Location and Interaction of Upper- and Lower-Troposphere Adiabatic Frontogenesis

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ABSTRACT

Both upper-air and surface frontogenesis have often been depicted as processes whose dynamics could be reduced to 2D balanced problems in which “self-sharpening” configurations could be highlighted. This paper reports on a 3D adiabatic simulation of a baroclinic wave life cycle. Great care has been devoted to the vertical resolution, allowing for a good description of both surface and upper-air frontogenesis. The authors introduce a kinematic diagnostic (Q’ vector) that permits the identification of frontogenetic areas in such complex 3D flows where classical, low–Rossby number balance conditions can be violated. Relations and specificity with respect to frontogenetic forcing diagnostics are discussed. First, Q’ is used for surface frontogenesis, where it describes well the actual frontal activity, including the complex warm-frontal seclusion process. Upper-air frontogenesis is also investigated, both in terms of this kinematic diagnostic or in terms of potential vorticity displacements on isentropic surfaces. Both types of diagnostics clearly distinguish between dynamics of the entrance zone of the northerly jet—where 2D concepts may usefully be applied—and those of the strongly curved zone near the trough axis. Classical cyclogenetic terms (stretching and tilting) as well as the separation of ageostrophic circulations in terms of natural components of the wind also lead to a clear dynamical separation. The cold front is shown to extend from the surface far into the troposphere. This is shown to be related to a singular property of the 3D flow. Parcels undergoing frontogenesis in the northwesterly upper-air flow are advected on top of those that were forced at the surface cold front in a southwesterly flow. The occurrence of a feedback process between these upper-air frontogenesis processes and the surface ones is then investigated. Stepwise vertical profiles of horizontal diffusion are used to force local frontogenesis. The resulting upper-air frontogenesis, despite its local efficiency, does not have any remote effect on the surface front, whose frontolysis in turn has no effect on the upper-air front. The feedback process is thus not occurring in our simulation.

1. Introduction

The dynamical processes associated with surface front self-sharpening phenomena were first exhibited by Hoskins and Bretherton (1972). They showed, in a 2D context, that if geostrophic advects of momentum and potential temperature become important enough, their frontogenetic effect can be relayed by ageostrophic cross-front circulations, leading to a non-linear increase of the forcing.

Although the strongest thermal gradients are found near the ground, important frontogenetic processes also act at higher levels. [See Keyser and Shapiro (1986) for a review.] Their practical importance comes from the creation of a large slope of the isentropic surfaces [a process that Newton and Trevisan (1984a,b) designated as clinogenesis], which allows high–potential vorticity (PV) air to be advected down to unusually low levels on a fairly large scale. This advection is thought to be partly responsible for strong surface cyclogenesis events (Uccellini et al. 1985; Hoskins and Berrisford 1988).

The success encountered in surface frontogenesis led different authors to apply the geostrophic–ageostrophic partition to upper-air frontogenesis. This technique was again successful in describing the “catastrophic” character of upper-air frontogenesis at the entrance of jet streaks advecting cold air (Shapiro 1981; Keyser and Pecnick 1985a,b). This thermal, geostrophic advection was shown to change the forced ageostrophic circulation from a classical, thermally direct circulation (air rising on the warm side and sinking on the cold side of the jet) to a pattern in which strong subsidence was found just beneath the jet axis [ascent needed for mass balance are found either on the warm-air side (Keyser and Pecnick 1985a) or on both sides (Shapiro 1981)]. This displacement of the subsidence was found to be a key factor for (i) the tilting of the horizontal vorticity toward the vertical and (ii) the tilt-
ing of the isentropes (cliniogenesis). The feedback loop is closed since both effects increase the ageostrophic circulation forcing, which in turn increases the cyclogenesis and cliogenesis.

However, this is not the whole story. Owing to the high wind speeds typically encountered, air parcels do not remain long in the jet entrance area. It seems rather unlikely that they can stay there for the 24–48-h period necessary for a strong upper-level front to be created in 2D models. If a rather small average speed of 30 m s⁻¹ is postulated in the entrance area, parcels will be displaced by about 2600 km in a day! On the other hand, some authors (Newton and Trevisan 1984a,b) have stressed the efficiency of strongly curved flows for upper-level frontogenesis.

The complexity of the processes associated with upper-level frontogenesis, which seems to support not only different geometries (straight jet entrance area and curved trough base) but also different dynamical balances (ageostrophic in the former—gradient wind in the latter), forced upon us the idea that a kinematic diagnostic, free from any dynamical and geometrical hypothesis, could be of value. The aim of such a frontogenesis diagnostic would be to point out those areas that are really efficient and for which a conceptual effort should be made to better understand the dynamics involved. This diagnostic is introduced in section 2.

The 3D, nonlinear development of normal-mode-type perturbations superimposed on simple zonal jets with meridional and vertical variations (crude reproduc- ing the westerly midlatitude jet) has received a great deal of attention from modellers since the pioneering work of Mudrick (1974). [See Polavarapu and Peltier (1990) and Schär and Wernli (1993) for a review of the scientific advances in this area.] This is because such experiments, despite the highly idealized initial condition from which they start, bear a strong resemblance to real extratropical perturbations. This is the reason why we performed one such simulation as a test bed for our diagnostic. To parallel other studies (Hoskins and West 1979; Davies et al. 1991), we use an f-plane, hydrostatic framework, with full representation of the tropopause and a weak upper-level jet. The numerical treatment is presented in section 3.

The description of surface frontogenesis in our experiment is presented in section 4. Upper-air frontogenesis is discussed in section 5. The existence of a feedback loop between these two processes is investigated in section 6. A summary of the results and the main conclusions are found in section 7.

2. A kinematic, three-dimensional frontogenesis diagnosis

a. Definition

Frontogenetic forcing was introduced by Petterssen (1936) in terms of the kinematic action of the nondivergent components of the flow on a passive scalar (taken as \( \theta \)) at a rigid lid (the ground). Important dynamic insights with respect to this problem have then been developed by making a distinction between the primary and secondary forcings, the latter being implicit through quasigeostrophic or semigeostrophic balance assumptions (Hoskins and Bretherton 1972; Hoskins and Draghici 1977). Nevertheless, it should be noted that such a distinction is useful mainly in 2D geometry (negligible surface front curvature), although Keyser et al. (1989) recently proposed a distinction in terms of irrotational and nondivergent components making 3D concepts easier to interpret.

Miller (1948) stressed that for upper-air frontogenesis, vertical velocity gradients should be of crucial importance in tilting strong vertical gradients of potential temperature into the horizontal (his Fig. 3). Straight accelerating jet streaks advecting cold air have been identified by Shapiro (1981) and Keyser and Pecnick (1985) as configurations for which upper-air frontogenesis could be described as a "catastrophe," where implicit ageostrophic frontogenetic circulations act as key terms of the nonlinear development. This process is fairly similar to the one occurring for surface frontogenesis, even if (due to the importance of vertical motions) the dynamical processes differ (cf. section 5d). Such configurations are nevertheless quite singular: the point here is that we want to draw a general picture of the frontogenetic processes in order to locate the areas where the dynamics should be carefully examined. That is why any dynamical separation will be avoided so that no presumption will be made as to the importance of the frontogenetical processes involved. The following Miller (1948) formulation is introduced in fully 3D geometry: for any scalar \( \alpha \) whose Lagrangian tendencies are noted \( \dot{\alpha} \), it can be written

\[
\frac{d}{dt} \nabla \alpha = -' \nabla u \cdot \nabla \alpha + \nabla \dot{\alpha},
\]  

(1)

where \( ' \nabla u \) is the transposed tensor of the velocity gradient; its Cartesian components are \( (\partial u_j / \partial x_i)_{\alpha=i,j} \).

We define the \( Q' \) frontogenetic vector as the total adiabatic frontogenetic forcing that occurs in the flow \( (\alpha = \theta, \dot{\alpha} = 0) \):

\[
Q' = -' \nabla u \cdot \nabla \theta
\]

\[
= \left( -\frac{\partial u_x}{\partial x} \cdot \nabla \theta, -\frac{\partial u_y}{\partial y} \cdot \nabla \theta, -\frac{\partial u_z}{\partial z} \cdot \nabla \theta \right). 
\]  

(2)

Under adiabatic assumption, frontogenesis (i.e., tightening of isentropes) occurs every time \( Q' \) points toward warm air \( (d[\nabla \theta]^2 / dt = 2Q' \cdot \nabla \theta) \). This is a purely kinematic result, as no assumption such as rigid-lid (zero vertical displacement) or nondivergent primary flow is being made.

An interesting formulation may be found if (2) is expanded following a quite classical approach in kin-
mathematics. [See Germain (1962) for an academic reference or Davies-Jones (1982) for a recent application to tornadogenesis.] When the partition of the velocity gradient tensor between its symmetric and antisymmetric parts is introduced, and when all these are translated in terms of rotational and rate of strain tensors, it turns out that

$$Q' = -\frac{1}{2} (\nabla u + (\nabla u) \cdot \nabla \theta) - \frac{1}{2} (\nabla u - \nabla u) \cdot \nabla \theta$$

$$= \frac{1}{2} \xi \wedge \nabla \theta - \frac{1}{3} \nabla \cdot u \nabla \theta - D \cdot \nabla \theta. \quad (3)$$

In the latter expression, $\xi$ is the 3D vorticity (twice the rotational vector) and the strain tensor is separated into its divergence/convergence and deformation $D$ parts; $D$ may be reduced to the canonical form $(\lambda_i \delta_{ij})_{1 \leq i,j \leq 3}$ on the principal directions $(e_i)_{1 \leq i \leq 3}$. Locally, $D$ transforms a material sphere into an ellipsoid of the same volume. The principal directions $(e_i)_{1 \leq i \leq 3}$ are the principal axes of this ellipsoid, and $(\lambda_i)_{1 \leq i \leq 3}$ are the rates of linear dilatation on these axes. Such a formulation indicates that only deformation (most efficient when the isentropes are normal to the maximum contraction axis) and convergence may be frontogetic, while vorticity only rotates the isentropes keeping their spacing constant.

In practice, only projections of $Q'$ will be displayed (on either the horizontal or vertical plane). These visualize the evolution of $\theta$ gradients in the plane in question:

$$Q'_{x_i} = \frac{d}{dt} \nabla_{x_i} \theta, \quad (4)$$

where $Q'_{x_i}$ is the $e_i$-orthogonal projection of $Q'$, and $\nabla_{x_i}$ is the gradient operator with $x_i$ being kept constant. The cross-isentropic component of $Q'_{x_i}$ measures the frontogenesis/frontolysis in the $e_i$-orthogonal plane, while its along-isentropic component measures the orientation changes.

Further separation in terms of the vertical and horizontal velocity contributions to frontogenesis may easily be recovered. They are

$$Q'_w = -\frac{\partial \theta}{\partial z} \nabla w \quad (5)$$

$$Q'_u = Q' - Q'_w \quad (6)$$

$$= -\frac{\partial \theta}{\partial x} \nabla u - \frac{\partial \theta}{\partial y} \nabla v. \quad (7)$$

Figure 1 illustrates $Q'$ for some 2D $(x, z)$ flow patterns. Even if the $Q'$ formulation allows a full 3D picture of the frontogenesis (including static stability evolution, cf. Figs. 1c and 1d), it should be kept in mind that clinogenesis is felt to be the most important dynamical process, because it allows the advection of high PV from stratosphere to troposphere, for example.

b. Relations of $Q'$ with frontogetenral forcing vectors

Even if horizontal projections of $Q'$ look a bit like classical $Q$ vectors (Hoskins et al. 1978), it should be remembered that $Q$ retain only the primary part of the frontogenetical function (i.e., the geostrophic, horizontal frontogenesis). Even if the secondary circulations can be determined from the primary circulations through the quasi- or semigeostrophic balance assumption, it is not straightforward to infer the total frontogenetical tendencies from a display of $Q$ and isentropes.

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1 A 2D quasigeostrophic version of this treatment has been independently derived by Schär and Wernli (1993); a part of it also appears in Mudrick (1974).
Davies-Jones (1991, hereafter DJ91) introduced a nice primitive equation discussion in terms of the 3D pseudovorticity vector. The horizontal component of this vector reduces to the horizontal thermal gradient under the thermal wind balance assumption (with opposite sign, and after some rescaling). A generalized, but still 2D, Q vector [Q*, Eq. (3.17) in DJ91] could be shown to be the mean of two vectors: one is a measure of thermal frontogenesis, and the other is a measure of vertical wind shear generation. This generalization is quite illuminating when it makes possible a primitive equation discussion of the necessary generation of (ageostrophic) thermal wind imbalance in frontogenetic flows starting from geostrophic equilibrium (DJ91, section 5). However, a generalized version of the quasigeostrophic $\omega$ equation forced by the divergence of the $\mathbf{Q}^*$ vector [Eq. (7.4) of DJ91] was possible only through both a new balance hypothesis and a further approximation $\hat{\mathbf{Q}}$ of the $\mathbf{Q}^*$ vector; $\hat{\mathbf{Q}}$ is the horizontal projection $\mathbf{Q}^*_{\omega}$ of our $\mathbf{Q}^*_{\omega}$ [cf. Eq. (7)] except for the divergent part of the wind field (which is omitted in DJ91, as required by a formulation of the forcing of an $\omega$ equation). As demonstrated in DJ91, $\hat{\mathbf{Q}}$ (and so our $\mathbf{Q}^*_{\omega}$) is the Keyser et al. (1988) version using a rigid-lid hypothesis of the vector frontogenetic function $\mathbf{F}$, whose $\mathbf{Q}^*$ is the 3D generalization.

Xu (1992) proposed the 3D $\mathbf{C}$ vector as a quasi-geostrophic (QG) forcing of secondary circulations. The vertical component was defined by Xu [1992, Eq. (2.3.c)] as a geostrophic forcing of the ageostrophic vorticity. Thus, the information about the barotropic part of the ageostrophic circulation, which is lost by Hoskins et al. (1978) Q-vector formulation, may be recovered. The $\mathbf{C}$ vector should not, however, be confused with our $\mathbf{Q}$. The vertical component of $\mathbf{Q}$ is a measure of the Lagrangian evolution of the static stability. It is shown later [Eq. (11)] that it is associated with the baroclinic part of the ageostrophic circulation—quite at variance with the vertical component of the $\mathbf{C}$ vector that is associated with the barotropic ageostrophic circulation. Moreover, $\mathbf{C}$ is a forcing for QG ageostrophic circulations, while $\mathbf{Q}$ is a kinematic measure of the 3D action of the flow on the $\theta$ distribution.

3. The numerical model and the experiment

a. Numerical techniques

The model comes directly from the mesoscale numerical weather prediction PERIDOT (Prévision à Échelane Rapprochée Intégrant des Données Observées et Télédéectées) system, which has been operationally run by Météo France between 1985 and 1993. [Imbard et al. (1986) describe it in detail, while J. Stein (1992) and V. Ducrocq (1993) show recent research undertaken with it.] It is a limited-area model, forced on its lateral boundaries using a simplified form of the Davies (1976) relaxation method. Some aspects have been modified for the simulations described in this paper. To allow free development in one direction, periodic conditions were introduced at the so-called zonal boundaries (channel-like geometry). The governing equations are the primitive equations, solved on an Arakawa C-type horizontal grid, with $\sigma = p/p_e$ as vertical coordinate. An implicit treatment is adopted for both gravity waves and vertical advection. The only diabatic parameterization used here is a Laplacian diffusion applied to $\theta$ and $\rho_u$, mainly to counter numerical noise production. No spherical effect is introduced ($f$-plane approximation, to allow comparison with semigeostrophic models). The water vapor content is kept at zero everywhere.

We used a horizontal grid of 98 $\times$ 52 points, with $\Delta x = \Delta y = 83.33$ km spacing (maximum zonal length resolved: 4000 km). Only 74 points (6000 km) in the north–south direction will be displayed; extra space to the north and to the south is kept only to prevent spurious reflection. Using the zonal periodicity, 1.5 times the maximum wavelength (again 6000 km) will be displayed in the west–east direction. The vertical grid includes 41 levels, with the tightest spacing in the PBL (7 levels from the ground to $\sigma = 0.84$), about 500-m spacing in the troposphere and lower stratosphere, and a model top at $\sigma = 0.005$. It should be noted that such a vertical resolution near the tropopause is a significant improvement over previous studies. The time step is 1200 s, and the horizontal viscosity $\nu = 83.33 \times 10^3$ m$^2$ s$^{-1}$ leads to an $e$-damping time of 2.5 h for a $2\Delta x$ wave.

b. The basic state and initial perturbation

The basic state (Fig. 2) is a zonal jet with smooth meridional and vertical variations. Our reference is the

![Fig. 2. Meridional cross section of the basic state: zonal wind (solid line at intervals of 5 m s$^{-1}$), and potential temperature (dashed lines at intervals of 4 K). The short dashed contour is for $\theta/\theta_0 = +1$ K (100 km)$^{-1}$ (see distance along the section). The tropopause is shaded for PV values between 1.5 and 2.5 PVU (1 PVU = 10$^{-6}$ K m$^2$ s$^{-1}$ kg$^{-1}$). Horizontal tick marks are at 250 km (equal to three model grid spaces). The X and Y refer to the horizontal axes (Fig. 3 and subsequent figures).](image-url)
Hoskins and West (1979) formulation in the troposphere. A stratospheric thermal extension is formulated in order to introduce a realistic tropopause definition and a decrease of the wind. The maximum intensity is moderate (a “jet” maximum of 28 m s\(^{-1}\)) and the surface wind is zero (uniform surface pressure). The temperature field is computed to be in thermal wind balance with the jet profile.

The most unstable 4000-km-wavelength mode for this basic state was found by using an iterative procedure starting from a random noise perturbation.\(^3\) The normal-mode character of the perturbation after 15 days of quasi-linear run was verified from the almost uniform zonal phase speeds and growing rates in the domain. For example, the phase speeds and growing rates of the meridional wind have a respective mean value of 8.3 m s\(^{-1}\) and 4.4 \(\times\) 10\(^{-6}\) s\(^{-1}\) and a respective standard deviation of 0.3 m s\(^{-1}\) and 0.3 \(\times\) 10\(^{-6}\) s\(^{-1}\), if weighted by the perturbation amplitude of each wave. The initial state for the life cycle experiments described in the following sections was the sum of the basic state and of this normal-mode perturbation scaled to have 1.7 m s\(^{-1}\) maximum amplitude for the meridional wind at ground level.

4. Surface frontogenesis

a. Frontal pattern

The results obtained (Fig. 3) are in good agreement with previously mentioned simulations using primitive equations (Mudrick 1974; Newton and Trevisan 1984; Takayabu 1986; Keyser et al. 1988). The main points are the following:

- A very strong\(^4\) cold front that forms from the southeast of the cyclone and then bends around the anticyclone to appear as a weak warm front on its western part (Fig. 3). This extension was already described in terms of advection of frontogenetic forcing in Hoskins and West (1979) and Takayabu (1986). Other aspects of this front are in agreement with 2D, semigeostrophic predictions: vertical tilting toward cold air of the direct, cross-frontal circulation (Fig. 5) and strong vorticity generation on the warm side of the front.
- A frontogenetic area that appears as a warm front on the north-northeastern part of the cyclone. During the first stages of the frontogenesis, this is more intense than the cold front (see Fig. 3a). But as the alongfront advections become more important (Fig. 4 and Takayabu (1986, Fig. 11)], particles undergoing frontogenesis are quickly removed from the warm front. This is in contrast with the situation near the cold front, where relative winds are quite weak. All this is in good agreement with Takayabu’s (1986) and Schär and Wernli’s (1993) findings. At the end of the simulation, this front is finally less marked than in the cold one (Fig. 3b), with an important curvature around the cyclone that seems far from being tractable in a classical, 2D context.

b. Frontogenesis diagnostics using the \(Q’\) vector

Keyser et al. (1988) introduced, at a rigid lid, a 2D natural partition of the frontogenesis function \(Q^\prime_{uc}\) = \(d\nabla \theta / dt\) into its magnitude and direction components. Their results may be recovered through 2D vector analysis similar to (3):

\[
Q^\prime_{uc} = -\nabla \cdot \mathbf{u} \cdot \nabla \theta
\]

\[
= \frac{1}{2} \zeta \kappa \wedge \nabla \theta - \frac{1}{2} \nabla_z \cdot \mathbf{u} \nabla \theta - D^\prime \cdot \nabla \theta,
\]

where \(\zeta\) is the vertical component of vorticity and \(D^\prime\) is the 2D isosurface deformation tensor. Such partitioning of frontogenetical forcings in natural components of the velocity tensor (namely vorticity, divergence, and deformation) are much more meaningful than the classical, 2D-originated partitioning between shear and deformation-induced frontogenesis.

Frontogenetical processes occur at ground level both at the cold front and at the warm front (see 300- and 304-K isotherms, Fig. 3). However, frontogenesis is more active at the cold front in the later stages of the simulation (see the cross-isentropic \(Q^\prime\) component in Fig. 6a).\(^5\) Between day 8 and day 9, the maximum surface thermal gradients go from 3.39 to 4.78 K (100 km)\(^{-1}\) at the cold front, while they go only from 2.77 to 3.02 K (100 km)\(^{-1}\) at the warm front. This is an indication that the warm front, unlike the cold front, does not really proceed from a “self-sharpening” process but rather quickly secludes. This important distinction between frontogenesis at the cold and warm front was first depicted and explained in terms of alongfront advections by Takayabu (1986). Keyser et al. (1988)\(^6\) could create patterns looking much like ours at day 7 with a purely kinematic model of passive scalar deformations (their Fig. 8). The important point, nevertheless, comes from the fact that at day 9 the kinematic symmetry is quite broken down in terms of frontogenetic vectors \(Q^\prime\) (Fig. 6a)—this being due to the positive frontogenetic feedback that occurs at the cold

\(^3\) The initial perturbation was taken to be monochromatic with a 4000-km wavelength, only amplitudes and phases being specified at random. From the Hoskins and West (1979) results, it was felt computertime consuming and useless to excite the 2000-km and following subharmonics.

\(^4\) Thermal gradients go from 0.8 K (100 km)\(^{-1}\) in the initial state to a local maximum of 5.63 K (100 km)\(^{-1}\) at day 10.

\(^5\) Note that due to the lower boundary condition \(w = 0\), \(Q^\prime = Q^\prime_{uc}\) and so may be interpreted as Davies-Jones’s \(Q^\prime\) forcing.

\(^6\) Keyser et al. (1988) referenced Doswell (1984) as the original work.
showing that only ageostrophic circulations can modify dry static stability.

The most striking feature of our simulation (Fig. 6b) is the increasing stability behind and at the cold front. This has an Eulerian influence on the static stability in this part of the system, as relative winds are small (Fig. 4). A residence time of $1.5 \times 10^3$ s can be estimated in the 1500-km area behind the cold front where relative winds are no stronger than 10 m s$^{-1}$ and the vertical frontogenesis larger than $70 \times 10^{-9}$ K m$^{-1}$ s$^{-1}$. This leads to an increase of the vertical stratification $\frac{\partial \theta}{\partial z}$ by an amount of 0.01 K m$^{-1}$. (This value is in agreement with those that can be deducted from Fig. 5a.) Destabilization is noticeable only at the northernmost part of the warm sector. The asymmetry between positive and negative effects may easily be interpreted in terms of the secondary ageostrophic circulations that become increasingly important at the front during the simulation. The cross-front ageostrophic winds convert horizontal gradients of potential temperature into static stability at the front (Fig. 1d), while the vertical space between isentropes is compressed by the subsidence behind the front (Fig. 1c). Only ahead of the front is the ascent stretching the isentrope spacing.

5. Upper-air frontogenesis

Surface and upper-air processes cannot be dynamically separated at the synoptic scale, as baroclinic instability has been convincingly described as a vortex interaction process between an upper-air potential vorticity anomaly and a surface temperature anomaly.

Fig. 4. Relative surface winds together with 1500-m vertical velocities (negative dotted, positive solid lines at intervals of 0.5 cm s$^{-1}$). The average system speed is 7 m s$^{-1}$ eastward.
this quasi-rectilinear trajectory, at the end of which a cutoff occurs. The respective properties of baroclinic wave life cycles that lead to such cutoff processes after a cyclonic or anticyclonic deviation have recently been discussed in detail by Thorncroft et al. (1993). Our simulation clearly is of their “cyclonic” type.

Fig. 5. Vertical CC' cross section through the surface front at day 9 (see Figs. 4 and 6). (a) Potential temperature (solid lines at intervals of 4 K) and vertical velocities (positive dashed, negative dotted lines at intervals of 0.5 cm s\(^{-1}\)), and (b) normal wind (solid lines at intervals of 5 m s\(^{-1}\)) and positive \(\partial \theta / \partial s\) [dashed lines at intervals of 1 K (100 km\(^{-1}\), \(s\) being distance along CC'). Tropopause, axes, and tick marks as in Fig. 2.

(Hoskins et al. 1985). Such an interaction, however, occurs at a fairly broad scale, which is confirmed by the sensitivity experiment described in section 6. That is the reason why upper-air mesoscale processes produced by scale contraction frontogenesis phenomena will be treated in a separate way from surface frontogenesis.

**a. Description**

A southeastward extrusion of stratospheric air with high PV occurs on the 325-K surface (Fig. 7), with a very marked anisotropic character—the cross-flow scale is much smaller than the along-flow one. A sudden and dramatic cyclonic change in direction follows

Fig. 6. Day 9, at the surface: (a) potential temperature at intervals of 4 K and horizontal frontogenetic \(Q'\) vector (arrow centered at grid points), and (b) ageostrophic winds and vertical component of the \(Q'\) frontogenetic vector (at intervals of \(25 \times 10^{-9}\) K m\(^{-1}\) s\(^{-1}\)).
If frontogenetic processes have been thoroughly studied in the quasi-2D entrance of the northwesterly jets [see Keyser and Shapiro (1986) for a review; more recent results are in Reeder and Keyser (1988, hereafter RK88) and Moore (1987)], it seems important to stress again that the story is far from being complete at the end of the quasi-2D regime in our simulation.

The frontogenesis linked with this high-PV extrusion also appears in Fig. 8 with the tightening of both isentropes and isotachs on the cyclonic side of the northwesterly jet, where a well-marked descent occurs. The important change of regime between the very anisotropic, quasi-2D regime upstream of the trough and the very curved regime in the trough again appears; important frontogenetic processes also occur in and lead to maximum tightening of isentropes just downstream of the trough. Numerous cross sections and 3D visualizations have been made using a $z$ vertical coordinate, which also show that the maximum downward extension of the 1-PVU surface also occurs at the trough base, with an absolute height of 7000 m (not shown).

b. Frontogenesis diagnostic using the $Q'$ vector

Again, a synoptic view of all frontogenetic processes (including ageostrophic and vertical advections) at 7000 m can easily be captured from a display of $Q'$ and $\theta$ (Fig. 9). Frontogenesis occurs in the quasi-2D and northwesterly part of the flow, but it is clearly more active just upstream of the trough base. Frontolysis then happens downstream of the trough, which is fully consistent with the observed location of maximum isentropic gradients. These frontogenesis processes are, however, much less intense than at the surface. (The across-isentropic component of $Q'$ is about $1.5 \times 10^{-10}$ K m$^{-1}$ s$^{-1}$ at 7000 m, one tenth of the surface value.) The along-isentropic component of $Q'$ indicates an important Lagrangian change in the isentropic orientation at the trough.

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Fig. 7. Potential vorticity (at intervals of 0.25 PVU) and relative wind field on the 325-K isentropic surface (day 10; the average system speed is 7 m s$^{-1}$ eastward).

Fig. 8. Day 10: (a) potential temperature (solid lines at intervals of 4 K) and pressure (dashed lines at intervals of 5 hPa) at 7000 m; (b) vertical velocities at 7000 m (positive dashed, negative dotted lines at intervals of 0.5 cm s$^{-1}$) and wind speed at 9000 m (solid lines at intervals of 5 m s$^{-1}$ starting at 25 m s$^{-1}$).
the very anisotropic entrance zone of the northwesterly jets (Keyser and Pecnick 1985a,b; RK88) and those stressing the strong curvature effects at the trough axis (Newton and Trevisan 1984a,b). Cammas and Ramond (1989) introduced a partition of the ageostrophic flow between along-flow \( (u_{an}) \) and cross-flow \( (u_{as}) \) components. They can be easily interpreted using the dissipation-free horizontal momentum equation

\[
\frac{du}{dt} + f_0 k \times u = 0 \tag{12}
\]

[cf. Holton (1979), Eqs. (3.2) and (3.3)]. Assuming quasi-horizontal trajectories and using natural components we get

\[
u_{an} = \frac{1}{f_0} \frac{dU}{dt} \tag{13}
\]

\[
u_{as} = -\frac{U^2}{for} \tag{14}
\]

where \( r \) is positive (negative) for cyclonic (anticyclonic) trajectory curvature. As expected (Fig. 10a), the entrance and exit zones of the northwesterly jet are associated with cross-isobaric components of the ageostrophic winds, while the strong curvature near the trough axis has upflow signature. No trace of inertial oscillation downstream of the ridge can be found here.\(^7\)

Following Cammas and Ramond (1989), we introduce these natural ageostrophic components in the expression for the divergence of the flow again expressed in terms of natural components [this is an essential distinction from the Keyser et al. (1989) formulation]:

\[
\nabla \cdot \mathbf{u} = \nabla \cdot \mathbf{u}_u
\]

(b)

\[
= \frac{\partial u_{as}}{\partial s} + \frac{\partial u_{an}}{\partial n} + u_{as} \frac{\partial \psi}{\partial n} - \frac{u_{an}}{r_s}, \tag{15}
\]

where \( \psi \) is the angle between the flow and some constant direction, and \( r_s \) is the streamline curvature radius.

The spatial correlation between the total divergence field \(h) at 9000 m and the vertical velocity patterns at 7000 m is always very good, especially in the northwesterly jet zone, up to the trough base (see Fig. 10b and Fig. 8b at day 10). In this zone, most of the divergence is explained by the terms (c) and (d) in (15): the residual (r) term appears to be negligible (Fig. 10b). Term (c) (Fig. 10c) is the diffuseness of the along-flow ageostrophic wind; we may associate it with the along-flow ageostrophic circulation created by cur-

\(^7\) Sanders et al. (1991) stressed the importance of such unbalanced oscillations in the development of observed intense upper-level fronts.
vature effects. The convergence maximum ($7.6 \times 10^{-6} \text{ s}^{-1}$) appearing just upstream of the trough axis is mainly the result of this effect. Term (d) (Fig. 10d), on the other hand, is the diffluence of the cross-flow ageostrophic wind. Its signature is maximum in the entrance and exit jet areas, with characteristic dipoles. (Refer to Fig. 8b for the jet position.) The first convergence maximum ($8.43 \times 10^{-6} \text{ s}^{-1}$) is mainly the result of this curvature effect.

Clearly, Cammas and Ramond’s (1989) diagnostic permits an objective separation between the two kinds of ageostrophic circulations, which gives a confirmation of the role of both the strongly curved parts of the flow and the linear acceleration area in the tropopause folding process. Such a diagnostic of the curvature effects following the streamlines is more in accordance with the “gradient wind thinking” than partitions in orthogonal planes.

However, the smallness of the residual term (d) has to be verified each time such a separation is made. Our treatment in natural components does not separate between the rotational and divergent part of the ageostrophic wind; this is done to represent vertical circulations among orthogonal vertical planes without internal cancellation of vertical velocity components (Keyser et al. 1989). Difficulties involved with retain-
ing the nondivergent part of the ageostrophic wind in our treatment do not seem to be crucial as far as only the interpretation of the divergence field is concerned. There is a straightforward similarity of Figs. 10c and 10d with the divergence associated with the gradient wind and a straight jet streak, respectively, which gives less importance to internally canceling contribution problems in this case.

d. Dynamical diagnostics

1) Rossby numbers

The importance of ageostrophic advections of momentum has been stressed in several studies of upper-air frontogenesis (Shapiro 1981; RK88). These terms are neglected in QG equations and retained in semi-geostrophic ones (Hoskins 1975). An expansion of the momentum equation in terms of a Lagrangian Rossby number (hereafter, \( R_{o_0} \)) was introduced in this latter reference, as opposed to the Eulerian classical expansion in terms of the advective Rossby number (\( R_{o} \)) leading to the QG equations (Snyder et al. 1991): \( R_{o_0} \) is defined using the rate of change of momentum, while \( R_{o} \) retains only the momentum advections. (See the appendix for more details.) The differences between the \( R_{o_0} \) and \( R_{o} \) fields are quite important in the elongated jet-entrance zone (Fig. 11). Characteristic values for \( R_{o_0} \) are 0.05 on the cyclonic side, while they are 0.15 for \( R_{o} \). On the anticyclonic side, these values are 0.15 and 0.3, respectively. This indicates both that the ageostrophic momentum approximation is better justified than the QG one in the jet-entrance zone and that the flow is in approximate geostrophic balance there. This is in agreement with numerous results from 2D models (e.g., RK88).

The occurrence of nonsmall Rossby numbers (\( R_{o_0} \approx 0.4 \) and \( R_{o} \approx 0.3 \)) near the trough has to be stressed, which makes the separation between primary and secondary flow difficult, at least on the basis of geostrophic balance. Such a result is in agreement with Keyser et al. (1989). This (geostrophically) unbalanced situation has to be contrasted with what has been encountered in the jet-entrance zone.

2) Vorticity diagnostics

The partition of the ageostrophic flow in natural components that is introduced in section 5c indicates that most of the upper-air divergence in the jet-entrance region comes from the cross-flow ageostrophic wind. Such symmetric configurations have been extensively studied in recent years by 2D simulations (Keyser and

---

Singlarities in Fig. 11b occur at points where \( u \approx 0 \). At such points, \( u \cdot \nabla u \to 0 \) but \( du/dt \) usually approaches nonzero values, which explains why singularities occur in Fig. 11b but are not found in Fig. 11a.

---

Fig. 11. Day 10, Rossby numbers at 9000 m (contours every 0.15) and vorticity generation at 7000 m (light shading larger than 10\(^{-10}\) s\(^{-1}\); heavy shading larger than 2 \( \times 10^{-10}\) s\(^{-1}\)): (a) \( R_{o_0} = |u \cdot \nabla u|/f_0|\mathbf{k} \times \mathbf{u}| \) and tilting of horizontal into vertical vorticity, and (b) \( R_{o} = |du/dt|/f_0|\mathbf{k} \times \mathbf{u}| \) and stretching. Jet streak as in Fig. 9b. Note that the maximum of the tilting occurs in a zone where the stretching is minimum.

Pecnick 1985a,b; RK88; Moore 1987). The authors have proposed an interesting feedback process to explain the dramatic tropopause folding occurring in these regions:

1) a displacement of the subsidence beneath the jet core due to the differential cold advection forcing term \( (\partial u/\partial y)(\partial \theta/\partial x) \) in the Sawyer–Eliassen (SE) diag-
nostic equation [see section 3b of the Keyser and Shapiro (1986) review], and
2) an efficient vorticity generation by tilting from the horizontal to the vertical component, leading to strong shears on the cyclonic side of the jet that in turn reinforces the subsidence by an increase of the SE forcing term (point 1).

This strong localized subsidence is also a key factor in upper-air clinogenesis, which seems mainly induced by horizontal shear in vertical velocity (Keyser and Pecnik 1985b).

The displacement of the subsidence pattern toward the jet core clearly happens in our simulation (Fig. 8b). Weak cold advection also can be detected in the entrance area of the jet (Fig. 8a). Therefore, following RK88, the vorticity generation terms were investigated in order to get some insight into the possible feedback processes occurring in our simulation (point 2). The inviscid, hydrostatic vorticity equation used is

\[
\frac{d(\zeta + f)}{dt} = - (\zeta + f) \nabla \cdot \mathbf{u}_h - \mathbf{k} \cdot \left( \nabla \cdot \frac{\partial \mathbf{u}_h}{\partial z} \right) + \mathbf{k} \cdot \left( \frac{\nabla \rho \cdot \nabla P}{\rho^2} \right) .
\]

(16)

The first rhs term is the stretching term; the second one is the tilting term. We verified that the last (solenoidal) term was negligible for the case shown. RK88 have shown that the tilting term was clearly overriding the stretching term when the feedback processes were acting in the jet-entrance area (their Fig. 7). This clearly is not the case in our simulation (Fig. 11). Maxima are $1.3 \times 10^{-18}$ and $2.7 \times 10^{-18}$ s$^{-1}$, respectively, for the tilting and stretching cyclogenetic terms in this area. This is not too surprising, since we are dealing with a rather modest upper-level frontogenesis and RK88 (their Fig. 7) found the feedback process for strong cold advections only.

The upstream part of the trough base is another area of vorticity generation. Tilting terms are important there. They are acting on the cyclonic side of the jet, with values of about $2 \times 10^{-18}$ s$^{-2}$ in a $800 \text{ km} \times 200$ km area, where there is almost no vorticity generation by stretching. This is in accordance with the Rossby numbers in this area (Fig. 11) and confirms that the dynamics of this very curved and frountogenetic part of the flow is far from QG.

6. Some investigation into upper-air–surface frontogenesis interaction

A striking feature when comparing surface and upper-air frontogenesis patterns is that they appear to be linked, since the maximum tropopause descent occurs directly over the surface cold front, with a tilt toward the cold air. This is reminiscent of results obtained with 2D models. Therefore, one might expect to see a positive coupling between surface and upper-air frontogenesis. After some depiction of the vertical signature of both upper-level and surface frontogenesis (section 6a), a dynamical proof of their noninteraction will be given (section 6c).

a. Q' in vertical cross sections

The vertical extension of frontogenesis can be explored using Q' displays in vertical cross sections [Q_p, where n designates the coordinate normal to the cross section; see Eq. (4)]. Two such cross sections

![Fig. 12. Day 10: AA' cross section (jet entrance, see Fig. 8) showing Q' every 4 K together with (a) positive \( \partial \theta / \partial s \) [dotted lines at intervals of 1 K (100 km)$^{-1}$] and Q_p vectors (s being the distance along AA', n across AA'). Note that clinogenesis occurs only at high levels. (b) Vertical velocities at intervals of 0.5 cm s$^{-1}$ and their frontogenetical contribution Q'. The vector orientation is consistent with the horizontal and vertical distance scales. Only vectors greater than 1/2 of the plotted scales are plotted. Tropopause, axes, and tick marks as in Fig. 2.](image)
have been produced at day 10. The first one ($AA'$, Fig. 12) crosses the quasi-2D entrance zone of the northwesterly jet (see Fig. 8b), while the second one ($BB'$, Fig. 13) crosses the area where a strong subsidence pattern is created by the along-flow ageostrophic wind (see Fig. 10c). On both cross sections, clinogenesis occurs since $Q_n'$ points horizontally to the warm side (the right side in the troposphere on Figs. 12 and 13). The vertical component gives information as to the static stability that particles are undergoing. Since the potential vorticity is conserved \[ \left\{ \frac{d}{dt} \left[ (\zeta + f) \frac{\partial \theta}{\partial z} \right] = 0 \right\}, \] such variations of static stability must be compensated for by isentropic vorticity variations of the opposite sign. The $Q'$ pointing down can thus be interpreted in terms of isentropic vorticity generation.

Frontogenetical processes are nascent only in the jet-entrance zone (Fig. 12) and are confined to the upper troposphere (8000–9000 m). The $Q_n'$ show that isentropes are tilted clockwise in this zone (clino genesis), while a strong isentropic vorticity generation can be found from the downward $Q_n'$ orientation. The former effect is mainly the result of the vertical velocity action (Fig. 12b) transforming static stability into barocliny \[ Q_n' \cdot \tau = - (\partial \psi/\partial s) (\partial \theta/\partial z) \] where $\tau$, $s$ refer to the cross section horizontal direction. Such a process has maximum efficiency just under the tropopause where both static stability becomes strong and vertical velocities keep their large tropospheric signature.

Things look much different at the jet exit, just upstream of the trough ($BB'$ cross section, Fig. 13). Particles arriving there have been frontogenetically forced for a longer period of time, and both the tropopause folding and the baroclinic zone are better defined. The vertical extent of the frontogenetical process is striking. The $Q_n'$ points to the right from the tropopause to the ground. This is an important fact that must be related to the large vertical extension of the baroclinic zone at the surface cold front, just downstream of the upper-level trough ($CC'$ cross section, Fig. 5). Hoskins and Heckley (1981) and Keyser and Pecnick (1987) have postulated that 2D processes are responsible for this characteristic. These authors have stressed that this is a fundamental difference between cold and warm fronts. Both studies underline the importance of the cold advection on the frontogenetical processes that take place at the cold front. However, the frontogenetical diagnostic reported in Fig. 13a traces back the origin of the upper-level and midtropospheric barocliny to the clinogenesis in the curved jet exit. Again, these processes are mainly the result of the action of vertical velocities (Fig. 13b). Yet the effect of the total wind (Fig. 13a) is smaller than this vertical velocity contribution. This shows that even if it is not dominant, the action of the horizontal wind cannot be neglected in the clinogenesis process.

Conceptual models convincingly depicting the frontogenetical dynamics in such baroclinic, curved zones are still lacking, the problem being not only to adapt the concepts derived from the usual balance approximations to these curved geometries but also to find new balance conditions for these large–Rossby number zones. An important fact that would be explained by such a conceptual model is, for instance, the vertical tilt of the vertical subsidence pattern (Fig. 13b), which is very reminiscent of the vertical tilt of the secondary circulations and which were well accounted for by the semigeostrophic theory in 2D geometry.

The upper- and lower-troposphere-filling frontogenetical processes occurring in the $BB'$ cross section appear to feed one another. To prove that this is not the case, we will use the horizontal viscosity as an effective (even if quite artificial) frontolysitical process whose efficiency will be demonstrated in the next subsection. This frontolysitical effect will then be used in a selective
way, either in the upper (above 4000 m) or in the lower troposphere (below 4000 m). In case of a feedback process, a remote effect should be felt by the other frontogenetical process (section 6c).

b. The frontotypical impact of horizontal diffusion

To test the impact of the horizontal diffusion on the simulation, we have run the model without it from day 5 to day 9 (experiment 552). It should be remembered at this point that hyperdiffusion is introduced in numerical models not only to counteract numerical noise production but also to parameterize the large-scale impact of the turbulent cascade going to molecular scales (viscous dissipation). The complete suppression of diffusion can thus be expected to lead to small-scale energy production, especially at fronts (Gall et al. 1987), which would be compensated for by viscous dissipation in a "perfect" model (i.e., without truncation effect). This is the reason why we present only results from run 552 up to day 9.

Surface fields indicate that the fronts are more vigorous at this date (Fig. 14a) than one day later in the reference run (experiment 55, Fig. 3b). This shows that frontogenesis is largely reduced by diffusion at day 10 in the reference simulation. The difference remains quantitative, however, since all qualitative features (relative position of fronts and of action centers, relative importance of the two observed fronts) remain the same. This is equally true for the upper-air (325 K) fields (Fig. 14b, to be compared with Fig. 7): PV is best conserved in the nondissipative case, but the aspect of the stratospheric extrusion is not fundamentally different. The 0.5-PVU line, for example, still undulates in a reversible way (no cutoff), indicating that confinement of the PV rearrangement north of this line is not an artifact of diffusion (Thornicroft et al. 1993).

The CC' cross section intersects both the surface cold front and the point of the maximum tropopause descent (Fig. 14). The flow there (Fig. 15) is very reminiscent of 2D studies, with a baroclinic zone extending from the ground throughout the entire troposphere, with a marked tilt toward cold air, its upper part being associated with a tropopause fold that can be seen clearly on the PV contours. The differences introduced by diffusion appear to be more quantitative than qualitative. Both the surface and upper-air fronts are dramatically enhanced when diffusion is removed, which is evident by comparison with all fields in Fig. 5: the PV contours show a greater descent of stratospheric values on a smaller horizontal scale; velocities normal to the cross section show a more pronounced contrast between the cyclonic and anticyclonic part of the jet, and more surface shear; and potential temperatures show greatly enhanced barocliny. Particular attention should be devoted to the vertical velocity pattern, even if it becomes somewhat noisy in the case without diffusion. In both cases, two dipoles of ascent/descent can be defined: one associated with the surface front, and the other with the upper-air front. The same could be said about the baroclinic zones that, rather than extending from the ground to the tropopause throughout the troposphere, seem to extend from both the ground and the tropopause in two distinct bands that marginally join. This reinforces us with the idea that each phenomenon supports its own dynamics without being greatly influenced by the other. This rather subjective opinion has already been proposed by many authors (Hoskins and Bretherton 1972; Buzzi et al. 1981; Key-
the applied diffusion (Fig. 16). The remote (surface) front keeps all its activity (cf. vertical velocity, thermal, and wind signatures) exactly as if it did not feel the upper-air change. This is confirmed by the horizontal patterns (not shown) that are dependent only on the local diffusion that we used and not on what happens in the other layer.

A companion experiment was run (experiment 553) in order to test the remote impact that the surface frontogenesis might have on the upper-air front. The surface frontogenesis was reduced by application of diffusion at levels where $\sigma > 0.611$, diffusion being kept at zero elsewhere. The results (Fig. 17) again indicate a local influence of the weakening of the surface front, without any remote influence on the upper-air front through a feedback process. If the surface front results from a quasi-2D, local process, the upper-air one is the result of a more complex 3D evolution. As was shown

c. Has frontogenesis a remote impact?

Having shown the sensitivity of frontogenesis to diffusion, we used it to reduce the upper-level frontogenesis while allowing the lower-troposphere dynamics to operate with zero diffusion. From day 5 to day 9, diffusion was applied at levels where $\sigma < 0.611$ ($z \geq 4000$ m) and kept at zero at lower levels (experiment 554). The upper-air frontal pattern alone is affected by

Fig. 15. Vertical $CC'$ cross section through the surface front at day 9 when diffusion is suppressed (run 552; see Fig. 5 for the same cross section in the reference run). (a) Potential temperature (solid lines at intervals of 4 K) and vertical velocities (positive dashed, negative dotted lines at intervals of 0.5 cm s$^{-1}$), and (b) normal wind (solid lines at intervals of 5 m s$^{-1}$) and positive $\theta$/$\partial s$ [dotted lines at intervals of 1 K (100 km)$^{-1}$, $s$ being for distance along $CC'$]. Tropopause, axes, and tick marks as in Fig. 2.

Fig. 16. Same as Fig. 15 but with diffusion acting at $\sigma < 0.611$ (run 554).
in section 5, the origin of the upper-air baroclinic zone observed in CC’ lies far upstream, in the quasi-2D region just downstream of the ridge. The frontogenetical process is continuous from this point and includes the strongly curved region in the trough (Fig. 9). If a ‘meeting point’ between the surface and upper-air frontogenesis exists, this appears to be almost fortuitous.

The fact that frontogenesis has been shown to be rather insensitive to remote effects might be related with the well-known property of small-scale features that they have only a marginal vertical penetration ("Rossby height"; e.g., see Hoskins et al. 1985). In that sense, the self-sharpening process might be thought of as a small-scale phenomenon whose influence cannot go very deep in the vertical. Such arguments start from the Poisson equation form of the forcing equation. [See, e.g., Gill (1982), Eq. (12.8.14), or Hoskins et al. (1985), Eq. (29), for the invertibility equation, or Keyser and Pecknick (1985b), Eqs. (2.5)–(2.7), for the Sawyer–Eliassen equation.] It is then easy to show that the response to the forcing is anisotropic in space, the aspect ratio of the vertical to the horizontal scale being crudely \( f/N \). However, this is not the whole story, because there is no reason why the horizontal scale of the response should be the same as that of the forcing. A limit example is nicely illustrated by a Dirac \( \delta \) functionlike forcing, which induces a remote response (Green function) far beyond its position and so on a scale that is infinitely larger than its zero-width support.\(^9\) The intrusion of stratospheric high-PV air in a folding tropopause may be seen as an intermediate situation between a Dirac distribution (where the scale of the response depends only on the "dielectric" properties of the basic state) and a sinuliike distribution (where the horizontal scale of the response is constrained to be that of the forcing, the vertical scale being related to it by the Rossby height relation).

7. Summary and conclusions

Further insights into fully 3D, geostrophically unbalanced frontogenesis is gained in this study by the use of a kinematic diagnosis. Using the adiabatic assumption, the \( Q' \) vector was defined to be the Lagrangian rate of change of the potential temperature gradient. It was shown to have a diagnostic expression [Eq. (2)]. An intrinsic separation of \( Q' \) into the local rotation, divergence, and deformation part of the velocity gradient tensor was recovered. This indicates that any frontogenetic process can be reduced to convergence and/or deformation processes. An alternative anisotropic separation of \( Q' \) into the vertical and horizontal wind velocity contributions to the velocity gradient tensor also was introduced.

The standard problem of the nonlinear evolution of the most unstable normal mode of a zonal jet with realistic vertical and meridional structure was then revisited. The vertical numerical treatment \((\sigma = \Omega/\sqrt{\rho_0} \text{ coordinate, with } 500\text{-m grid spacing at most})\) enabled a precise description of both surface and upper-air frontogenesis, together with their possible interaction.

Both upper-air and surface frontogenetic processes occurring in this experiment were highlighted by this diagnostic. At the surface, cold and warm frontogenesis were found to be of quite different character. Cold frontogenesis was found to be in good agreement with 2D, semigeostrophic Eady simulations. Warm frontogenesis, on the contrary, was occurring at a curved front (substantial alongfront \( Q' \) component) where self-sharpening processes were not found to be strongly active. The vertical frontogenetic signature (static stability evolution) was found to be most important at the cold front, which resulted in an increased stability (this result is, however, suspected to be very sensitive to the adiabatic assumption).

The \( Q' \) diagnostic revealed that most of the upper-air frontogenesis occurred in the very curved zone at the trough base, even if the area highlighted by many 2D simulations or diagnostics (Shapiro 1981; Keyser and Pecnick 1985a,b) was found to have some action in the entrance zone of the northwesterly jet, upstream of the trough. Some key factors revealed by these 2D studies were recovered, such as a strong subsidence occurring just beneath the jet-entrance axis in a cold advection zone. Following Cammas and Ramond (1989), we have separated the divergence in terms of natural components of the ageostrophic wind, thereby proving that the ageostrophic circulation in the entrance area of the jet could be described within a 2D frame-

\(^9\) The analogy between electrostatics and the invertibility problem was recently discussed in detail in Bishop and Thorpe (1993).
work. Unlike what occurred in 2D simulations (Reeder and Keyser 1988), vorticity was, however, shown to be generated mainly by stretching processes, even if tilting of horizontal toward vertical vorticity proved to be nonnegligible. No clear feedback between subsidence forcing and cyclogenesis could therefore be invoked in our experiment.

The frontogenetically efficient zone at the base of the upper trough was shown to be noneustrophically balanced, with strong tilting of horizontal into vertical vorticity. Its frontogenetic character was also shown to act through a thick layer of the troposphere. This is proposed to be an alternative explanation to previous 2D descriptions (Hoskins and Heckley 1981; Keyser and Pecnack 1987) for the deep baroclinic extension of the cold front that is downstream of the frontogenetic trough base for upper-air parcels.

As both surface and upper-air frontogenesis occur in almost the same vertical plane (with an upward tilt toward cold air), a constructive cooperation between the two, by ageostrophic transverse circulation, could be imagined. However, the uncoupled character of upper-air and surface frontogenesis was confirmed by our sensitivity study, using vertically varying horizontal diffusion. As each of the two phenomena was quantitatively strongly influenced by the value of this parameter, we forced a stepwise vertical modulation to act selectively on the net local frontogenesis. If each frontogenesis process was fed by the other in the reference experiment without any diffusion, an impact of local diffusion of one front should occur on the remote front. This was not observed, and the noninteraction of the upper-air front with the surface one is thus proved. This demonstration is, of course, limited to the adiabatic framework of our experiment. Coupling processes between the jet streak dynamics and the low-level ones have been shown through diabatic processes by Keyser and Johnson (1984) or Sortais et al. (1993), among others. The limited strength of our jet streak (41 m s\(^{-1}\)) might also be recalled.

Beyond the ad hoc introduction of diffusion that was used artificially in order to modify the frontogenetic phenomena, a more fundamental problem is posed by the strong quantitative influence of this dissipative process on the frontal scales, both at the surface and at upper levels. It is well known that a free-slip lower boundary condition is very crude, and more or less realistic boundary layer parameterizations of momentum transfers have long been introduced in atmospheric models. The intense response of upper-air frontogenetic flows to the horizontal diffusion reveals that much work also has to be done to improve our knowledge of dissipating processes at higher levels, since the formulation introduced in the models greatly influences the results produced at the resolved scales. This pleads the case of an extensive study of dissipating processes in the 3D, frontogenetical context.

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APPENDIX

Advective and Lagrangian Rossby Numbers

As stressed by Hoskins (1975), the semigeostrophic (SG) theory assumes only that "the Rossby number, defined as the ratio of the magnitudes of the rate of change of momentum and of the Coriolis force, is small." We note \( R_{O} = \| u \| / f_{o} \| k \wedge u \| \) —the "Lagrangian" Rossby number. Thus, SG makes a far less restrictive assumption than QG, which states that each term of the rate of change of momentum (namely, the Eulerian tendency and the advection) is separately small as compared to the Coriolis force (see Gill 1982, for example).

This claim was recently claimed to be false in Snyder et al. (1991). The authors, using the same expansion in terms of Lagrangian Rossby numbers \( R_{O} \) as Hoskins (1975), claim that the \( O(R_{O}) \) expansion of the momentum equations leads to the QG, not the SG system. Such expansions often lead to nonunique solutions, depending on the assumptions made by the different authors. In the Snyder et al. (1992) case, a rigorous expansion of both the Eulerian velocity \( v \) field and the particle position \( x \) is proposed. This should lead to an expansion of their Eq. (B5') different from their Eq. (B5')

\[
\delta x_{0} = v_{0}[x_{0}(a, t), t] \tag{A1}
\]

\[
\delta x_{1} = v_{1} \{ [x_{0}(a, t), t] + x_{1}(a, t) - \nabla v_{0}[x_{0}(a, t), t]. \tag{A2}
\]

The implied modification of the approximate Eulerian momentum equation is of the same order as the difference between QG and SG. So, we will refer in the following to the original Hoskins (1975) Lagrangian expansion. An advective Rossby number is introduced in section 5d as \( R_{O} = (\| u \| \| \nabla u \| / f_{o} \| k \wedge u \|) \); QG (SG) requires \( R_{O} \approx 1 \) (\( R_{O} \leq 1 \)). Both Rossby numbers are displayed in Figs. 11a and 11b. The numerators were computed as

\[
\| \frac{du}{dt} \| \approx \| -f_{o}k \wedge u_{a} \|
\]

\[
\| u \cdot \nabla u \| \approx \| -f_{o}k \wedge u_{a} - \frac{\partial u}{\partial t} \|
\]

where Eq. (12) was used. The time derivative was computed using finite differences between two consecutive model time steps (1200 s). The robustness of the method was tested by computing
\[ \| \mathbf{u} \cdot \nabla \mathbf{u} \| = \left( \| \mathbf{u} \cdot \nabla u \|^2 + \| \mathbf{u} \cdot \nabla v \|^2 \right)^{1/2} \]

\[ \frac{\partial \mathbf{u}}{\partial t} \approx \nabla \| \mathbf{u} \| + \mathbf{u} \cdot \nabla \mathbf{u} \].

Vertical advections of momentum were neglected in the last equation. The fields computed using the last two equations were almost indiscernible from Figs. 11a and 11b (not shown).

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