Reply

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We thank Professor Staley for his comments (Staley 1992; hereafter referred to as S) concerning the work described in Hirschberg and Fritsch (1991a,b; hereafter referred to as HFa and HFb, respectively) and appreciate the opportunity to elaborate upon the applicability of the height-tendency equation used in our observational study. The height-tendency equation is formed in HFb by simply taking the temporal derivative of the hypsometric equation. This operation yields

$$\frac{\partial Z_B}{\partial t} = \frac{R}{g}\int_{P_B}^{P_T}\frac{\partial T_v}{\partial t}\,d\ln p + \frac{\partial Z_T}{\partial t}, \qquad (1)$$

where P_B and P_T are some fixed pressure levels, Z_B and Z_T are the heights at P_B and P_T, respectively, and T_v is virtual temperature. Hence, a profile of instantaneous height tendency $\partial Z_B/\partial t$ can be calculated if a profile of instantaneous virtual-temperature tendency $\partial T_v/\partial t$ and a value for the top boundary condition $\partial Z_T/\partial t$ are available. In HFb, we expand $\partial T_v/\partial t$ via the thermodynamic equation and successfully use real data to evaluate the effects of various temperature change mechanisms on the height tendencies associated with a rapidly developing cyclone.

To further demonstrate the efficacy of (1) as a height-tendency equation we show here results from an analytic model discussed in Hirschberg and Fritsch (1991c). This model yields instantaneous three-dimensional fields of height tendency by solving the geostrophic vorticity equation for simple atmospheric fields. The model boundary conditions contain no rigid-lid assumption. Specifically, the top and bottom boundary conditions are such that the vertical motions vanish at $p = 0$ and 1000 mb, respectively. Figure 1 shows the vorticity equation–derived height-tendency profile for a representative set of model parameters. The corresponding height-tendency profile calculated independently from (1) using Simpson's rule (see HFb for procedure) with the model-generated profile of instantaneous temperature tendency $\partial T/\partial t$ and 50-mb height tendency $\partial Z_T/\partial t$ is also shown in the figure. Obviously, the profiles are indistinguishable, which shows the efficacy of (1) if accurate temperature-tendency profiles and top boundary conditions are available for the calculation.

Ultimately, the potential weakness of using this integral form or any other form of a height or pressure tendency equation is the a priori knowledge or assumption concerning the top boundary condition. Practical applications of (1) demand that some finite height level P_T be chosen where either $\partial z/\partial t$ is known or can be assumed to be trivial. Often in practice, a lid condition, where the height tendency (or pressure tendency in the case of pressure-tendency equations) at some high level is assumed to be zero, is used as this top boundary condition. There are many examples in the literature that document this practice. As pointed out in HFb, Godson (1948) used observed 12-h pressure changes at the 15 000-ft level as the basis for estimating the value of the top boundary condition $\partial z(P_T)/\partial t$ of his so-called "isobaric tendency equation," which is essentially equivalent to our equation. He further noted that Raethjen (1939) suggested that $(g\rho)^{-1}\partial p/\partial t = \partial z/\partial t$ should vanish near 30 km or approximately 10 mb. More recently, Boyle and Bosart (1986) and Pauley and Smith (1988), for example, used height-tendency equations based on the vorticity and thermodynamic equations in which both upper and lower boundary height tendencies at $p = 100$ mb and pressure surfaces near the ground, respectively, were assumed to be zero.

In HFa and HFb we argue that a vanishing top boundary condition $[\partial z(P_T)/\partial t = 0]$ can be assumed for all practical purposes to lie at some high but finite pressure level for any particular system of interest, which occurs within some limited geographic region and temporal interval. Specifically, we claim that if the $\partial z/\partial t = 0$ or lid level is high enough and the atmosphere is found to be virtually unaffected by disturbances be-

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low, then the effects of any unrepresented processes that occur above this level are negligible. A theoretical basis for this reasoning can be found, for example, in Charney and Pedlosky (1963), who show how unstable tropospheric baroclinic waves can be vertically trapped. In the cyclone case we studied, the 12-h height changes through the 48-h period of interest were found to be significantly diminished by 50 mb. We, therefore, assumed that the lid condition could be applied at this level and are confident that we did not lose much quantitative or qualitative insight from the results obtained from the instantaneous height-tendency equation (1) applied below this level.

The scale analysis presented in S, which shows that the instantaneous height tendency $\partial z/\partial t$ at a given latitude depends only on the scale of the horizontal wind, that is,

$$\frac{\partial z}{\partial t} \sim \frac{fV^2}{g}, \qquad (2)$$

confirms the validity of a high-level lid assumption. As S points out, according to (2), height tendencies at a given latitude should be a maximum near the tropopause where the wind velocities maximize, a result consistent with an analysis of the height-tendency profiles presented in HFb. Conversely, height tendencies should vanish where the horizontal wind velocities vanish. Such a level in the stratosphere can be estimated if we use thermal-wind arguments and assume that the mean temperature gradient in the troposphere and

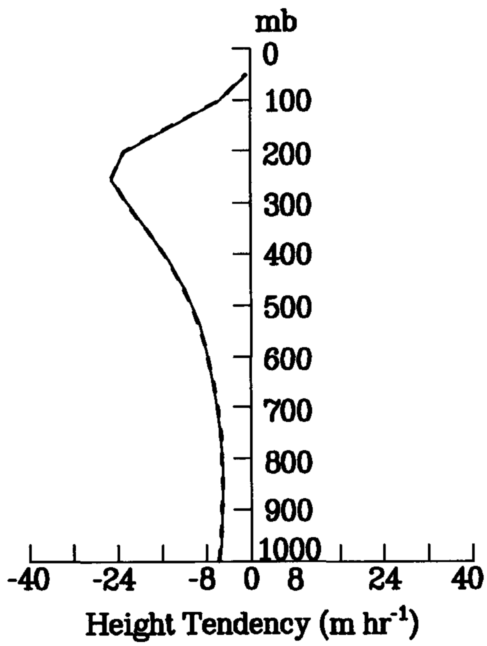

FIG. 1. Vertical profiles of height tendency (m h^{-1}) obtained from solution of the analytic model vorticity equation (solid) and integral form of the height-tendency equation (dashed).

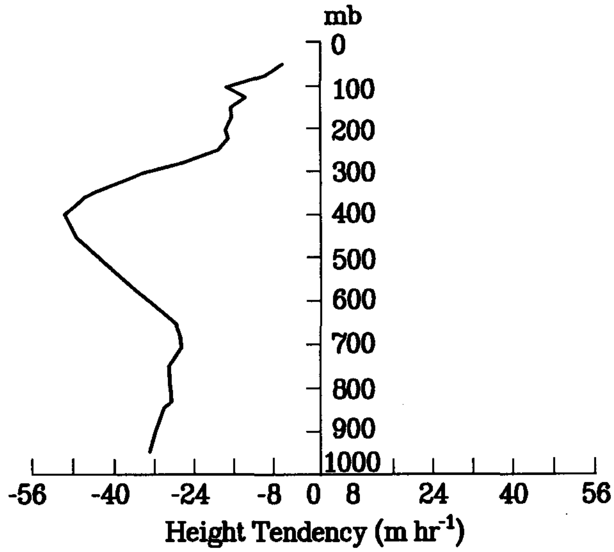

FIG. 2. Vertical profile of the maximum model-generated height tendencies (m h^{-1}) occurring in the vicinity of the ERICA IOP 5A cyclone at the time of most rapid sea level deepening as simulated by NORAPS.

stratosphere are equivalent but of opposite sign and that the wind velocity at the ground is trivial. After little manipulation, the stratospheric level where the winds should vanish, that is, where the thermal wind is equal but opposite to the thermal wind at tropopause level, is given by

$$P_{\rm van} = \frac{P_{\rm trp}^2}{P_{\rm grd}}, \qquad (3)$$

where $P_{\rm van}$ is the stratospheric isobaric level where the wind and height tendencies vanish, $P_{\rm trp}$ is the tropopause level, and $P_{\rm grd}$ is ground level. For typical values of $P_{\rm trp}$ = 225 mb and $P_{\rm grd}$ = 1000 mb, $P_{\rm van}$ = 51 mb. Therefore, on the basis of this simple analysis, the assumption of a lid level at 50 mb is not unreasonable.

Other independent work also suggests the existence of a sufficiently high level at which height or pressure tendencies have little effect on tropospheric development. Jusem and Atlas (1991) found in a numerical study of a rapidly developing extratropical cyclone off of the Argentinean coast that the variation of the free model top (10 mb) had practically no influence on lower-level pressure tendencies. Preliminary results from a numerical study by the first author (Hirschberg and Langland 1992) of the rapidly developing extratropical cyclone observed during the Experiment on Rapidly Intensifying Cyclones over the Atlantic (ERICA) intensive observation period (IOP) 5A confirm these findings. Figure 2 depicts a profile of height falls obtained during a 36-h simulation of ERICA IOP 5A with the U.S. Navy Operational Regional Atmospheric Prediction System (NORAPS) model (see Hodur

1987). Note that NORAPS contains a free upper surface. Specifically, the figure shows a profile of the maximum model-generated height tendencies occurring over a 30 × 30 grid box (1800 km square) near the surface cyclone at the time of most rapid sea level deepening. The dramatic fall off of the model height tendencies above 100 mb suggests that the high-level lid assumption is not unrealistic and furthermore that such an assumption will not affect the evaluation of lower stratospheric and tropospheric height and pressure tendencies to any appreciable degree.

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