The Accuracy of Divergence Estimates Calculated Using the Linear Vector Point Function Method and Three Profilers

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ABSTRACT

The effects of spatial, combined spatial and temporal sampling errors, and wind measurement errors on profiler-derived divergence estimates computed using the linear vector point function method are examined. Analysis indicates that divergence errors are minimized when the ratio between the spacing of the profilers and the sampled wavelength ($\Delta x/L_x$) is between 0.15 and 0.24 and the ratio between the profiler sampling time to the timescale of the weather system ($\Delta t/T$) is less than 0.055.

When $\Delta x/L_x \leq 0.24$, synoptic-scale divergence smaller than $\pm 1.0 \times 10^{-5}$ s^{-1} cannot be measured, because the error in the profiler wind estimates is larger than the horizontal velocity gradients. The expected errors in divergence calculations given typical profiler spatial and temporal sampling strategies are examined.

1. Introduction

Zamora et al. (1987) demonstrated that upper-tropospheric divergence could be computed using the linear vector point function (LVPF) method and hourly averaged wind profiler measurements. The calculated divergence patterns evolved in a manner that was consistent with the passage of large-scale weather systems over the profiler network.

Schaefer and Doswell (1979) show that divergence computed using wind observations and the line integral method (Ceselski and Sapp 1975) is more accurate than divergence calculated using finite differences on wind components that have been mapped onto a regular grid. Doswell and Caracena (1988) explored the theoretical basis for the results noted by Schaefer and Doswell (1979) and showed that line integral or LVPF methods make full use of the available observations when these techniques are used in derivative estimation. Davies-Jones (1993) demonstrated that the LVPF and line integral techniques are formally equivalent.

Zamora et al. (1987) showed that the kinematic properties of the wind field and the translational wind components could be estimated in a computationally efficient manner using the LVPF method rather than the line integral approach. Given that divergence estimated using the LVPF method makes optimal use of observational data, understanding how errors affect the LVPF-derived diagnostics becomes important. These errors are caused by spatial sampling error, temporal sampling error, and profiler measurement errors.

In this paper we examine the accuracy of divergence calculation made using the LVPF technique and a triad of wind profilers when both sampling and observational errors are considered. Section 2 reviews the LVPF technique. Section 3 examines the effect of spatial sampling errors. Section 4 considers temporal sampling errors. In section 5 we examine the effect of combined spatial and temporal sampling errors. Our findings are summarized in section 6.

2. The LVPF method

If the horizontal velocity field is linear, then the field can be expressed by the linear terms of a Taylor's series expansion:

$$u(x, y) = u(x_0, y_0) + \left.\frac{\partial u}{\partial x}\right|_0 (x - x_0) + \left.\frac{\partial u}{\partial y}\right|_0 (y - y_0)$$
$$v(x, y) = v(x_0, y_0) + \left.\frac{\partial v}{\partial x}\right|_0 (x - x_0) + \left.\frac{\partial v}{\partial y}\right|_0 (y - y_0), \tag{1}$$

where x, y, u, and v are the Cartesian position and velocity coordinates (Petterssen 1936). Given the linear system of six equations in six unknowns and six wind components measured at the vertices of a triangle, the LVPF algorithm (Zamora et al. 1987) computes the translational wind components, divergence, vorticity, shearing deformation, and stretching deformation. Saucier (1955, 316–319) examined the linear velocity decomposition problem in detail. Zamora et al. (1987)

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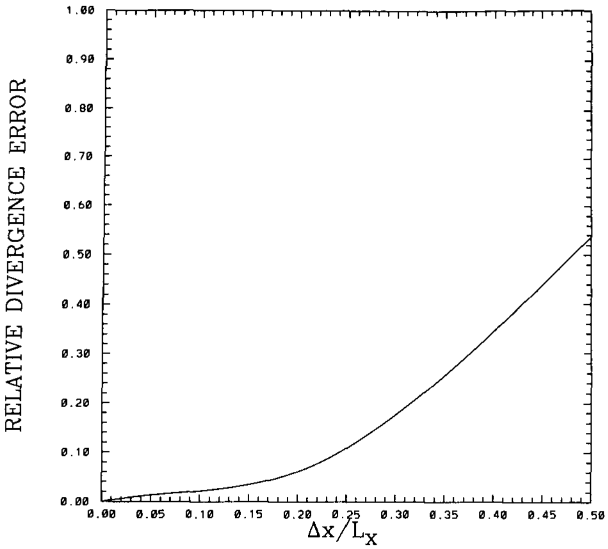

FIG. 1. Relative divergence error as a function of nondimensional distance $\Delta x/L_x$.

discussed the numerical details of the technique and its computational efficiency.

3. Spatial sampling errors

We examined the effect of spatial sampling errors on the LVPF-computed divergence empirically by sampling an analytic velocity field using the LVPF method, where the distance Δx between vertices in an equilateral triangle changed from 1 to 500 km. The velocity field is

$$u(x, y) = \sin(kx + ky + \phi)$$
$$v(x, y) = \sin(kx + ky + \phi), \quad (2)$$

where $k = 2\pi/L_x$ and L_x is a characteristic wavelength. The phase ϕ of the analytic field varied from 0 to $\pi/4$ in steps of $\pi/128$. The divergence values at each phase shift were then averaged because the estimated divergence depends on the phase. Equation (2) was evaluated at the centroid and the midpoint of each triangle side. These four values were averaged together and used for the analytic or true divergence. The relative divergence error is given by

$$\frac{|\mathrm{div}_a - \mathrm{div}_{\mathrm{LVPF}}|}{|\mathrm{div}_a|}, \quad (3)$$

where div_a is the analytic value or true value and $\mathrm{div}_{\mathrm{LVPF}}$ is the mean LVPF divergence estimate. The relative error as a function of nondimensional distance ($\Delta x/L_x$) is shown in Fig. 1.

Assuming perfect wind measurements, relative errors in the divergence estimates smaller than 10% (an arbitrarily chosen threshold) are possible using the LVPF technique as long as the sampling distance to wavelength ratio is less than 0.24 (Fig. 1). If the ratio is less than 0.18, the errors are smaller than 5%. Thus, if the wavelength of the disturbance passing over the profiler triad is 4.1 times greater than the distance between profilers, the computed divergence will be within 10% of the true value. On the other hand, when the wave is sampled at the Nyquist interval ($\Delta x/L_x = 0.5$), the divergence error is greater than 50%.

Given a sampling distance to wavelength ratio of 0.24 and the spacings of the profilers in typical network triangles, we can estimate the wavelengths for which the profiler-derived divergence will be within 10% of the true value. The National Oceanic and Atmospheric Administration (NOAA) Wind Profiler Demonstration Network (Chadwick 1988) has stations spaced at approximately 150- and 400-km intervals. The stations surrounding Lamont, Oklahoma (LMN), are approximately 150 km apart (Fig. 2). The remainder of the stations in the network are about 400 km apart. During the STORM Fronts Experiment Systems Test (STORMFEST) five boundary layer wind profilers (Ecklund et al. 1988) were placed at 50-km intervals. For profiler triangles 50, 150, and 400 km on a side, the sampled wavelengths should be greater than or equal to 200, 600, and 1600 km, respectively, if estimates of divergence are to have relative errors less than or equal to 10%.

4. The effect of profiler measurement errors

The results of the previous section might suggest that representative divergences can be calculated for wavelengths smaller than 200 km if the profiler triangle is smaller than 50 km on a side. This might allow us to observe the divergent component of the wind associated with mesoscale weather systems, for example, in addition to the divergent component of the velocity

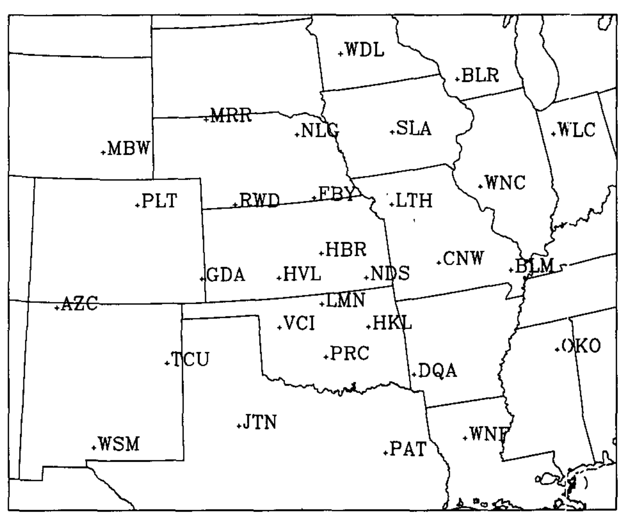

FIG. 2. The NOAA demonstration wind profiler network.

field associated with synoptic- and planetary-scale waves. If the wind measurements made by the radars were perfect, this strategy would be useful. However, errors in the profiler measurements also limit the accuracy of the computed divergence because the uncertainty in a single profiler wind estimate can be as large as the velocity gradient between profilers. The imprecision of wind profiling radar measurements is a function of the signal-to-noise ratio (S/N) and the width of the Doppler spectrum (Strauch et al. 1984). Strauch et al. (1987) found standard deviations of the radar horizontal wind components of ± 1.3 m s^{-1} at low S/N and 0.88 m s^{-1} at high S/N. We define low S/N as less than or equal to 0.0 dB and high S/N as greater than 0.0 dB.

Using the simplifications introduced by Davies-Jones (1993), the LVPF horizontal divergence and its error can be written as

$$\delta + \delta_e = \frac{1}{2A}[(u_1+\epsilon)(y_2-y_3) + (u_2+\epsilon)(y_3-y_1) + (u_3+\epsilon)(y_1-y_2) + (v_1+\epsilon)(x_3-x_2) + (v_2+\epsilon)(x_1-x_3) + (v_3+\epsilon)(x_2-x_1)], \quad (4)$$

where δ is the horizontal divergence, δ_e is the divergence error, ϵ is the profiler wind component standard deviation, and $A = 0.5(x_2y_3 - x_1y_3 - x_2y_1 - x_3y_2 + x_1y_2 + x_3y_2)$ is the area of the triangle formed by the observing stations.

Assuming the errors in the measured wind components are random (not correlated), the rms divergence error is given by

$$\delta_e = \left\{\frac{\epsilon^2}{A^2}[(y_2-y_3)^2 + (y_3-y_1)^2 + (y_1-y_2)^2 + (x_3-x_2)^2 + (x_1-x_3)^2 + (x_2-x_1)^2]\right\}^{1/2}. \quad (5)$$

The solutions of (5) are shown in Fig. 3.

In the previous section, we found that as long as the station spacing ratio to characteristic wavelength was less than or equal to 0.24, the expected divergence spatial sampling error is 10%. When profiler measurement errors are considered at this ratio, the divergence error is on the order of $\pm 7.0 \times 10^{-6}$ s^{-1} at high S/N and $\pm 1.0 \times 10^{-5}$ s^{-1} at low S/N. Placing the profilers closer together reduces the spatial sampling error but *increases* the uncertainties introduced by the measurement errors.

5. The effect of spatial and temporal sampling errors

Temporal sampling errors can occur when the phase speed of a wave is such that it can cross the observing network without being sampled at least three to four times in the time domain. We tested the effect of combined spatial and temporal sampling errors on the divergence field using a velocity field of the form

$$u(x, y, t) = \sin(kx + ky + \sigma t + \phi)$$
$$v(x, y, t) = \sin(kx + ky + \sigma t + \phi), \quad (6)$$

where $\sigma = \pi/T$ and T is a characteristic timescale. The phase ϕ was varied from 0 to $\pi/2$ in increments of $\pi/64$, and the results were averaged over all phases, as before. The relative divergence error for $\Delta x/L_x$, 0.08, and 0.24 is shown in Fig. 4.

The divergence estimates computed in Zamora et al. (1987) compared favorably with the synoptic weather patterns and distribution of precipitation over the profiler triangle. The profilers were placed 400 km apart. For a 400-km station spacing, 5000-km wavelength ($\Delta x/L_x = 0.08$), a 48-h timescale, and hourly wind measurements ($\Delta t/T = 0.02$), the relative divergence error is less than 4%. When the station spacing-to-wavelength ratio is 0.24, the relative divergence error is smaller than 10% for $\Delta t/T \leq 0.055$. Thus, for profiler averaging times of 60, 30, 20, and 15 min, the timescale of the wave should be greater than 18.2, 9.1, 6.1, and 4.5 h, respectively, assuming the spatial sampling criterion ($\Delta x/L_x = 0.24$) is met.

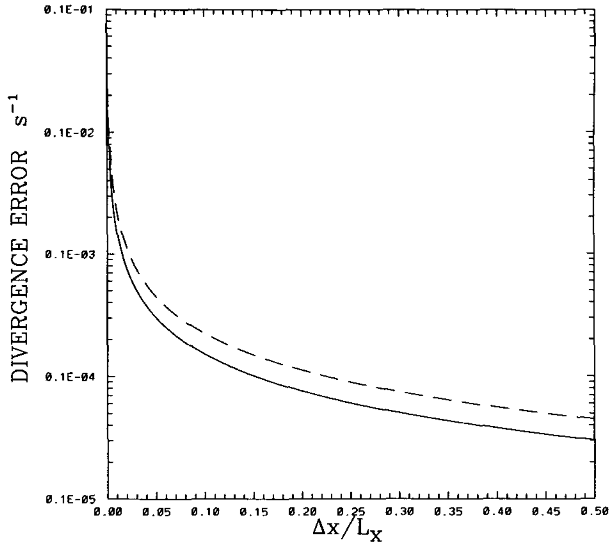

FIG. 3. Divergence error caused by profiler measurement error as a function of nondimensional distance $\Delta x/L_x$. The dashed curve shows errors when S/N $\leq$ 0.0 dB, and the solid curve shows errors at S/N > 0.0 dB.

6. Summary and conclusions

We have examined the effects of spatial sampling errors, profiler measurement errors, and combined spatial and temporal sampling errors on divergence inferred using a single triad of wind profilers using simple

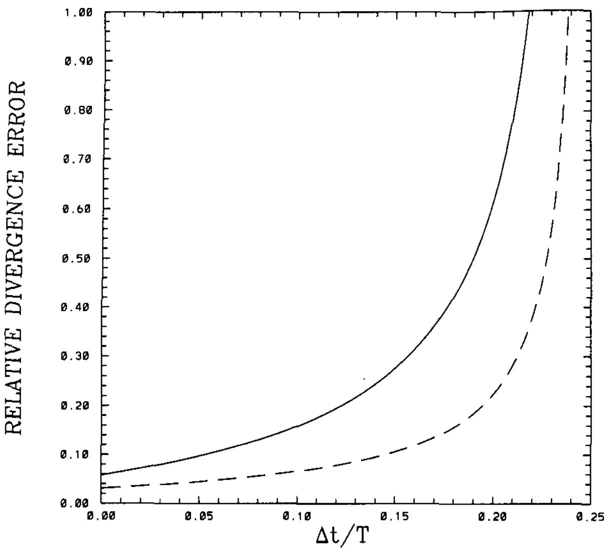

FIG. 4. Relative divergence error as a function of nondimensional time $\Delta t/T$. The dashed curve gives the results for $\Delta x/L_x = 0.08$, and the solid curve gives the results for $\Delta x/L_x = 0.24$.

analytic velocity fields. If the LVPF approach does not sense the divergence associated with these wind fields, it seems unlikely that complex divergence patterns associated with the actual wind field will be recovered using this technique.

Our findings indicate that *minimizing spatial sampling errors by reducing the distance between profilers is limited by the measurement errors inherent in profiler wind estimates.* Divergence less than $\pm 1.0 \times 10^{-5}$ s^{-1} cannot be measured when the ratio of the profiler station spacing to wavelength is smaller than 0.24, depending on the S/N. Our analysis indicates that optimum divergence estimates are obtained from a single triad of profilers when $\Delta x/L_x$ is between 0.15 and 0.24 and $\Delta t/T$ is less than 0.055.

Doswell and Caracena (1988) have shown that derivatives of the wind field can be estimated using the data and the LVPF technique without mapping the wind components to a grid. The fidelity of the derivative estimates is controlled in part by the scales of motion sensed by the individual triangles. The average distance between the vertices of the triangles that is used in the estimation of the divergence field determines the scales of motion that can be resolved by the analysis. The results of this paper can be used to determine the accuracy of the divergence estimates returned by the individual triangles.

Caracena (1987) has developed a method of evaluating derivatives without interpolating to a grid using a Gaussian weighting function to specify how the wind components vary in space. Conceptually, both the LVPF and Caracena methods evaluate derivatives without gridding the data. The LVPF method assumes that wind field varies linearly between the observations. Caracena uses the weight function to describe the spatial variation between the sample points. Future work should compare both approaches and their utility in derivative estimation.

We should work toward improving the accuracy of wind profiler velocity estimates. Better radar antennas and finer signal processor resolution might increase the accuracy of the Doppler radar velocities. Very accurate wind measurements will allow closer placement of profilers in space and, hence, better estimates of divergence using the LVPF technique.

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