Estimation of Atmospheric Predictability by Circulation Analogs

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ABSTRACT

An empirical predictability study is presented based on 700-hPa Northern Hemispheric circulation analogs. A linear relationship between the initial root-mean-square difference of analog pairs and the time taken for the error to reach a certain limit value is used to extrapolate the predictability with initial errors considerably smaller than those in the present database. The relationship, first used in predictability experiments with the NMC numerical weather prediction (NWP) model, conforms to the experimental data in that the error growth depends not only on the magnitude of the error but also, to a lesser extent, on the initial error.

Despite the fact that earlier error growth studies did not reflect this dependence on the initial error predictability results with two state-of-the-art numerical models using different analysis methods, and those derived here by the linear relationship mentioned above from circulation analogs are gratifyingly similar. These estimates indicate that given the present observational error (about 12 m rms) and spatial resolution of the data, in the NH winter, the atmosphere seems to have 17–18 days of predictability before the initial difference reaches 95% of the saturation level (random error). In present models, the forecast error reaches the same 95% level at around ten and a half days. Since the climate mean as a forecast has considerably less error than a random forecast, from a forecaster's point of view it is more appropriate to use the climate error as a reference level (71% of the saturation level). With the same conditions as above and using this alternative error reference level, the atmosphere might have a predictability of nine days, while the two models considered currently exhaust predictability at close to six days, leaving considerable room for improvement. Note that these atmospheric predictability estimates were obtained without considering a possible enhancement of predictability due to interactions with the slowly changing ocean and other geospheres. Hence, these estimates can be considered as lower limits to atmospheric predictability.

Comparing the predictability estimates gained from twin model experiments to those from observational data is a special, complex method of model verification. Keeping in mind the uncertainties in the observational studies, one can ascertain that the models produce quite similar error growth characteristics to those of the real atmosphere. Hence, the NWP models are quite good on the time and spatial scales for which they were designed. However, there are some indications that they probably could not be reliably used to answer the theoretical questions regarding the gain in predictability with very small initial errors or with very high spatial resolution. Some kind of dynamic–empirical approach based on the interactions between different scales of motion is required to enhance current knowledge on these topics.

1. Introduction

Although predictability studies might guide modelers, it is probably the unique theoretical nature of the question rather than practical concerns that inspires many researchers. Since the late 1950s there has been an increasing emphasis on different aspects of atmospheric predictability. Lorenz (1969a) mentions three possible research techniques: a dynamical approach using numerical model experiments; an empirical approach that is based on naturally occurring analogs; and a dynamical–empirical approach that employs a system of equations to describe the spectral distribution of errors. This last method (Lorenz 1969b) has been neglected except for some recent work by Schubert and Suarez (1989). For a long time, in the first two approaches, investigators focused on estimating the doubling time of the initial error between two initially very close atmospheric states (e.g., Charney et al. 1966; Smagorinsky 1969; Lorenz 1969c). Dalcher and Kalnay (1987; hereafter called DK) recently pointed out that the estimated "doubling time of small errors" is not a good measure of error growth. Because the true state of the atmosphere is not known (due to uncertainties in the analysis and model initialization) it cannot be measured directly; and the results gained by extrapolation are very sensitive to the parameterization in the empirical models of error growth. Hence, the emphasis has shifted to the estimation of error growth at finite times. This error growth is well defined by the available (either model or observational) data. Instead

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of addressing atmospheric predictability in general, recent studies tend to focus on the limit of predictability after a certain value of error has been reached (e.g., the present day analysis error—DK—or the error of a one day forecast—Schubert and Suarez 1989). Some new details of atmospheric predictability—e.g., its dependence on the two-dimensional wavenumber—have also been investigated in these and other recent studies.

Despite the diversity of questions posed in predictability studies, they usually share a common feature. The majority of them do not consider the possible effect of the slowly varying boundary conditions on atmospheric predictability in the extratropics. It is assumed that specifying sea surface temperature, soil moisture or sea ice/snow cover can actually improve atmospheric forecasts. Of course, due to the complexity of the interacting systems it is very difficult to unravel how much extra time we could gain in atmospheric predictability if we were able to study the atmosphere as it interacts with the neighboring geospheres, and we could specify a relatively small initial error not only in the atmosphere but also in the other systems (ocean, land surface processes, etc.).

This study does not attempt to address the effect of the neighboring geospheres on atmospheric predictability. Considering the apparently very high dimensionality of the coupled ocean–atmosphere system, the shortness of the available observational records seems to be prohibitive. As atmospheric general circulation models become more realistic, it is possible that at least some aspects of this extra predictability can be studied with them. For a better understanding realistic ocean–atmosphere coupled models will be needed. In the present experiments, however, only the initial error in the atmosphere is specified. Consequently there is some (exactly unknown) "noise" in the other systems in these atmospheric experiments. This is because there is probably a range of boundary conditions that are possible with a particular atmospheric flow pattern. Each atmospheric flow pattern in this dataset is, therefore, accompanied with a particular but here unspecified set of boundary conditions from a range of possible boundary condition realizations. This noise in the neighboring systems probably degrades predictability. Hence, all the predictability estimates given below should be considered as lower limits to atmospheric predictability in this respect. Note also that the data are not stratified according to different spatial scales, frequency ranges, or initial conditions. Hence, the results reveal an average picture of atmospheric predictability from which the predictability of particular cases or phenomena may depart considerably.

In the following section a simple atmospheric predictability experiment is made following Chen (1989). He compared 1-, 2-, 3-, and 4-day lagged 30-day National Meteorological Center (NMC) Numerical Weather Prediction (NWP) model forecasts to estimate the internal (maximum) predictability of the atmosphere, assuming that the NMC model is a perfect model of the atmosphere. He found that the initial root-mean-square (rms) error (i.e., the rms distances between successive 30-day forecasts) and the limit of predictability (number of days before the error between the lagged runs reaches the saturation level equal to random errors) are linearly related. Instead of an error growth model, he then used this linear relationship to estimate the atmospheric predictability with very small initial errors. The same procedure is used here except that the "model" is the real atmosphere. Current investigation includes experimentation with atmospheric circulation analogs. Unlike Lorenz (1969c) who used an empirical error growth model to estimate the error growth with very small initial errors, the limit of predictability (in days) with analogs in different initial error ranges is observed. From this experimental data in section 2 a linear relationship is established between the initial errors and the limit of predictability. This linear relationship is used to extrapolate the predictability with (much) smaller initial errors. In section 3, the results are compared to those of other investigators and some comments follow about the method used here and about other methods. Conclusions are given in section 4.

2. Empirical predictability results

The following analysis is based on the results presented in Fig. 1. A detailed discussion of the database and the analog forecasting methods used to create this figure can be found in Toth (1990). Briefly, analog pairs of 700-hPa Northern Hemisphere (NH) extratropical daily circulation patterns were chosen from 33 Januarys (1950–82) using the rms difference between the base case and candidate analogs from different years. Then the best analogs were stratified into six groups according to their initial error [rms difference between the analogs on the 358-point equal-area grid of Barnston and Livezey (1987) at time step 0]. At this step only those analog pairs that initiate within the first ten days of January were considered.[1] The limit of predictability was defined as the intersection of the curves representing the growing average difference between the analogs in time and 95% of the error saturation level (118.5 m). The saturation level was estimated as the average difference between two randomly chosen atmospheric states. The reason to choose a limit value slightly below the saturation level is to reduce the effect of sampling fluctuations. [In their predictability study, DK used 95% while Schubert and Suarez (1989) used 90% of the climate variance that translates

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[1] Note that the number of independent pairs is considerably smaller than the 209 analog pairs that remained after this reduction. First, there are many cases when the same pair occurred twice switching only the role of base case–best analog status. And also some time dependence in the series of analog pairs is suspected that would be very difficult to quantify.

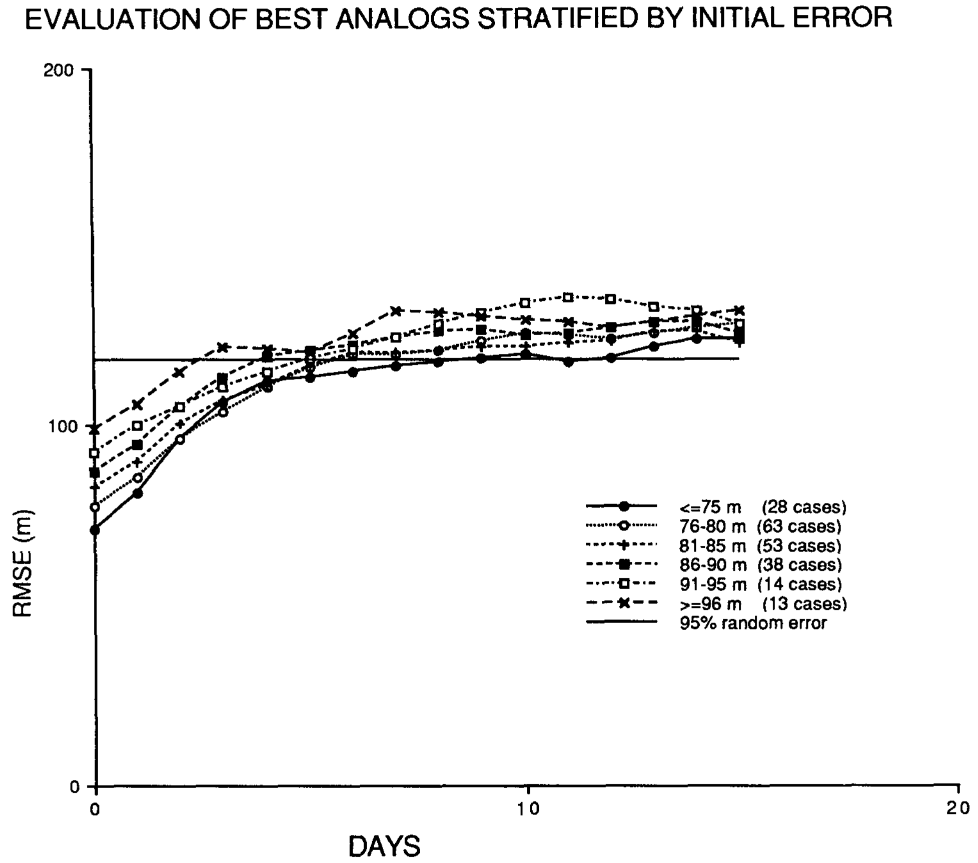

FIG. 1. Averaged error growth between 700-hPa Northern Hemispheric extratropical winter circulation analogs with different initial error ranges. The 95% of the estimated saturation level (averaged error between randomly chosen atmospheric states) is also shown.

into 97.5% and 95% of the rms random error, respectively.] However, here the results are not very sensitive to the choice of this limit value.

Once having the results of "twin" experiments with different initial errors, the method of Chen (1989) was followed and the initial rms was error plotted against the period of time at which the initial error reaches the 95% saturation level (limit of predictability). For an 118.5 m initial error, a 0-day predictability was assumed. The experimental data along with the fitted linear curve are shown in Fig. 2. For comparison, similar data from the NMC medium-range model's twin experiments, recalculated for the same 95% saturation level of those experiments (123.5 m) from Chen (1989) are also plotted. Note that in the linear fit the data points on the abscissa (95% random error, 0-day predictability) were treated as the other (real) data points in the figure since there is probably some sampling error in the estimate of random error level, too.

The parameters of the curve for the empirical (analog) experiment are the following. The ordinate intersection (predictability limit extrapolated to infinitesimal initial errors) would be 19.1 days; however, the curves are drawn only for initial errors larger than 12 m for which the assumed linear relationship still holds. The slope is −0.164 and the correlation coefficient is −0.954 (significant at the 0.001 probability level). Experiments were carried out with and without incorporating the data point on the abscissa in the fitting of the linear curve and with varying weighting factors on the data points (dependent on the number of analog pairs they represent). The sampling fluctuations in these cases led to parameters of the linear curve that were usually within ±15% of the parameter values presented here. Moreover, when the curves in Fig. 1 were smoothed with a running 3- or 5-point mean (employing triangular weights on the 3 or 5 points) the results remained basically the same. The data points for the analog and numerical experiments overlap only in the 70–80 m initial error range. This is because there are no truly good hemispheric analogs in our relatively short (30–40 yr) historical records (see e.g., Ruosteenoja 1988). Nevertheless the fitted linear curves in Fig. 2 are surprisingly close. The next section evaluates the results and compares them to the predictability estimates made by other methods.

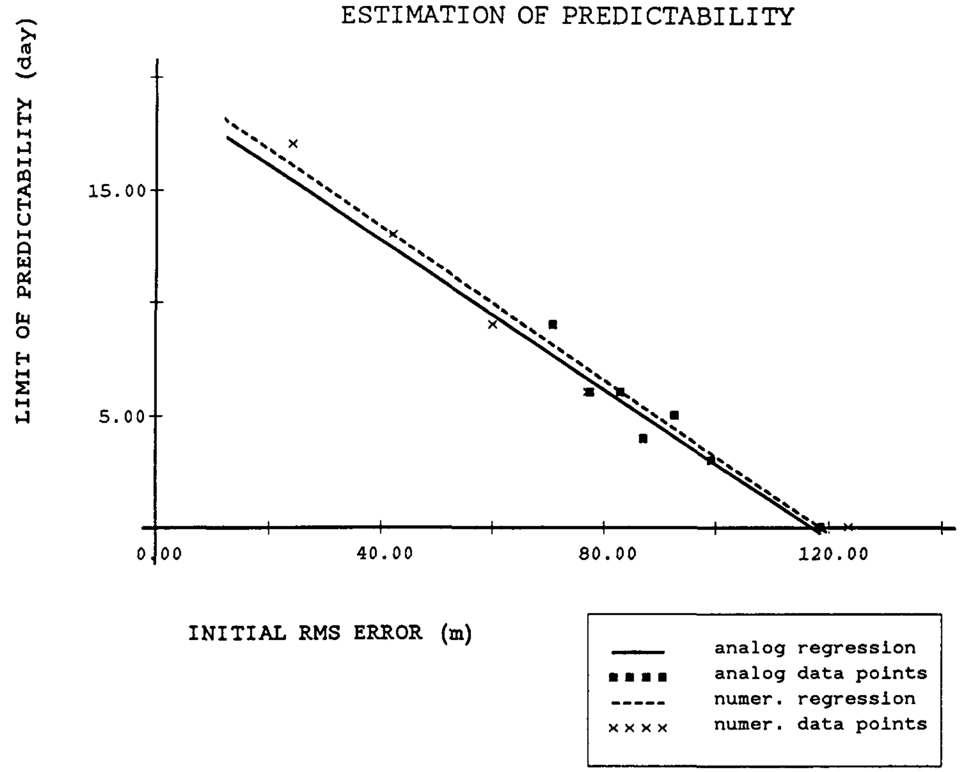

FIG. 2. Estimation of predictability (the time period in which a certain initial error reaches 95% of the saturation level) by the linear method of Chen. Northern Hemisphere extratropical winter data points from an empirical analog study (Fig. 1) and from NMC NWP twin model experiments are presented as squares and crosses, respectively. The corresponding fitted linear curves (solid and dashed lines, respectively) are also shown.

3. Discussion

The empirical results are now compared with those reported by Chen (1989), Lorenz (1982), and Dalcher and Kalnay (1987). All of these NWP studies estimated internal and external predictability. By internal predictability, after DK, we refer to the predictability of the real atmosphere. In other words, how an initial error between two observed atmospheric states evolves in time. In contrast, by external predictability we refer to the evolving difference between an atmospheric model prediction and the atmosphere itself. To estimate internal (intrinsic) atmospheric predictability the above authors assumed that the NWP model used is a perfect model of the atmosphere. Consequently, the time evolution of the difference between two initially close states in the model approximates how the difference, if observed in the real atmosphere, would evolve. They used the same model to measure external predictability but did not assume perfectness. By comparing model predictions to the appropriate verifying observed analysis fields they estimated how far ahead a prediction, on average, is better than a random or climate mean forecast. This limit value indicates both an intrinsic limit of atmospheric predictability and model deficiencies. Chen (1989) used 108 NMC 30-day NWP model runs and the corresponding 500-hPa analyses on the Northern Hemisphere (20°–80°N), from the 1986/87 winter. The previous section presented the way the internal predictability was estimated in his study. To calculate the external predictability, first the systematic error of the forecasts was removed, and then the time was observed at which the experimental forecast error curve reached the critical value (95% of the saturation level or 71% of the same level that is equivalent to the error of the climate mean forecast). Once systematic error is eliminated the estimated external predictability refers to a "geared" numerical model's performance. Since the systematic error cannot be totally removed operationally from the forecasts the external predictability is, strictly, an overestimation.

The other two studies, those of Lorenz (1982) and DK used an identical database; a 100-day ECMWF set of 10-day 500-hPa forecasts. In the main part of his study, Lorenz used the raw global forecast and observed data from the 1980/81 NH winter. To study internal predictability through the comparison of rms differences between forecasts lagged 1, 2, . . . , 9 days apart, he fitted a parabolic empirical error growth model to the experimental data, satisfying the requirement that the errors saturate at a certain level. To evaluate internal and external predictability, he used only NH data; the forecasts were modified to have spherical harmonic coefficients with the same temporal mean and standard deviation as the observed data. Again, this adjustment cannot be performed completely in an operational mode.

In their study, DK removed the systematic error and investigated only the random error variances (the square of rms). They enhanced the empirical error growth models employed in earlier studies by adding a term representing errors resulting from model deficiencies. So the same model can be used for estimating the internal and external predictability.

What is new in the linear model used by Chen (1989) and applied to observed data in section 2 is now discussed. In computing their error growth model's parameters both Lorenz and DK assumed that in twin model experiments the growth rate at lead time T is determined only by the error magnitude at the same lead time (and not at all by the value of initial error at $T = 0$). Experimental data from neither the NMC nor the ECMWF model supports this assumption. If one carefully studies the thin curves in Figs. 3 or 4 [that are the reproductions of Fig. 9 of Chen (1989) and Fig. 1 of Lorenz (1982)], there is a difference in the slope of the internal error growth curves. The larger the initial error, the steeper the curves. Although the empirical error growth models use a more complex expression to depict the internal error curves, the first part of these curves, as Chen (1989) noted, are close to linear. In the curves with the lowest initial errors there is a slight deviation from linearity at short (1- or 2-day) lead times. This deviation can be at least partly attributed to the uncertainty in observations. First of all, the heavy line in Fig. 4 should start at about 12 m at day 0, reflecting the uncertainty in the analysis. Another observation is that the second dot on the heavy line in the same figure, which stands for the difference between a 1-day forecast (exactly known) and the verifying analysis (that has about 12 m error in it) should be in a somewhat lower position. This is because if one has two maps (the forecast map and the state of the atmosphere that is not known exactly) and then we add a random error (that is the error of the observations with a variance that is close to the rms difference) to one of the maps (the result is the verifying analysis with approximately 12 m error), the rms difference between the new pair will be considerably larger than the rms between the original pair. The additional dots on the heavy line probably have negligible errors due to this effect.

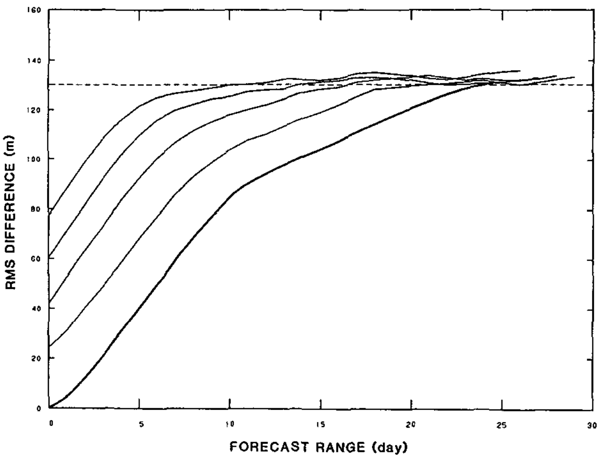

FIG. 3. Upper (thin, continuous) curves: growth of error difference for various magnitudes of initial error. The curves show the average distance between 1-, 2-, 3-, and 4-day lagged 500-hPa NH wintertime forecasts from NMC NWP model experiments. The dashed line is the random error level of the model. (The heavy line is an estimate for infinitesimal initial error.) For further details, see source of reproduction (Chen 1989).

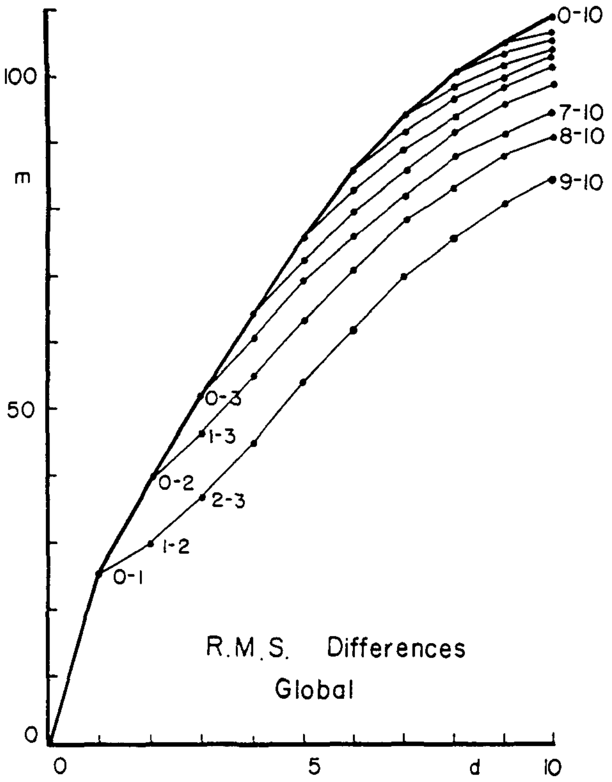

FIG. 4. Thin curves: same as in Fig. 3 but for ECMFW NWP model runs and for global domain. The forecasts are lagged up to nine days. Heavy line: external error growth curve as measured by the average distance between model forecasts and corresponding atmospheric analyses. For further details, see source of reproduction (Lorenz 1982).

To summarize, there are two features in Figs. 3 and 4 to point out. First, as mentioned above and illustrated in Fig. 5, the slope of the linear part of the internal error curves (thin lines) increases with increasing initial error. Second, the curves change shape and turn into asymptotic curves at increasing error values. Both characteristics work in the same direction, resulting in different predictability limit values estimated on the basis of internal error growth curves with *different initial errors.* Although the internal error curves of Fig.

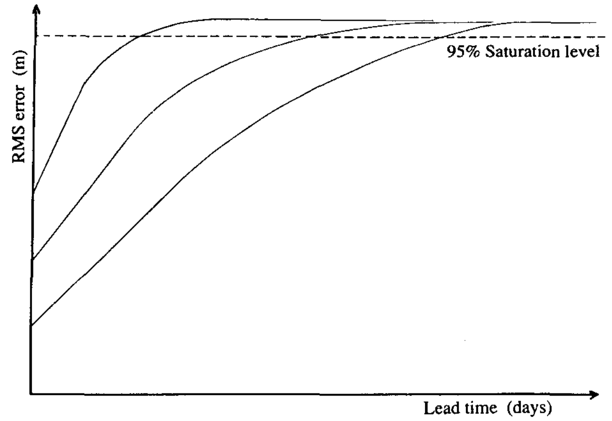

FIG. 5. A schematic drawing to enhance the characteristics of error growth curves from NWP twin model experiments and observational analogs. The error growth rate is not only a function of the error magnitude but also of the initial error. Note that the slope of the linear part of the curves increases with increasing initial error and the asymptotic part begins at higher and higher error values.

1 in this article display more sampling fluctuations due to fewer available cases, they show basically the same features.[2] One can only guess about the possible causes of the phenomena. First the structure of the same magnitude of error, particularly its wavenumber or geographical distribution, might be different with various initial errors; and second, the difference (or similarity) in the histories of the cases examined might also have an effect, at least resulting in different errors in fields other than that (700 or 500 hPa) studied here. And the greater similarity in those other fields can lead to a slower error growth in cases when the histories of the two circulation patterns are similar (i.e., the initial error is smaller). This reasoning, if true, explains why the curves in Fig. 5 or the thin curves in Figs. 3 and 4 are not superposable.

No matter how the effect of the initial error on the predictability limit is explained, the effect is reflected in Chen's (1989) linear model through its construction but is not examined by Lorenz (1982) and DK. As Lorenz and DK considered only the magnitude of the error at 500 hPa for calculating the error growth, they averaged the data from the thin curves in Fig. 4 when computing the parameters of their error growth models. This might lead to underestimating internal atmospheric predictability. However, other factors in the error growth models, especially at the error range close to the saturation level (without corresponding experimental data from the 10-day forecasts in those two studies) might compensate for this effect.

As noted earlier, experimental data on error growth at very small initial values are lacking. Also, it is believed that the linear method cannot be used for extrapolating atmospheric predictability for arbitrarily small initial errors. Hence, Table 1 intercompares different predictability studies with a 12 m initial error. The first column shows that from the analog experiments and the linear extrapolation of Chen (1989), the estimated time a 12 m initial rms difference between two atmospheric states would reach 95% of the saturation level (118.5 m given in parentheses) is 17.14 days. Column 2 presents the internal and external predictability using the NMC NWP model's data and the same method from Chen (1989). Columns 3, 4, and 5 contain predictability results from ECMWF NWP model runs. Since Lorenz (1982) estimated doubling times he omitted predictability results relating to particular limit values such as the climate error. He gave as an example that 10-day forecasts with a perfect model would be as reliable as 7-day predictions with the presently available NWP model. His examples are included in Table 1, because the 3-day advantage with a perfect model at that time range can be compared to the outcome of other studies at the climate mean error level. In column 4 results are based on the same experimental data, using Chen's (1989) linear model. As the ECMWF dataset contains 10-day forecasts, predictability can be estimated by this method only for the climate mean error level (77.8 m and 83.8 m rms error) for internal and external predictability, respectively. The latter value was estimated using the ratio between the saturation level for internal and external experiments in the NMC model (Chen 1989). The internal and external saturation levels here, and for the NMC results, differ because the model forecasts have smaller variance than the observed climatological variance. In their work, DK presented their predictability results for 95% of the error variance. To estimate the 95% level of the rms random error in column 5 (that is comparable with the other results), their computed parameters were used here to adjust their results. Note that in their error growth model the internal and external saturation levels were set at the same value. This might result in slightly overestimating internal and underestimating external predictability. DK presented no estimate for internal predictability at the climate mean error level. Table 1 does not include the result of (among others) Schubert and Suarez (1989) with a less realistic 2-layer general circulation model who arrived at an internal predictability of about 24 days at the 95% saturation limit value.

Despite the fact that the results in Table 1 come from three different sources (observational, NMC, and ECMWF NWP) and through different analysis meth-

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[2] There is a striking example that also shows that it is not only the magnitude of the difference between two atmospheric states that determines the error growth. As an extreme case, consider and compare two atmospheric circulation time series, starting one day apart. The initial error in 700-hPa height, on the average, is around 40 m (in the rms sense). But in this case, the error will fluctuate around the same value and will never grow (until a dramatic change in the earth's climate occurs).

TABLE 1. Estimate of atmospheric predictability (in days) by observed circulation analogs (first column), NMC NWP model data (second column), and ECMWF NWP data (last three columns). The predictability was determined as the time period in which an initial difference of 12 m between two observed or two model circulation patterns (internal predictability), or between an observed and a model circulation pattern (external predictability), would reach 95% (upper two rows) or 71% (lower two rows) of the estimated random error level (the latter level is equal to the climate mean error). The appropriate error levels (in meters) are indicated in parentheses. Columns 1, 2, and 4 were obtained by the linear method of Chen, while columns 3 and 5 were derived from the empirical error growth model of Lorenz (1982) and Dalcher and Kalnay (1987), respectively. Note that Lorenz provides only two examples with unspecified error limit values. Hence, the numbers in column 3 (marked by asterisk) cannot be directly compared to those in the other columns.

	Source	Observational	NMC	ECMWF	ECMWF	ECMWF
Type of predictability	Analyzing method	Linear method	Linear method	Erorr growth (Lorenz)	Linear method	Error growth (Dalcher & Kalnay)
95% Internal		17.14 (118.5)	17.29 (123.5)	13.5*	—	18.2 (101.2)
95% External		—	10.55 (133.0)	10*	—	10.4 (101.2)
71% Internal		—	8.85 (91.9)	10*	9.15 (77.8)	—
71% External		—	5.86 (99.0)	7*	5.85 (83.8)	5.5 (75.3)

ods, and that there are unavoidable uncertainties, they are surprisingly similar. So one can confidently infer that in the NH winter, with the estimated 12 m (rms) initial observational error and using existing NWP models, the lead time for which our forecasts on average perform better than the climate mean is slightly less than six days. At this error level with a perfect model with the same initial error and spatial resolution, one could gain three additional days in predictability. On the other hand, if one considers 95% of the random error as a reference level, the present models reach this error level within ten and a half days. A perfect model would achieve about seven additional days in predictability.

4. Conclusion

An atmospheric predictability experiment was presented based on circulation analogs (Fig. 1). Compared to NWP identical twin experiments, such an empirical study has the advantage of working with observational data, and avoids assuming that a NWP model is a perfect model of the atmosphere. Indeed, the empirical study offers a means of verifying such an assumption. On the other hand, we are constrained to use and experiment only with a limited amount of observed data with a limited range of certain variables.

In this study a linear relationship [found by Chen (1989) between the initial rms difference (error) of twin experiments and the time the error reaches a reference level (time of predictability)] was applied to estimate and extrapolate atmospheric predictability for small initial errors. Although the linear relationship was established on NWP twin model experiments and in a partly different range of initial errors, it applies as well to analogs (observed "twin" experiments, Fig. 2). The linear model uses some information neglected by earlier error growth studies; namely, .that the error growth is not independent of the initial error (Fig. 5). Experimental data, both from model runs (NMC and ECMWF twin experiments) and from an observational study, show that larger initial errors are associated with faster error growth. Despite the fact that this was ignored and also that the analyzing techniques are different, the predictability results presented for observational data (analogs) and for different model experiments are similar. It is concluded from Table 1 that the external (or lower) boundary of predictability of the atmosphere in the Northern Hemisphere winter, with the approximately 12 m rms observational error, and with the available models is slightly less than six days, in reference to the climate mean error (71% of random saturation level). Predictability with a perfect model (internal or upper boundary of predictability) with the same initial error and spatial resolution would be around nine days. With the same conditions but with a different, 95% random error limit value, external predictability is about ten and a half days while the internal predictability is around 17–18 days. The comparison of the values of internal predictability inferred from observational analogs and derived from state-of-the-art NWP models can be regarded as a specific verification of the models. The fact that the predictability estimates are almost the same (they of course have a certain amount of uncertainty) gives further evidence—beyond the traditional weather forecast verification figures—that the models perform quite well on the time and spatial scales they were designed for. This is true even if there is considerable room for improvement. It is important to note that the above predictability experiments were derived without considering the possible effects of the atmosphere's interaction with the ocean and other geospheres. With these interactions it is expected that the atmosphere may have an enhanced predictability compared to the estimates presented here.

It is stressed that the values given above refer to predictability with an approximate *12 m initial error* and with the *spatial resolution* used in the models. Given these limitations, the numbers in Table 1 are so close that considerable further refinement of these predictability results is unlikely. Nevertheless a lot of questions are still open. First, what is the effect of the interacting neighboring spheres (ocean, etc.) on atmospheric predictability? What is the explanation of the dependence of the error growth on the initial error? However, the most interesting and challenging problem is likely the following: "How much could we gain in predictability by reducing the initial error by a certain amount?", and (probably not an entirely independent question), "How much could we gain by increasing the spatial resolution in our models (and observations)?" The present NWP models probably could not reliably answer these questions. As Van den Dool (1989, personal communication) pointed out, results with NWP models (e.g., Schubert and Suarez 1989) indicate that our models, with arbitrarily small initial errors, have an infinite predictability. This conflicts with our assumption from the theory of nonlinear dynamical systems. A dynamical–empirical-type approach (Lorenz 1969b) might work better. Without a sound representation of interactions between the different scales of motions (including the very small scales as well), advance in predictability studies focusing on the very small initial error range is unlikely.

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