The Role of Surface Heat and Moisture Fluxes Associated with Large-Scale Ocean Current Meanders in Maritime Cyclogenesis

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ABSTRACT

A structurally simple analytic quasi-geostrophic model is used to investigate the role of diabatic processes resulting from surface fluxes of heat and moisture associated with ocean current meanders in midlatitude maritime cyclogenesis. The combined effects of sensible heat transfer and convective latent heating resulting both from surface water vapor fluxes and large-scale moisture convergence can contribute both directly and indirectly to cyclogenesis, the extent of the contribution being dependent on a number of cooperative processes interacting within the cyclogenesis. Latent and sensible heating associated with shallow convection in a moist adiabatic environment in which the large scale convergence supplies ample moisture to the cumulus clouds represent the conditions under which the greatest enhancement of diabatic model cyclogenesis occurs; but cyclogenesis induced by differential vorticity advection is 90° out of phase (with respect to the large-scale tropospheric thermal wave) with direct diabatic intensification associated with sensible heating. The indirect effects of surface fluxes associated with the moistening and destabilization of the boundary layer and the amplification of the overall atmospheric baroclinity have greater potential to enhance model cyclogenesis than direct diabatic processes under conditions of strong adiabatic forcing. Preliminary observational evidence suggests that the occurrence of explosive cyclogenesis over the western Atlantic Ocean is marginally more likely when cyclones propagate across the positions of mean warm sea surface temperature anomalies along the Gulf Stream boundary than in the absence of such features. The existence of such anomalies is neither a necessary nor a sufficient condition for explosive cyclogenesis to occur, but may enhance the development in some cases.

1. Introduction

The intensification of maritime cyclones is a topic of increased research interest from both theoretical and observational points of view. Sanders and Gyakum (1980) showed that explosive deepening is a characteristic of the vast majority of the deepest cyclones, and that such deepening is primarily a maritime event. Those authors also demonstrated that such storms exhibit a relationship to the large scale flow that is consistent with adiabatic-baroclinic processes, but that the forcing excluding diabatic effects is insufficient to account for observed rates of deepening. Sanders (1986a) reiterated this finding by showing a high correlation between upper-level vorticity advection over surface cyclones and simultaneous surface-deepening rates. He concluded that the explosive maritime cyclone probably represents a fundamentally baroclinic disturbance in which the surface response to upper-level forcing is exceptional. Roebber (1984) suggested through a statistical analysis that the mechanism(s) involved in the deepening process is in some fundamental way distinct from that of ordinary baroclinic instability. Gyakum et al. (1989) and Roebber (1989) have extended and refined those results, showing a dramatic difference in the character of the statistical distribution of deepening rates for oceanic and continental cyclones. Thus, it might be more relevant to focus on the distinctions between land-based and maritime cyclones than explosive versus nonexplosive events as defined by storm central pressure falls.

The climatology of explosive cyclones shows that the position of the midpoint of the period of maximum 24-hour deepening most often corresponds with regions of strong mean sea surface temperature (SST) gradients such as those associated with the Kuroshio current off Japan and the Gulf Stream off North America (Sanders and Gyakum 1980; Roebber 1984; Sanders 1986a), areas that are also conducive to the development and intensification of less intense storms (Colucci 1976; Whittaker and Horn 1984; Roebber 1984). Physically, the differences between continental and oceanic areas are related to boundary layer processes, such as fluxes of heat, moisture and momentum. Surface fluxes over oceanic regions can affect the cyclogenetic environment both directly, through diabatically induced low-level...
convergence, and indirectly, through mechanisms as diverse as reduced surface drag, weaker static stability, increased convective and stable condensational heating and the creation or intensification of low-level baroclinic zones through differential heating along coastal regions. Vertical motions associated with these mechanisms further feed back into the cyclogenesis process by modifying the three-dimensional temperature and moisture structure of the atmosphere.

Some of these indirect processes have been studied. Numerous authors have investigated the contribution of convective latent heating to storm development (Tracton 1973; Rasmussen 1979; Reed 1979; Bosart 1981; Gyakum 1983a,b; Pauley and Smith 1988). Liu and Elsberry (1987) conducted a heat budget analysis of a particular rapid storm development in the Pacific Ocean. Their work suggested that surface sensible heat fluxes may have played an important role in the initial period of rapid intensification of the storm, and that latent heat release at midlevels was crucial to the second period of explosive deepening. Orlanski (1986) theorized that surface heat fluxes from the ocean help to organize meso-α cyclones by reducing the static stability of the atmosphere in the lowest kilometer. The organized convergence of surface moisture leads to condensational heating by convection which promotes explosive development of the wave. Nuss and Anthes (1987) used a mesoscale baroclinic channel-flow numerical model to examine how a number of physical factors modify cyclogenesis, including the effects of surface fluxes of heat and moisture. Their results indicated that surface heating in phase with the planetary boundary layer thermal structure is the potential to enhance cyclogenesis while surface fluxes associated with a meridional SST gradient opposed intensification. Chen and Dell’osso (1987) found that for a particular case of East Asian explosive cyclogenesis, the direct role of sensible heating was less important than its indirect contribution to convective latent heating. Pedlosky (1975), using an idealized but nonlinear coupled ocean–atmosphere model, showed that cyclogenesis over long time periods is enhanced in the presence of oceanic thermal anomalies through the gradual intensification of the latitudinal atmospheric temperature gradient. Further, the anomaly is itself intensified in these interactions, leading to a positive feedback in the coupled air–sea system. This diabatic effect leads to an enhancement of the baroclinic zone over a period much greater than the life-cycle of an individual storm.

The direct contribution of surface fluxes has received least attention. Emanuel (1986a) has suggested a positive feedback loop process involving surface processes, dubbed Air–Sea Interaction Instability (ASII), to explain the dynamics of tropical cyclones and to a lesser degree, that of polar lows and explosive cyclones (Rasmussen and Lystad 1987). Emanuel (1986b) showed that the cyclogenetic potential from this mechanism is large, but that considerable time is necessary for this thermodynamic potential to be converted to storm intensification through boundary layer processes. Davis and Emanuel (1988) showed that systematic underpredictions of oceanic cyclogenesis by the National Meteorological Center Limited-area Fine-mesh Model (LFM II) were well correlated with the amount of heating obtainable from the ocean surface, and suggested that this behavior resulted from an inadequate representation of the effects of evaporation and sensible heating from the sea surface on surface cyclogenesis.

The idea that surface fluxes of sensible heat and water vapor might play a direct role in explosive cyclogenesis has been dismissed by the argument that the heat transfer from ocean to atmosphere associated with midlatitude cyclones is confined largely to the colder northwesterly flow in the wake of the storm (Petterssen et al. 1962) and therefore is not critical to its continued evolution. Petterssen (1956) showed that the direct impact of surface heating on cyclone development is related to the pattern of heating as expressed by the Laplacian of the heat transfer. Danard and Ellenton (1980) found no significant direct contribution of heat and water vapor input from the sea surface during four cases of rapid east coast cyclogenesis, finding that the strongest diabatic vorticity tendencies occurred behind the storms. Those authors suggested that the preconditioning of the atmosphere by these fluxes was probably important to the subsequent developments.

The location and effect of heat sources and sinks on cyclone development, however, are complex functions of the coupled air–sea system and need not be small. Patterns of upward and downward heat transfer introduced at the surface will result in corresponding patterns of cyclonic and anticyclonic vorticity tendency. In the mean, ocean currents such as the Kuroshio and Gulf Stream do not present such patterns directly along their boundaries where maximum cyclone intensification is observed. In individual cases, though, these streams are characterized by meanders of warm and cold water along the stream fronts, in analogy to the baroclinic instability prevailing in westerly atmospheric flow (Saltzman and Tang 1975; Newton 1978). Thus, the physical mechanism exists under special circumstances for the transfer of surface-derived heat energy from the ocean to the developing midlatitude cyclone in a way that may directly enhance its development.

Halliwell and Mooers (1983) noted that shoreward excursions of the Gulf Stream meander, such as occurred during the winter of 1977/78, seem to contribute to the intensity of offshore cyclones. Pandolfo (1985), using order-of-magnitude estimates of potential vorticity generation rates, proposed that SST meanders and rings in the Gulf Stream and Kuroshio current could play a significant direct role in explosive cyclogenesis. Ample theoretical and observational evidence exists in the literature to suggest that the Great Lakes in wintertime are a factor in increased cyclogenesis in that region (Petterssen and Calabrese 1959;
Roebber (1984; Whittaker and Horn 1984). The Great Lakes, however, are not a region of explosive cyclogenesis. These land-locked lakes are of a size comparable to oceanic rings and meanders, but represent isolated pools of relatively warm water, in contrast to the conditions offshore, where the SST features are localized “hot spots” embedded within an overall region characterized by a warm and moist lower boundary. Thus, the unexceptional cyclogenetic activity in the Great Lakes is not clear proof that diabatic effects are unimportant in producing substantial pressure falls over 24 hour periods; but if diabatic effects are important, this observation is suggestive that spatial and temporal factors may have significant influence.

Accordingly, a number of aspects concerning the role of the underlying sea surface in maritime cyclogenesis need to be investigated. These include the relative importance of time–space scale relationships and phasing between the geopotential wave, the air temperature wave and the SST wave in contributing to direct and indirect cyclogenesis and the impact of variations in and interactions between properties of the storm and the underlying surface such as thermal gradients and amplitudes, large-scale static stability, and the vertical distributions of heat and moisture. With the advent of large scale experiments designed to study cyclone development in greater detail such as GALE/CASP (Dirks et al. 1988; Stewart et al. 1987) and ERICA (Hadlock and Kreitzberg 1988), it seems an appropriate time for additional effort in these areas.

This study, using a variation on the analytic quasi-geostrophic model of Sanders (1971), will attempt to determine whether and under what conditions cyclogenesis can be directly enhanced by surface fluxes and whether such conditions are consistent with published observations of the evolution and structure of explosive cyclones. The somewhat dated analytic quasi-geostrophic modeling approach is chosen in order to take advantage of its relative simplicity for the purpose of explicitly examining the impact of physical factors on development, recognizing that certain limits to the analysis will thereby be imposed. The simplified atmospheric structure, the crude representation of the diabatic heating and the diagnostic nature of the solutions mean that we will be limited to rather broad inferences concerning the interactions between the relative roles of the various surface flux related processes. The complex, interactive nature of the cyclogenesis problem suggests, however, that a simplified approach such as this may provide conceptual guidance as an aid to understanding the results of prognostic mesoscale primitive equation models, which incorporate considerably more sophisticated representations of diabatic processes within complex atmospheric circulations. Such models have in recent years shown increasing success at simulating explosive cyclogenesis (Anthes et al. 1983; Sanders 1986b; Sanders 1987; Kuo and Reed 1988).

The paper is organized as follows. Section 2 gives the basic background of the model. Readers interested in the model sensitivity analysis are referred to section 3. Some preliminary climatological information concerning the impact of cyclone-meander interactions on cyclone intensity is presented in section 4. Details of the model solutions are provided in the Appendix.

2. Analytic quasi-geostrophic model

Table 1 provides a summary of the parameters used to represent the physical structure of the atmosphere and the temperature of the sea surface. The thermal structure of the model atmosphere is represented by

\[
T(x, y, p) = T_m(p) - \{ ay + \frac{T}{p} \sin(x + \lambda) \cos y \},
\]

(1)

where

\[
T_m(p) = T_m(p_0) \left( \frac{p}{p_0} \right)^b,
\]

(2)

and the geopotential structure at the surface pressure level, \(p_0\), is taken to be

\[
\Phi(x, y, p_0) = -\frac{\Phi_0}{\sin k x} \cos y.
\]

(3)

The parameter \(\lambda\) represents the phase lag of the temperature field relative to the surface geopotential field, such that \(\lambda = 0\) and \(\lambda = L/2\) correspond to cold core and warm core cyclones, respectively. The geopotential low at \(p = p_0\) is fixed at \(x = \pi/4, y = 0\). At pressure levels above \(p_0\), the geopotential field is found by hydrostatic integration such that

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<th>Table 1. Model parameters.</th>
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<td><strong>Cyclone parameters</strong></td>
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<th><strong>Sea surface temperature parameters</strong></th>
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<td>(T_S)</td>
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<td>(k_s)</td>
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\[
\Phi(x, y, p) = \Phi(x, y, p_0) + \int_{p_0}^{p} \frac{\partial \Phi}{\partial p} dp
\]
\[
= \Phi_m(p) - \Phi \sin kx \cos ky - R \ln \left( \frac{p_0}{p} \right) \\
\quad \times \left[ ay + \hat{T} \sin (x + \lambda) \cos ky \right],
\]
where
\[
\Phi_m(p) = R \int_{p}^{p_0} T_m(p) d \ln p
\]
\[
= \frac{RT_m(p_0)}{b} \left[ 1 - \left( \frac{p}{p_0} \right)^k \right].
\]

The sea surface thermal structure corresponds to a circular Gaussian anomaly superimposed upon a mean SST field with a constant latitudinal temperature gradient, specified by
\[
T_S(x, y) = T_m - (a_S y - \hat{T}_S e^{-k_S(x-x_0)^2+y^2}),
\]
where the coordinate \(x_0\) corresponds to the position of the warm perturbation core, \(k_S\) is a size factor and the other terms are comparable to the respective atmospheric terms except that the subscript `S` refers to the sea surface.

The atmospheric structure differs from Sanders (1971) primarily in that the model domain is centered upon the surface geopotential low \((x = L/4, y = 0)\) rather than the thermal wave. Adjustments of the phase parameter \(\lambda\) shift the position of the thermal wave within the model domain so that the impact of phase relationships on cyclone development can be directly studied. Additionally, the vertical structure of the air temperature wave is specified such that no change in the intensity of horizontal temperature contrasts through the model troposphere is allowed. The validity of this assumption may be questioned with respect to the initial stages of explosive cyclones, however, which frequently appear early in their life histories as rather shallow disturbances (Anthes et al. 1983; Rogers and Bosart 1986). A priori, it is recognized that this simplification will lead to somewhat smaller computed geopotential tendencies than would occur if the thermal gradient was concentrated at the lowest levels (Gyakum 1983a). Figure 1 shows the geometry of the model structure for the case of a surface low situated at the inflection point of the tropospheric thermal wave \((\lambda = L/4)\), directly above a warm SST meander \((x_0 = L/4)\).

The problem is then formulated through the quasi-geostrophic omega equation in \(p\)-coordinates, generalized to include diabatic effects (Haltiner 1971):
\[
\left( \sigma \nabla^2 + f_0^2 \frac{\partial^2}{\partial p^2} \right) \omega = f_0 \frac{\partial}{\partial p} (\mathbf{V} \cdot \nabla \eta)
\]
\[
- \nabla^2 \left( \mathbf{V} \cdot \nabla \frac{\partial \Phi}{\partial p} \right) - \frac{Rg}{pC_p} \nabla^2 Q,
\]
specifying the vertical profile of sensible heating rate (Fig. 2). The surface sensible heat flux is

$$H = \rho C_v C_h |V_0|(T_S - T_A),$$  \hspace{1cm} (12)$$

where $C_h$ is the bulk transfer coefficient for sensible heat flux. The requirement that the total heating through the atmospheric column resulting from the surface fluxes is equal to the bulk transfer at the surface results in

$$H = \int_{p_S}^{p_0} Q_h(p) dp = \int_{p_S}^{p_0} A_h |V_0|(T_S - T_A)(\frac{p}{p_0})^{p'} dp,$$  \hspace{1cm} (13)$$

where $p_S$ is the pressure level at which the sensible heating is cut off. This condition, combined with (12), yields:

$$Q_h(p) = \frac{(r + 1)\rho C_v C_h}{p_0^{r+1} - p_s^{r+1}} |V_0|(T_S - T_A)p'.$$  \hspace{1cm} (14)$$

The latent heating rate, $Q_L(p)$, is defined somewhat differently since water vapor becomes thermodynamically important only where and when it condenses. The transfer of water vapor at the surface (evaporation) is defined by

$$E = \rho C_e |V_0|(q_s - q_A),$$  \hspace{1cm} (15)$$

where $C_e$ is the bulk transfer coefficient for water vapor, $|V_0|$ is the surface wind speed, $q_s$ is the saturation specific humidity of the sea surface at temperature $T_S$ and $q_A$ is the specific humidity of the overlying air.

To parameterize the magnitude and vertical distribution of convective condensational heating, a wave-CISK type of process is assumed where the total vertically integrated heating is related to the rate of convective precipitation out of the column. This use of an upright type of convective parameterization is justified primarily on the basis of the overall simplicity of the model, since it is recognized that convection in mid-latitude cyclogenetic events is probably better characterized by moist slantwise ascent (Emanuel 1983; Sanders and Bosart 1985; Gyakum 1987; Emanuel 1988). The results of Nordeng (1987) suggest, however, that the impact of this simplification on surface cyclogenesis rates will be small, since we will diagnose intensification rates assuming the existence of convection. As discussed by Nehrkorn (1986), the implicit assumption in this kind of wave-CISK formulation is that the space and time scales of large-scale and convective motions are clearly separated so that the convective heating is considered to respond immediately and in situ to the larger scale forcing. Further limitations include the existence of “negative heating” in regions of low-level subsidence, and the sensitivity of the results to the details of the parameterization (Nehrkorn 1986; Davies and de Guzman 1979). With these caveats in mind, we proceed by writing

$$-\int_{p_e}^{p_T} \frac{Q_L(p)}{gL_o} dp = \text{precipitation},$$  \hspace{1cm} (16)$$

where $p_e$ is the condensation level and $L_o$ is the latent heat of vaporization. The total moisture supply to the convective process is assumed to arise from moisture convergence below the condensation level plus surface evaporation:

$$M = E + \int_{p_0}^{p_e} \nabla \cdot \left( \frac{\rho V}{g} \right) dp.$$  \hspace{1cm} (17)$$

A precipitation efficiency parameter is defined after Nehrkorn (1986):

$$\epsilon = \frac{\text{precipitation}}{M}.$$  \hspace{1cm} (18)$$

![Figure 2](image_url)  

**FIG. 2.** Sample vertical profiles of sensible heating rate, $h(p) = (r + 1)p^{p'}/(p_0^{r+1} - p_s^{r+1})$, for $r = 0, 2, 4$ and $6$ and $p_S = 700$ mb.
Thus, $\varepsilon$ represents the efficiency of the convective system in processing the synoptic-scale moisture supply into precipitation. For the purposes of this study, $\varepsilon$ is set equal to 1, recognizing that this may represent an overestimate since part of the moisture supply is used to moisten the atmospheric column. The convective heating is parameterized in the standard fashion, with its magnitude proportional to the low-level vertical velocity plus surface evaporation using an a priori specification of vertical distribution (e.g., Holton 1979). The heating resulting from the convective condensation of surface-derived water vapor, $Q_{LE}(p)$, can then be written as

$$Q_{LE}(p) = \varepsilon g L_v C_e |V_0| (q_s - q_a) h(p), \quad (19)$$

where $h(p)$ is the vertical heating profile such that

$$\int_{p_T}^{p_e} h(p) = 1. \quad (20)$$

The heating resulting from horizontal moisture convergence below the condensational level, $Q_{LM}(p)$, is

$$Q_{LM}(p) = \varepsilon L_v q(p_c) \omega(p_c) h(p), \quad (21)$$

where $q(p_c)$ is the specific humidity of the air and $\omega(p_c)$ is the vertical $p$-velocity at the condensation level. Thus, $Q_{LM}(p)$ contains components from each of the adiabatic, diabatic and frictional solutions of (7). The function $h(p)$ is chosen in order to allow for a range of realistic vertical profiles:

$$h(p) = K\left(\frac{p}{p_c}\right)^\alpha \left(1 - \frac{p}{p_T}\right)^2 \quad p_c \geq p \geq p_T$$

$$= 0 \quad \quad \quad p > p_c, \quad (22)$$

where $K$ is the normalization factor satisfying (20). A number of profiles are displayed in Fig. 3. It can be seen that the choice of $\alpha$ and $p_c$ completely determines the form of the profile, with large values of $\alpha$ corresponding to heating maxima at low levels.

Considerable discussion of the bulk transfer coefficients $C_h$ and $C_e$ can be found in the literature. Tisdale and Clapp (1963) pointed out that the coefficients are not constant, but are rather a function of stability, taking on larger values for more unstable lapse rates. Those authors noted that large random errors might be expected when bulk formulas are used in individual synoptic cases. Frequently, the transfer coefficients are replaced by estimates of the surface drag coefficient which in turn is expressed as a function of wind speed (Thompson and Pinker 1981; Agee and Howley 1977). Heat and water vapor, unlike momentum, are not transferred through form drag on surface waves. The wind speed dependence is based upon the argument that higher wave states and spray resulting from increased wind speed lead to increased transfer surface area; but Smith (1980), in an analysis of a series of measurements from an offshore stable platform, showed that the sensible heat transfer coefficient varied more strongly as a function of surface temperature difference than wind speed. Friehofer and Schmitt (1976) argued that the vapor flux coefficient should be approximately 16% larger than that for sensible heat, based on the ratio of the coefficient of diffusion of water vapor to air to the coefficient of diffusivity of nonturbulent air. Blanc (1985) examined the variation in ten published bulk transfer schemes for a large set of North Atlantic weathership observations and found differences on the order of 75% between them.

For the purposes of this study, the mean sensible heat transfer coefficient derived from the review conducted by Blanc (1985) will be used, recognizing that variations as large as 75% in the model instantaneous intensification rates associated with surface fluxes can result from the choice of the transfer coefficient alone. Thus,

$$C_h \approx 1.3 \pm 1.0 \times 10^{-3}$$

$$C_e \approx 1.16 C_h.$$

The use in the model of a domain average transfer coefficient independent of stability means that downward heat fluxes will tend to be overestimated. In general, one might expect that such errors will tend to result in larger values of the Laplacian of the surface heating, and therefore, diabatic cyclogenesis rates will be overstated. As shown by the form of the diabatic vorticity tendency solutions (see Appendix), however, the actual impact of the simplification is a complex function of the spatially integrated air–sea temperature field and by extension, most of the parameters of the model.

In the study of intense cyclones, frictional effects cannot be neglected. To account for these effects in the model, the surface stress is taken as

$$\tau_0 = \rho C_D |V_0| V_0. \quad (23)$$

The drag coefficient, $C_D$, is based on the expression of Smith (1980) for maritime conditions:

![Graph](https://example.com/graph.jpg)
\[ C_D = (0.61 + 0.063 |V_0|) \times 10^{-3}. \]  
(24)

The level \( p_0 \) is assumed to be at the top of the surface layer such that

\[ V_0(x, y) = 0.7V(x, y, p_0). \]  
(25)

The following balance is assumed in the surface layer:

\[ f_0 \nabla \cdot \mathbf{V} = -gk \cdot \nabla \times \frac{\partial \eta}{\partial p}. \]  
(26)

Replacement of the divergence in (26) with the vertical derivative of \( \omega \) and integrating both sides from the surface to \( p_0 \) results in the following lower boundary condition:

\[ \omega(p_0) = -\frac{g}{f_0} k \cdot \nabla \times \tau_0 \]

\[ = -\frac{\rho g}{f_0} \left[ \frac{\partial}{\partial x} C_D |V_0| v_0 - \frac{\partial}{\partial y} C_D |V_0| u_0 \right]. \]  
(27)

The solution of (7) with the right-hand side set to zero, subject to the above boundary condition at \( p_0 \), \( \omega(p_T) = 0 \) and appropriate conditions in \( x \) and \( y \) determines the form of the frictional vertical motion. The details of the solutions to (7) for the adiabatic, frictional and diabatic processes are provided in the Appendix.

Once the field of vertical motion is determined, it is possible to derive the instantaneous geopotential tendency by solving the geostrophic vorticity tendency equation for \( x \):

\[ \nabla^2 \chi = -f_0 V \cdot \nabla \eta + \frac{\partial \omega}{\partial p}, \]  
(28)

where \( \chi \) is the time rate of change of geopotential height, \( \eta \) is the absolute vorticity, and the other terms are as before. The form of the solutions to (28) at the low center for each of the forcing terms is provided in the Appendix.

3. Sensitivity tests

Since the model is diagnostic rather than prognostic, potentially important time-dependent interactions between cyclone parameters cannot be directly modeled. Processes such as surface heating may over time exhibit contradictory effects with respect to cyclogenesis, at once destabilizing the atmosphere yet acting to destroy the thermal perturbation upon which the amplification depends. The storm development rate is itself a non-linear function of storm intensity (\( \Phi \)). As the surface-derived heating is released, the circulation amplifies which in turn acts to increase the heat transfer from the sea surface (see Appendix). Adiabatic effects may act independently or as a function of the disturbance amplitude to intensify the storm, hastening the positive feedback process. Brakes on development include the advection of planetary vorticity and frictional dissipation, the latter of which increases rapidly with increasing storm intensity. Within the context of the present study, it will be possible to comment only upon the tendencies of the system at a given point in time, and how sensitive the instantaneous system response is to a particular set of atmospheric conditions.

Diagnostic model results were obtained for a range of values of the model parameters. Computations were carried out for the first five terms of the summations in both \( x \) and \( y \) for the geopotential tendency solutions (see Appendix), as tests showed that the solutions were dominated by the low frequency modes. A model structure corresponding to the 26 March 1970 continental cyclone modeled by Sanders (1971) was used to check the adiabatic and frictional components of the model, and forms the basis for the sensitivity tests reported in this section. The parameters diagnosed for the 26 March 1970 cyclone have the following values:

- Cyclone wavelength (\( L \)) 2900 km
- Geopotential perturbation amplitude (\( \Phi \)) 1020 m² s⁻²
- Thermal-geopotential phase lag (\( \lambda \)) 0.25 L
- Meridional temperature gradient (\( a \)) \( 1.12 \times 10^{-5} \) K m⁻¹
- Thermal perturbation amplitude (\( T \)) 7.25 K
- Tropospheric mean temperature (\( T_0 \)) 250 K
- Static stability parameter (\( \gamma \)) 0.144
- Coriolis parameter (\( f_0 \)) 9.2 \( \times 10^{-5} \) s⁻¹
- Surface pressure level (\( p_0 \)) 1000 mb
- Tropopause pressure level (\( p_T \)) 250 mb

The instantaneous adiabatic model geopotential tendency converted to equivalent 12 hour deepening rate of \(-8.3 \) mb \((-9.6 \) mb with no friction) compares well with the observed intensification of \(-8 \) mb. The base case corresponds to a situation of moderate adiabatic forcing. The choice of a continental cyclone to test the response of oceanic cyclones to surface fluxes is of no consequence, since the lower static stabilities and decreased storm wavelengths which are observed in oceanic storms will be accounted for by varying the basic structural parameters in the analysis.

The dependence of the cyclone growth rate on wavelength (\( L \)) and stability (\( \gamma \)) for the adiabatic and frictional solutions is displayed in Fig. 4. All other parameters were fixed at the values indicated above. The wavelength of maximum instability increases with increasing stability, ranging from approximately 2250 km for low static stability to about 3500 km for very stable atmospheres. Sanders (1971) argued qualitatively that this tendency in his model is supported by observational experience that anticyclones are larger than cyclones and that low hydrostatic stability tends to favor small synoptic-scale disturbances. The rate of frictional dissipation is only weakly dependent on static stability.
for scales typical of midlatitude cyclones, but the rate of filling increases rapidly for storms of small wavelength. The effect of this characteristic is to provide a short-wave limit to intensification of disturbances while the wavelength of maximum instability is increased slightly. For the conditions displayed in Fig. 4, the limiting wavelength ranges from approximately 1000–1500 km, while the most unstable wavelengths vary between 2500–4000 km, depending on static stability, scales generally consistent with midlatitude experience. The frictional dissipation is also strongly dependent on disturbance amplitude (Fig. 5), so the position of the most unstable wavelength and the short-wave cutoff are functions of the stage of storm development as well as the dynamical parameters controlling intensification. The effect of the meridional temperature gradient on adiabatic intensification was also examined (Fig. 6). The rate of intensification is almost linearly proportional to the temperature gradient, although a tendency for the wavelength of maximum instability to shift towards larger scales with increasing thermal gradient is also apparent.

Of considerable importance to the question of diabatic influences is the sensitivity of the complete solution to the thermal–geopotential phase lag parameter (λ). The frictional solution does not depend on this parameter, while the adiabatic terms are maximized at the inflection point of the tropospheric thermal wave (λ = L/4). It might be expected that the diabatic solution will depend strongly on the phase and scale relationships between the warm SST perturbation, the air temperature perturbation and the geopotential perturbation. Halliwell and Mooers (1983) reported that meanders of the Gulf Stream downstream of Cape Hatteras were most energetic at wavelengths of approximately 330 km, but were energetic across a wide spectrum ranging from less than 200 km up to 1000 km. Typical phase speeds ranged from 5–10 cm s⁻¹, so that a meander would travel less than 10 km in 24 hours, essentially stationary relative to cyclones passing through the region. As reported by Lai and Richardson (1977), warm rings form from pinched-off meanders north of the Gulf Stream. In the centers of warm meanders and rings lies warm Sargasso Sea water; thus, temperatures more than 10°C warmer than the surrounding water north of the Gulf Stream are not unusual. The size of warm rings varies but typically they are small, on the order of 100 km diameter.

The following values of the SST and diabatic heating parameters were used to examine the diabatic solution:

SST perturbation diameter¹ 100 to 400 km

¹ The SST perturbation diameter so-defined is represented by twice the distance from the warm core to the point at which the anomaly falls to 20% of the core value.
SST perturbation amplitude $T_s$ 5°C
meridional SST gradient $(a_s)$ $1.5 \times 10^{-5} ^\circ C \text{ m}^{-1}$
sensible heating vertical extent $(p_s)$ 600 mb
sensible heating profile parameter $(r)$ 2
condensation level $(p_c)$ 900 mb
latent heating profile parameter $(\alpha)$ 4

The above values for the SST field thus represent fairly strong perturbations ranging in size from warmings to that of the larger scale meanders. The heating profile parameters represent moderately intense convection, with condensational heating maxima occurring at midlevels in the atmosphere and vertical mixing of the surface sensible heating up to 600 mb. Intense but shallow convection in association with surface cyclogenesis was well documented in a polar low (Shapiro et al. 1987) under conditions of significant surface heat and moisture fluxes. Recent observational experience in ERICA suggests that shallow convection does occur in association with intense midlatitude cyclones, although whether this can be considered to be generally representative under such conditions remains an open question.

Figure 7 shows intensification rates due to sensible and condensational heating from surface-derived water vapor as a function of storm wavelength and SST perturbation diameter. The phasing relative to the tropospheric thermal wave was chosen so as to maximize sensible heating induced cyclogenesis rates, as will be discussed below. The sensible heating effect is a complex function of perturbation size both in determining the storm wavelength of maximum instability and the value of the maximum intensification rate. Intensification rates take on a maximum value for intermediate storm wavelengths for a fixed perturbation size, a result most clearly seen for the conditions shown in Fig. 7 corresponding to SST perturbations on the order of 300 to 400 km in diameter. In contrast, the condensational heating effect resulting from surface-derived water vapor is essentially independent of perturbation size, and increases with increasing storm wavelength up to 2500 km. This response results from the spatial pattern of sea–air specific humidity difference, which tends to be positive through most of the cyclone domain, and thus is not weighted strongly by the impact of the local SST perturbation.

Figure 8 shows the model instantaneous diabatic intensification rate as a function of the phase lag parameter $(\lambda/L)$ for the surface-derived flux components and the total diabatic heating (sensible heating plus convective condensational heating from the total moisture convergence including surface evaporation). The rate of intensification due to surface fluxes is a complex function of the phase relationships between the atmospheric thermal and geopotential waves and that of the underlying SST. The effect due to sensible heating resulting from an underlying warm SST perturbation is to favor intensification of cyclones on the cold side of the inflection point of the tropospheric thermal wave and weaken cyclones on the warm side of this position. This transition occurs somewhat more deeply in the warm ridge for the total diabatic heating effect. These phase dependencies are modulated somewhat, but are also qualitatively true, for situations in which the SST perturbation is upstream or downstream of the cyclone. Thus, for the range of values shown in Fig. 8, diabatic heating results in a trend with respect to the atmospheric thermal–geopotential structure lagged by nearly a quarter wavelength from that evidenced by adiabatic cyclones. As discussed earlier, the adiabatic forcing is substantial, subject to the phasing of the surface cyclone relative to the tropospheric thermal wave. Thus, for conditions of strong adiabatic forcing, the direct diabatic effect has a small relative influence on intensification rates. Since the static stability in the atmosphere is a function of time and space, however, a feedback loop not directly incorporated into the model is the destabilization of the atmosphere and consequent nonuniform reductions in static stability that may occur with surface heat and moisture fluxes and latent heating associated with convective and stable condensation. Indirect effects such as reductions of static stability (Fig. 4) and enhancement of the meridional temperature gradient (Fig. 6) may be significant in terms of overall cyclogenesis rates in individual cases.

The diagnostic diabatic model results are also quite sensitive to the form of the heating profiles specified by the parameters $p_s$, $r$, $p_c$, and $\alpha$ in (14) and (22). Figure 9 shows the sensitivity of the sensible and surface-derived condensational heating solutions to these
parameters. The sensible heating effect is strongly dependent on the maximum vertical extent of the heating, with conditions most conducive to cyclogenesis occurring for heating confined to low levels \( (p_S \rightarrow p_0) \). The vertical profile of the sensible heating as expressed by the value of \( r \) has a stronger impact on intensification rates when the heating is allowed to penetrate to mid-levels of the atmosphere. Under those conditions, concentration of the sensible heating at the lowest levels \( (r \text{ large}) \) is the most favorable situation for cyclogenesis. Thus, deep convection, which might allow for greater vertical extension and more uniform mixing of the surface sensible heating \( (r \text{ small}) \) appears to be less effective at promoting cyclogenesis, although for the conditions examined, the overall variation between instantaneous cyclogenesis rates for the different vertical profiles is less than 2 mb per 24 hours. The results for the surface-derived convective condensational heating also suggest that heating resulting from shallow convection \( (p_c \rightarrow p_0, \alpha \text{ large}) \) is most favorable for intensification. This effect diminishes as the convective condensation level is raised \( (p_c, \text{ decreasing}) \), that is, elevated convection appears to have little impact on surface cyclogenesis rates, regardless of the relative location of the heating maximum. The stronger response of the diabatic solutions to low-level heating is in agreement with numerous other studies of the effects of convective condensation (e.g., Bannon 1966; Davies and de Guzman 1979).

The sensitivity of diabatic cyclogenesis rates associated with sensible heating and convective condensational heating of surface-derived water vapor to the air–sea thermal structure was examined by varying a particular thermal parameter about a basic state while holding the other parameters constant (Table 2). The basic state corresponds to a mean surface air temperature and SST of 278 K, and air and SST perturbation amplitudes of 2.5 K, meridional air temperature gradient of 1.0 K 100 km\(^{-1}\), and a SST gradient of 1.5 K 100 km\(^{-1}\). Variations of \( \pm 2 \) K about the mean, \( \pm 2.5 \) K in the perturbation values, and \( \pm 0.5 \) K 100 km\(^{-1}\) in the air and SST gradients were allowed. The values in the table correspond to the percentage increase (positive) or decrease (negative) in deepening rate from the basic state. The phasing of the atmospheric thermal structure was such that the cyclone was positioned at the inflection point \( (\lambda = L/4) \) of the tropospheric thermal wave, and the SST perturbation was positioned at the surface cyclone center \( (x = L/4) \).

Intensification rates due to sensible heating increase

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Value</th>
<th>Sensible (% change)</th>
<th>Condensational (% change)</th>
</tr>
</thead>
<tbody>
<tr>
<td>( T ) (K)</td>
<td>0</td>
<td>0</td>
<td>-85</td>
</tr>
<tr>
<td>5</td>
<td>0</td>
<td>85</td>
<td></td>
</tr>
<tr>
<td>( T_a ) (K)</td>
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<td>-100</td>
<td>-7</td>
</tr>
<tr>
<td>5</td>
<td>-100</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>( T_m ) (K)</td>
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<td>-200</td>
<td>-3</td>
</tr>
<tr>
<td>280</td>
<td>-200</td>
<td>8</td>
<td></td>
</tr>
<tr>
<td>( T_m' ) (K)</td>
<td>276</td>
<td>200</td>
<td>-6</td>
</tr>
<tr>
<td>280</td>
<td>200</td>
<td>6</td>
<td></td>
</tr>
<tr>
<td>( a ) (K 100 km(^{-1}))</td>
<td>0.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>1.5</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
<tr>
<td>( a_c ) (K 100 km(^{-1}))</td>
<td>1.5</td>
<td>0</td>
<td>0</td>
</tr>
<tr>
<td>2.0</td>
<td>0</td>
<td>0</td>
<td></td>
</tr>
</tbody>
</table>
strongly with increasing amplitude of the SST perturbation, while increases in the air temperature perturbation have no effect for a cyclone situated at the inflection point of the tropospheric thermal wave (not shown in Table 2, increases are cyclogenetic on the cold side of the tropospheric thermal wave and cycloytic on the warm side). Changes in the meridional temperature gradients of the SST and the air have no direct impact on sensible heating induced cyclogenesis associated with surface fluxes due to model symmetry, although changes in the latter will influence total diabatic intensification rates by increasing moisture convergence through enhanced adiabatic vertical motions. Surprisingly, increases in the domain average SST and decreases in the domain average air temperature are cycloytic. This results from the requirement that the amplitude of the sensible heating induced vertical motion is proportional to the integrated Laplacian of the heating field. Since the wind varies throughout the model domain, a change in the mean air temperature or SST does not simply add a constant in the heating field. Mathematically,

\[
\nabla^2(V \Delta T) = V \nabla^2 \Delta T + \Delta T \nabla^2 V + 2(\nabla V \cdot \nabla \Delta T) = \Delta T \nabla^2 V, \tag{29}
\]

where \(\Delta T\) represents the mean sea temperature minus the mean air temperature. For \(\Delta T > 0\), this term induces sinking motion just above the low due to the sign of the Laplacian of the wind field. The mean SST field, as long as it attains a value greater than that of the mean air temperature, will oppose intensification through sensible heating, although this effect may be more than compensated in the real atmosphere by resultant reductions of effective static stability, an effect that must be externally imposed on the analytic model. This process acts to mask the relationship of the amplitude of the SST perturbation to intensification. To the extent that one expects the mean value of the air temperature to approach that of the sea surface, however, this effect would be minimized.

The surface-derived convective condensational effect shows a different character in terms of the response to variations in the temperature fields. Intensification rates resulting from this effect are most sensitive to changes in the air temperature perturbation due to the nonlinear response of specific humidity differences (\(\Delta q\)) to temperature changes. As the air temperature perturbation increases, larger local values of \(\Delta q\) occur in the cold air that are not compensated by corresponding decreases of \(\Delta q\) in the warm air, which leads to an overall increase of the Laplacian integrated over the cyclone domain. This asymmetric response results in increasing intensification rates with increasing amplitude of the tropospheric thermal wave.

Sanders and Gyakum (1980) found no strong correlation between deepening rates and sea surface temperature for a sample of midlatitude explosive cyclones.

The limited direct cyclogenetic response to air–sea temperature differences and the large degree of case-to-case variability indicated by the markedly different results for different air–sea thermal phasing suggests that a similar lack of correlation between intensification rates and air–sea temperature difference at the cyclone center would be found. If indirect effects such as the destabilization of the boundary layer air ahead of the cyclone are important, one might expect to find some correlation between air–sea temperature differences downstream of cyclones and subsequent cyclogenesis rates. Unfortunately, the author is not aware of any studies that have examined such relationships.

The overall direct effect of surface fluxes is to favor amplification of storms of relatively shorter wavelengths than purely adiabatic disturbances, but only under highly specific circumstances. Even under such conditions, that is, heating that is distributed in a manner consistent with that resulting from relatively shallow convection, and a storm that is exposed to the diabatic effects for sufficient duration (at least 12 hours) while positioned upstream of the inflection point in the tropospheric thermal wave and in close proximity to a fairly large and intense SST perturbation, the diabatic enhancement of model deepening rates is rather modest.

Haltiner (1967), within the context of a linear two-level model, found that sensible heat exchange tended to reduce the instability of short and medium waves. That study, which was based on a mean SST field of higher temperature than the overlying air, is consistent with the present study for the case of mean sea–air temperature fields. Nuss and Anthes (1987) also found that a mean underlying SST field tends to reduce cyclone instability, but they found that a sinusoidal sea surface temperature pattern in-phase with the air temperature wave tends to enhance cyclogenesis, their suggestion being that this pattern indirectly leads to the development by increasing thermal advection.

Indirect interactions are not specifically incorporated into the analytic model employed here, yet some inferences can be drawn from the form of the solutions. Adiabatic intensification in the analytic model arises primarily from differential vorticity advection associated with that part of the flow attributed hydrostatically to the atmospheric thermal perturbation. In that sense, a diabatic effect that increases the amplitude of the thermal perturbation could indirectly enhance cyclogenesis, net deepening or filling, dependent upon whether the adiabatic process more than offsets the direct diabatic weakening of the cyclone indicated in Fig. 8.

Mullen and Baumhefner (1988) found that sensible heating, within the context of a GCM simulation of a series of explosive cyclones, exerted a sizable positive influence on cyclone intensification, representing about one-half the total deepening rate produced by the model’s parameterized diabatic processes. These results
showed considerable case-to-case variability, however, indicating that contributions by specific processes in any individual case depend strongly on the details of the physical structure. We suggest that these results primarily reflect the efficiency of the indirect process in enhancing cyclogenesis rather than a dominant direct role. Within the context of the diagnostic model, the impact of indirect effects is highly dependent on the spatial relationship of the surface fluxes to the overall cyclone structure. Strong sensible heating ahead and to the right of the surface cyclone can be cyclogenetic by lowering the effective static stability of the atmosphere in the vicinity of the disturbance, and by amplifying the tropospheric thermal wave upon which the adiabatic development depends. Sensible heating in other regions may be cyclolytic by reducing the overall baroclinicity. The model results, while idealized, suggest that direct processes do not contribute strongly to intensification in the explosive stages of development, although such processes may be important in terms of the initial organization of lower tropospheric thermal vorticity.

4. Explosive cyclone/sea surface temperature climatology

The theoretical results of this paper have suggested that the direct effect of surface fluxes on maritime cyclogenesis is relatively small except under artificially extreme circumstances. Other results of the diagnostic model suggest, however, that surface fluxes may still strongly influence an existing storm through indirect effects associated with the moistening and destabilization of the boundary layer in the path of the cyclone. In this section, we test this assertion in a broad sense by postulating that cyclogenetic effects associated with storm/SST perturbation interactions will result in statistically discernible differences in the frequency of explosive cyclogenesis for storms that propagate across the position of SST anomalies.

Charts of monthly sea surface temperature anomalies were obtained from back issues of Climatic Perspectives (a publication of the Canadian Climate Centre, Atmospheric Environment Service) and compared to monthly Atlantic cyclone tracks published in the Mariners Weather Log. A warm anomaly was defined by the warmer water within the 1°C anomaly isotherm. The exceptionally warm water so defined presumably appears in the analyses as a result of quasi-steady warm meanders in the Gulf Stream averaged over monthly periods. Storms were studied in the area bounded by a line running from 30°N, 70°W to 45°N, 50°W to the east and the east coast of North America to the west. The location, approximate size and magnitude of existing SST anomalies within this region were documented for seven winter months between January 1980 and February 1982. The breakdown of cyclone activity and intensity as a function of storm track relative to the warm SST anomalies was also documented and is presented in Table 3. An explosive development was defined based on the Sanders and Gyakum (1980) 24 hour criterion, and was considered to occur in conjunction with a SST anomaly only in the instance that the storm traversed the anomaly position during its explosive deepening period.

The data show that 8 of 19 storms developed explosively as they traversed warm SST anomalies. Several of the 11 nonexplosive storms that interacted with SST perturbations developed into nonetheless intense disturbances with winds exceeding 50 knots. In contrast, without encountering a SST anomaly, only 8 of 28 storms underwent rapid development. This strong and sometimes explosive development in association with warm sea surface temperature anomalies and the reduced frequency of similar development outside of those areas is suggestive, but the differences are small enough that we cannot conclude to any high degree of confidence that this enhanced frequency is other than coincidental (the difference is statistically significant at confidence levels less than 90% based on Fisher’s Exact Test). There appears to be a greater likelihood of explosive development if a cyclone traverses a warm SST anomaly than if it does not. Whether such correspondence is merely an artifact of the small data sample or is a consequence of surface fluxes requires more detailed analysis of individual cases. Strictly, one would suppose that interactions with SST anomalies will affect all storms to a certain degree, the extent of this influence likely depending on the details of the atmospheric structure in each case. Examination of the adiabatic-dynamical and diabatic forcing in storms that develop versus storms that do not develop in association with meanders would help to answer whether such interactions are physically meaningful.

Evidence that the diabatic influences discussed in this paper are not in themselves sufficient for explosive cyclogenesis in the absence of large-scale adiabatic support is provided by a comparison between the months of February 1981 and February 1982. Pronounced anticyclogenesis over the western Atlantic at upper levels (Dickson 1981) restricted the entire spectrum of cyclone development in February 1981 (3 storms, none explosive). The circulation in February 1982 was characterized by an exceptionally strong zone of atmospheric thermal contrast over the Atlantic (Dickson 1982) and cyclone activity was intense (5 storms, 4 of which developed explosively during at least one 24 hour period). This is particularly suggestive since the sea surface temperature anomaly patterns were strikingly similar in the two months, with strong warm anomalies occurring southeast of Cape Cod. Three of four cyclones that tracked through the warm anomaly position exhibited explosive development in February 1982. In contrast, two storms in February 1981 tracked through the anomaly position but neither developed explosively. These results are consistent with
the view that surface-based diabatic influences associated with SST perturbations are neither sufficient nor necessary for explosive development, but rather may simply enhance the overall development.

5. Summary

The theoretical and limited observational findings reported here suggest that greater attention to the underlying sea surface structure may be required to fully understand the phenomenon of maritime cyclogenesis. The model results indicate that surface fluxes of sensible heat and water vapor arising from storm interactions with larger scale warm ocean current meanders can contribute both directly and indirectly to the amplification of maritime cyclones, but the extent of that contribution depends critically on a number of processes acting in cooperation. Thus, a synergism is suggested in which a combination of adiabatic and diabatic processes may interact over time to amplify the storm to a degree much greater than the sum of the individual instantaneous forcings would indicate. At the same time, such interactions may preclude significant development by quickly destroying the instabilities upon which these processes depend. The conditions under which the diabatic heating modulates cyclogenesis are complex functions of the temporal and spatial relationships between the atmospheric geopotential and thermal structure and the underlying sea surface. Important model findings include: 1) shallow convection appears to be most effective at promoting direct diabatically induced surface cyclogenesis; 2) strong adiabatic forcing, as represented by substantial differential vorticity advection above the surface low, tends to occur out of phase (with respect to the tropospheric thermal wave) with direct diabatic cyclogenetic processes associated with surface fluxes; and 3) the direct effects of diabatic heating appear less critical to the outcome than the indirect effects associated with the moistening and destabilization of the boundary layer and the amplification of the overall atmospheric baroclinicity.

Detailed diagnostic analysis of individual cases of maritime development from an integrated air–sea perspective is required to validate and extend the results of this study. Such an analysis should involve the application of existing mesoscale numerical models in combination with detailed observational data. A number of aspects of the problem are in particular need of research attention. What are the magnitudes and three-dimensional distributions of diabatic heating in mid-latitude maritime cyclones? How do surface fluxes modify large-scale static stability, the tropospheric thermal wave and phase relationships between the air–sea thermal waves and the storm pressure (geopotential) field? How do these processes feed back into the evolution of the cyclone?

This synergistic interaction between cyclogenetic processes complicates understanding, yet also suggests a means for organizing otherwise disconnected investigations on the subject of maritime cyclogenesis. It is hoped that this work will encourage additional research in a number of these areas.

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APPENDIX

Detailed Quasi-geostrophic Model Solutions

Equation (7) can be written as

$$\left( p^2 \frac{\partial^2}{\partial p^2} + \frac{\gamma R T_0}{f_0^2} \nabla^2 \right) \omega(x, y, p)$$
\[
\omega(x, y, p) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{i=1}^{4} g_{mn}(x, y) A_{mn}(p),
\]

where

\[
\begin{align*}
g_{m1}(x, y) &= \sin \sigma_m x \sin \sigma_n y \\
g_{m2}(x, y) &= \cos \sigma_m x \sin \sigma_n y \\
g_{m3}(x, y) &= \sin \sigma_m x \cos \sigma_n y \\
g_{m4}(x, y) &= \cos \sigma_m x \cos \sigma_n y
\end{align*}
\]

and \( A_{mn} \) is found by solving

\[
p^2 \frac{\partial^2 A_{mn}}{\partial p^2} - \frac{\gamma R T_0}{f_0^2} (\sigma_m^2 + \sigma_n^2) A_{mn}
\]

\[
= 4 \int_{-L/2}^{L/2} \int_{0}^{L} f(x, y, p) g_{mn}(x, y) \, dx \, dy,
\]

where \( f(x, y, p) \) represents each of the individual forcing terms on the right hand side of (A1). For the fifth term of (A1), the solution technique is analogous to the above, except that the equations are in two dimensions \((y, p)\).

The vertical boundary conditions specified are that the vertical motion vanish at the top and bottom boundaries. Additional vertical boundary conditions are required for the sensible heating and convective condensational heating solutions since their forcing terms in (A1) apply only within certain pressure ranges. The sensible heating forcing is nonzero for \( p > p_c \) while the condensational heating forcing is nonzero for \( p < p_c \). Thus, for sensible heating, the additional boundary conditions applied are that the vertical motion obtained from the forced and nonforced solutions of (A1) match at \( p = p_c \) along with the first derivatives. Similarly, the convective condensational heating vertical motion obtained from the forced and nonforced solutions of (A1) are required to match at \( p = p_c \), along with the first derivatives.

The frictional solution is found by solving (A1) for no forcing and assuming the same boundary conditions as above with the exception that the lower boundary condition is specified by (27). This results in
\[ \omega_F(x, y, p) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \hat{\omega}_{F1} \sin \sigma_m x \cos \sigma_n y F_F(p) \right) \]

where

\[ \hat{\omega}_{F1} = -\frac{4pgL^2}{T_0} \int_{-L/2}^{L/2} \int_0^L \left[ \frac{\partial}{\partial x} C_D |V_0| |v_0| \right] \sin \sigma_m x \cos \sigma_n y dy dx \] (A7)

\[ \hat{\omega}_{F2} = -\frac{4pgL^2}{T_0} \int_{-L/2}^{L/2} \int_0^L \left[ \frac{\partial}{\partial y} C_D |V_0| |v_0| \right] \cos \sigma_m x \sin \sigma_n y dy dx \] (A8)

\[ F_F(p) = \frac{\left( \frac{p}{p_0} \right)^{n_{ms}} - \left( \frac{p_T}{p_0} \right)^{n_{ms}}}{1 - \left( \frac{p_T}{p_0} \right)^{n_{ms}-n_{ms}}} \] (A9)

\[ r_{mn} = \frac{1}{2} \left[ 1 + 4 \gamma R T_0 \left( \sigma_m^2 + \sigma_n^2 \right) \right]^{1/2} \] (A10)

\[ s_{mn} = 1 - r_{mn}. \] (A11)

The forcing represented by the fourth and fifth terms of (A1) do not lead to vertical motion at the cyclone center and therefore do not contribute through (28) to intensification of the low; the solutions for these terms will not be shown. The application of the above boundary conditions to (A5) for each of the other adiabatic forcing terms (denoted by the subscripts 1 to 3) and the diabatic forcing terms (denoted by the subscripts \( H \), \( E \) and \( M \) for sensible heating, convective condensational heating associated with surface fluxes and horizontal moisture convergence, respectively) of (A1) leads to the following solutions for the vertical motion:

\[ \omega_1(x, y, p) = -\frac{2g \Delta T}{\gamma T_0} F_1(p) + \frac{2 \gamma T_0}{\gamma^2 T_0^2} G_1(p) \times \cos(k + \lambda) \cosy \] (A12)

where

\[ F_1(p) = p \ln p \]

\[ + \frac{\left[ p_0 \left( \frac{p}{p_0} \right)^n - p \left( \frac{p_0}{p} \right)^n \right] \left[ \ln p_0 - n \left( \frac{p_T}{p_0} \right)^n \ln p_T \right]}{\left( \frac{p_T}{p_0} \right)^{(n_{ms}-n_{ms})} - 1} - p \left( \frac{p_0}{p} \right)^n \ln p_0 \] (A13)

\[ G_1(p) = p \]

\[ + \frac{\left[ p_0 \left( \frac{p}{p_0} \right)^n - p \left( \frac{p_0}{p} \right)^n \right] \left[ 1 - \left( \frac{p_T}{p_0} \right)^n \right]}{\left( \frac{p_T}{p_0} \right)^{(n_{ms}-n_{ms})} - 1} - p \left( \frac{p_0}{p} \right)^n \] (A14)

and

\[ \omega_2(x, y, p) = \frac{2g \Delta T \ln p_0}{\gamma T_0} F_2(p) \cos(k + \lambda) \cosy \] (A15)

where

\[ F_2(p) = p + \frac{\left[ p_0 \left( \frac{p}{p_0} \right)^n - p \left( \frac{p_0}{p} \right)^n \right]}{\left( \frac{p_T}{p_0} \right)^{(n_{ms}-n_{ms})} - 1} - p \left( \frac{p_0}{p} \right)^n \] (A16)

and

\[ \omega_3(x, y, p) = -\frac{\beta T}{2g T_0} F_3(p) \cos(k + \lambda) \cosy \] (A17)

where \( F_3(p) = F_2(p) \) and \( r_1 = r_{mn}, s_1 = s_{mn} \) for \( m = n = 1 \); and

\[ \omega_H(x, y, p) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \hat{\omega}_{H1} \sin \sigma_m x + \hat{\omega}_{H2} \cos \sigma_m x \right) \cos \sigma_n y F_H(p) \] (A18)

where

\[ \hat{\omega}_{H1} = -\frac{4pgRC_H(r_1+1)}{f_0^2 L^2 (p_0^{r_1+1} - p_T^{r_1+1})} \left[ r(r_1+1) - \frac{\gamma R T_0}{f_0^2} (\sigma_m^2 + \sigma_n^2) \right] I_{H1} \] (A19)

\[ I_{H1} = \int_{-L/2}^{L/2} \int_0^L \nabla^2 |V_0| |T_S(x, y) - T(x, y, p_0)| \sin \sigma_m x \cos \sigma_n y dy dx \]
\[ \hat{\omega}_{H2} = -\frac{4\rho g R C_b(r + 1) p_{r+1} - \gamma R T_0}{f_0^2 L^2 p_r - \gamma R T_0 \left( \sigma_m^2 + \sigma_n^2 \right)} I_{H2} \]  
\[ I_{H2} = \int_{-L/2}^{L/2} \int_0^L \nabla^2 [V_0 \left( T_s(x, y) - T(x, y, p_0) \right) \cos \sigma_m x \cos \sigma_n y] dx dy \]  
\[ F_H(p) = \begin{cases}  
p_{r+1} + C_{H1} p_r - C_{H2} p^{r-1}, & p \geq p_s \\
p_{r-1} - p_r + C_{H3} \left( p_{r-1} - p_r \right), & p \leq p_s 
\end{cases} \]  
\[ C_{H1} = p_s \frac{p_0 - p_r}{p_0} \left( p_0 - p_s \right) \frac{r_m - r_{mn}}{r_m - r_{mn}} - 1 \]  
\[ C_{H2} = p_s \frac{r_{mn} - r_m}{r_{mn} - r_{mn}} - p_s \frac{r_{mn} - r_m}{r_{mn} - r_{mn}} \]  
\[ C_{H3} = p_s \frac{r_{mn} - r_m}{r_{mn} - r_{mn}} - p_s \frac{r_{mn} - r_m}{r_{mn} - r_{mn}} \]  
\[ \omega_E(x, y, p) = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \hat{\omega}_{E1} \sin \sigma_m x + \hat{\omega}_{E2} \cos \sigma_m x \right) \cos \sigma_n y F_E(p), \]  
where  
\[ \hat{\omega}_{E1} = -\frac{4\rho g R K L C_b \epsilon}{f_0^2 L^2 C_p \left( \alpha + t - 1 \right) \left( \alpha + t - 1 \right) - \gamma R T_0 \left( \sigma_m^2 + \sigma_n^2 \right)} I_{E1} \]  
\[ I_{E1} = \int_{-L/2}^{L/2} \int_0^L \nabla^2 [V_0 \left( q_s(x, y) - q(x, y) \right) \sin \sigma_m x \cos \sigma_n y] dx dy \]  
\[ \hat{\omega}_{E2} = -\frac{4\rho g R K L C_b \epsilon}{f_0^2 L^2 C_p \left( \alpha + t - 1 \right) \left( \alpha + t - 1 \right) - \gamma R T_0 \left( \sigma_m^2 + \sigma_n^2 \right)} I_{E2} \]  
\[ I_{E2} = \int_{-L/2}^{L/2} \int_0^L \nabla^2 [V_0 \left( q_s(x, y) - q(x, y) \right) \cos \sigma_m x \cos \sigma_n y] dx dy \]  
\[ K = \frac{1}{p_c} \left\{ 1 - \left( \frac{p_T}{p_c} \right)^{a+1} \right\}^{-1} \left\{ 1 - \left( \frac{p_T}{p_c} \right)^{a+2} \right\}^{-1} \left\{ 1 - \left( \frac{p_T}{p_c} \right)^{a+3} \right\}^{-1} \]  
\[ B_t = 1 \quad \text{for} \quad t = 1 \quad \text{and} \quad t = 3 \]  
\[ B_t = -2 \quad \text{for} \quad t = 2 \]  
(A20)  
(A21)  
(A22)  
(A23)  
(A24)  
(A25)  
(A26)
\[ F_{E1}(p) = C_{E3} \left[ p^{m_{nes}} - p_0^{m_{nes}} \left( \frac{p}{p_0} \right)^{m_{nes}} \right] \] \[ [p > p_c] \]

\[ = p_c \left( \frac{p}{p_c} \right)^{a+t} + C_{E3} p^{m_{nes}} + C_{E3} p_0^{m_{nes}} \quad [p < p_c] \]

\[ C_{E1} = p_c \left( \frac{p}{p_c} \right)^{a+t} p_T^{m_{nes}} - p_T^{m_{nes}+t_{mn}} \]

\[ + \frac{p_c \left( \frac{p}{p_c} \right)^{a+t} p_T^{t_{mn}} - p_c^{m_{nes}} + C_{E3} \left[ p_0 \left( \frac{p}{p_c} \right)^{m_{nes}+t_{mn}} - 1 \right]}{p_c^{m_{nes}} - p_T^{m_{nes}+t_{mn}}} \]

\[ C_{E2} = \frac{\left( \alpha + t - r_{mn} \right) \left[ 1 - \left( \frac{p}{p_T} \right)^{m_{nes}} \right] p_c^{m_{nes}} + \left( \alpha + t - s_{mn} \right) \left[ 1 - \left( \frac{p_T}{p_c} \right)^{m_{nes}} \right] p_c^{m_{nes}}}{(s_{mn} - r_{mn}) (p_c^{m_{nes}+t_{mn}} - p_T^{m_{nes}+t_{mn}}) (p_T^{m_{nes}+t_{mn}} - p_0^{m_{nes}+t_{mn}})} \]

\[ C_{E3} = \frac{p_c \left( \frac{p}{p_c} \right)^{a+t} p_T^{m_{nes}+t_{mn}} p_T^{t_{mn}}}{(p_c^{m_{nes}+t_{mn}} - p_T^{m_{nes}+t_{mn}}) (p_T^{m_{nes}+t_{mn}} - p_0^{m_{nes}+t_{mn}})} \]

and

\[ \omega_m(x, y, p) = \sum_{t=1}^{3} \left[ \omega_1(x, y, p_c) + \omega_2(x, y, p_c) + \omega_3(x, y, p_c) \right] F_{M_1}(p) \]

\[ + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{t=1}^{3} \left[ \hat{\omega}_{1F} F_{F}(p_c) + \hat{\omega}_{1H} F_{H}(p_c) \right] \sin n \sigma_x \cos n \sigma_y G_{M_1}(p) \]

\[ + \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \sum_{t=1}^{3} \left[ \hat{\omega}_{2F} F_{F}(p_c) + \hat{\omega}_{2H} F_{H}(p_c) \right] \cos n \sigma_x \cos n \sigma_y G_{M_1}(p) \]

where

\[ F_{M_1}(p) = - \frac{RKL_q q(p_c) B_{L \epsilon}}{f_0^2 C_e \left[ (\alpha + t - 1)(\alpha + t) - \frac{\gamma RT_0}{f_0^2} \right]} \]

\[ G_{M_1}(p) = - \frac{RKL_q q(p_c) B_{L \epsilon}}{f_0^2 C_e \left[ (\alpha + t - 1)(\alpha + t) - \frac{\gamma RT_0}{f_0^2} (\sigma_m^2 + \sigma_n^2) \right]} \]

The terms resulting from the adiabatic forcing terms four and five have been omitted since they do not lead to intensification of the model low.

The geopotential tendency is evaluated at the surface low by solving (28) at x = L/4, y = 0, p = p_0. Since vorticity advection is identically zero at the surface low, the geopotential tendency arises as a result of divergence associated with vertical motion. The solutions of (28) are of the form

\[ \chi \left( \frac{L}{4}, 0, p_0 \right) = \frac{f_0^2}{2k^2} \frac{\partial \omega}{\partial p} \bigg|_{x=L/4, y=0, p=p_0} \]

\[ \chi \left( \frac{L}{4}, 0, p_0 \right) = - \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} \left( \frac{f_0^2}{2k^2} \frac{\partial \omega}{\partial p} \right) \bigg|_{x=L/4, y=0, p=p_0} \]

for the adiabatic vertical motion terms and

\[ \chi \left( \frac{L}{4}, 0, p_0 \right) = \frac{f_0^2}{2k^2} \frac{\partial \omega}{\partial p} \bigg|_{x=L/4, y=0, p=p_0} \]

for the diabatic and frictional vertical motion terms.
REFERENCES


