Comments on "Use of Multiquadric Interpolation for Meteorological Objective Analysis"

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Nuss and Titley (1994) demonstrate that multiquadric interpolation using hyperboloid radial basis functions is a significantly more accurate means of dealing with typical meteorological data distributions than either the Cressman (1959) or the Barnes (1973) objective analysis schemes. The mathematical formulation of their scheme is similar to that developed by Caracena (1987) in that both use matrix methods to compute the effective weights of observations, and all observations influence the interpolations to arbitrary grid points. That is, there is no artificial radius of influence of the basis function as is commonly applied in the Cressman and Barnes schemes. This difference alone should not be a reason for concern in comparing the accuracy of their scheme with the Barnes scheme, provided that the radius of influence of the latter's Gaussian weight function is chosen with due consideration for its possible effect on the resulting analyses. I find that Nuss and Titley violated a very basic consideration in this regard.

This is not to say that the multiquadric interpolation scheme and other recently developed schemes (e.g., Bratseth 1986; Caracena 1987; Pedder 1993) do not have real advantages over the traditional Barnes two-pass analysis scheme, just that Nuss and Titley's error comparisons with the Barnes scheme very likely are exaggerated.

Nuss and Titley test their scheme and the Cressman and Barnes schemes on four typical distributions of observations of an analytic function composed of Gaussian waves in two dimensions whose effective wavelengths vary between approximately 0.5 and 1.0 times the analysis domain size. Each of the four observation sets is composed of 150 data points arrayed 1) pseudouniformly; 2) biased toward one side of the domain, as in a land–sea situation; 3) in one-dimensional swaths, as though obtained from a polar-orbiting satellite; and 4) as a dense cross centered in the domain, as might be obtained along aircraft flight paths (see their Fig. 2).

They indicate that the mean observation spacing varies from 7 to 27 grid units in their 101 × 101 gridpoint domain. They use these numbers to set the smoothing length scale in Barnes' Gaussian weight function—that is, to four-thirds the mean observation spacing. Except for the uniformly distributed example, this choice violates the advice of many who have tried to develop more generalized applications of the Barnes scheme (Koch et al. 1983; Pauley and Wu 1990). For example, I presume that their mean spacing of 7 grid units applies to the fourth example, the dense cross of aircraft data with about one in eight data points distributed in the four quadrants of the domain. The distribution of the separation distances between observations is so skewed that the mean value is of little use for designing an appropriate weight function. At the distances that the sparsely distributed observations lie from each other and from the dense observations in the cross, the sparse observations essentially do not "see" the rest of the data field (the half-power point of the weight function lies only 7.8 grid units away). The analysis in the sparsely observed quadrants probably is composed of a series of stair steps centered at each data point, each step having an "elevation" of the observed value at that data point; for an analysis result that demonstrates this effect, see Fig. 3 in Barnes (1994a).

An equally serious error in the application of the Barnes scheme to the other three observation distributions is the decision to limit the influence of any observation to 25 grid units, arbitrarily setting the weight function to 0 for greater distances. For example, by setting the smoothing length scale to four-thirds of the largest mean observation spacing (to 36 grid units), the authors inadvertently throw away the influence of observations that make up 0.4 of the effective weight at any given grid point [$\exp(-625/1296) \doteq 0.62$]. The error generated by this misapplication of the Barnes analysis may be comparable with the error that would be generated in the multiquadric scheme had the authors arranged the elements of the **Q** matrix in order of increasing separation distances, and then set to 0 a significant number of the elements in the off-diagonal corners, artificially creating a band-diagonal matrix.

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The authors make the same mistake in their search for an optimal value of the tuning parameters when comparing the Barnes analysis obtained from a 25-station observation set of the same analytic function. Because of the sparser sampling, they set the smoothing length scale to 50 grid units, but they also truncate the weight function at 50 grid units where it has a value equal to 0.37. These choices cannot produce a reasonable analysis on the first pass, and a second pass cannot improve upon it significantly. Judging by the wavelengths represented in their "Gaussian hills" analytic function, and applying results similar to those obtained in Barnes (1994a), I believe that a better two-pass analysis could have been obtained using a scale length set to about 27 grid units with the radius of influence extended to at least 80 grid units.

As for the rest of the article, the authors have been careful to provide useful information regarding multiquadric analysis variability as a function of the free parameters. Users of their scheme should carefully consider the consequences of the various choices in these parameters, especially as demonstrated in the authors' Figs. 8 and 9. With regard to the authors' experiments using other real data distributions (Figs. 12 and 13), I agree with most of the comments the authors make concerning the difficulty of tuning a two-pass Barnes analysis to produce reasonable resolution in data-dense areas without also producing questionable details in data-sparse areas. It is largely for this reason that I recommend (Barnes 1994b) that two-pass analyses be abandoned in favor of the better noise suppression capability of a scheme using three or more passes with a fixed weight function.

Nuss and Titley are to be commended for bringing to the attention of analysts in the meteorological community the apparent advantages of multiquadric methods. They demonstrate that results obtained by that method are less sensitive to errors arising from grossly nonhomogeneous data distributions than are results obtained by either the Cressman or Barnes schemes. However, in their tests involving five distributions of observations of an analytic function, I am certain that they have overstated the degree of the advantage over the Barnes (1973) scheme by comparing their method with poorly designed two-pass analyses.

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