Is Buoyancy a Relative Quantity?

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(Manuscript received 14 April 2003, in final form 25 September 2003)

ABSTRACT

Basic concepts of buoyancy are reviewed and considered first in light of simple parcel theory and then in a more complete form. It is shown that parcel theory is generally developed in terms of the density (temperature) difference between an ascending parcel and an ''environment'' surrounding that parcel. That is, buoyancy is often understood as a *relative* quantity that apparently depends on the choice of a base-state environmental profile. However, parcel theory is most appropriately understood as a probe of the static stability of a sounding to finite vertical displacements of hypothetical parcels within the sounding rather than as a useful model of deep convection.

The *thermal* buoyancy force, as measured by the temperature *difference* between a parcel and the base state, and vertical perturbation pressure gradient force together must remain independent of the base state. The vertical perturbation pressure gradient force can be decomposed to include a term due to thermal buoyancy and another due to the properties of motion in the flow. Some thought experiments are presented to illustrate the ambiguous relevance of the base state.

It is concluded that buoyancy is not a relative quantity in that it cannot be dependent on the choice of an essentially arbitrary reference state. Buoyancy is the static part of an unbalanced vertical pressure gradient force and, as such, is determined locally, not relative to some arbitrary base state outside of a parcel. This has direct application to the diagnosis of buoyancy from numerical simulations—done properly, such a diagnosis must include not only the thermal buoyancy term but also the perturbation pressure gradient force due to buoyancy.

1. Introduction

In order to clarify our intentions for asking the provocative question embodied in our title, we begin with some definitions. The notion of buoyancy in atmospheric science has its roots in the so-called Law of Archimedes, taught in basic physics courses. In the recently revised *Glossary of Meteorology* (Glickman 2000, p. 106), buoyancy is defined as the following:

1. That property of an object that enables it to float on the surface of a liquid, or ascend through and remain freely suspended in a compressible fluid such as the atmosphere.
Quantitatively, it may be expressed as the ratio of the specific weight of the fluid to the specific weight of the object; or, in another manner, by the weight of the fluid displaced minus the weight of the object.
2. (Or buoyant force, buoyancy force; also called Archimedean buoyant force.) The upward force exerted upon a parcel of fluid (or an object within the fluid) in a gravitational field by virtue of the density difference between the parcel (or object) and that of the surrounding fluid.

This Archimedean view of buoyancy, as given in the second of the two definitions, is what is widely accepted as the definition of ''buoyancy'' in atmospheric science. Such a definition explicitly describes buoyancy as a *relative* quantity; that is, buoyancy is commonly understood as a density *difference* between a parcel and its surrounding fluid (called the ''environment'' of the parcel). This definition asserts that the buoyant force on a parcel is defined in these relative terms; that is, a comparison of the parcel properties to those in the surrounding fluid. For the purposes of this paper, we are going to define buoyancy in a way similar to that proposed by Davies-Jones (2003), but we will not address the issue of condensate products within the parcel. Hence, we define positive buoyancy as *the statically forced part of the locally nonhydrostatic, upward pressure gradient force*. The details of this definition will be clarified in what follows.

At this point, however, the *Glossary* definition for

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buoyancy begs the question of what is meant by a ''parcel'' of air. Turning once again to the *Glossary* (Glickman 2000, p. 556), a parcel is defined as the following:

> An imaginary volume of fluid to which may be assigned various thermodynamic and kinematic quantities.
> The size of a parcel is arbitrary but is generally much smaller than the characteristic scale of variability of its environment.

In atmospheric science, the governing equations are derived in a manner analogous to other disciplines involving fluid flow, whereby the so-called continuum hypothesis is applied to various physical laws presumed to apply to fluids. As discussed in, for example, Batchelor (1967, 4–6), this avoids the additional complexity associated with the molecular nature of real fluids. Measurements within fluids are assumed to apply to some volume of essentially arbitrary size (i.e., a parcel) containing enough molecules to allow the application of the continuum hypothesis. The application of physical laws governing the parcel's momentum and regulating mass continuity is developed in terms of integrals over the parcel volume (e.g., Salby 2003). Since the integrals vanish and apply to an indefinite volume, their integrands can be assumed to vanish identically. The integrands make up the familiar system of coupled partial differential equations used in fluid dynamics.

2. Preliminary discussion of parcel theory

A greatly simplified treatment of buoyancy called ''parcel theory'' is widely taught in meteorology. Its formal development is reviewed in the next section. Within the particular confines of parcel theory, it is generally assumed that any changes of field variables *within* the imaginary parcel volume are small, as suggested by the *Glossary* definition of a parcel. The notion of a parcel as a volume within which field variables are essentially constant is not *necessary* to the development of the governing equations, as just noted, but it is commonly associated with parcel theory. In fact, derivatives of field variables are also defined for parcels, which implicitly recognizes the existence of field variability *within* parcels.

This common simplification to a purely Archimedean understanding of buoyancy, in terms of homogeneous parcels embedded within, but not interacting with, a homogeneous environment can be misleading. As shown recently by Davies-Jones (2003), this is an abstraction of the real situation that oversimplifies the situation for air parcels involved in convection. As our ability to observe and describe processes within convective clouds and their immediate surroundings improves with enhanced observations and increasingly sophisticated numerical simulation models, the notions of simple parcel theory can be an impediment to enhanced understanding provided by these new tools. A complete definition of buoyancy with respect to atmospheric flows is distinctly non-Archimedean, an idea developed in some detail by Das (1979) and recently reiterated by Davies-Jones (2003).

Parcel theory explicitly equates a *base* (or *reference*) *state* with the environment, and the perturbation with the parcel. However, the use of such a base state is a source of some difficulty if we *define* buoyancy in such terms. Traditional parcel theory is one-dimensional; that is, it only considers the vertical momentum equation, so there is no place within this one-dimensional context for the concept of an environment. All that exists in parcel theory is a sounding, which is what is often referred to as the environment. Parcel theory then specifically considers the behavior of parcels drawn from levels or layers *within* that sounding that are displaced hypothetically in the vertical along adiabatic trajectories on a thermodynamic diagram. The apparent basis for equating the sounding with the environment is that the vertically displaced parcel is considered somehow to be embedded within the environment represented by the original sounding, but any interaction with that environment is ignored. Moreover, the environment is explicitly assumed homogeneous. Therefore, the *implicit* model associated with parcel theory has the vertically displaced parcel rising adiabatically in its own column of infinitesimal diameter within a horizontally homogeneous environment of infinite extent, but not directly interacting with that environment in any way.

Parcel theory's abstract view of convection is rather different from a *practical* one-dimensional cloud model. In all one-dimensional cloud models used for various purposes (e.g., Warner 1970; Holton 1973; Ryan and Lalousis 1979; Kessler 1985; Ferrier and Houze 1989), there is some attempt to account for the environment around convection without explicitly adding horizontal dimensionality. If a model includes such factors as lateral entrainment or the size of a simulated convective element, it is *implicitly* incorporating horizontal dimensionality. Such models (see references above) are sometimes referred to as ''1.5-dimensional'' models. These are not considered herein since they incorporate horizontal dimensionality in a parameterized way in order to achieve a practical simulation in one dimension.

A truly one-dimensional cloud model within a vertical column of infinitesimal horizontal extent is mathematically equivalent to a slab model (horizontally homogeneous) of infinite horizontal extent. However, there is a challenge with such a one-dimensional model: there can be no meaningful incorporation of the mass continuity equation in such a context. Mass continuity is an essential property of fluids, so its absence from parcel theory and any other one-dimensional model is decidedly problematic. In a way comparable to what is done in cloud models with horizontal as well as vertical dimensionality, a one-dimensional model would be initiated by inserting the one-dimensional analog to an initiating bubble into the sounding: an air column of finite vertical extent embedded within the original sounding but with different density. This one-dimensional bubble of rising air cannot displace the air above it laterally; the bubble must force the air above it upward as well. Similarly, no air can flow in from the sides in the wake of such a rising one-dimensional bubble of air; the rising one-dimensional bubble would have to stretch the air column below it, as well as compress the air above it.[1]

Therefore, we propose that the mathematic development associated with parcel theory, to be reviewed in the next section, is not appropriately thought of as a *model* of an atmospheric fluid flow. Rather, it can be thought of as a *probe*: it probes the relationship between air parcels hypothetically displaced along adiabatic trajectories (dry and/or moist) and the original sounding, equating buoyancy with the diagnosed temperature difference between the parcel and the sounding. The classical application of this probe is to assess the static stability of finite-amplitude vertical displacements (Sherwood 2000; Schultz et al. 2000). Therefore, parcel theory explores the nature of the environment (the base state) and can only be thought of as a very simple model of convection in which the vertical displacements of the air parcels have no effect on the base state. It is dependent wholly on the choice of the base state: change that base state and the results of applying parcel theory are changed. Whatever practical value this highly idealized model of deep convection might have, it often leads to physical misunderstandings and incomplete diagnosis of the role of buoyancy in convection, as we will discuss below.

Aircraft penetrations through clouds (e.g., Fig. 1) suggest that considerable variability exists within clouds and within the surrounding environment as well. In Fig. 1, note that the aircraft was within cloudy air through most of the traverse shown. The so-called weak echo region (WER) is characterized by an updraft core about 2–3 km across, with nearly constant equivalent potential temperature (θ_e), and so *might* conceivably be modeled as horizontally homogeneous, but notable variability is present elsewhere within the cloud. Do parcels within the WER respond to their immediate neighbors with similar properties or do they respond to more distant parcels within the cloud that have distinctly different properties? Or do they respond to parcels *outside* the cloud? Aircraft observations have shown that at a visual cloud boundary (as in Fig. 2), there is not necessarily an abrupt transition to quasi-homogeneous cloud air. If we assess buoyancy as the simple difference between a homogeneous parcel and its homogeneous surroundings, this creates a dilemma: *Which* environmental air parcels do we choose when the surroundings are inhomogeneous?

For pedagogical purposes, we certainly admit that it can be useful to simplify the problem by treating both the parcel and its environment as homogeneous and noninteracting, thereby reducing buoyancy to its purely Archimedean form. This is apparently intended to allow prior experience that students might have with such things as hot-air balloons and solid objects floating in water to clarify the concept of atmospheric buoyancy. However, this also creates difficulties, because real convecting fluids behave differently from solid bodies floating in tanks of homogeneous fluids, or gases enclosed in physical containers (e.g., balloons) while embedded within an external fluid. Air parcels have no physical reality, and no well-defined boundaries separate them from their surroundings.

As we attempt to simulate clouds numerically and learn more about the behavior of real clouds with detailed observations, the simple parcel theory model of a homogeneous parcel displaced vertically within a homogeneous environment must yield to a more complete and accurate understanding of buoyancy. In our experience, the simplified development of buoyancy using simple Archimedean arguments has become entrenched to the extent that it is now reflected in such reference works as the American Meteorological Society (AMS) *Glossary* in spite of occasional recognition that it is incomplete and potentially misleading (e.g., Das 1979; section 1.2 of Emanuel 1994; section 7.2 of Houze 1993; Davies-Jones 2003).

3. A review of parcel theory's development

The starting point for consideration of buoyancy in parcel theory is the vertical momentum equation in height (z) coordinates:

$$\rho \frac{dw}{dt} = -\frac{\partial p}{\partial z} - \rho g - \rho g(q_l + q_i) + 2\rho\Omega u \cos\phi + \rho F_z, \quad (1)$$

where w is the vertical component of motion, ρ is density, p is pressure, g is the acceleration due to gravity, q_l is the mixing ratio of liquid condensate, q_i is the mixing ratio of ice condensate, Ω is the angular rate of the earth's rotation, ϕ is the latitude, u is the zonal component of motion, and F_z is the vertical component of any external forces (e.g., viscosity). It is common in theoretical treatments to ignore the last two terms, and we shall do so from here on. We shall also ignore the contributions from condensate loading for simplicity; Davies-Jones (2003) includes condensate loading in his development. For some assumed state of pure hydrostatic balance (the base state, denoted by an

[1] Owing to the absence of an appropriate mass continuity equation, such a model is analogous to that of a mass suspended vertically between two springs, with the static stability of the air above and below the one-dimensional ''bubble'' corresponding to the spring constants. Such a model is conceptually quite different from the traditional model of parcel theory.

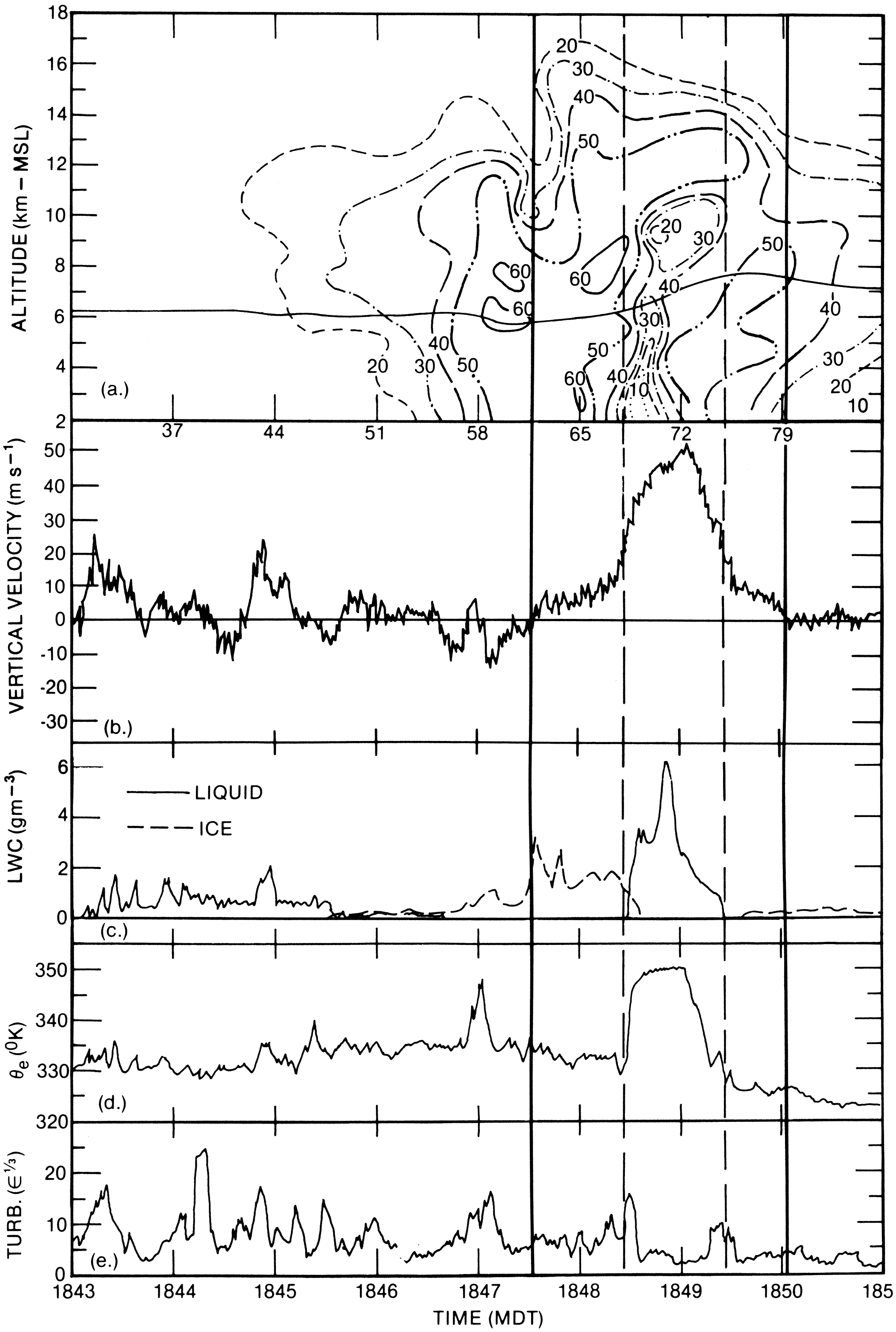

FIG. 1. Summary of T-28 armored aircraft data during a thunderstorm penetration between 1843 and 1851 MDT on 2 Aug 1981: (a) vertical section of reflectivity along the penetration path of the T-28 aircraft, including a line showing T-28 altitude; radar contours are in dB*Z*, and an approximate horizontal scale in km is indicated; (b) vertical air velocity; (c) liquid water and ice mass concentrations; (d) equivalent potential temperatures; and (e) intensity of turbulence. The dashed vertical lines represent the approximate boundary of the WER, while the solid vertical lines outline the total region of updraft associated with the WER. [Fig. 9 in Musil et al. (1986).]

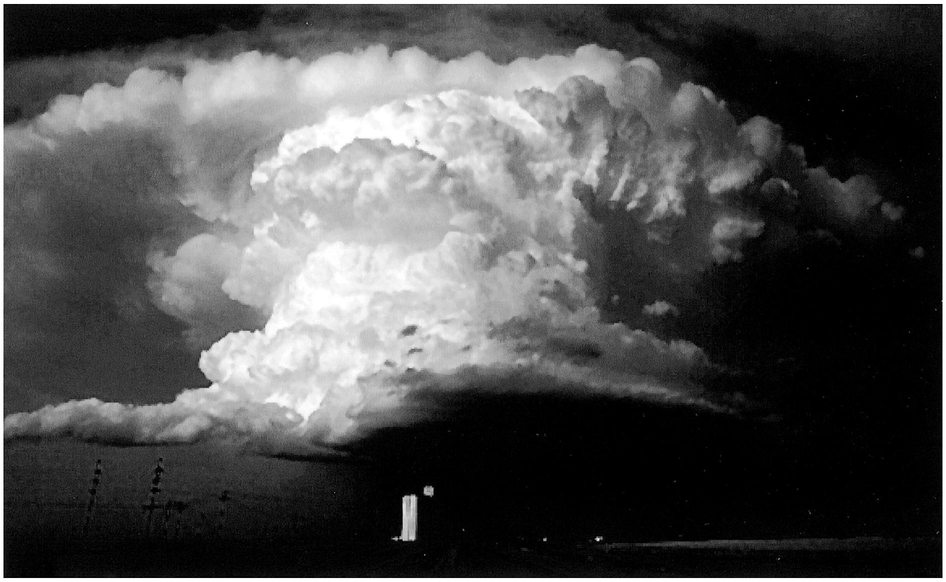

FIG. 2. Cumulonimbus cloud during the afternoon of 18 May 1990, in the Texas panhandle, showing the complex ''bubbly'' appearance typical of cumuliform clouds. (Photograph copyright 1990 by C. Doswell.)

overbar), the lhs of (1) is zero, resulting in the well-known simple relationship:

$$\frac{\partial \overline{p}}{\partial z} = -\overline{\rho}g. \tag{2}$$

Next, it is customary that the pressure and density are decomposed into a hydrostatic basic state and a perturbation from that base state, denoted by ( )′:

$$p = \overline{p} + p', \qquad \rho = \overline{\rho} + \rho'. \tag{3}$$

Consequently, using (3) and the simplified form of (1), it can be seen that

$$\rho\frac{dw}{dt} = -\left(\frac{\partial \overline{p}}{\partial z} + \frac{\partial p'}{\partial z}\right) - (\overline{\rho} + \rho')g. \tag{4}$$

Making use of (2) in (4) results in

$$\frac{dw}{dt} = \underbrace{-\frac{1}{\rho}\frac{\partial p'}{\partial z}}_{i} - \underbrace{\frac{\rho'}{\rho}g}_{ii}. \tag{5}$$

Term *i* on the rhs of (5) is associated with the vertical gradient of the perturbation pressure, and term *ii* is associated with buoyancy [cf. *Glossary of Meteorology* (Glickman 2000, p. 106, second definition, first paragraph)].

In many textbook treatments of parcel theory (e.g., Hess 1959; Holton 1992; Dutton 1976; Emanuel 1994), another step is done. This involves a replacement of ρ in the denominator of both terms in (5) with $\overline{\rho}$. Such a step implicitly involves making either an anelastic [$\overline{\rho} = \overline{\rho}(z)$] or Boussinesq [$\overline{\rho}$ = constant] approximation and linearizing the equation by treating the perturbation as if it is small compared to the basic state, allowing the neglect of any high-order terms. When this is done, the result is identical to (5), except that $\overline{\rho}$ replaces ρ. Also common in textbook developments of parcel theory is the neglect of term *i* in (5) [e.g., as in Holton (1992, p. 54), Dutton (1976, p. 70), or Emanuel (1994, p. 6)], confining the effect of buoyancy only to term *ii*, which Kessler (1985) calls the *thermal buoyancy* (a convention we will follow hereinafter).

The remaining term, term *ii* on the rhs of (5), is not yet in its most commonly used form, however. The equation of state is simply

$$p = \rho R T_v, \tag{6}$$

where T_v is the virtual temperature, and R is the gas constant for dry air. The base state can be assumed to very nearly satisfy the equation of state, such that

$$\overline{p} \cong \overline{\rho}R\overline{T}_v,$$

and if products of perturbations are neglected, then

$$\frac{\rho'}{\overline{\rho}} \cong \frac{p'}{\overline{p}} - \frac{T'_v}{\overline{T}_v}.$$

It is traditional (e.g., Emanuel 1994, 7–8) at this point to neglect the contribution of pressure perturbations to the density, such that

$$\frac{\rho'}{\overline{\rho}} \cong -\frac{T'_v}{\overline{T}_v}. \tag{7}$$

Therefore, if we temporarily ignore the first term on the rhs of (5) and substitute from (7), using the fact that $T'_v = T_v - \overline{T}_v$, the following result is obtained:

$$\left.\frac{dw}{dt}\right|_B = g\frac{T_v - \overline{T}_v}{\overline{T}_v} \equiv B, \qquad (8)$$

where B is a commonly used form of the expression for the vertical acceleration due to buoyancy as the difference between the parcel temperature and that of a hydrostatic basic state. Sometimes, B is used to denote term *ii* of (5) also. For many purposes, the virtual correction is also neglected; however, see Doswell and Rasmussen (1994) for an assessment of this simplifying assumption. In any case, B is the contribution to buoyancy due to the density difference between the parcel and the base state. For deep moist convection, it is assumed that the parcel follows a moist adiabat once condensation is attained during finite parcel displacements. Thus, the temperature difference between a saturated ascending parcel and any base state (e.g., a sounding) is likely to be a complicated function of height. If B is integrated between the level of free convection and the equilibrium level, the result is the so-called convective available potential energy (CAPE).

Now (8) is a traditional formulation of parcel theory (e.g., Hess 1959, 95–103), but we will show in the next section that (8) is more than just a simplification of (5). In textbook developments of parcel theory, (8) is used to develop the notions of (a) the Brunt–Väisälä frequency,[2] as well as (b) measures of static stability associated with buoyancy (see Schultz et al. 2000). In deriving (8), perturbation pressure is ignored *twice*: once in deriving (7)—which Emanuel (1994, 7–8) suggests may not be too bad an assumption—as well as earlier, in assuming buoyancy includes only the *thermal* buoyancy. It is widely accepted that thermal buoyancy, B, is *the* buoyancy, in fact. It is this latter notion with which we are concerned.

Finally, it should be noted that if the so-called Exner function

$$\pi \equiv \left(\frac{p}{p_0}\right)^{\kappa},$$

where $\kappa \equiv R/c_p$ and c_p is the specific heat of air at constant pressure, is employed as the pressure variable, then it is possible to derive a perturbation buoyancy formulation comparable to (8) that does not ignore the pressure fluctuations; see Hane et al. (1981, p. 566) for this development.

4. A description of buoyancy that is independent of the base state

If the buoyant forces on a parcel inside a convective cloud truly depend on the temperature *difference* between the parcel and some imposed base-state profile, how does a parcel within that cloud ''know'' its temperature (density) *relative* to the surrounding fluid? The quantitative evaluation of thermal buoyancy alone apparently depends on the rather arbitrary choice of *which* environmental profile is used as the base state in the calculation. Bryan and Fritsch (2000) have raised similar questions. In a real fluid including the effects of mass continuity, the existence of horizontal density variations is communicated within the fluid by pressure gradient forces, but in a one-dimensional model, this is not possible. In models with one or more horizontal dimensions, the existence of horizontal density gradients implies the presence of solenoidal contributions to horizontal vorticity. Thus, the thermal buoyancy gradient contributes to vertical motion through the solenoidal generation process and mass continuity. Solenoids cannot be included in parcel theory or any other one-dimensional model.

Associated with this issue is confusion about just what the base state represents. It is possible to define a base state in which the *nonkinetic* energy (i.e., the sum of potential, internal, and latent energies) is minimal after mass rearrangements [see Eq. (6.6) of Emanuel 1994]. Emanuel shows that when considering the available potential energy of the whole system (not just that of the parcel), the actual amount of energy available can be substantially less than that indicated by parcel CAPE. This is related to a similar concept developed by Wang and Randall (1996). However, these notions have not yet gained widespread acceptance. The base state is generally chosen more or less arbitrarily.

To consider what this implies, consider a thought experiment involving the operation of a three-dimensional numerical cloud model of the sort pioneered by Schlesinger (1975) and Wilhelmson and Klemp (1978). Fully compressible cloud models of this sort represent the current state of the art in quantitative assessment of convection. Thus, they are a critically important context for our understanding of buoyancy and convection beyond simple parcel theory. Our thought experiments herein are aimed at elucidating the implications of the choice of a base state.

In the typical simulation of deep, moist convection, the initial state is at rest and is horizontally homogeneous everywhere, except within an initiating ''bubble'' that encompasses several grid points and, hence, can be thought of as comprising several parcels.[3] The initiating bubble incorporates a density (i.e., virtual temperature) perturbation from the base state that can be due to either a temperature perturbation or a moisture perturbation, or both. This density perturbation is chosen to provide positive thermal buoyancy at the initial time ($t = 0$). The initiating bubble is usually not homogeneous but has some spatial structure within. If, as is typically the case for cloud model simulations, the hydrostatic base

[2] There are some issues with this development, as well, as discussed in Schultz et al. (2000).

[3] In a gridpoint model, each point represents a surrounding volume within which the governing equations apply and so can be thought of as representing a single parcel.

state is assumed to be identical to some horizontally homogeneous vertical thermodynamic profile used to initialize the model, then the only nonzero perturbations within the domain at $t = 0$ are those within the boundaries of the initiating bubble. Since the distribution of density within the initiating bubble differs from that of the hydrostatic base state, there *must* be a pressure perturbation associated with the bubble. The introduction of the perturbation has ''replaced'' the air of the hydrostatic base state and the equation of state requires a change in pressure. This change in the pressure field produced by introducing the buoyant bubble is usually neglected in the initialization of cloud models with buoyant bubbles, creating an imbalance at the initial instant. Such an initial pressure perturbation cannot be determined using the hydrostatic relationship, which assumes that $dw/dt = 0$ initially. The initial pressure imbalance is related to concepts described by van Delden (2000) and Fiedler (2002)—in a multidimensional compressible model, it produces a ''convective adjustment'' mediated by acoustic waves. The acoustic waves rapidly accomplish this adjustment, owing to their high propagation velocities,[4] such that this initial imbalance rapidly disappears and has little effect on the ensuing simulation (G. Bryan 2001, personal communication).

Most numerical cloud models and even mesoscale models employ the artifice of a base state primarily to reduce truncation error. They introduce this base state, which typically is hydrostatic, horizontally homogeneous and time independent, to avoid a direct evaluation of the pressure gradient force that otherwise would involve a relatively small difference between two relatively large numbers. The base state in cloud models almost invariably is chosen to be the same as the sounding used to initialize that model, although if the environment is *not* horizontally homogenous (as in a cloud-resolving mesoscale model), the choice of *which* vertical profile to use as the base state is not clear.

If the model's initial state is changed, the outcome of the simulation will certainly be changed; the course of the simulation surely depends on the initial conditions. Indeed, changing the initial conditions has been the technique of choice when using numerical cloud models to explore the impact on convection of such external parameters as CAPE and environmental vertical wind shear (e.g., Weisman and Klemp 1982, 1984). The use of cloud models initiated with horizontally homogeneous initial conditions (and an initiating bubble) is an abstraction that is very distinct from parcel theory or a purely one-dimensional model because numerical simulations with horizontal as well as vertical dimensionality necessarily enforce mass continuity in some form. Numerical simulation models always include a mass continuity equation and enforce that mass continuity statement to whatever accuracy is permitted by their numerical methods. The key point is that in such models the bubble is capable of interacting with its environment in a physically realistic way.

In such numerical cloud model simulations, it is at least logically possible to choose a *different* hydrostatic base state than the sounding used to initialize the model's horizontally homogeneous initial conditions. If so, the resulting perturbation fields would be nonzero nearly *everywhere* within the model domain, even though the ''full field'' initial conditions (the base state plus the perturbation) would have to remain the same. If the full field is altered, the simulation is for a *different* atmospheric initial state. Changing the static stability of the sounding, as probed by parcel theory, properly *should* affect the outcome of a simulation. Physically, the net vertical accelerations due to buoyancy must be independent of any arbitrary choice for a model's base state, but they certainly do depend on the full field. Therefore, buoyancy must not depend on an arbitrary choice of the base state. The point is that if the base state used in the simulation differs from that of the sounding that specifies the horizontally homogeneous initial conditions, then the buoyant acceleration cannot depend only on this essentially arbitrary choice of the base state, because the full field (including the initiating bubble) must be the same no matter what base state is chosen.

In our thought experiment, after the very first time step following $t = 0$, the environment outside the initiating bubble will no longer be horizontally homogeneous—the processes set in motion by the initiating bubble will begin to modify the bubble's surroundings. Indeed, with time it will become increasingly difficult to *define* a boundary between the initiating bubble and its environment. Even if the base state and the environment are identical at $t = 0$ (except within the initiating bubble), this will no longer be the case after the very first time step. When the base state is held constant (as it is in most numerical simulations) after $t = 0$ during the simulation, the differences between the base state and the simulated environment generally increase with time, albeit not without limit. Typically in the simulations, the base state is used as the reference to diagnose the thermal buoyancy, B, and this diagnosis is used to represent the contributions due to buoyancy. Therefore, even in the usual case of choosing the initial horizontally homogeneous environment as the base state, its relevance to the forces experienced by parcels during the subsequent evolution is increasingly unclear after the initial time. Fortunately, as we will show, numerical simulations are generally not affected by the choice of the base state.

For models that are *not* initiated with horizontally homogeneous conditions, such as convection-resolving mesoscale models, this argument is even more pertinent. If a horizontally homogeneous base state is used in a

[4] If a model uses a mass continuity equation that is *not* fully compressible, acoustic modes are not admissible, but in such a case, the modified pressure field caused by introducing the initiating bubble could be found using a diagnostic pressure equation.

simulation that begins with inhomogeneous initial conditions, as is often the case for mesoscale models, nonzero perturbations will exist over most of the domain at $t = 0$ and thereafter. Changing the base state for such a simulation changes the values of the perturbations (but not the patterns), thereby changing the thermal buoyancy, but choosing a different base state can not change the initial full field. It is usually the case that the choice of base state is not very important, although there might be some purely numerical issues with choosing it properly to keep the truncation error reasonably low. A sufficiently unusual base state could create *numerical* difficulties but should not alter the physics, assuming the initial full field is not changed. For the purposes of this discussion, the preceding discussion illustrates that when doing a diagnostic analysis of the simulations, the thermal buoyancy (B) calculated with reference to the base state gives only *part* of the total buoyancy.

To see that neglecting term *i* in (5)—or, in effect, using (8)—results in an incomplete description of buoyancy, let us return to our thought experiment. At $t = 0$, a density perturbation from the base state is introduced within some part of an atmospheric column initially at rest. We have said that the presence of a density perturbation implies a change in the pressure profile. That is, as discussed in Emanuel (1994, p. 385), the perturbation pressure is $p' = p'_b + p'_d$, where p'_b is the contribution due to *buoyancy*, arising from the air within the perturbation volume having a different density than the hydrostatically balanced base state, and p'_d is the *dynamic* contribution to perturbation pressure arising from flow field differences created by the perturbation.[5] In our unphysical thought experiment, $p'_d = 0$ at $t = 0$ because there is initially no flow, but there must still be an acceleration—since p'_b is not hydrostatic, p'_b cannot be diagnosed *hydrostatically*, as noted earlier.

Equation (5) includes the impact of perturbation pressure gradient force, but it combines the specific contribution from thermal buoyancy with that due to the flow. Virtually all numerical models use (5) in some form or another and so do not ignore term *i* in (5). However, consider the following rearrangement of (5):

$$\begin{aligned}\frac{dw}{dt} &= -\frac{1}{\rho}\left(\frac{\partial p'_b}{\partial z} + \frac{\partial p'_d}{\partial z}\right) + B \\ &= \underbrace{-\frac{1}{\rho}\frac{\partial p'_d}{\partial z}}_{i} + \underbrace{\left(-\frac{1}{\rho}\frac{\partial p'_b}{\partial z} + B\right)}_{ii}.\end{aligned} \quad (9)$$

[5] Observe that for a finite-volume perturbation (e.g., an initiating bubble), there are also *lateral* boundaries to that volume, and along these lateral boundaries, there are horizontal perturbation pressure gradient forces acting to accelerate the horizontal winds. The aspect ratio (i.e., the height relative to the width) of the perturbation volume becomes an important issue when buoyancy is considered in simulations involving more than one (vertical) space dimension, as discussed in Das (1979) or Kessler (1985).

Note that (9) is virtually identical to Eq. (11.5.17) in Emanuel (1994) and is similar to descriptions found in Rotunno and Klemp (1985), among others. This rearranged form highlights the physically distinct contributions to the vertical accelerations. As noted before, term *i* of (9) is due to the dynamic perturbation pressure (a function of the flow field), which is clearly independent of the choice of a thermodynamic base state. Term *ii* of (9) combines the traditional thermal buoyancy term with the buoyant contribution to the perturbation pressure gradient force. We believe that this complete definition of buoyancy is not widely recognized—strictly speaking, ''buoyancy'' is term *ii* of (9), not just B. The *partitioning* of buoyancy into the two separate contributions included in term *ii* of (9) depends on the base state, but the sum of the two terms cannot be affected by changing the reference state. In fact, Davies-Jones (2003) has shown this to be the case.

When thermal buoyancy is altered by changing the base state, it must follow that the perturbation pressure gradient force due to buoyancy $(-1/\rho)(\partial p'_b/\partial z)$ is also changed so as to compensate for the changes to B. There is no alternative if the total forcing due to buoyancy is to be independent of the base state. A special case showing this compensation between the two parts of term *ii* in (9) is developed in the appendix. In general, the connection between thermal buoyancy and the perturbation pressure gradient force depends on the form of the mass continuity equation used; Davies-Jones (2003) has shown that the buoyancy does not depend on the base state for an anelastic system.

5. Issues with defining the base state

Numerical simulation models of atmospheric flow generally involve the solution of a system of coupled partial differential equations, including some form of mass continuity. The forces on air parcels at points are local rather than being defined relative to some arbitrary base state that exists outside the parcel somewhere. Partitioning the vertical pressure gradient force into an unaccelerated, hydrostatically balanced base state and a perturbation is an artifice that is primarily done for pedagogical reasons or for computational accuracy rather than being a physical reality. The atmosphere cannot know anything about such a partitioning; air parcels accelerate only in response to the local force balance.

When doing observations-based diagnostic studies, the contribution to $\partial p'/\partial z$ from buoyancy can, in principle, be found from B (see Emanuel 1994, p. 385) by solving an appropriate diagnostic equation for pressure. However, the choice of a mass continuity equation affects the resulting diagnostic pressure equation. For a Boussinesq fluid, with density ρ_0, this is simply

$$-\frac{1}{\rho_0}\nabla^2 p' = |\mathbf{D}|^2 - |\boldsymbol{\zeta}|^2 - \frac{\partial B}{\partial z}, \quad (10)$$

where $|\mathbf{D}|$ is the magnitude of the local resultant deformation, and $\boldsymbol{\zeta}$ is the local three-dimensional vorticity vector. The first two terms on the rhs of (10) cannot be determined, in general, from a single observed sounding, but it is possible to determine $\partial B/\partial z$ if a local sounding is used as the reference state and parcel theory is used to assess the stability of vertically displaced parcels. With the thermal buoyancy estimated this way, a solution of (10) for that part of the perturbation pressure due to buoyancy could be found, given appropriate boundary conditions. From the solution, $\partial p'_b/\partial z$ could be diagnosed. For cumulonimbus convection, the Boussinesq approximation is not quantitatively accurate, although it might be qualitatively acceptable. An anelastic version of the diagnostic pressure equation in generally regarded as an improved representation for cloud models, and Davies-Jones (2003) has developed an expression for buoyancy that explicitly incorporates the effects of horizontal density variation. Nevertheless, the Boussineq form of the diagnostic pressure equation contains the most important physical contributions to perturbation pressure.

Houze (1993, p. 225) has shown that for a one-dimensional model, which is equivalent to a slab of infinite horizontal extent (see his Fig. 7), the vertical perturbation pressure gradient due to buoyancy exactly cancels the effect of *B.* Thus, the classical development of parcel theory only in the vertical dimension ignores the vertical perturbation pressure gradient term because if it is included, the model makes the apparently nonintuitive prediction of no acceleration. Although both perturbation pressure gradient terms usually oppose *B*, the dynamic term diminishes as the diameter of the convecting element decreases [see the discussion by Houze (1993)] and under certain circumstances actually enhances the effect of thermal buoyancy, as noted by Rotunno and Klemp (1982) and many others.

Doswell and Rasmussen (1994) have asserted that CAPE generally is not an accurate predictor of vertical motion in storms. There are many issues besides the choice of a base state that can alter the accuracy of the vertical motion estimates derived from CAPE: the choice of which parcel to lift, the neglect of dynamic pressure perturbation effects in sheared environments, whether moist adiabatic ascent is pseudoadiabatic or reversible moist adiabatic, the complex topic of entrainment, and the impact of freezing on the process, to name only some of them. Apart from these, however, horizontal variability of the thermodynamic variables in the severe convective environment has been of concern in trying to estimate CAPE (Brooks et al. 1994; Weisman et al. 1998), and specification of a ''representative'' sounding has long been recognized as a troubling issue. Davies-Jones (2003) has demonstrated, furthermore, that convection responds to horizontal variations in the environment in a complex way.

Parcel theory–based parameters such as CAPE can still be viewed as meaningful descriptors of the environment in which deep convection takes place and are valid when used as predictors of convection, but should *not* be understood as representing a complete description of the contribution due to buoyancy in deep convective storms. The *buoyant* contribution to the perturbation pressure generally opposes that due to the *thermal* buoyancy and so considering *B* alone typically overestimates the magnitude of any contribution from buoyancy.

Numerical cloud simulation models have demonstrated that the association between CAPE and the vertical motion of severe convection is open to question, in large part because of the importance of *dynamic* vertical perturbation pressure gradient force contributions [i.e. p'_d in (10), above] to updrafts in sheared environments (e.g., Rotunno and Klemp 1985). Overall, this is an issue outside the scope of this article, but it is nevertheless an important limitation of the use of CAPE as an estimator of vertical motion.

6. Conclusions

The absence of a meaningful form of mass continuity in a one-dimensional model precludes the possibility of such a model making a physically accurate prediction. Parcel theory neglects the vertical perturbation pressure gradients and so produces an inaccurate mental picture of the actual processes of convection, as well as inaccurate predictions of the accelerations. We have asserted that parcel theory is designed to probe the stability characteristics of that very base state. Change the base state, and the results of a parcel theory analysis *should* change because it is concerned with the sounding, not any convection that might ensue. Multidimensional numerical models that employ some form of (5) and enforce mass continuity do not depend *physically* on the choice of the base state, although their numerical aspects might exhibit some such dependence.

The diagnostic pressure equation for models that are not fully compressible is generally elliptical, so a point source involving the thermal buoyancy on the right-hand side implies some ''action at a distance'' regarding the diagnosed pressure distribution. This is indeed what has been shown for the case of an anelastic mass continuity equation by Davies-Jones (2003). The effect of the surrounding density distribution falls off with distance. However, to say that buoyancy is a relative quantity in the sense that it is defined relative to some hypothetical environment is a drastic oversimplification of the situation, to the point of being misleading.

At any point in a fluid, the accelerations attributable to the density distribution are manifested by unbalanced pressure gradient forces. The pressure field that drives the vertical (and horizontal) accelerations responds to the density distribution, but the resulting accelerations are still evaluated locally (at points) as the sum of the forces in the momentum equations. Changing the base state changes the diagnosed thermal buoyancy term (*B*), but it also alters the diagnosed perturbation pressure field (and, hence, the perturbation pressure gradient terms) in a way that compensates for the changes in the diagnosis of B caused by changes to the base state. The details of that compensation depend on what mass continuity equation is used, but in the end, the physics of buoyancy cannot depend on the base state used to partition the full field into buoyant and nonbuoyant (i.e., hydrostatic) components. The choice of what hydrostatic base state to use is arbitrary and so cannot have physical meaning.

The pedagogical development of parcel theory needs modification to emphasize the limitations imposed by pure parcel theory that lead to the erroneous interpretation of buoyancy as a quantity relative to some base state. In fluids, Archimedean notions of buoyancy are incomplete and misleading. Buoyancy is the statically forced part of the locally nonhydrostatic, pressure gradient force, not just the thermal buoyancy. Use of B alone in forecasting might be successful as a parameterized expression of the actual buoyancy, but it is not the complete buoyancy because it depends on the choice of a base state.

Therefore, any *diagnosis* of buoyancy from model simulations that does so with reference to a constant (also usually horizontally homogeneous) base state needs to include not only the thermal buoyancy (term B) but also the contributions from the vertical perturbation pressure gradient term due to buoyancy. The perturbation pressure due to the flow should be diagnosed separately.

Acknowledgments. We appreciate many helpful discussions on this general topic with David Mechem, Erik Rasmussen, Evgeni Fedorovich, Alan Shapiro, and Louis Wicker that have clarified our presentation. David Schultz and Kerry Emanuel made several useful suggestions early in the development of this concept. George Bryan, Peter Bannon, and Mike Fritsch helped correct some errors within an early version of the manuscript. George Bryan also generously discussed his numerical simulation results with us. Andy Detwiler and Ms. Connie Crandall obtained a clean copy of Fig. 1 for us. Our anonymous reviewers also contributed numerous valuable suggestions for improving the manuscript.

APPENDIX

Compensation between the Perturbation Pressure and Relative Density Terms to Produce Independence from the Reference State

Consider the special case of an isothermal, hydrostatic atmosphere. The temperature is T_0, so the hydrostatic equation can be integrated for the pressure and density:

$$p(z) = p_0 \exp\left(-\frac{gz}{RT_0}\right),$$

$$\rho(z) = \rho_0 \exp\left(-\frac{gz}{RT_0}\right), \tag{A1}$$

where p_0 and ρ_0 are the surface pressure and density, respectively, and z is the height above the surface. For an isothermal, hydrostatic base state, given by $\overline{T} = T_0 + \psi$, the pressure and density profiles are given by

$$\overline{p}(z) = \overline{p}_0 \exp\left(-\frac{gz}{R\overline{T}}\right),$$

$$\overline{\rho}(z) = \overline{\rho}_0 \exp\left(-\frac{gz}{R\overline{T}}\right). \tag{A2}$$

Therefore, the perturbation pressure and density can be shown to be

$$p'(z) = p_0 \exp\left(-\frac{gz}{RT_0}\right) - \overline{p}_0 \exp\left[-\frac{gz}{R(T_0 + \psi)}\right],$$

$$\rho'(z) = \rho_0 \exp\left(-\frac{gz}{RT_0}\right) - \overline{\rho}_0 \exp\left[-\frac{gz}{R(T_0 + \psi)}\right]. \tag{A3}$$

Now consider the two terms in part *ii* of Eq. (10). The first is the perturbation pressure gradient term; differentiating the expression for p' in (A3) with respect to z yields

$$-\frac{1}{\rho}\frac{\partial p'}{\partial z} = g\left\{1 - \frac{\overline{\rho}_0}{\rho_0}\exp\left[\frac{gz\psi}{RT_0(T_0 + \psi)}\right]\right\}, \tag{A4}$$

whereas the relative density term is given by

$$-\frac{\rho'}{\rho}g = \left\{\frac{\overline{\rho}_0}{\rho_0}\exp\left[\frac{gz\psi}{RT_0(T_0 + \psi)}\right] - 1\right\}g. \tag{A5}$$

It is easy to see that the two separate contributions to buoyancy [i.e., Eqs. (A4) and (A5)] are equal in magnitude but of opposite sign, yielding a zero sum, which must be the case since both the base state and the actual atmosphere are hydrostatic. In this special case, the two terms are equal and opposite no matter what the choice of ψ might be, including the trivial case where $\psi = 0$.

Obviously, showing this in more general cases is considerably more complicated, but the principle remains the same. Changes to the base state always result in changes to the two parts of term *ii* in (10) that will be equal in magnitude and have opposite sign.

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