreciable terminal velocities of their own. The adjustment scheme proposed below is needed to compute the amount of condensation/deposition (evaporation/sublimation) of cloud water and cloud ice.

2. Procedure

Two major assumptions involved in the proposed saturation adjustment scheme are the following:

1) The saturation vapor mixing ratio (q_{vs}) is defined as a mass weighted combination of the saturation values over liquid water (q_{ws}) and ice (q_{is}) between the temperature range T_0 and T_{00} . Thus,

$$q_{vs} = (q_c q_{ws} + q_i q_{is}) / (q_c + q_i) \quad (1)$$

For an initial state where the temperature exceeds T_0 (0°C), only liquid water is allowed to be present. For an initial state where the temperature is below T_{00} (it typically ranges from -30° to -40°C), only cloud ice is produced. The choice of T_{00} is simply academic in this study. However, aircraft observations can provide valuable guidance for selecting an appropriate value. The saturation vapor mixing ratio is defined with respect to liquid water for temperatures above T_0 and with respect to ice at temperatures below T_{00} .

2) Under super- (or sub-) saturated conditions condensation and deposition occur in proportions that depend linearly on the temperature in the range T_0 to T_{00} . Excess water vapor is assumed to condense/deposit into cloud water/cloud ice instantaneously when mixed-phase supersaturation occurs. The cloud water/cloud ice is assumed to evaporate/sublimate instan-

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taneously if the air is subsaturated. Precipitating raindrops (snow and graupel) evaporate (sublimate) only if the cloud water (cloud ice) is exhausted before saturation is attained. Thus,

$$dq_v = q_v - q_{vs} \tag{2a}$$

$$dq_c = dq_v \cdot \text{CND} \tag{2b}$$

$$dq_i = dq_v \cdot \text{DEP} \tag{2c}$$

where CND is $(T - T_{00}) / (T_0 - T_{00})$ and DEP is $(T_0 - T) / (T_0 - T_{00})$, T is temperature and dq_v , dq_c and dq_i are the changes in q_v , q_c and q_i . These assumptions were first used by Lord et al. (1984).

The saturation adjustment is as follows. First, the potential temperature, water vapor, cloud water and cloud ice are updated from $t - dt$ to $t + dt$, taking into account only the dynamical terms (advection and diffusion). These new values of the potential temperature, water vapor, cloud water and cloud ice at time $t + dt$ are indicated by θ^* , q_v^* , q_c^* and q_i^* , respectively. The saturation mixing ratios over water and ice corresponding to θ^* are then calculated using Tetens's formula,

$$q_{ws}^* = b \exp[a_1(\bar{\pi}\theta^* - 273.16) / (\bar{\pi}\theta^* - 35.86)] \tag{3a}$$

$$q_{is}^* = b \exp[a_2(\bar{\pi}\theta^* - 273.16) / (\bar{\pi}\theta^* - 7.66)] \tag{3b}$$

where $b = 3.8/P$, a_1 and a_2 are 17.2693882 and 21.8745584, respectively. \bar{P} and $\bar{\pi}$ are, respectively, the

dimensional and nondimensional pressure at each grid point.

The condensation/deposition (evaporation/sublimation) rate must be determined so that θ^* and q_v^* are in equilibrium; i.e., $q_v^{t+dt} = q_{vs}^{t+dt}$. Of course any evaporation (sublimation) will be limited by the availability of q_c^* (q_i^*). We require the adjustment to proceed moist adiabatically; thus

$$d\theta = \theta^{t+dt} - \theta^* = (L_v dq_c + L_s dq_i) / (C_p \bar{\pi}) \tag{4a}$$

$$q_v^{t+dt} = (q_c^* q_{ws}^{t+dt} + q_i^* q_{is}^{t+dt}) / (q_c^* + q_i^*) \tag{4b}$$

where C_p , L_v and L_s are the specific heat of air at constant pressure, the latent heat of evaporation and latent heat of sublimation, respectively. By substituting $\theta^{t+dt} = \theta^* + d\theta$ into (3a) and (3b) and following a similar approximation of Soong and Ogura (1973), thus, retaining only the terms of the first order of $d\theta$, we get the following expression for q_{vs}^{t+dt} :

$$q_{vs}^{t+dt} = q_v^* - r_1 + r_2 d\theta \tag{5}$$

We define

$$r_1 = q_v^* - (q_c^* q_{ws}^* + q_i^* q_{is}^*) / (q_c^* + q_i^*) \tag{6a}$$

$$r_2 = (A_1 q_c^* q_{ws}^* + A_2 q_i^* q_{is}^*) / (q_c^* + q_i^*) \tag{6b}$$

$$A_1 = (237.3 a_1 \bar{\pi}) / (\bar{\pi}\theta^* - 35.86)^2 \tag{6c}$$

$$A_2 = (265.5 a_2 \bar{\pi}) / (\bar{\pi}\theta^* - 7.66)^2 \tag{6d}$$

$$A_3 = (L_v \text{CND} + L_s \text{DEP}) / (C_p \bar{\pi}). \tag{6e}$$

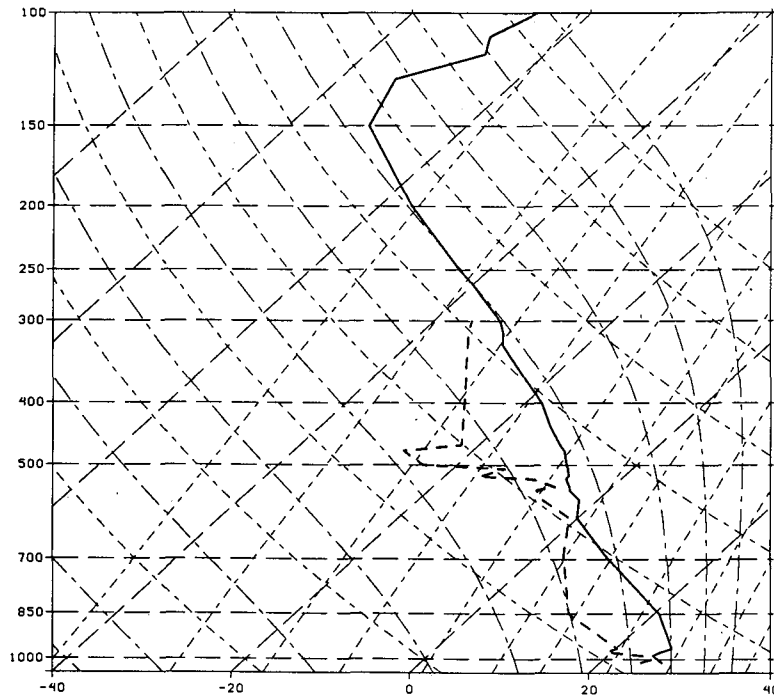


FIG. 1a. The initial temperature and mixing ratio of water vapor taken from Cape Hatteras, North Carolina at 2400 UTC June 29 1986.

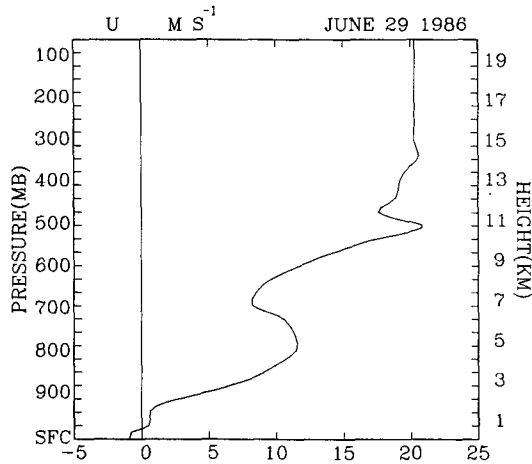


FIG. 1b. The u -component of the wind (wind normal to the convective line).

Then

$$\theta^{t+dt} = \theta^* + r_1 A_3 / (1 + r_2 A_3) \quad (7a)$$

$$q_v^{t+dt} = q_v^* - r_1 / (1 + r_2 A_3). \quad (7b)$$

It should be noted that this saturation adjustment technique does not preclude the possibility of ice supersaturation. This can result because we make use of a mass-weighted saturation mixing ratio which is larger than the saturation mixing ratio for ice. In this context, depositional growth of ice, which is taken relative to this weighted saturation mixing ratio, could be underestimated. If, however, the saturation adjustment is one of the last microphysics processes to be computed, then much of the supercooled water above the -10°C iso-

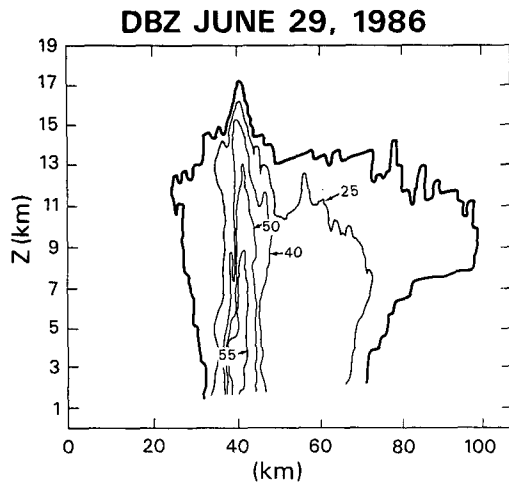


FIG. 2. Radar reflectivity observed during COHMEX (From NASA Spandar Radar at Wallops Island, Virginia). This cross section is normal to the convective line and this radar presentation may indicate the strongest convective event during the system. The contour labels are 10, 25, 40, 50 and 55 dbZ.

TABLE 1. List of key parameters in the two-dimensional sensitivity tests.

Runs	T_{00} ($^\circ\text{C}$)	q_{vs} between T_0 and T_{00}
1	-40	weighted q_{ws} and q_{is}
2	-25	weighted q_{ws} and q_{is}
3	-40	q_{is}
4	-40	q_{ws}

therm will be depleted in some way (such as by scavenging by precipitation sized particles). Thus, the weighting of the saturation mixing ratio in these regions will favor the value for ice and in circumstances where only cloud ice remains, then the weighted saturation mixing ratio will be the ice value.

3. Case study

The temperature, mixing ratio and wind profiles associated with a very strong thunderstorm observed over land during COHMEX (Cooperative Huntsville Meteorological Experiment) on 29 June 1986 is used as an initial condition for the model (Fig. 1). The convective event was observed in southeastern Virginia and was roughly aligned from west-southwest to east-northeast. The observed cloud top exceeded 17 km and the peak radar reflectivities from the facility at Wallops Island were above 55 dbZ (Fig. 2).

The x -direction of the model is chosen to be perpendicular to the convective line. A stretched vertical coordinate is used. The grid interval is 200 m at the lowest level and about 1000 m at the highest level. There are 34 vertical grid points for a model depth of 20 km, and 128 horizontal grid points, with a grid spacing of 1 km. The cloud is initiated with a warm thermal (1.25°K with 12 km radius) as prescribed by Klemp and Wilhelmson (1978).

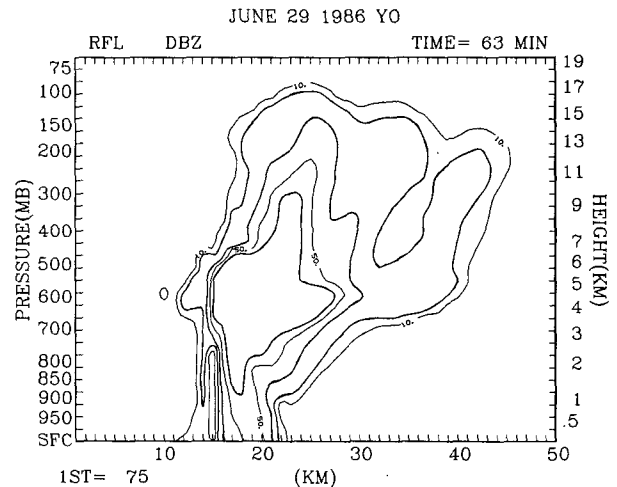


FIG. 3. As in Fig. 2 except obtained from a two-dimensional model simulation. The contour labels are 10, 25, 40, 50 and 55 dbZ.

A total of four numerical experiments was made to test the variations in key parameters used in (1) and (2). The first and second experiments were designed to test the sensitivity of the selection of T_{00} . The third and fourth experiments were focused on the selection of saturation vapor mixing ratio assumed between the temperature range T_0 and T_{00} . Table 1 lists the key parameters in the sensitivity tests.

4. Results and discussion

The strongest vertical velocity is 31 m s^{-1} at 61 minutes of simulation time in Run 1. Reflectivity estimates generated by the model are calculated using the method of Smith et al. (1975). Figure 3 shows a result generated from Run 1 which represents the most vigorous convective activity during the 90 minutes of simulation time. The model predicted cloud width is narrower than the observation. This is probably caused by the two-dimensionality of the model and by the single thermal initialization. However, the strongest convective activity occurred in a narrow region as was observed. The 40 dbZ contour also extends to nearly the 15 km level and the 50 dbZ contour to nearly the 12 km level. Overall, the model results agree well with observations.

The iterative saturation adjustment method generally guarantees that the cloudy area (defined as the area which contains q_c and/or q_i) is saturated (100% relative humidity, RH). The evaluation of the proposed scheme, then, can be easily made by estimating the RH inside the cloudy area. Table 2 shows the averaged RH and its standard deviation obtained from Run 1. The averaged RH in the cloudy area is further divided into super- (greater than 100% RH) and sub- (less than 100% RH) saturated areas. The nearly 100% RH inside the cloudy area and small deviation from the saturated mixing ratio suggest that the proposed scheme is reasonably accurate. Even though the simulated vertical velocity is very strong ($29\text{--}31 \text{ m s}^{-1}$), the numerical requirement for stability makes the time step small (10 s). This results in a small dq_c for the time step which in turn minimizes error. Furthermore, the error caused by the scheme in heat exchange, $d\theta$, is even smaller (less than 0.10°C). This is much less than observational error. The most significant error occurred in the region near the -40°C isotherm, because of the different definition of q_{vs} used below and above it. Nevertheless,

TABLE 2. Averaged relative humidity (%) and the standard deviations from the mean inside the modeled cloudy area.

	Relative humidity (%) in cloudy area*		Deviation from saturated mixing ratio (g g^{-1})
Mean	100.29	99.91	4.2×10^{-6}
Std dev	0.44	0.21	8.1×10^{-6}

* Cloudy area is defined as $q_c + q_i > 1.0 \times 10^{-4} \text{ g g}^{-1}$.

TABLE 3. The accumulated water content, ice content and surface precipitation generated by four experiments. The surface precipitation is space-averaged (128 km) over the 90-min simulation time. The water and ice content are also vertically averaged over the 90 min simulation time.

Runs	Precipitation ($\text{mm h}^{-1} \text{ grid}^{-1}$)	Water content ($\text{g m}^{-2} \text{ grid}^{-1}$)	Ice content ($\text{g m}^{-2} \text{ grid}^{-1}$)
1	4.67	416.0	368.4
2	4.63	422.4	401.9
3	4.81	430.7	385.0
4	4.66	416.7	361.8

the error is automatically compensated by the model over the next few time steps of integration.

The averaged precipitation at the surface varies only slightly between the four experiments (Table 3). The relatively small difference in the sensitivity tests may be due to the fact that most convective updrafts originate in the lower troposphere. These updrafts transport relatively abundant moisture from the lower troposphere. A small difference (less than 0.0004 g g^{-1}) in q_{vs} with respect to water and ice in the 0° and -40°C layer will not have a significant effect in terms of total precipitation at the surface. It was also found that the predicted radar reflectivity patterns (e.g., Fig. 3) are very similar to each other. The accumulated water and ice content generated by these experiments are also shown in Table 3. It was found that the ice content generated by Runs 2 and 3 is 5 to 10% more than that generated by Run 4 at the dissipating stage of the simulated cloud. Based on the sensitivity tests, the saturation adjustment scheme is not sensitive to the parameters involved in (1) and (2) in the convective region of vigorous upward motion. But it is probably sensitive to the choice of parameters in simulating cirrus (also stratiform) clouds with a high cloud base. Note also that the parameters used in (1) and (2) are also less sensitive than most other parameters used in an ice-phase scheme (e.g., McCumber et al. 1987) with respect to the modeled radar reflectivity pattern, the stratiform cloud structure and the vertical profiles of the hydrometeors.

This proposed scheme can be implemented for a non-hydrostatic cloud model with bulk parameterized microphysical processes (e.g., Klemp and Wilhelmson 1978; Lin et al. 1983; Lord et al. 1984; and many others) which treats the microphysical processes fairly realistically (although by no means perfectly). The performance of this scheme has been shown to be reasonably accurate in simulating tropical and midlatitude convective events. Some modification of parameters used in (2) can also be made including the relationship between the growth of hydrometeors and the strength of vertical motion of an air parcel (e.g., Heymsfield and Hjelmfelt 1984).

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