Local Climatic Guidance for Probabilistic Quantitative Precipitation Forecasting

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ABSTRACT

The predictand of a probabilistic quantitative precipitation forecast (PQPF) for a river basin has two parts: (i) the basin average precipitation amount accumulated during a fixed period and (ii) the temporal disaggregation of the total amount into subperiods. To assist field forecasters in the preparation of well-calibrated (reliable) and informative PQPFs, local climatic guidance (LCG) was developed. LCG provides climatic statistics of the predictand for a particular river basin, month, and period (e.g., 24-h period beginning at 1200 UTC and divided into four 6-h subperiods). These statistics can be conditioned on information entered by the forecaster such as the probability of precipitation occurrence and various hypotheses regarding the precipitation amount and timing.

This article describes two probability models of the predictand, details guidance products, and illustrates them for the Lower Monongahela River basin in Pennsylvania. The first model provides marginal climatic statistics of the predictand on an ''average'' day of the month. The second model conditions the statistics on the timing of precipitation within the diurnal cycle. The resultant characterization of the precipitation process allows the forecaster to decompose the complex assessment of a multivariate PQPF into a sequence of feasible judgmental tasks.

1. Introduction

An earlier article (Krzysztofowicz et al. 1993) described a methodology that had been formulated to aid a field forecaster in preparing probabilistic quantitative precipitation forecasts (PQPFs) for river basins. This methodology has been tested operationally in the Weather Service Forecast Office (WSFO) in Pittsburgh, Pennsylvania, since August 1990 and is currently being developed into an operational prototype system for dissemination to other offices of the National Weather Service. The primary purpose of the PQPF is to provide an input to a hydrologic model, which produces forecasts of river stages, particularly during flood conditions.

The local climatic guidance (LCG) is an interactive computer software that provides long-term statistics of the predictand. These statistics can be used in the preparation and verification of PQPF. This article describes two probability models of the predictand, details guidance products, and illustrates them for a river basin in Pennsylvania. As a descriptor of the precipitation process, the LCG has three distinguishing features: (i) the predictand is a time series of basin average precipitation amounts, (ii) the time series is modeled via the disaggregation approach, and (iii) the guidance has a probabilistic format compatible with the format of PQPF. These features require probability models that in many respects depart from the classical ones. In particular, the second model exploits the conditional disaggregative invariance of the precipitation process, which implies that the multivariate distribution of the predictand is given by a product of conditional distributions. The inference is then conditioned on the timing of precipitation within the diurnal cycle, which by itself is a significant predictor of the total amount and its temporal disaggregation. The resultant characterization of the precipitation process allows the forecaster to decompose the complex assessment of a multivariate PQPF into a sequence of feasible judgmental tasks.

2. Methodological background

a. Purposes

The LCG serves four purposes. First, it allows the forecaster to become familiar with statistics of the predictand, basin average precipitation process, which is not directly observable, unlike the point precipitation process. Second, it reinforces the frame for forecaster's

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judgments because the format of LCG is identical with the format of PQPF. Third, it provides the forecaster with climatic (a priori) estimates of the predictand, which play an important role in assessing final (a posteriori) estimates that are well calibrated (reliable). Fourth, it provides statistics necessary for a comparative verification of PQPFs.

The guidance is formulated for a river basin and each month of the year, under the assumption that monthly climatic statistics are stationary across the years.

b. Predictand

Let W denote the basin average precipitation amount accumulated during a fixed period (e.g., 24 h). Let W_i denote the basin average precipitation amount accumulated during the ith subperiod, $i \in \{1, \ldots, n\}$, where n is the number of subperiods. Thus,

$$W \geq 0;\ W_i \geq 0,\ i = 1, \ldots, n;$$
$$W_1 + \cdots + W_n = W. \tag{1}$$

Given a precipitation event, $W > 0$, define for each i a variate $\Theta_i = W_i/W$ representing a fraction of the total amount accumulated during subperiod i. Thus,

$$0 \leq \Theta_i \leq 1,\ i = 1, \ldots, n;$$
$$\Theta_1 + \cdots + \Theta_n = 1. \tag{2}$$

The vector of fractions $(\Theta_1, \ldots, \Theta_n)$ defines the temporal disaggregation of the total precipitation amount accumulated during a period into n subperiods. The predictand for a river basin is the vector $(W; \Theta_1, \ldots, \Theta_n)$. Note that only n variates must be forecast because one of the fractions can always be expressed in terms of the remaining fractions through the unit sum constraint.

Classical models represent the precipitation process in a time series form $(W_1, \ldots, W_n)$. Our model employs a disaggregative form $(W; \Theta_1, \ldots, \Theta_n)$. In principle, the two forms are equivalent. However, the disaggregation approach offers several advantages (Krzysztofowicz et al. 1993). In particular, it is compatible with the thought process of many forecasters who prefer to decompose the predictand into two parts: (i) the total precipitation amount W and (ii) the temporal disaggregation $(\Theta_1, \ldots, \Theta_n)$ of the total.

c. Events and probabilities

Throughout the paper, a distinction is maintained between a random variable (a variate), denoted by an uppercase letter, and a realization (an observation, a fixed magnitude), denoted by a lowercase letter. For instance, at the time of forecast preparation, the basin average precipitation amount is uncertain and hence is treated as a random variable W. The forecaster may consider a fixed magnitude ω and then assess the probability P of event $W > \omega$ in which the basin average precipitation amount W will exceed magnitude ω, symbolically $P(W > \omega)$. Given a record of past realizations of the basin average precipitation amount, a climatic estimate of the probability $P(W > \omega)$ is equal to the number of realizations exceeding ω divided by the total number of realizations in the record.

d. Data and samples

The source data consist of hourly precipitation amounts recorded by rain gauges within or near the basin of interest from 1948 to 1993. For each hour, the basin average precipitation amount is approximated by a weighted sum of rain gauge observations, with weights determined via the Thiessen method (Linsley et al. 1975).

For a given basin, the LCG is defined in terms of five specifications: (i) month (or season consisting of any number of consecutive months), (ii) the beginning hour (0000, . . . , 2300) UTC, (iii) the duration of the guidance period in hours, (iv) the number of subperiods n, and (v) the duration of each subperiod in hours. A realization of the random vector $(W_1, \ldots, W_n)$ is obtained by summing up the hourly basin average precipitation amounts over the appropriate subperiods. The number of realizations is thus equal to the number of periods with complete hourly observations. Realizations are assumed to form a random sample. The option of clustering months into seasons serves chiefly to increase the sample size.

e. Test cases

In the operational testing of the PQPF methodology by the WSFO in Pittsburgh, the predictand is the 24-h basin average precipitation amount and its disaggregation into four 6-h subperiods. Forecasts are made twice a day for 24-h periods beginning at 0000 and 1200 UTC, for two river basins. The Upper Allegheny River basin above the Kinzua Dam covers 5853 km^2 (2260 square miles) in Pennsylvania and New York. The Lower Monongahela River basin above Connellsville covers 3429 km^2 (1324 square miles) in Pennsylvania and Maryland.

Whereas general conclusions drawn in this paper are based on the complete LCG for both basins, data analyses and guidance products are illustrated with the LCG for the Lower Monongahela River basin, month of March, 24-h period beginning at 1200 UTC, and disaggregation into four 6-h subperiods: 1200–1800, 1800–0000, 0000–0600, and 0600–1200. Hourly basin average precipitation amounts were estimated from observations at six stations with the following weights: Sines Deep Creek (0.24), New Germany (0.11), Confluence (0.26), Glencoe (0.10), Connellsville (0.14), and Boswell (0.15). All amounts are reported herein in inches.

TABLE 1. Structure of the local climatic guidance.

Guidance segment	Hypothesis input by forecaster
I. Guidance without the timing hypothesis	
A. Exceedance function of total amount	
1) Unconditional	
(a) Climatic probability of precipitation	
(b) Forecast probability of precipitation	PoP
2) Conditional on threshold amount	$W > r$
B. Fractions statistics of temporal disaggregation	
II. Guidance with the timing hypothesis	
A. Precipitation timing tree	
B. Timing-dependent exceedance function	
1) Conditional on timing	$T = t$
2) Conditional on timing and threshold amount	$T = t$, $W > r$
C. Timing-dependent fractions statistics	$T = t$

f. Guidance structure

The structure of the LCG is shown in Table 1. There are two main segments, each based on a different model. The first model describes the total precipitation amount W and the vector of fractions $(\Theta_1, \ldots, \Theta_n)$ independently, and for each provides marginal climatic statistics. This guidance is to be used in introductory training, to familiarize the forecaster with statistics of the predictand on an ''average'' day of the month, and in operation, when the forecaster is highly uncertain about the timing of precipitation within the period.

The second model describes all possible timing patterns of the precipitation and for each pattern provides conditional climatic statistics of the predictand $(W; \Theta_1, \ldots, \Theta_n)$. According to the model, the problem is decomposed into three tasks: (i) forecasting the precipitation timing; (ii) forecasting the total amount, conditional on timing; and (iii) forecasting the temporal disaggregation of the total amount, conditional on timing. This guidance is to be used in training, to familiarize the forecaster with the nature of stochastic dependence among the elements of the predictand, and primarily in operation.

3. Guidance without the timing hypothesis

a. Probability of precipitation and distribution of amount

Two elements are estimated from climatic data. The first element is the probability of the event $W > 0$:

$$\pi = P(W > 0). \tag{3}$$

This can be interpreted as the probability of measurable basin average precipitation occurring during the guidance period, on any day of the month. The second element is the distribution G of the basin average precipitation amount W, conditional on precipitation occurrence, $W > 0$. For any fixed amount $\omega \geq 0$,

$$G(\omega) = P(W \leq \omega | W > 0). \tag{4}$$

To develop flexible guidance, the distribution G must be modeled parametrically. Herein, G is chosen to be a member of the Weibull family of distributions:

$$G(\omega) = 1 - \exp[-(\omega/\alpha)^{\beta}], \ \omega \geq 0, \tag{5}$$

where $\alpha > 0$ is the scale parameter and $\beta > 0$ is the shape parameter. This model is supported by investigations of Selker and Haith (1990), who showed that (5) fits well to empirical distributions of daily amounts recorded on wet days at stations throughout the northeastern United States and stratified by months, and Hershenhorn and Woolhiser (1987), who found (5) to be the best among five alternative models for daily amounts of summer precipitation recorded at stations in Arizona. An additional advantage of (5) is the existence of a closed form inverse. Specifically, for any probability p the equation $p = G(\omega)$ can be solved for $\omega = G^{-1}(p)$, where

$$G^{-1}(p) = \alpha[-\ln(1 - p)]^{1/\beta}, \ 0 < p < 1. \tag{6}$$

The parameters α and β were estimated by the method of linear regression (Weibull 1951; Schütte et al. 1987). For the Monongahela Basin, guidance period 1200–1200 UTC, and every month, Table 2 shows sample size and estimates of parameters π, α, and β.

b. Unconditional exceedance function

Given π and G, two guidance products can be derived. The first product is the unconditional exceedance

TABLE 2. Monthly statistics of the 24-h basin average precipitation amount W; Monongahela Basin, from 1200 UTC.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
Sample size	620	653	725	726	825	870	937	902	921	871	728	762
π	0.55	0.57	0.60	0.61	0.57	0.56	0.62	0.52	0.46	0.41	0.50	0.54
α	0.14	0.12	0.15	0.17	0.17	0.18	0.19	0.19	0.19	0.15	0.14	0.13
β	0.91	0.99	0.93	0.94	1.00	0.96	0.97	1.01	0.99	0.94	1.04	0.98
$\omega_{75\|0}$	0.03	0.03	0.04	0.04	0.05	0.05	0.05	0.05	0.05	0.04	0.04	0.04
$\omega_{50\|0}$	0.09	0.08	0.10	0.11	0.12	0.12	0.13	0.13	0.13	0.10	0.10	0.09
$\omega_{25\|0}$	0.19	0.17	0.21	0.24	0.23	0.25	0.26	0.26	0.26	0.22	0.19	0.18

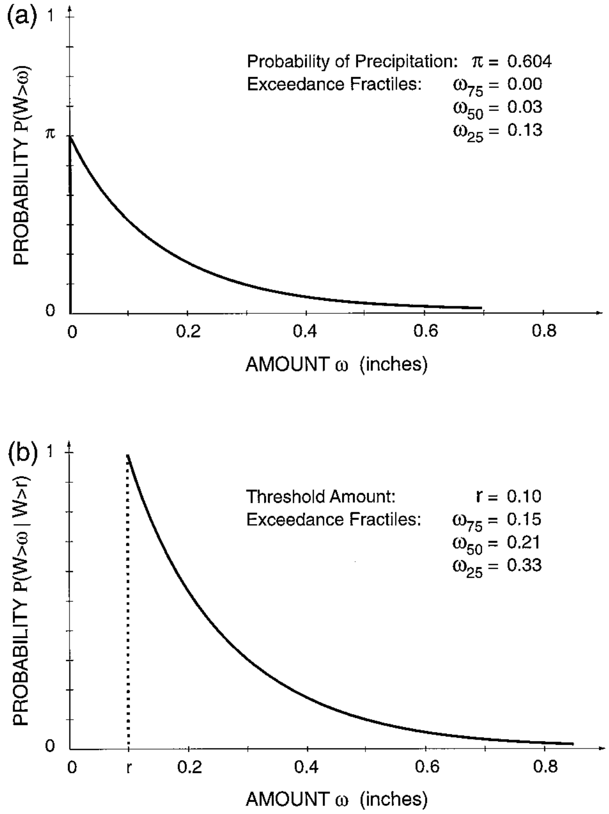

FIG. 1. Exceedance functions of the 24-h basin average precipitation amount W: (a) unconditional and (b) conditional on the event $W > r$; Monongahela Basin, March, from 1200 UTC.

probability $P(W > \omega)$. This is the probability that on any day of the month, the basin average precipitation amount W accumulated during the guidance period will be greater than ω. For any $\omega \geq 0$, the total probability law yields

$$\begin{aligned} P(W > \omega) &= P(W > \omega \mid W = 0)P(W = 0) \\ &\quad + P(W > \omega \mid W > 0)P(W > 0) \\ &= \pi[1 - G(\omega)], \end{aligned} \tag{7}$$

because $P(W > \omega | W = 0) = 0$ and $P(W > \omega | W > 0) = 1 - P(W \leq \omega | W > 0)$.

For any probability p, such that $0 < p < 1$, the $100p\%$ exceedance fractile of W is an amount ω_{100p} such that $P(W > \omega_{100p}) = p$. If $p \geq \pi$, then $\omega_{100p} = 0$ because $P(W > \omega) \leq \pi$ for every $\omega \geq 0$. If $p < \pi$, then the equation $\pi[1 - G(\omega_{100p})] = p$ yields

$$\omega_{100p} = G^{-1}\left(1 - \frac{p}{\pi}\right), \tag{8}$$

where G^{-1} denotes the inverse of the distribution G.

The guidance product is illustrated in Fig. 1a. It displays the probability of precipitation occurrence π and the unconditional exceedance function—a plot of probability $P(W > \omega)$ for all values of $\omega \geq 0$. In addition, three exceedance fractiles are highlighted: the 75% exceedance fractile ω_{75}, the 50% exceedance fractile ω_{50}, also called the median, and the 25% exceedance fractile ω_{25}. These exceedance fractiles are for the same probabilities that are used in the forecasting procedure (p = 0.75, 0.50, 0.25); thus, they constitute the climatic PQPF.

This guidance product can be updated in real time by replacing the climatic probability of precipitation occurrence π with a forecast probability of precipitation, PoP, for a specified basin and the current guidance period. Given this PoP, and no other information, the probability that the basin average precipitation amount W will exceed ω is $P(W > \omega) = (\text{PoP})[1 - G(\omega)]$. Guidance of this type was investigated and advocated by Wilks (1990).

c. Conditional exceedance function

The second guidance product is the conditional exceedance probability $P(W > \omega | W > r)$. It offers guidance in situations wherein the forecaster is certain that precipitation will occur and the basin average precipitation amount W will exceed a threshold r, say $r = 0.2$ inches. For any $r \geq 0$ and any $\omega > r$, Bayes's theorem yields

$$\begin{aligned} P(W > \omega \mid W > r) &= \frac{P(W > r \mid W > \omega)P(W > \omega)}{P(W > r)} \\ &= \frac{P(W > \omega)}{P(W > r)} \\ &= \frac{1 - G(\omega)}{1 - G(r)}, \end{aligned} \tag{9}$$

because $P(W > r | W > \omega) = 1$ for any $r < \omega$.

For any p and r, such that $0 < p < 1$ and $r \geq 0$, the $100p\%$ conditional exceedance fractile of W is an amount $\omega_{100p|r}$ such that $P(W > \omega_{100p|r} | W > r) = p$. The equation $[1 - G(\omega_{100p|r})]/[1 - G(r)] = p$ yields

$$\omega_{100p|r} = G^{-1}(1 - p[1 - G(r)]). \tag{10}$$

The guidance product is illustrated in Fig. 1b. It displays the threshold amount r input by the forecaster and the conditional exceedance function—a plot of probability $P(W > \omega | W > r)$ for the specified r and all values of $\omega > r$. In addition, three conditional exceedance fractiles are highlighted: ($\omega_{75|r}$, $\omega_{50|r}$, $\omega_{25|r}$). Their values are shown in Table 2 for threshold amount $r = 0$ and every month.

d. Fractions statistics

The temporal disaggregation of the total precipitation amount accumulated during a wet guidance period on any day of the month is completely characterized by an $(n - 1)$-variate distribution of the vector of fractions $(\Theta_1, \ldots, \Theta_n)$. The dimensionality of this distribution

TABLE 3. Monthly statistics of fractions (Θ_1, Θ_2, Θ_3, Θ_4); Monongahela Basin, from 1200 UTC.

Month	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$P(\Theta_1 = 1)$	0.14	0.12	0.12	0.11	0.09	0.09	0.08	0.12	0.11	0.11	0.10	0.15
$P(\Theta_2 = 1)$	0.06	0.07	0.05	0.07	0.10	0.13	0.14	0.13	0.09	0.05	0.06	0.04
$P(\Theta_3 = 1)$	0.03	0.04	0.04	0.04	0.05	0.08	0.06	0.05	0.05	0.04	0.02	0.05
$P(\Theta_4 = 1)$	0.12	0.13	0.11	0.10	0.08	0.07	0.08	0.09	0.09	0.12	0.15	0.10
$P(\Theta_1 = 0)$	0.40	0.41	0.42	0.41	0.45	0.52	0.56	0.52	0.44	0.45	0.43	0.41
$P(\Theta_2 = 0)$	0.51	0.48	0.47	0.40	0.34	0.35	0.36	0.39	0.41	0.44	0.46	0.50
$P(\Theta_3 = 0)$	0.51	0.52	0.44	0.46	0.45	0.46	0.51	0.53	0.48	0.49	0.53	0.47
$P(\Theta_4 = 0)$	0.44	0.46	0.41	0.45	0.51	0.54	0.56	0.55	0.48	0.44	0.43	0.47
$E(\Theta_1)$	0.32	0.32	0.30	0.27	0.26	0.22	0.22	0.25	0.29	0.28	0.28	0.32
$E(\Theta_2)$	0.19	0.24	0.22	0.27	0.32	0.34	0.36	0.33	0.26	0.24	0.24	0.20
$E(\Theta_3)$	0.19	0.18	0.20	0.22	0.22	0.25	0.21	0.20	0.21	0.21	0.18	0.21
$E(\Theta_4)$	0.29	0.26	0.28	0.25	0.20	0.20	0.21	0.22	0.25	0.27	0.30	0.27

precludes its direct use in judgmental forecasting. Instead, the guidance characterizes each fraction Θ_i in terms of its (i) marginal probability function, which specifies $P(\Theta_i = 0)$, $P(0 < \Theta_i < 1)$, and $P(\Theta_i = 1)$; and (ii) marginal distribution statistics such as the mean $E(\Theta_i)$, the 75% exceedance fractile $\theta_{i(75)}$, the median $\theta_{i(50)}$, and the 25% exceedance fractile $\theta_{i(25)}$. To obtain the fractiles, the marginal distribution of Θ_i is modeled parametrically (see appendices A and B). The association between fractions is characterized in terms of the correlation matrix $\{\text{cor}(\Theta_i, \Theta_j): i = 1, \ldots, n; j = i + 1, \ldots, n\}$. For every month and fraction, Table 3 reports estimates of $P(\Theta_i = 0)$, $P(\Theta_i = 1)$, and $E(\Theta_i)$.

The guidance product takes the form of n box plots, as shown in Fig. 2. Atop the boxes are the marginal probability functions, which apprise the forecaster of the frequency with which fractions take on the extreme values. For instance, when precipitation does occur during the period, the first subperiod has, on average, a larger chance of being wet than dry, $P(\Theta_1 > 0) = 0.584$ versus $P(\Theta_1 = 0) = 0.416$. But the chance of all precipitation occurring in the first subperiod is considerably smaller, $P(\Theta_1 = 1) = 0.121$.

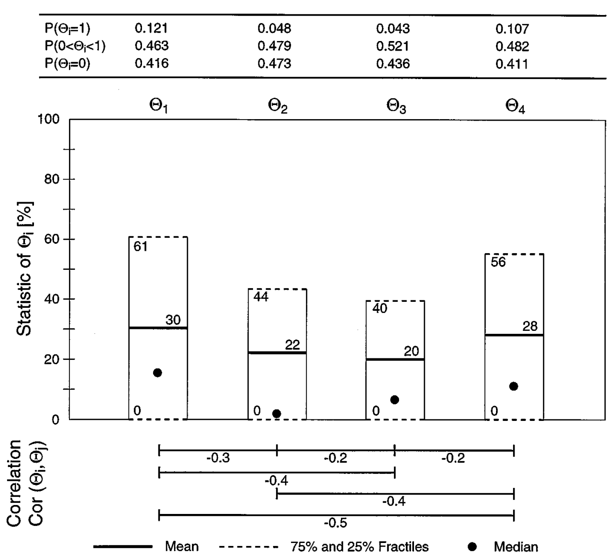

FIG. 2. Statistics of fractions (Θ_1, Θ_2, Θ_3, Θ_4) that define the temporal disaggregation of the 24-h basin average precipitation amount W; Monongahela Basin, March, from 1200 UTC.

The median $\theta_{i(50)}$ of fraction Θ_i, shown inside a box, is an estimate such that $P(\Theta_i \leq \theta_{i(50)}) = 0.50$. The medians do not necessarily sum up to one, but the means always do; $E(\Theta_1) + \cdots + E(\Theta_n) = 1$. The inequalities $\theta_{i(50)} < E(\Theta_i)$ for $i = 1, 2, 3, 4$, which can be inferred from Fig. 2, reflect skewness of the marginal distributions of fractions: on more than half of precipitation events a fraction is smaller than its climatic mean. The box itself represents the 50% credible interval of Θ_i; that is, $P(\theta_{i(75)} < \Theta_i \leq \theta_{i(25)}) = 0.50$. Furthermore, $P(\Theta_i \leq \theta_{i(75)}) = 0.25$ and $P(\Theta_i > \theta_{i(25)}) = 0.25$. Loosely speaking, the 50% credible interval conveys the variability of Θ_i. It, thus, provides guidance on the extent to which an estimate of Θ_i on a particular day can depart from the climatic median $\theta_{i(50)}$.

Beneath the boxes are the correlations: $\text{cor}(\Theta_1, \Theta_2) = -0.3$, $\text{cor}(\Theta_1, \Theta_3) = -0.4$, and so on. The correlations are usually negative, which is a typical trait of fractions (Krzysztofowicz and Reese 1993). As a measure of the strength of association between fractions, the correlation offers guidance on how to simultaneously adjust several fractions. For example, when the forecaster takes the vector of climatic means (30%, 22%, 20%, 28%) as a starting estimate and increases the estimate of Θ_4, from 28% to 48%, then he must decrease estimates of other fractions by 20% so that the sum of the estimates remains 100%. The correlations tell him that Θ_4 is associated most strongly with Θ_1 and Θ_2, whereas most weakly with Θ_3. Thus, in the absence of other information, most of the decrease should be allocated to estimates of Θ_1 and Θ_2.

e. Monthly variations

Tables 2 and 3 summarize monthly variations in precipitation statistics. The probability of precipitation occurrence π (within the Monongahela Basin, during a 24-h period beginning at 1200 UTC) has two maxima, in March–April and July, and a minimum in October. The exceedance fractiles of the basin average precipitation amount per occurrence ($\omega_{75|0}$, $\omega_{50|0}$, $\omega_{25|0}$) follow a unimodal cycle, which reaches a maximum in July–August–September and the minimum in February. Of the two Weibull parameters, it is the scale parameter α that tracks the cycle of fractiles, whereas the shape parameter β appears independent. The fact that the probability of occurrence, π, and the distribution of the amount per occurrence, G, follow different annual cycles underscores the importance of providing the climatic guidance in a decomposed form (7).

The temporal disaggregation of the 24-h basin average precipitation amount also exhibits variations, most prominently between the cool season (October–March) and the warm season (April–September). The probability of all 24-h precipitation occurring in the ith subperiod, $P(\Theta_i = 1)$, has two distinct cycles. For $i = 1$ (1200–1800 UTC) and $i = 4$ (0600–1200 UTC), this probability reaches a maximum during the cool season and a minimum during the warm season; for $i = 2$ (1800–0000 UTC) and $i = 3$ (0000–0600 UTC), this probability reaches a maximum during the warm season and a minimum during the cool season. The mean of a fraction, $E(\Theta_i)$, follows the cycle of probability $P(\Theta_i = 1)$. On the other hand, the probability of the ith subperiod being dry, when the precipitation does occur during the 24-h period, $P(\Theta_i = 0)$, follows a reverse cycle of probability $P(\Theta_i = 1)$. Together, these results are qualitatively similar to the climatic frequency of precipitation events over eastern and central United States documented by Wallace (1975): during summer the frequency reaches a maximum in the late afternoon, early evening, or around midnight ($i = 2, 3$); during winter the frequency is the highest at nighttime or in the early morning ($i = 4, 1$).

4. Guidance with the timing hypothesis

a. Precipitation timing tree

When precipitation occurs during a period, $W > 0$, it occurs in one or more of the n subperiods. The occurrence or nonoccurrence of precipitation in subperiods defines the timing T. There are $2^n - 1$ possible timing patterns. They can be formally defined in terms of fractions ($\Theta_1, \ldots, \Theta_n$), as shown in Table 4 for $n = 4$. A particular pattern is identified by the concatenation of indices of subperiods that are wet. For example, if all precipitation for the period occurs in subperiods 1 and 3, then $\Theta_1 > 0$, $\Theta_3 > 0$, $\Theta_1 + \Theta_3 = 1$, and $T = 13$.

Another descriptor of precipitation is its duration D. It counts the number of wet subperiods. It may be refined to also indicate whether the wet subperiods are consecutive (C) or nonconsecutive (N). In the example from Table 4, $D = 3$ denotes three wet subperiods, which may result from any of the four timing patterns ($T = 123, 234, 124, 134$), whereas $D = 3C$ denotes three consecutive wet subperiods, which may result from any of the two timing patterns ($T = 123, 234$).

TABLE 4. Definition of precipitation duration and timing pattern for four subperiods.

Duration index* d	Timing pattern t	Definition in terms of fractions	
1	1	$\Theta_1 = 1$	
1	2	$\Theta_2 = 1$	
1	3	$\Theta_3 = 1$	
1	4	$\Theta_4 = 1$	
2C	12	$\Theta_1, \Theta_2 > 0$	$\Theta_1 + \Theta_2 = 1$
2C	23	$\Theta_2, \Theta_3 > 0$	$\Theta_2 + \Theta_3 = 1$
2C	34	$\Theta_3, \Theta_4 > 0$	$\Theta_3 + \Theta_4 = 1$
2N	13	$\Theta_1, \Theta_3 > 0$	$\Theta_1 + \Theta_3 = 1$
2N	14	$\Theta_1, \Theta_4 > 0$	$\Theta_1 + \Theta_4 = 1$
2N	24	$\Theta_2, \Theta_4 > 0$	$\Theta_2 + \Theta_4 = 1$
3C	123	$\Theta_1, \Theta_2, \Theta_3 > 0$	$\Theta_1 + \Theta_2 + \Theta_3 = 1$
3C	234	$\Theta_2, \Theta_3, \Theta_4 > 0$	$\Theta_2 + \Theta_3 + \Theta_4 = 1$
3N	124	$\Theta_1, \Theta_2, \Theta_4 > 0$	$\Theta_1 + \Theta_2 + \Theta_4 = 1$
3N	134	$\Theta_1, \Theta_3, \Theta_4 > 0$	$\Theta_1 + \Theta_3 + \Theta_4 = 1$
4	1234	$\Theta_1, \Theta_2, \Theta_3, \Theta_4 > 0$	

* C = consecutive subperiods, N = nonconsecutive subperiods.

The guidance product takes the form of the precipitation timing tree shown in Fig. 3. The tree displays all possible realizations of the duration D and timing T, and the probabilities of observing them. Branches emanating from a node signify events that are mutually exclusive and collectively exhaustive—their probabilities sum up to one. The first node specifies the unconditional probability function of D. The subsequent nodes specify the conditional probability functions of D and T. All probabilities in the tree are calculated from the unconditional probability function of timing T, estimated from climatic frequencies and shown in the last column of Fig. 3. The calculations employ the addition and multiplication laws of probability. For example,

$$P(D = 3) = P(T = 123) + P(T = 234) + P(T = 124) + P(T = 134),$$

$$P(D = 3C \,|\, D = 3) = \frac{P(T = 123) + P(T = 234)}{P(D = 3)},$$

$$P(T = 123 \,|\, D = 3C) = \frac{P(T = 123)}{P(T = 123) + P(T = 234)}.$$

The tree provides guidance in three ways. First, it shows the climatic probability of observing each timing pattern. For instance, for the Monongahela Basin for the 24-h period beginning at 1200 UTC in March, the four most likely timing patterns, which comprise approximately 50% of all precipitation events, are $T = 1234$, 1, 4, and 12. Equally useful may be the knowledge of the four least likely patterns, which are $T = 24$, 13, 124, and 3.

Second, the tree offers guidance to sequential inference about the duration and timing of precipitation. For

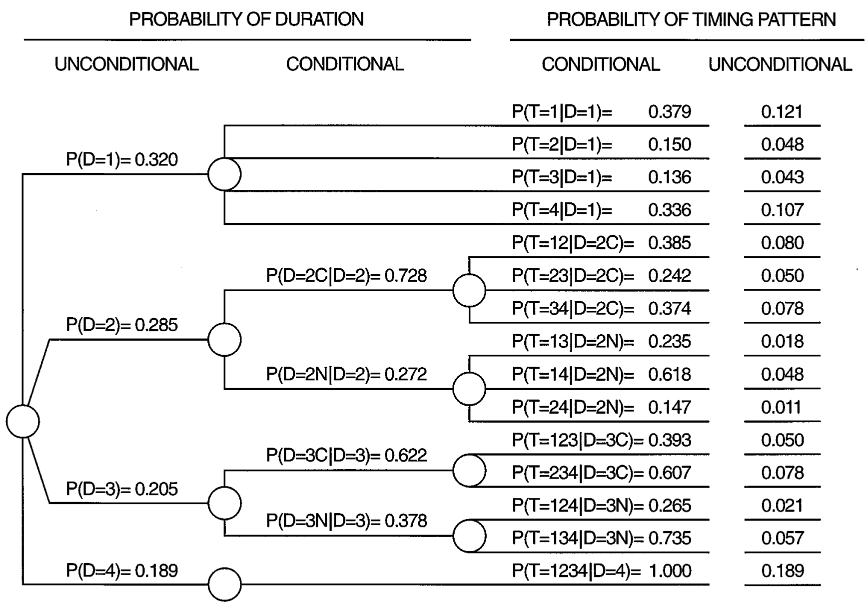

FIG. 3. Precipitation timing tree for a 24-h period divided into four 6-h subperiods; Monongahela Basin, March, from 1200 UTC.

example, Fig. 3 enables the forecaster to make the following inferences. If precipitation occurs during the 24-h period, then the most likely duration is one subperiod, $P(D = 1) = 0.320$; and if the precipitation is of such a short duration, then subperiod 1 is the most likely to receive it, $P(T = 1|D = 1) = 0.379$. If precipitation covers two subperiods, then it is 2.68 times more likely that these subperiods are consecutive rather than nonconsecutive, $P(D = 2C|D = 2) = 0.728$ versus $P(D = 2N|D = 2) = 0.272$; and if two consecutive subperiods are wet, then the most likely are subperiods 1 and 2 because $P(T = 12|D = 2C) = 0.385$.

Third, when the forecaster estimated the timing or wishes to examine implications of various hypotheses about the timing, the tree leads to a menu for selecting timing-dependent guidance products. For each value of T shown in the tree, the forecaster can retrieve the exceedance function of the total precipitation amount and the fractions statistics of the temporal disaggregation.

b. *Conditional disaggregative invariance*

Let t denote a realization of precipitation timing T, which in the case of $n = 4$ subperiods can take any of the 15 patterns listed in the second column of Table 4. The timing pattern $T = t$ indicates the fractions that are positive; the remaining fractions are zero. Thus, by conditioning the inference on the timing pattern $T = t$, the dimensionality of the predictand $(W; \Theta_1, \ldots, \Theta_n)$ can be reduced if the duration D is less than n subperiods. Is there any other advantage of such a conditioning?

One of the most important statistical properties of point and areal average precipitation processes is the *conditional disaggregative invariance*. It was investigated by Krzysztofowicz and Pomroy (1997) and can be stated as follows. Conditional on precipitation timing $T = t$, the vector of fractions $(\Theta_1, \ldots, \Theta_n)$ is stochastically independent of the total precipitation amount W. In other words, conditional on timing $T = t$, the joint distribution of the predictand $(W;\Theta_1, \ldots, \Theta_n)$ is given by the product of the distribution of the amount W and the distribution of fractions $(\Theta_1, \ldots, \Theta_n)$.

The practical implication of the conditional disaggregative invariance is formidable. It allows us to decompose the problem into three tasks: (i) forecasting the precipitation timing T; (ii) forecasting the total amount W, conditional on timing $T = t$; and (iii) forecasting the temporal disaggregation $(\Theta_1, \ldots, \Theta_n)$, conditional on timing $T = t$. Tasks (ii) and (iii) can be performed independently of one another, and this reduces the complexity of judgments required on the part of forecasters. The next two sections describe guidance to these tasks.

c. *Timing-dependent exceedance function*

Conditional on timing pattern $T = t$, the distribution of the basin average precipitation amount W accumulated during the guidance period is defined for all $\omega \geq 0$ by

$$G_t(\omega) = P(W \leq \omega|T = t). \tag{11}$$

This distribution is also implicitly conditioned on the occurrence of precipitation, $W > 0$, which is implied by the consideration of timing pattern $T = t$. Statistical analyses of climatic data did not reject the hypothesis that distribution G_t conforms to the Weibull model (5).

Figure 4a shows several timing-dependent exceed-

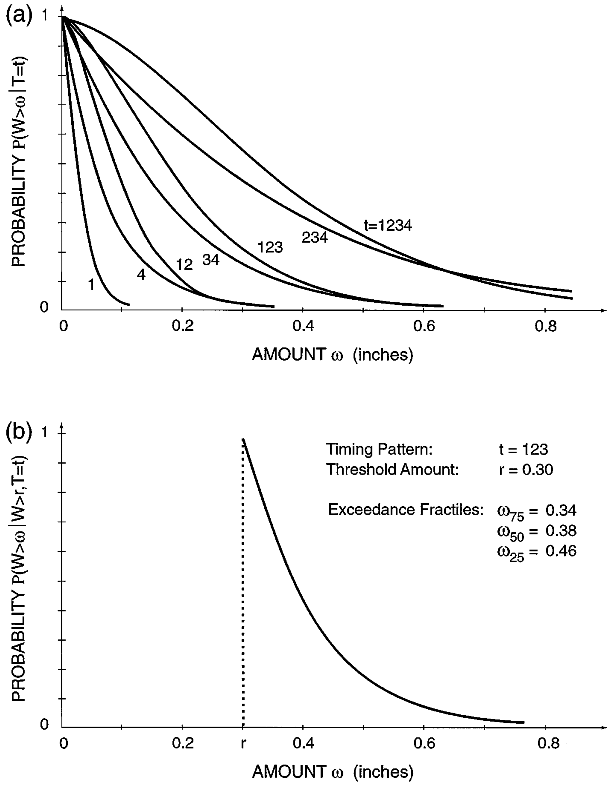

FIG. 4. Timing-dependent exceedance functions of the 24-h basin average precipitation amount W: (a) conditional on timing $T = t$ and (b) conditional on timing $T = t$ and event $W > r$; Monongahela Basin, March, from 1200 UTC.

ance functions. They confirm the well-known stochastic dependence between the amount W and duration D. To wit, for timing patterns $T = 1$, 12, 123, and 1234, whose corresponding durations are $D = 1$, 2, 3, and 4, the exceedance functions form a dominating sequence; in other words, for any fixed probability p, the conditional exceedance fractiles $\{\omega_{100p|t}$: $t = 1, 12, 123, 1234\}$ of precipitation amount W increase with precipitation duration D. Figure 4a also reveals that the exceedance functions conditional on different timing patterns may be distinct, even though the corresponding durations are identical. Such is the case for $T = 1$, 4 with $D = 1$, for $T = 12$, 34 with $D = 2$, and for $T = 123$, 234 with $D = 4$. This implies that the timing T of precipitation within the diurnal cycle is more informative for predicting the precipitation amount W than the duration D of precipitation within the 24-h period.

When the forecaster is certain that the timing T will follow pattern t and the basin average precipitation amount W will exceed threshold r, the exceedance probability can be conditioned on both hypotheses. For any $r \geq 0$, $\omega > r$ and any t, the derivation analogous to (9) yields

$$P(W > \omega \,|\, W > r,\, T = t) = \frac{1 - G_t(\omega)}{1 - G_t(r)}. \qquad (12)$$

The conditional exceedance fractiles of W can be found from an expression analogous to (10).

The guidance product, illustrated in Fig. 4b, displays the conditional exceedance function and highlights the three conditional exceedance fractiles. In the example, $t = 123$; thus, the product was derived from an exceedance function of W, which appears in Fig. 4a. By comparing the two functions, one can judge the effect of conditioning the exceedance probabilities on the threshold amount r.

d. Timing-dependent fractions statistics

Conditional on timing pattern $T = t$, the temporal disaggregation of the total precipitation amount, whatever this amount might be, is completely characterized by a distribution of the vector of fractions. The timing pattern $T = t$ implies the number of wet subperiods $D = d$, where $d \in \{1, \ldots, n\}$. Only fractions for the wet subperiods are positive, whereas the remaining fractions are equal to zero. Because one of the positive fractions can always be expressed in terms of the remaining positive fractions through the unit sum constraint, the distribution of $(\Theta_1, \ldots, \Theta_n)$ conditional on $T = t$ is $(d - 1)$-dimensional. For each value of duration D and timing T, Table 4 lists positive fractions in the case of $n = 4$ subperiods. The guidance provides statistics of these fractions.

For timing patterns with duration $D = 1$, there is only one positive fraction having a constant value of one, so the case is trivial. For any other timing pattern, each positive fraction Θ_i is characterized in terms of conditional marginal distribution statistics: the conditional mean $E(\Theta_i|T = t)$ and the 100p% conditional exceedance fractiles $\theta_{i(100p|t)}$, such that $P(\Theta_i > \theta_{i(100p|t)}|T = t) = p$ for $p = 0.75$, 0.50, and 0.25. To obtain the fractiles, the conditional marginal distribution of Θ_i is modeled parametrically (see appendices A and B). The association between any two positive fractions is characterized in terms of the conditional correlation $\text{cor}(\Theta_i, \Theta_j|T = t)$.

The guidance product takes the form of box plots. The statistics displayed in the plots and their interpretation are the same as in the unconditional guidance discussed in section 3d. The statistical behavior of conditional fractions is somewhat different, however, and is worth a commentary.

Figures 5, 6, and 7 illustrate the guidance product for three timing patterns. In general, for timing patterns with duration $D = 2$, there are two positive fractions, say (Θ_i, Θ_j), having these properties. First, the conditional exceedance fractiles satisfy the relation

$$\theta_{i(100p|t)} + \theta_{j(100(1-p)|t)} = 1. \qquad (13)$$

In particular, $\theta_{i(75|t)} + \theta_{j(25|t)} = 1$, $\theta_{i(50|t)} + \theta_{j(50|t)} = 1$, and $\theta_{i(25|t)} + \theta_{j(75|t)} = 1$. Second, the conditional correlation is always -1 because fractions are related through the unit sum constraint, $\Theta_i + \Theta_j = 1$. Consequently, the forecaster must assess only one fraction whereas the

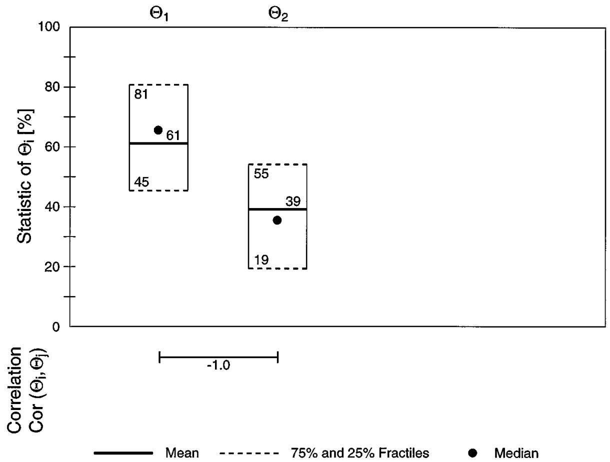

FIG. 5. Timing-dependent statistics of fractions that define the temporal disaggregation of the 24-h basin average precipitation amount W for timing pattern $T = 12$; Monongahela Basin, March, from 1200 UTC.

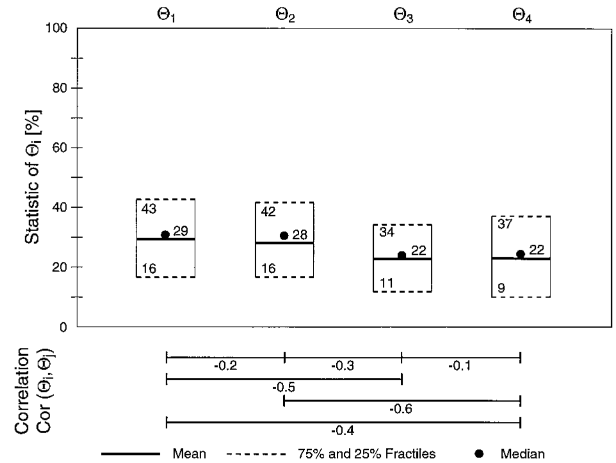

FIG. 7. Timing-dependent statistics of fractions that define the temporal disaggregation of the 24-h basin average precipitation amount W for timing pattern $T = 1234$; Monongahela Basin, March, from 1200 UTC.

assessment of the other fraction can be found from the constraint.

For timing patterns with duration $D = 3$ there are three positive fractions, say Θ_i, Θ_j, and Θ_k, satisfying the constraint $\Theta_i + \Theta_j + \Theta_k = 1$. Consequently, the forecaster must assess only two fractions. The correlations have predominantly negative signs. (In fact, in all data analyses performed thus far, we have not encountered a positive correlation although it is a theoretical possibility.)

Finally, let us turn to an empirical observation that underscores the advantage of guidance with the timing hypothesis: precipitation timing is a significant predictor of the temporal disaggregation. Specifically, when the forecaster is certain that the timing T will follow pattern t, and the distribution of the vector of fractions $(\Theta_1, \ldots, \Theta_n)$ is conditioned on the timing pattern $T = t$, then the uncertainty about the fractions is reduced considerably. The reduction of uncertainty can be appraised by comparing Figs. 5, 6, and 7 with Fig. 2. A particular measure of uncertainty is the width of the 50% credible interval, defined as the difference between the 25% exceedance fractile and the 75% exceedance fractile. In Fig. 2 for unconditional fractions, the widths are (61%, 44%, 40%, 56%). In Figs. 5, 6, and 7 for fractions conditional on $T = 12$, the widths are (36%, 36%, 0%, 0%); for fractions conditional on $T = 123$, the widths are (30%, 25%, 19%, 0%); and for fractions conditional on $T = 1234$, the widths are (27%, 26%, 23%, 28%). As these examples pinpoint, and Table 5 confirms in general, a significant portion of uncertainty about the fractions $(\Theta_1, \ldots, \Theta_n)$ can be eliminated by solely predicting timing T.

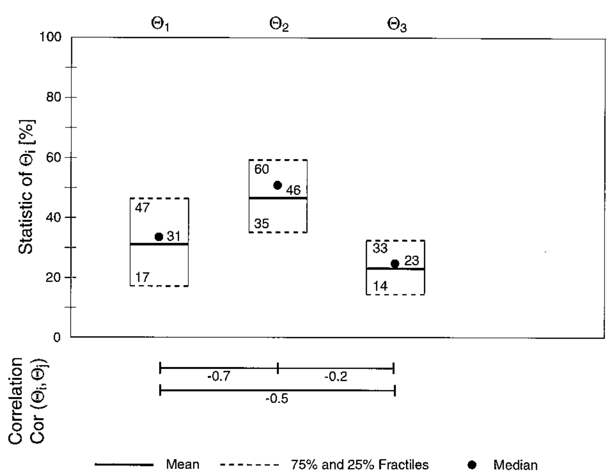

FIG. 6. Timing-dependent statistics of fractions that define the temporal disaggregation of the 24-h basin average precipitation amount W for timing pattern $T = 123$; Monongahela Basin, March, from 1200 UTC.

TABLE 5. Reduction of uncertainty about fractions achieved by the conditioning on precipitation timing; Monongahela Basin, March, from 1200 UTC.

Timing pattern t	Width of the 50% credible interval (%)			
	Θ_1	Θ_2	Θ_3	Θ_4
12	36	36	0	0
23	0	34	34	0
34	0	0	28	28
13	33	0	33	0
14	42	0	0	42
24	*	*	*	*
123	30	25	19	0
234	0	39	33	33
124	26	28	0	19
134	39	0	32	28
1234	27	26	23	28
Unconditional	61	44	40	56

* Insufficient sample size.

TABLE 6. Monthly probability functions of precipitation duration D within a 24-h period divided into four 6-h subperiods; Monongahela Basin, from 1200 UTC.

Probability	Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
$P(D = 1)$	0.35	0.35	0.32	0.32	0.32	0.36	0.36	0.38	0.33	0.33	0.33	0.35
$P(D = 2)$	0.29	0.32	0.29	0.27	0.30	0.29	0.36	0.32	0.30	0.30	0.33	0.28
$P(D = 3)$	0.22	0.19	0.20	0.22	0.19	0.22	0.19	0.21	0.22	0.22	0.20	0.24
$P(D = 4)$	0.14	0.14	0.19	0.19	0.19	0.13	0.09	0.09	0.15	0.15	0.14	0.13

5. Operational aspects

a. Guidance conditional on storm type

One would naturally expect the timing T and duration D of precipitation to be stochastically associated with the storm type. Convective precipitation tends to be associated with timing patterns having duration $D = 1$ or $D = 2$, whereas stratiform precipitation tends to be associated with timing patterns having duration $D = 3$ or $D = 4$. This association is reflected in the probabilities attached to the timing tree. Table 6 offers an example wherein $P(D = 1)$ and $P(D = 2)$ peak in summer, reflecting a higher frequency of convective precipitation, whereas $P(D = 3)$ and $P(D = 4)$ peak in winter and spring, reflecting a higher frequency of stratiform precipitation. Monthly variations in the unconditional probabilities of timing patterns, $P(T = t)$, reflect even more subtle shifts in the frequency of storm type. For instance, stratiform precipitation may be associated with timing patterns $T = 1$ or $T = 4$ ($T = 12$ or $T = 34$) whenever the 24-h guidance period covers only the end or the beginning of a storm lasting longer than one subperiod (two subperiods).

In general, then, the timing T can be thought of as a proxy variable for storm type. In addition, the timing-dependent guidance for PQPF is compatible with the concept of type-dependent guidance, which has been suggested to us on several occasions. However, the development of such a guidance faces an obstacle because climatic precipitation records do not contain information about the storm type. The forecaster can partly compensate for this inadequacy by adhering to three-stage inference: (i) forecast storm type; (ii) forecast precipitation timing T, conditional on storm type; and (iii) use timing-dependent guidance from LCG.

b. Guidance for total storm

Operational PQPFs are prepared for a fixed period divided into fixed subperiods, and the LCG is compatible with this scheme. One implication is that duration of precipitation D within the period is not synonymous with duration of the storm.

Let a storm be defined as a sequence of consecutive wet subperiods. Given the LCG with specifications as in Fig. 3, the timing patterns $T = 2$, 3, and 23 arise from storms that occur totally during the designated subperiods. However, other timing patterns may arise from storms that begin before, and/or terminate after, the guidance period 1200–1200 UTC. For instance, timing pattern $T = 1$ arises from a storm that either occurs during 1200–1800 UTC, or begins before 1200 UTC and terminates between 1200 and 1800 UTC. Timing pattern $T = 134$ arises from two storms. The first storm has two possible timings as in the first example; the second storm either occurs during 0000–0600 and 0600–1200 UTC, or begins between 0000 and 0600 UTC and terminates after 1200 UTC.

Guidance for a total storm covering any number of consecutive subperiods can always be obtained by a suitable selection of LCG specifications, as described in section 2c. In general, the expected storm interval is bracketed by the fixed forecast subperiods, one dry subperiod is added at each end, and the resultant total interval specifies the beginning hour and the duration of the guidance period. To illustrate, suppose that at 1200 UTC the forecaster expects a storm beginning between 0000 and 0600 and ending between 0600 and 1200 UTC of the next day. To obtain guidance for such a storm it suffices to specify 1800 UTC as the beginning hour of the 24-h guidance period and then select guidance products for timing pattern $T = 23$.

c. Guidance with timing uncertainty

The two segments of the LCG summarized in Table 1 provide climatic statistics of the predictand for two situations. When the precipitation timing T cannot be predicted with probabilities sharper than the climatic probabilities $P(T = t)$ attached to the precipitation timing tree (Fig. 3), the forecaster should use guidance without the timing hypothesis. When the precipitation timing T can be predicted with probability one, or close to one, the forecaster should use guidance with a specific timing hypothesis. In reality, there will be intermediate situations in which it is appropriate to forecast timing T in terms of a probability function. In such situations, the forecaster should examine statistics of the predictand conditional on each probable timing pattern and then judgmentally integrate all information. A model that allows the forecaster to explicitly assess the probability function of timing T and then provides statistics of the predictand conditional on this assessment would be a natural extension of the models described herein.

d. Using the guidance

In operational forecasting, the LCG is to be used in the *integration phase*—the last phase in a scheme for

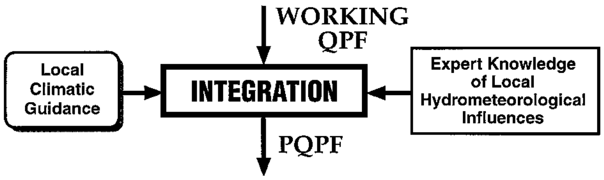

FIG. 8. Integration phase in a scheme for judgmental processing of information from multiple sources into the PQPF for a river basin.

processing information from multiple sources into the PQPF for a river basin (Krzysztofowicz et al. 1993). When entering this phase, the forecaster has already extracted all information from observations, model outputs, and guidance products that is relevant to probabilistic forecasting of precipitation. This information includes not only estimates of precipitation amounts but also judgments of the associated uncertainties. It may be viewed as a ''working QPF,'' which resides in the forecaster's mind and must yet be further processed.

To produce the final PQPF for a river basin, the forecaster must integrate information from three sources, as shown in Fig. 8: (i) his working QPF and the judgment of the associated uncertainty, (ii) his expert knowledge of local hydrometeorological influences, and (iii) prior estimates of the exceedance fractiles and expected fractions from the LCG.

The task may be conceptualized as a *judgmental revision process.* Estimates from the working QPF are revised based on the prior estimates specified by the LCG. When the forecaster judges that the uncertainty associated with his working QPF is small, his posterior estimates should tend toward the working QPF. When the forecaster judges that the uncertainty is large, his posterior estimates should tend toward the climatic estimates. It is the skill in performing this revision that is necessary to produce well calibrated (reliable) PQPFs.

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APPENDIX A

Restricted Marginal Distribution of Fraction

a. Choice of distribution

To obtain exceedance fractiles of a fraction, unconditional or conditional on timing pattern, marginal distribution of the fraction is modeled parametrically.

Let Θ denote a fraction (with the subscript identifying the subperiod omitted to simplify notation). The first step in modeling is to construct a parametric distribution H of Θ restricted to $0 < \Theta < 1$; for any fixed magnitude θ, such that $0 < \theta < 1$, $H(\theta) = P(\Theta \le \theta | 0 < \Theta < 1)$. Empirical testing of several models suggested the following rule for choosing a suitable parametric distribution: when $E(\Theta) \le 1/2$, use logWeibull I distribution; when $E(\Theta) > 1/2$, use logWeibull II distribution. Both models arise from transforming Θ into variable Z, whose distribution is assumed to be Weibull.

b. LogWeibull I

The transform $Z = -\ln\Theta$ is especially suitable when Θ has a positively skewed distribution. The resultant model is

$$H(\theta) = \exp\left[-\left(\frac{-\ln\theta}{\alpha}\right)^{\beta}\right], \quad 0 < \theta < 1. \tag{A1}$$

Equation $p = H(\theta)$ has an inverse $\theta = H^{-1}(p)$, where

$$H^{-1}(p) = \exp[-\alpha(-\ln p)^{1/\beta}], \quad 0 < p < 1. \tag{A2}$$

c. LogWeibull II

The transform $Z = -\ln(1 - \Theta)$ is especially suitable when Θ has a negatively skewed distribution. The resultant model is

$$H(\theta) = 1 - \exp\left\{-\left[\frac{-\ln(1 - \theta)}{\alpha}\right]^{\beta}\right\}, \quad 0 < \theta < 1. \tag{A3}$$

Equation $p = H(\theta)$ has an inverse $\theta = H^{-1}(p)$, where

$$H^{-1}(p) = 1 - \exp\{-\alpha[-\ln(1 - p)]^{1/\beta}\}, \quad 0 < p < 1. \tag{A4}$$

APPENDIX B

Exceedance Fractiles of Fraction

a. Unconditional fraction

For each fraction Θ_i, estimate (i) the marginal probability function, which specifies $p_{i0} = P(\Theta_i = 0)$ and $p_{i1} = P(\Theta_i = 1)$; and (ii) the marginal distribution H_i restricted to $0 < \Theta_i < 1$, which for any fixed magnitude θ_i specifies the probability $H_i(\theta_i) = P(\Theta_i \le \theta_i | 0 < \Theta_i$

< 1). The marginal distribution of Θ_i can now be constructed as follows:

$$P(\Theta_i \leq \theta_i) = \begin{cases} p_{i0}, & \theta_i = 0, \\ p_{i0} + (1 - p_{i0} - p_{i1})H_i(\theta_i), & 0 < \theta_i < 1, \\ 1, & \theta_i = 1. \end{cases} \tag{B1}$$

For any p, such that $0 < p < 1$, the 100p% exceedance fractile of Θ_i is the magnitude $\theta_{i(100p)}$ such that $P(\Theta_i > \theta_{i(100p)}) = 1 - P(\Theta_i \leq \theta_{i(100p)}) = p$. The solution of this equation takes the following form:

$$\theta_{i(100p)} = \begin{cases} 0, & p \geq 1 - p_{i0}, \\ H_i^{-1}\left(\dfrac{1 - p_{i0} - p}{1 - p_{i0} - p_{i1}}\right), & p_{i1} < p < 1 - p_{i0}, \\ 1, & p \leq p_{i1}, \end{cases} \tag{B2}$$

where H_i^{-1} is the inverse of the restricted marginal distribution H_i. When H_i is modeled as a logWeibull distribution, H_i^{-1} is specified by either (A2) or (A4).

b. Timing-dependent fraction

Conditional on timing pattern $T = t$, fraction Θ_i, such that $0 < \Theta_i < 1$, has marginal distribution $H_{i|t}$, which for any fixed magnitude θ_i specifies the probability $H_{i|t}(\theta_i) = P(\Theta_i \leq \theta_i | T = t)$. For any p, such that $0 < p < 1$, the 100p% conditional exceedance fractile of Θ_i is the magnitude $\theta_{i(100p|t)}$ such that $P(\Theta_i > \theta_{i(100p|t)} | T = t) = 1 - H_{i|t}(\theta_{i\,(100p|t)}) = p$. The solution of this equation yields

$$\theta_{i(100p|t)} = H_{i|t}^{-1}(1 - p). \tag{B3}$$

When $H_{i|t}$ is modeled as a logWeibull distribution, $H_{i|t}^{-1}$ is specified by either (A2) or (A4). For timing patterns with two positive fractions, the conditional exceedance fractiles are obtained via (B3) for the first fraction and via (13) for the complementary fraction. For timing patterns with more than two positive fractions, (B3) is used for every fraction.

REFERENCES

Hershenhorn, J., and D. A. Woolhiser, 1987: Disaggregation of daily rainfall. *J. Hydrol.,* **95,** 299–322.

Krzysztofowicz, R., and S. Reese, 1993: Stochastic bifurcation processes and distributions of fractions. *J. Amer. Statist. Assoc.,* **88,** 345–354.

——, and T. A. Pomroy, 1997: Disaggregative invariance of daily precipitation. *J. Appl. Meteor.,* in press.

——, W. J. Drzal, T. R. Drake, J. C. Weyman, and L. A. Giordano, 1993: Probabilistic quantitative precipitation forecasts for river basins. *Wea. Forecasting,* **8,** 424–439.

Linsley, R. K., Jr., M. A. Kohler, and J. L. H. Paulhus, 1975: *Hydrology for Engineers.* McGraw-Hill, 82–84.

Schütte, T., O. Salka, and S. Israelsson, 1987: The use of the Weibull distribution for thunderstorm parameters. *J. Climate Appl. Meteor.,* **26,** 457–463.

Selker, J. S., and D. A. Haith, 1990: Development and testing of single-parameter precipitation distributions. *Water Resour. Res.,* **26,** 2733–2740.

Wallace, J. M., 1975: Diurnal variations in precipitation and thunderstorm frequency over the conterminous United States. *Mon. Wea. Rev.,* **103,** 406–419.

Weibull, W., 1951: A statistical distribution function of wide applicability. *J. Appl. Mech.,* **73,** 293–297.

Wilks, D. S., 1990: Probabilistic quantitative precipitation forecasts derived from PoPs and conditional precipitation amount climatologies. *Mon. Wea. Rev.,* **118,** 874–882.