On the Statistical Analysis of Cyclone Deepening Rates

PAUL J. ROEBBER

Department of Meteorology, McGill University, Montreal, Quebec, Canada

3 January 1989 and 14 April 1989

ABSTRACT

Statistical analysis of cyclone deepening rates has been used in the past to infer distinctions between physical processes operative in cases of explosive cyclogenesis and lesser storms. This note attempts to qualify the conclusions of the previous study by analyzing cyclone deepening data from a new perspective. The results suggest that the debate concerning the relative normality of these distributions is essentially irrelevant. Significant statistical evidence is provided to suggest that midlatitude maritime cyclogenesis exhibits a fundamentally different character from continental events, and that this distinction is evident across a wide spectrum of storm intensities.

1. Introduction

Explosive cyclones have been characterized by the observation that their development typically represents an exceptionally strong surface response to a given level of large-scale baroclinic forcing (Sanders and Gyakum 1980; Sanders 1986). The implication is that such a process represents behavior distinctly different in some physically meaningful way from that of less extreme cyclogenesis. This idea was examined by Roebber (1984), who hypothesized that such behavior would be manifested by the statistical distribution of cyclone maximum 12 and 24 hour central pressure change. The underlying premise of this hypothesis is that cyclone intensification can be characterized by a linear combination of physical processes that are quasi-independent in space and time, and thus, the Central Limit Theorem can be used to interpret the resultant distribution of deepening rates. Under this assumption, non-Gaussian behavior of the statistics suggests a mixture of two or more distinct sets of cyclone intensification processes within the same sample.

There are a number of potential qualifications of this mathematical conception in its application to observational reality. Nonindependence between cyclone events (e.g., preconditioning, modification of the large-scale circulation) and between processes within a cyclogenetic event (e.g., feedbacks) represent means by which this interpretation may be invalidated. It might be possible to suppose quasi-independence between the majority of cyclones given a sufficiently large data sample, particularly in the light of the arguments of Lorenz (1969) and Leith (1971) concerning the inherent limits to predictability of atmospheric motions, and by extension, deterministic connections between individual events. However, independence between cyclogenetic processes within a single event is more difficult to satisfy a priori. Low-level and midlevel processes certainly interact in the course of cyclone intensification. Frictional effects depend in a nonlinear fashion on the intensity of a storm. In addition, self-amplification processes such as air–sea fluxes (Emanuel 1986) suggest potentially strong interdependence between cyclogenetic processes.

A further limitation concerns the physical limit to the numer of processes active in individual cyclogenetic events. The Central Limit Theorem indicates that if deepening rates arise as a sum of processes, then no matter what the probability distributions of the separate processes may be, their sum will have a distribution that will tend more and more towards Gaussian as the number of process components increases (Draper and Smith 1981). The critical number of processes required for approximate normality is dependent upon the distributions of the component processes. This number might be as small as 1 (in the case of a normally distributed process), but it might also be substantially larger. Within the context of a finite number of physical processes, it is arguable that a particularly dominant forcing mechanism may skew the overall distribution of deepening rates. Finally, it is impossible (not simply improbable) for cyclone intensification rates to extend beyond finite limits, although such is suggested by a truly Gaussian distribution. Practically, the point is that the probability of occurence becomes so small at extreme intensification rates that the expected recurrence time of such an event exceeds the time scale of climatic change. Thus, while exact normality is unobtainable, quasi-normal empirical distributions are possible and can be verified, provided there is sufficient information

Corresponding author address: Dr. Paul J. Roebber, Dept. of Meteorology, McGill University, 805 Sherbrooke Street West, Montreal P.Q. H3A 2K6, Canada.

concerning the tails of the distribution to reliably assess the data. In summary, non-Gaussian behavior of the deepening rate statistics may imply mixed cyclone intensification process sets within the sample, strong process interactions, an insufficient number of physical processes, insufficient data, or some combination of these effects; the exact nature of which we will be unable to specify through such an analysis.

Despite these limitations, there is information to be gained through the statistical analysis approach. Gyakum et al. (1989), using an eight year cold season dataset in the Pacific basin, showed that the higher moments of the statistical distribution of cyclone deepening rates do not change substantially in a northward progression of latitudinal bands, until the land and ice-choked areas between 60° and 70°N are reached. This distinction between the distributions of over-water cyclone deepening rates and those over land suggests that the underlying ocean surface may play a key role in these departures. However, there is some question as to the relevance of that high-latitude result, characterized by mainly filling storms, to the case of midlatitude, primarily deepening cyclones.

In this note, it is argued that the primary issue is to distinguish clearly the nature and character of the statistics of cyclone deepening in midlatitude maritime and continental regions and that the discussion of normality or the lack thereof in either region is essentially irrelevant. Of particular interest to the notion of explosive cyclogenesis is the relative behavior of the tails of the deepening distributions. The database compiled by Roebber (1984) is used to study these questions in greater detail.

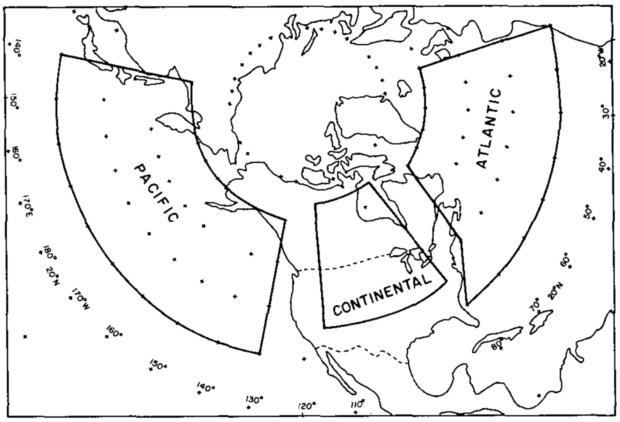

FIG. 1. Boundaries of the three regions (Atlantic Ocean, Pacific Ocean, continental) used in the statistical analysis of cyclone deepening rates.

2. Statistical analysis

As reported in Roebber (1984), a database was compiled consisting of surface cyclone life histories for the period February 1980 through January 1981. All surface lows (excluding thermal lows and tropical storms) that were analyzed on at least two successive National Meteorological Center (NMC) 12 hourly "front half" hemispheric surface charts were tracked for the extent of their identifiable existence, and latitude–longitude coordinates and analyzed central pressure as a function of time were recorded. In this study, the data were condensed to reflect the position and magnitude of the maximum 24 hourly pressure falls (minimum 24 hourly pressure rises) for every cyclone in the sample. The pressure changes were then normalized geostrophically to a reference latitude at 45°N by use of the formula

$$\Delta P_{45^\circ} = \frac{\Delta P_{\Phi} \sin 45^\circ}{\sin\Phi}, \qquad (1)$$

where Φ is the mean latitude of the storm during the deepening period. This adjustment is made on dynamical grounds as well as considerations of the relative geostrophic wind.[1]

The adjusted pressure changes obtained from (1) were analyzed for the regions depicted in Fig. 1, corresponding to one continental and two oceanic locations. The continental region was chosen in the interior of North America (35° to 65°N, 80° to 115°W) in order to remove the maritime influence from the statistics, while individual data points within the designated oceanic regions that were located over land were eliminated. Although it might be expected that a 12 hour period would correspond more closely to the interval of most rapid intensification, only the 24 hour adjusted rates were used in the study in an effort to minimize variability in the statistics associated with observational and analysis error.

Frequency distributions of the 24 hour adjusted maximum deepening rates for the Atlantic, Pacific and continental domains of Fig. 1 are shown in Fig. 2. The Atlantic and Pacific samples show rough qualitative agreement, with broad stretching of the intensification rates along the maximum deepening tail. Basic descriptive statistics related to the deepening distributions for each region are provided in Table 1. The data are tabulated for the Atlantic and Pacific regions, a dataset composed of the two oceanic regions combined (the justification of which will be shown below), and the continental region. The variables displayed include the standard statistics (mean, standard deviation etc.), as well as the p-values at the 10th, 25th, 50th (median), 75th and 90th percentiles, which are distribution free

[1] It can be shown that this geostrophic adjustment of pressure falls represents a latitude-dependent scaling factor for storm vorticity change provided the vorticity at the beginning of the deepening period is small, the fall in storm central pressure is not embedded within a region of generally falling or rising pressure and the storm maintains an approximately constant size through the period.

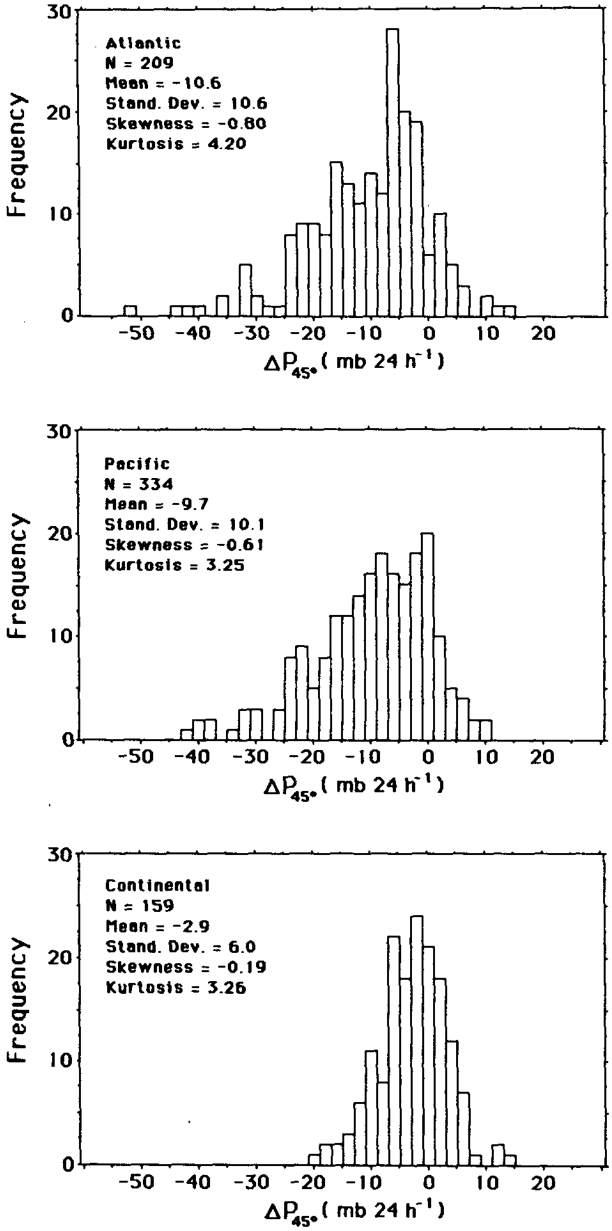

FIG. 2. The frequency distribution of maximum (minimum) continental cyclone deepening (filling) rates in 2 mb/24 h intervals for the regions: (a) Atlantic, (b) Pacific, and (c) continental depicted in Fig. 1. The data are for the period February 1980 through January 1981, inclusive.

measures of the data. The sample statistics for the Pacific region are in reasonable agreement with the findings of Gyakum et al. (1989) for their larger dataset, although our data suggests slightly more skewness towards strong deepening. The Atlantic sample is broadly similar, in most respects, the apparently greater kurtosis[2] representing the most significant departure. The continental sample, in contrast, is characterized by a considerably weaker mean deepening, and minimal (within one standard deviation) skewness towards stronger deepening. Overall, the continental sample shows substantially less variance in cyclone deepening rates.

We wish to formally assess the relative correspondence of the deepening rate distributions in each of the three regions. The χ^2 test, based on classical statistics, is frequently used for this purpose. However, this test *presupposes* that the Central Limit Theorem holds for the sample population. It can be shown that this statistic is quite nonrobust against nonnormality, does not necessarily improve with increasing sample size and is increasingly unreliable towards the remote tail regions (Bradley 1968). The unreliability in the tail region is usually reduced by grouping data points until frequencies above an arbitrary minimum threshold value are achieved, but at the expense of a loss of information. Instead, we apply the Kolmogorov–Smirnov maximum deviation test for identical populations (hereafter referred to as KS; see Bradley 1968). This test is distribution free, that is, no assumption concerning the underlying distribution of the sample statistics is required, and examines the "goodness of fit" of every observation. The null hypotheses we wish to test are that the distributions of cyclone deepening rates are identical in the Atlantic, Pacific and continental regions. The KS test requires that we compute the maximum deviation between the cumulative frequency distributions of the two regions to be tested across the range of maximum pressure falls represented in the data, and compare that value to the maximum expected deviation under the null hypothesis that the underlying distributions are identical.

Parameters obtained from the KS test are summarized in Table 2. The parameters shown are the maximum deviation between the two compared cumulative frequency distributions (maxΔF), the deviations at several points along the maximum deepening tail [-30, -20, and -10 (mb 24 h^{-1})], and the maximum expected deviation at three levels of significance (1%, 5% and 10%). The results suggest to a reasonable degree of certainty that we cannot reject the hypothesis that Atlantic and Pacific cyclone deepening rates are identically distributed. Based on this information, the Atlantic and Pacific data were pooled to form an oceanic region sample for comparison with the continental data. The KS statistic shows that it is reasonable to conclude that the character of the distribution of cyclone deepening rates is significantly different in the oceanic and continental regions.

It is useful to examine the influence of observational error on the maritime statistics to ascertain whether our results are strongly a function of the quality of the

[2] Kurtosis is a measure of the peakedness of a distribution, while skewness is a measure of the assymetry of the data. A normal distribution has coefficients of kurtosis and skewness of 3.0 and 0.0, respectively.

TABLE 1. Basic sample statistics for the Atlantic, Pacific, Oceanic (Atlantic and Pacific combined) and continental datasets.

	Atlantic			Pacific		
ΔP statistic	Total	Cold	Warm	Total	Cold	Warm
Sample Size	209	109	100	334	204	130
Mean	−10.6	−14.1	−6.8	−9.7	−12.1	−6.0
Std. dev.	10.5	11.4	7.9	10.1	10.6	7.9
Maximum	−51.7	−51.7	−25.0	−41.3	−41.3	−30.4
Minimum	14.8	10.2	14.8	13.0	13.0	8.6
10th percentile	−23.7	−30.9	−18.0	−22.7	−24.7	−17.2
25th percentile	−16.7	−20.6	−12.0	−16.4	−18.4	−10.2
50th percentile	−8.5	−13.5	−5.6	−8.4	−11.0	−4.4
75th percentile	−3.9	−5.4	−2.6	−2.2	−4.7	−0.9
90th percentile	1.5	−2.1	2.5	2.0	1.6	2.3
Skewness	−0.80	−0.70	−0.14	−0.61	−0.36	−0.81
Kurtosis	4.20	3.54	3.06	3.25	3.08	3.48
	Oceanic			Continental		
Sample Size	543	313	230	159	69	90
Mean	−10.1	−12.8	−6.3	−2.9	−2.9	−2.9
Std. dev.	10.3	10.9	7.9	6.0	6.9	5.2
Maximum	−51.7	−51.7	−30.4	−20.0	−20.0	−16.6
Minimum	14.8	13.0	14.8	13.5	12.9	13.5
10th percentile	−23.0	−26.2	−17.9	−10.7	−12.4	−10.0
25th percentile	−16.4	−19.8	−10.8	−6.7	−6.7	−6.6
50th percentile	−8.4	−11.8	−5.1	−2.3	−2.1	−2.4
75th percentile	−2.7	−5.1	−1.1	1.1	1.7	0.0
90th percentile	1.7	0.0	2.3	4.7	5.2	2.8
Skewness	−0.69	−0.51	−0.51	−0.19	−0.30	0.01
Kurtosis	3.69	3.37	3.23	3.26	2.89	3.36

available data. This was done by subdividing the Pacific region into two areas roughly east (180° to 140°E) and west (180° to 150°W) of the date line, which as shown by Roebber (1984), correspond approximately to the limits of relatively sparse versus relatively dense ship data coverage, respectively. The KS test shows that the two regions cannot be rejected as identically distributed, that is, data differences appear to have minimal impact on the maritime statistics. Furthermore, the relatively high quality data region along the Kuroshio current is shown to exhibit cyclone deepening distributions of a significantly different form than the continental sample. These results are entirely consistent with Gyakum et al. (1989) that maritime cyclone deepening rate statistics are of a fundamentally different character than those over land and that these differences are probably related to physical processes rather than the nature of the data.

The seasonal dependence of the cyclone deepening statistics was examined by separating the samples into cold (October–March) and warm seasons (April–September). In the oceanic sample, statistically significant slippage of the median cyclone deepening rate occurred, from −11.8 mb 24 h^{-1} in the cold season to −5.1 mb 24 h^{-1} in the warm season. A qualitative comparison of the differences in the seasonal distributions with this seasonal slippage corrected was attempted by removing the median difference from the cold distribution (in effect, "sliding" the cold season data down the ΔP axis to a position coincident with

TABLE 2. Kolmogorov–Smirnov test statistics.

Hypothesis	Max ΔF	ΔF_{-30}	ΔF_{-20}	ΔF_{-10}	ΔF_α $\alpha = 0.01$	ΔF_α $\alpha = 0.05$	ΔF_α $\alpha = 0.10$	Result
Atlantic = Pacific	0.092	0.018	0.013	0.021	0.144	0.120	0.108	Accept at all 3 levels
Oceanic = Continental	0.355	0.046	0.162	0.316	0.147	0.123	0.110	Reject at all 3 levels
Good Data = Poor Data	0.097	0.021	0.067	0.079	0.206	0.172	0.154	Accept at all 3 levels
Good Data = Continental	0.416	0.059	0.197	0.375	0.206	0.172	0.129	Reject at all 3 levels
Continental = Gauss	0.083	0.000	0.002	0.009	0.129	0.108	0.097	Accept at all 3 levels
Continental = Cauchy	0.114	0.027	0.057	0.036	0.129	0.108	0.097	Accept at $\alpha = 0.01$
Continental = Uniform	0.128	0.000	0.000	0.066	0.129	0.108	0.097	Accept at $\alpha = 0.01$
Continental = Double Exp.	0.134	0.000	0.009	0.022	0.129	0.108	0.097	Reject at all 3 levels

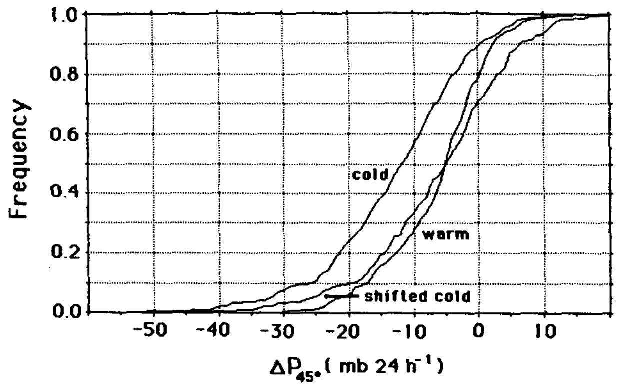

FIG. 3. The cumulative frequency distributions of cyclone deepening rate for the oceanic region (Atlantic and Pacific combined). Plotted are the cold season and warm season data along with the cold season distribution adjusted to the warm season median deepening rate (labelled shifted cold).

the warm season median). When the seasonal median difference is removed, the distributions of oceanic cyclone deepening rates are qualitatively similar in the two periods, with only slight differences in the tail region arising as a result of a higher frequency of exceptionally strong cases of cold season cyclogenesis. Figure 3 presents the oceanic warm and cold season cumulative frequency distributions of cyclone deepening rates, along with the cold season distribution corrected to the warm season median value (labelled shifted cold). The figure suggests that the processes that act to produce this "stretching" of the deepening tail are active throughout the year in maritime cyclogenesis, albeit more strongly so in the cold months, the main difference in the seasonal distributions arising as a result of generally stronger cyclogenesis across the entire spectrum of cyclones.

No corresponding slippage of the median cyclone deepening rate in the continental sample is detectable, the primary seasonal differences evidently resulting in increased variation in cyclone deepening rates in the cold season (Table 1). Overall, there is surprisingly little seasonal contrast in continental maximum deepening rates. Of some interest is the slight suggestion of skewness towards stronger deepening in the cold season. This skewness is within one standard deviation of symmetry about the mean, so it may be that this feature is an aberration of the relatively small sample. However, the possibility that this skewness is a feature of cyclone deepening rates irrespective of location cannot be completely dismissed and has considerable bearing on the physical interpretation based on the Central Limit theorem. Despite the presence of this skewness, it can be shown that the continental deepening distribution can be quite well approximated by the Gaussian curve. This approximately Gaussian nature of the distribution is relatively insensitive to the details of the regional boundaries; that is, shifting the region of interest within the continental interior does not change this conclusion. These results suggest that the departure found by Gyakum et al. (1989) for high-latitude storms is equally valid for predominantly deepening, midlatitude storms. Thus, the statistical argument advanced by Roebber (1984) appears to be supported by these data.

However, the "true" underlying form of continental cyclogenesis rates is open to debate. It can be shown that true forms as distinct as the Cauchy, the double exponential, and the uniform distribution cannot be reliably distinguished from the Gaussian distribution for samples of moderate size (Breiman 1973). This lack of discrimination is largely due to the inability of the statistical procedures to easily distinguish tail behaviors for small to moderate sample sizes. The goodness-of-fit tests are broad spectrum procedures, and in that sense, are heavily weighted by the fit in the bands about the median value of the distribution. Other tests can be devised to concentrate on specific tail features, yet it seems apparent that without a significantly expanded database, the discrimination ability of these tests will also be limited. The "fit" of the above distributions in the tail region to the continental sample is displayed in Fig. 4. The fits were accomplished by using the continental data sample mean and standard deviation as approximations for the midpoint and variance of the fitted distributions (Breiman 1973). The Gaussian distribution provides the best overall fit to the sample data based on the KS test, and also shows minimal deviations along the maximum deepening tail. However, at less stringent rejection levels (e.g. 99% confidence level, $\alpha = 0.01$ in Table 2), the sample can also be represented by the Cauchy and uniform distributions, despite less satisfactory performance in the tail region. It seems clear that an *unqualified* reiteration of the Central Limit argument would be ill-advised at this stage. Despite this, the author believes that the statistical comparisons presented in this paper provide additional justification of the view that explosive cy-

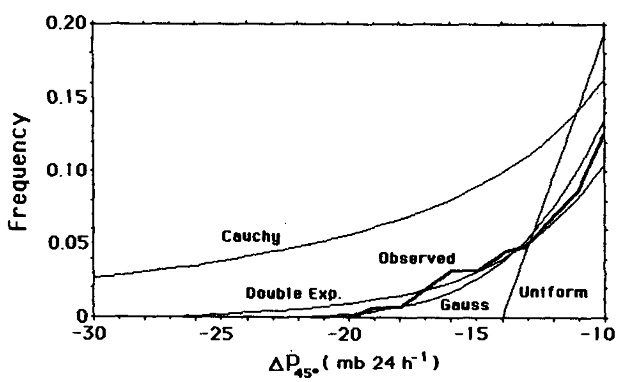

FIG. 4. The cumulative frequency distributions of the continental data sample (bold line) and the best-fit Gaussian, Cauchy, uniform, and double exponential curves for the deepening tail region.

clogenesis, as represented by the rapid deepening tail, represents physically distinct behavior that is fundamentally related to the nature of the marine boundary layer. Additional comments concerning this point are provided in the summary section.

3. Summary

Evidence is provided that substantiates the findings of Gyakum et al. (1989) that the character of the statistical distribution of deepening rates differs significantly between oceanic and continental samples. An important and unresolved question is the physical basis of the distinction between oceanic and continental cyclone development. Gyakum et al. (1989) and the results of the current study indicate that practical differences in data quality over the ocean do not substantially effect the character of the deepening distribution. Physically, the differences between the regions are fundamentally related to boundary layer processes, specifically, fluxes of heat, moisture and momentum. These fluxes can affect the cyclogenetic environment in a variety of ways, some direct and immediate, as when surface fluxes near the cyclone center induce vorticity spinup through enhanced horizontal convergence (Petterssen 1956), and some indirect and remote, perhaps prior to the cyclogenesis itself. Obvious influences of these fluxes include the reduced differential roughness and weaker static stability observed over oceans. The enhanced moisture supply to the condensational process both through convection and stable latent heating represent other important means by which cyclogenesis may be enhanced in maritime regions. Air–sea interactions that create or enhance low-level baroclinic zones along coastal regions, providing a suitable environment for a strong surface response to the forcing of a mobile upper level trough may be critical to oceanic cyclogenesis. Some indication has been provided by the statistics that the stretched tail behavior of oceanic cyclogenesis is not restricted to the cold season. This suggests that the concept of explosive cyclogenesis as represented by a specific maximum value of cyclone deepening is misleading. Specific case study and diagnosis of a large sample of individual oceanic cyclones is required to determine whether this tail behavior arises as a result of physical processes active to varying degree in *all* oceanic cyclogenetic events or specific forcing mechanisms that are operative in only a subset of these cases. This question is in many ways more general than that concerning explosive storms as presently defined, yet its resolution is ultimately tied to understanding the nature of the more extreme wintertime events.

Acknowledgments. I wish to thank Dr. John Gyakum and Mr. Tim Bullock for many interesting and useful discussions concerning the nature and implications of Gaussian versus non-Gaussian distributions of cyclone deepening rates. This work has been supported with a graduate stipend funded by a Canadian Atmospheric Environment Service (AES) subvention.

REFERENCES

Bradley, J. V., 1968: *Distribution-free statistical tests.* Prentice-Hall, 388 pp.

Breiman, L., 1973: *Statistics: With a view towards applications.* Houghton Mifflin, 399 pp.

Draper, N., and H. Smith, 1981: *Applied Regression Analysis, Second Edition.* Wiley and Sons, 709 pp.

Emanuel, K. A., 1986: An air–sea interaction theory for tropical cyclones. Part I: Steady-state maintenance. *J. Atmos. Sci.,* **43,** 585–604.

Gyakum, J. R., J. R. Anderson, R. H. Grumm and E. L. Gruner, 1989: North Pacific cold-season surface cyclone activity: 1975 through 1983. *Mon. Wea. Rev.,* **6,** 1141–1145.

Leith, C. E., 1971: Atmospheric predictability and two-dimensional turbulence. *J. Atmos. Sci.,* **28,** 145–161.

Lorenz, E. N., 1969: The predictability of a flow which possesses many scales of motion. *Tellus,* **21,** 289–307.

Petterssen, S., 1956: *Weather Analysis and Forecasting, Vol. 1. Motion and Motion Systems.* McGraw-Hill, 428 pp.

Roebber, P. J., 1984: Statistical analysis and updated climatology of explosive cyclones. *Mon. Wea. Rev.,* **112,** 1577–1589.

Sanders, F., 1986: Explosive cyclogenesis in the west-central North Atlantic Ocean, 1981–1984. Part I: Composite structure and mean behavior. *Mon. Wea. Rev.,* **114,** 1781–1794.

——, and J. R. Gyakum, 1980: Synoptic-dynamic climatology of the "bomb." *Mon. Wea. Rev.,* **108,** 1589–1606.