# Reply

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In the comments on our paper (Nuss and Titley 1994), Barnes (1995) claims that we violated some basic principles in applying a two-pass Barnes (1973) analysis scheme and have overstated the relative advantage of multiquadric interpolation or mathematically similar schemes in comparison to the Barnes scheme. We admit that our application of the two-pass Barnes scheme was rather simplistic to make objective comparisons relatively easy and that it probably failed to take full advantage of the available observations in each case. However, we also rather simplistically applied the multiquadric scheme and will demonstrate in this reply that we did not overstate the advantage of the multiquadric scheme over the Barnes scheme if both schemes are optimized but actually understated the advantage in some of the situations examined in Nuss and Titley.

In Nuss and Titley, four data distributions were generated from an analytic function of Gaussian hills and valleys to test the performance of various interpolation schemes under a variety of sampling arrangements that occur in meteorological applications (see our original Fig. 2). Of these distributions, Barnes (1995) states that "except for the uniformly distributed example, this choice (a smoothing length scale equal to four-thirds the mean observation spacing) violates the advice of many who have tried to develop more generalized applications of the Barnes scheme. . . ." We concur with this point. Barnes (1995) correctly points out the problems that arise in the Barnes scheme for nonuniformly spaced observations such as our aircraft data distribution (our fourth example). The mean observation spacing in the aircraft sample of 7 grid units is very highly skewed to the closely spaced points in the cross of aircraft sample points, and consequently, the random points away from this cross are largely ignored in our application of the Barnes scheme in Nuss and Titley. This is due in part to using a smoothing length scale that is too short for the sparse data off the cross of dense points, and it is compounded by the use of an arbitrary radius of influence limit of 25 grid units. However, for the root-mean-square error and maximum error comparisons we made (see our Fig. 3), this problem is not as serious as Barnes (1995) states.

To demonstrate that we have not unfairly represented the Barnes scheme by making less than ideal choices of the tuning parameters in our original paper, some further comparisons using the aircraft and other data distributions have been done. The aircraft data distribution will be used to demonstrate the results of these further tests. In these further comparisons, the smoothing length scale was systematically increased from its original setting of four-thirds the mean observation spacing (7 grid units for the aircraft data sample) and the radius of influence was set to 150 grid units so that all observations were included in the analysis at each point. The root-mean-square (rms) error and maximum deviation were again used as the method for measuring the fit of the interpolated analysis to the analytic function. For the aircraft data distribution, the rms error and maximum deviation decreased somewhat as larger values of the smoothing length scale were used. The rms error and maximum deviation reached minimum values of 0.12 and 0.28, respectively, for a length scale of 21 grid units. For length scales greater than this, both the rms error and the maximum deviation increased, reaching values comparable to those for the 7 grid unit length scale by 41 grid units. Similar behavior occurred using the other data distributions, although the optimal length scale was much closer to our original value of four-thirds the mean observation spacing for the more uniformly scattered samples. These results are consistent with comments later in Nuss and Titley that state that the Barnes scheme must be tuned to the data-sparse regions not the data-dense regions in order to avoid problems in the data-sparse regions.

To better illustrate the impact of these changes in the smoothing length scale on the resultant analysis, surface plots of the analytic and derived functions were plotted. Figure 1 shows the analytic Gaussian hills and valleys function that we used for these comparisons. The resultant Barnes analysis using the original smoothing length scale of four-thirds the mean obser-

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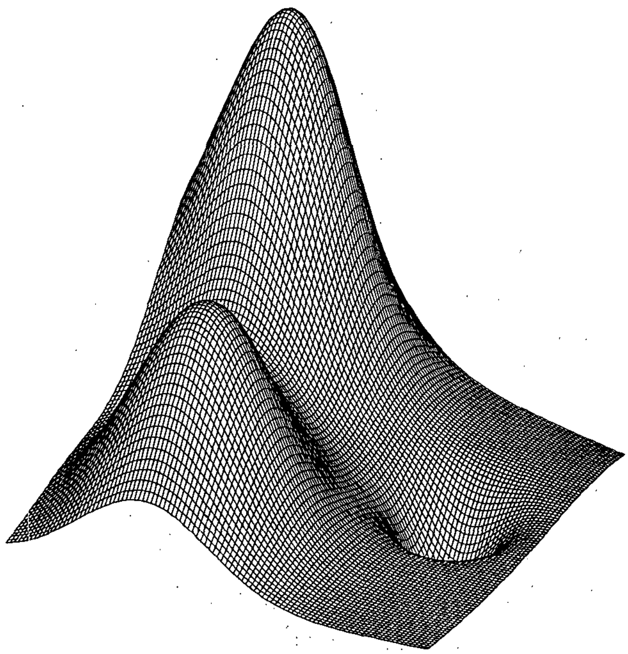

FIG. 1. Surface rendition of the Gaussian hills and valleys function used for analysis tests. The vertical relief represents the magnitude of the function at a particular point in the $x-y$ domain.

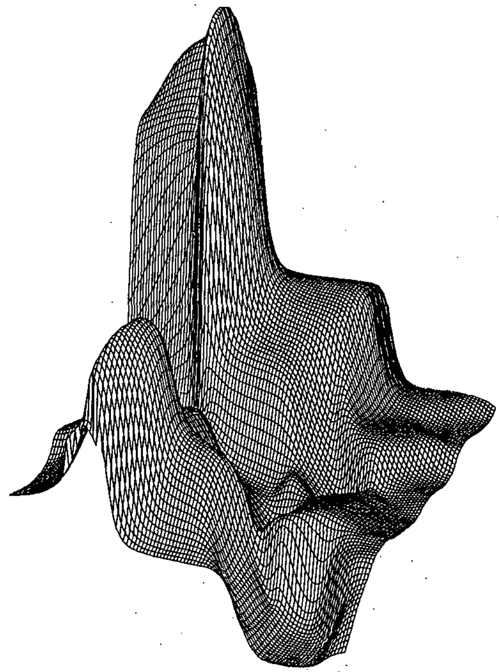

FIG. 2. Surface rendition of the Barnes analysis of the Gaussian hills and valleys function for the 150-point aircraft data distribution and a smoothing length scale of 7 grid units.

vation spacing is shown in Fig. 2. The sharp edges in Fig. 2 arise from the problems that Barnes (1995) has pointed out in the choice of tuning parameters used in Nuss and Titley. The Barnes analysis using the more optimal values for the smoothing length scale and radius of influence is shown in Fig. 3. Figure 3 clearly shows a much smoother field that more closely resembles the analytic Gaussian hill and valleys function (Fig. 1). These differences are even more evident if the deviation away from the analytic function is plotted. Figure 4 compares the deviations for the two Barnes analyses and clearly demonstrates that the larger smoothing length scale is a better choice.

To be completely fair, the multiquadric interpolation for this data distribution was also done again to find the best fit to the analytic function it could achieve. To this end, the multiquadric constant was varied from 0.1 to 0.0001, and the rms error and maximum deviation were calculated for each value used. The result was that the value of 0.025 for the multiquadric parameter produced the best fit for this data distribution, reducing the rms error to 0.035 with a maximum deviation of 0.16. The resultant surface and deviation plots are shown in Fig. 5. Figure 5a shows a clearly superior rendition of the analytic function as compared to either Barnes scheme (Figs. 2 and 3). Figure 5b shows that the deviations for the multiquadric analysis are generally very small except in regions that were simply not sampled by the observations at all (the bump on the right edge of the plot). The relative rms error advantage of the multiquadric analysis versus the Barnes scheme in this comparison, using presumably the lowest error settings for the tunable parameters, is far greater than stated in Nuss and Titley.

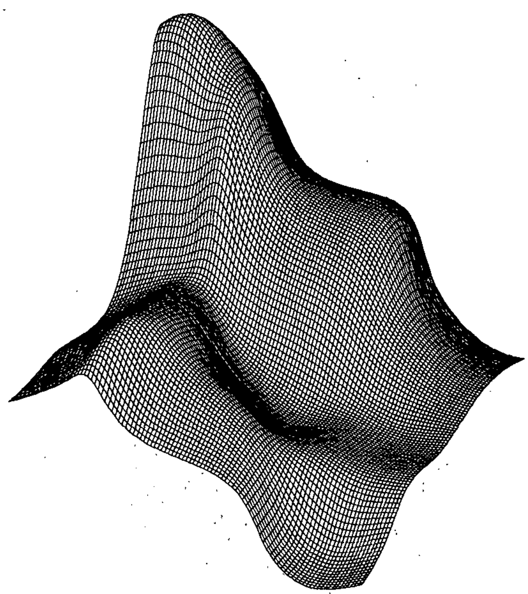

FIG. 3. Same as Fig. 2 except using a smoothing length scale of 21 grid units.

The large rms error differences between the multiquadric and Barnes schemes for the aircraft and other data distributions is primarily due to the limitations of the Barnes scheme in making use of both dense and sparse data within the same domain as discussed in Nuss and Titley. For more uniformly spaced data distributions the relative difference is smaller as seen in our original paper (see original Fig. 3). The basic problem for the Barnes scheme in fitting the observations is that a smoothing length scale must be chosen to produce an acceptable analysis over the entire domain. As this length scale gets too small, analysis jumps occur as seen in Fig. 2. As this length scale gets too large, the smaller-scale features are smoothed beyond recognition as in Fig. 3. Error increases in either direction around the length scale of lowest error. For the multiquadric scheme, the tuning parameter (multiquadric constant) is simply included to make the matrix more diagonally dominant to produce a well-conditioned matrix. As this parameter gets too large, the matrix becomes too ill conditioned to produce an acceptable solution. As this parameter gets ever smaller, the matrix becomes ever increasingly diagonally dominant. In our tests, we found that over several orders of magnitude the variations caused by smaller and smaller values of the multiquadric parameter produced almost no effect on the rms error of the resultant analysis. Consequently, one can simply choose a small value for this parameter and be assured that an acceptable analysis will result using multiquadric interpolation. I find that not to be true of the Barnes scheme, which requires much fiddling to find the right smoothing length scale for a given data distribution.

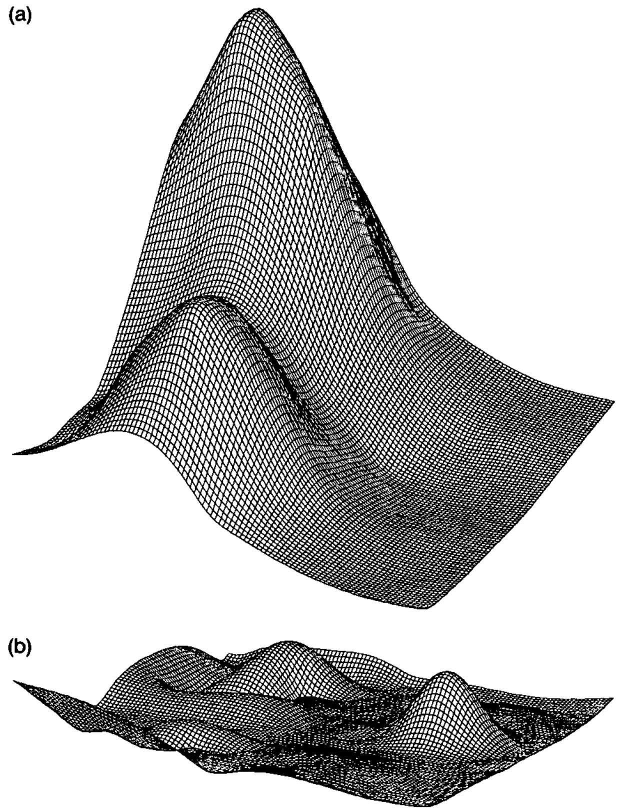


FIG. 5. Surface rendition of the (a) multiquadric analysis of the Gaussian hills and valleys function and (b) deviations from the analytic function for the 150-point aircraft data distribution and a multiquadric constant of 0.025.

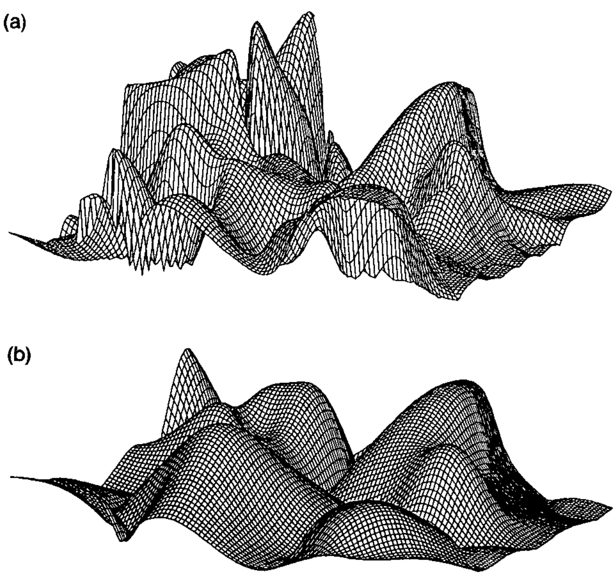


FIG. 4. Surface rendition of the deviations between the analytic function (Fig. 1) and either the Barnes analysis with a smoothing length scale of (a) 7 or (b) 21 grid units.

The Barnes scheme and future improvements to it may further reduce the differences in rms errors with the multiquadric scheme but the Barnes scheme seems inherently limited by its length scale. Within this limitation, it can produce useful analyses, but the relative advantage of the multiquadric scheme is anything but overstated in Nuss and Titley.

## REFERENCES

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