NOTES AND CORRESPONDENCE

Extracting Envelopes of Nonzonally Propagating Rossby Wave Packets

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ABSTRACT

Previously developed techniques that have been used to extract envelopes of Rossby wave packets are based on the assumption of zonally propagating waves. In this note a method that does not require such an assumption is proposed. The advantages of the new technique, both on analytical and real-world examples, are demonstrated.

1. Introduction

Techniques to extract envelopes of Rossby wave packets are useful tools in studying the dynamics of storm tracks (e.g., Chang and Yu 1999; Chang 2000). Such techniques can also be utilized in the practice of numerical weather prediction to track the origin of important forecast changes that occur because of localized changes in the initial conditions (e.g., Szunyogh et al. 2002). Such forecast changes are important because of their rapid speed of propagation and their potentially large magnitude. These properties often make the tracking of such changes feasible in the short and early medium-range forecasts (up to about 5 days). For instance, on rare occasions a current forecast can be of lower quality than the forecast issued at the previous analysis time (e.g., 6 or 12 h earlier); such cases usually occur when a localized group of bad observations is assimilated or the analysis scheme has difficulties assimilating a localized group of good observations. On the other hand, a sudden forecast improvement can occur after the assimilation of observed information in a region where model errors led to unusually large degradation of the forecasts started at earlier analysis times. "Rossby wave thinking" has been successfully used at the European Centre for Medium-Range Weather Forecasts (ECMWF) to "blacklist" observing stations that were determined to have reported bad observations, to detect problems with the data assimilation system, and to identify regions where model errors were significant (Persson 2000).

Rossby wave packet–type propagation can be tracked subjectively [visually inspecting the difference

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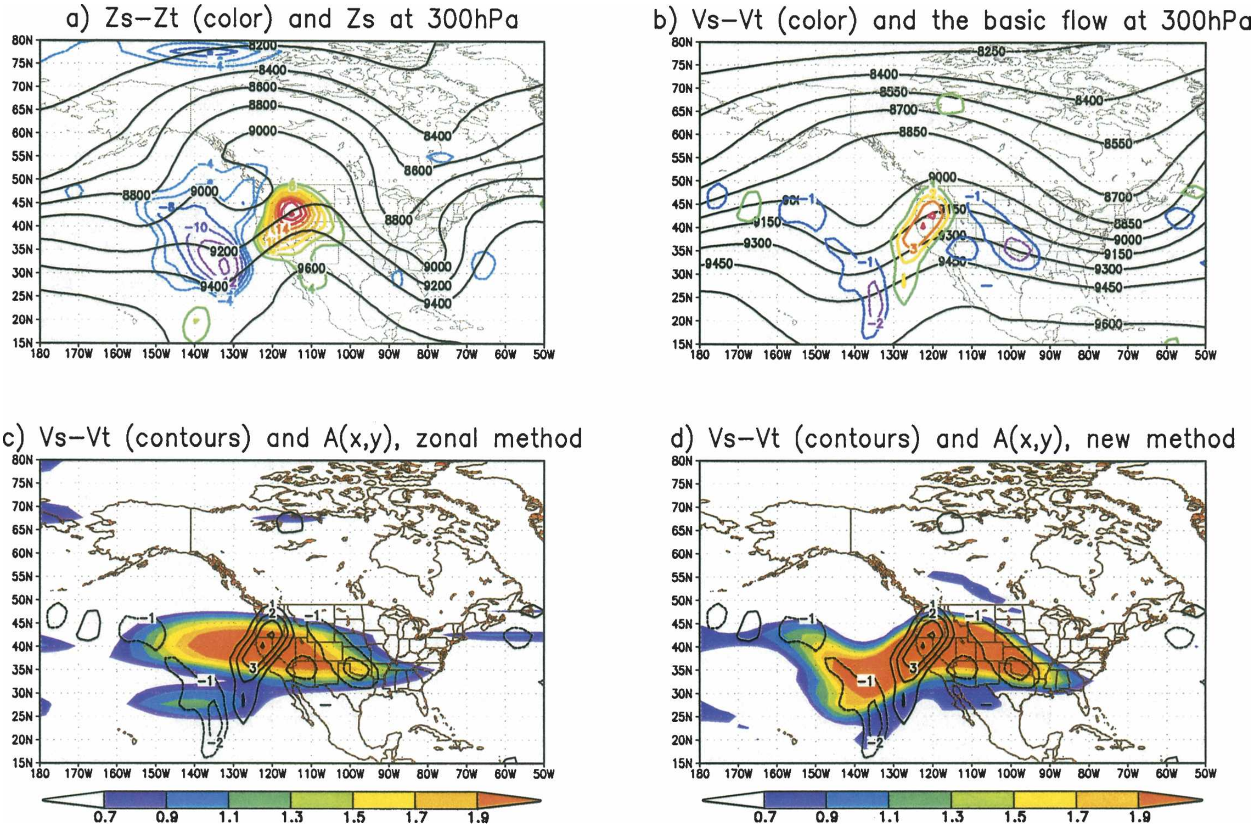

FIG. 1. Propagation of the difference between a targeted forecast that uses the additional observations from dropsondes in the northeast Pacific at initial time and a standard forecast that does not use this information. The forecasts are initiated at 0000 UTC 25 Jan 2000 and the 12-h forecast lead time is shown. (a) The geopotential height for 12-h targeted forecast (black contours) and the difference in the geopotential height at the same level between the standard and targeted forecasts (color contours). (b) The difference in the meridional wind component at 300-hPa level between the standard and targeted forecasts ϕ (color contours), and the basic flow (black contours), defined by a 20-day mean of the National Centers for Environmental Prediction operational analysis centered at the forecast verification time. (c) The packet envelope identified by the method of ZEA03 is shown in shaded colors and (d) the envelope recovered by the method introduced here is shown in shaded colors. The original signal ϕ is overlaid in black contours in (c) and (d).

between two forecasts (Persson 2000)], by plotting Hovmöller diagrams (e.g., Persson 2000; Szunyogh et al. 2000, 2002), or by employing techniques for extracting the envelopes of the wave packets at a given atmospheric level to provide a two-dimensional picture of the their propagation (e.g., Chang and Yu 1999; Chang 2000; Szunyogh et al. 2002; Zimin et al. 2003). This paper is concerned with the latter approach: extracting Rossby wave envelopes from forecast data. Techniques for accomplishing this have been based on the assumptions that (i) the atmospheric flow can be decomposed into a large-scale basic flow that is slowly varying in time, and a transient component that may include synoptic-scale Rossby waves, and (ii) when Rossby waves are present their propagation is guided by the basic flow.

In Zimin et al. (2003, hereafter ZEA03), we proposed a Hilbert transform–based technique (Gabor 1946) to extract the packet envelopes along latitude circles. This technique was later utilized at ECMWF to aid their error-tracking efforts (F. Grazzini 2004, personal communication). While the technique has performed well on most occasions, there has also been a small number of cases with less satisfactory results. For example, the technique sometimes indicated multiple wave packets at nearby latitudes, although all other information suggested a single wave packet (F. Grazzini 2004, personal communication).

We illustrate the problem by examining a case of targeted weather observations from the Winter Storm Reconnaissance 2000 field program (Szunyogh et al. 2002). In this case, the two forecasts we compare are started at the same analysis time and the forecast differences are due to differences in the analyses within a localized region over the northeast Pacific Ocean. The targeted observations collected in that region were assimilated only into one of the forecasts. Figure 1a shows that at the 12-h-forecast lead time, the added observations led to a forecast in which there was a deepening of a shallow trough off the coast of California and amplification of a downstream ridge over the western part of the United States. This pattern suggests that the dominant part of the forecast influence propagated in the form of a packet of synoptic-scale Rossby waves. To support this conclusion, we show that the magnitude of the difference between the two forecasts is comparable to the magnitude of the envelope extracted from the difference field in the wavenumber range 4–11. We first attempt to achieve this goal by applying the technique of ZEA03 to the scalar field ϕ defined by the difference between the meridional component of the wind at the 300-hPa level in the two forecasts (this difference field is shown by color contour lines in Fig. 1b).

The ZEA03 method recovers the packet envelope from a scalar atmospheric field $\phi(s)$. Appropriate to computations on a grid, we assume s to take on discrete values, $s = j\delta$ where $j = -N, \ldots, 0, \ldots, N$. The Fourier transform of $\phi(s)$ is defined by

$$\hat{\phi}(k) = \frac{1}{2N+1} \sum_{j=-N}^{N} \phi(j\delta) e^{-2\pi i[kj/(2N+1)]},$$
$$(k = -N, \ldots, N),$$

where the integer k denotes the longitudinal wavenumber. The quantity $\hat{\phi}(k)$ is then multiplied by a filter function $f(k)$:

$$f(k) = \begin{cases} 1, & \text{for} \quad 0 < k_{\min} \le k \le k_{\max} \\ 0, & \text{otherwise.} \end{cases}$$

This filter removes all wavenumber components except for a band between positive wavenumbers $k_{\min}$ and $k_{\max}$ ($k_{\min} = 4$ and $k_{\max} = 11$ in our example). Then the wave packet envelope $A(s)$ along the latitude circle is computed as the magnitude of the inverse Fourier transform of $2f(k)\hat{\phi}(k)$. Note that, since the filter retains only positive k components, the inverse transform of $f(k)\hat{\phi}(k)$ is complex.

In our example, one would expect, based on Fig. 1b, that if a detectable packet envelope were found in the required wavenumber range, it would be a coherent envelope whose maxima would overlap with the maxima of ϕ. Instead, the extracted envelope has two distinct zonally elongated centers: one primary center at around 37.5°N and a secondary center at around 27.5°N. We conjecture that this split of the extracted envelope is an artifact of the ZEA03 technique, which occurs when the basic flow guiding the evolution of the wave packet is strongly nonzonal. To better understand our reasoning for making this conjecture, we note that by applying the Fourier transform along latitude circles, we implicitly assume that the basic flow guiding the evolution of the wave packet is zonal. We also note that the alternative technique used in the literature (Chang and Yu 1999; Chang 2000), that is, the method of complex demodulation (Bloomfield 2000), makes the same assumption, and has the additional shortcoming that it distorts the packet envelopes when wave packets of similar carrier wavenumbers coexist at the same latitude (ZEA03).

In what follows, we provide an analytical example to demonstrate that incorrectly assuming zonality of the basic flow can lead to an artificial split of the packet envelope, and we also introduce an improved algorithm that does not require the assumption of a zonal basic flow. We note that the new algorithm retains the assumption that the flow can be reasonably decomposed into a slowly varying basic component and a transient part, and that the Rossby waves tend to propagate along the basic flow. Phenomenology indicates that, while these assumptions are not exactly true, they are reasonable approximations. We also show that the technique based on these assumptions can markedly improve upon the algorithms based on the more restrictive assumption of a zonal basic flow.

2. Analytical example

In this example, the angle between the direction of the basic flow and the zonal direction is 45°, and the wave packet envelope modulates a spatially sinusoidal wave (Fig. 2). More precisely, the wave packet has the following functional form:

$$\begin{aligned}\phi(x, y) &= A(x, y) \sin[k(x + y) + \gamma] \\ &= \exp[-\alpha(x + y)^2 - \beta(x - y)^2] \\ &\quad \sin[k(x + y) + \gamma],\end{aligned}$$

where $\alpha = 1/350$, $\beta = 1/40$, $k = 7(2\pi/144)$, and $\gamma = 1$. All variables are specified on the 73×144 grid with x varying from -71 to $+72$, and y varying from -36 to $+36$. Figure 2c demonstrates that the technique of ZEA03 distorts the shape of the wave packet envelope $A(x, y)$, indicating two zonally elongated centers of the wave packet. On the other hand, as shown in Fig. 2d, the envelope recovered with the technique to be described in section 3 is essentially identical to the envelope of the original signal (Fig. 2b).

3. Our new technique

In the new method we apply a filter algorithm similar to that of ZEA03 to an atmospheric variable $\phi(s)$ defined along a segment of a streamline of the basic flow

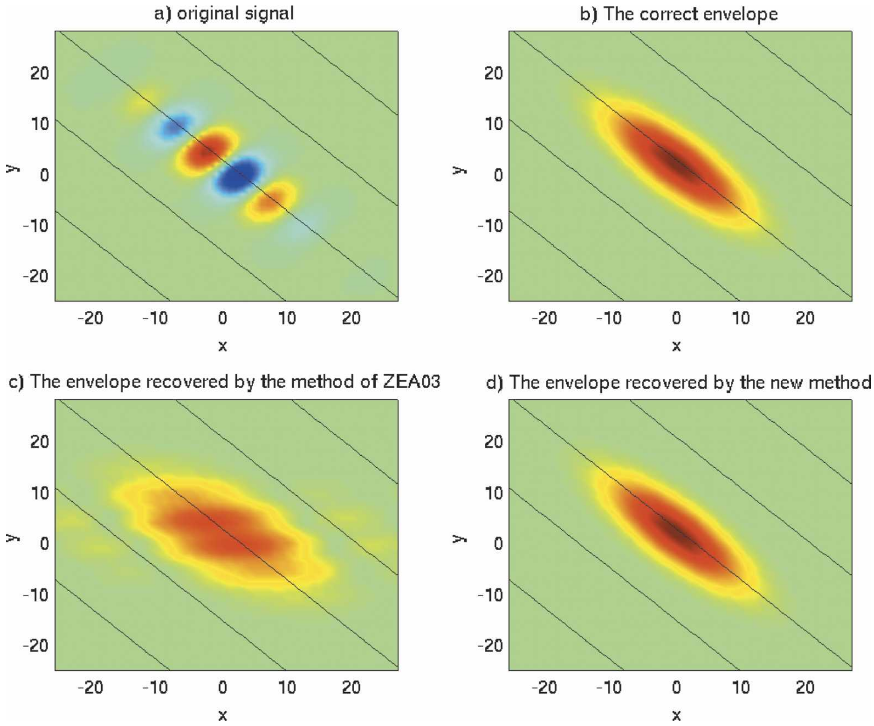

FIG. 2. (a) The wave packet $\phi(x, y)$ and (b) the packet envelope $A(x, y)$, defined by Eq. (1). The wave packet amplitude is shown in color and the basic flow is shown by black straight lines. The packet envelopes recovered (c) by the technique of ZEA03 and (d) by the technique described in this paper are also shown.

instead of along a latitude circle. Suppose we have an atmospheric variable $\phi(x, y)$, and the basic flow evaluated at a fixed geopotential height has zonal and meridional components $u(x, y)$ and $\upsilon(x, y)$. We assume that all variables are specified on a rectangular grid. The algorithm contains three steps as follows:

1) For each grid point (x_0, y_0) we find a piecewise-linear approximation of a streamline defined by (u, υ) in the neighborhood of (x_0, y_0). If x is measured in units of longitude and y is measured in units of latitude, then the point (x_1, y_1) that lies a distance δ in the direction of the streamline from (x_0, y_0) is

$$x_1 = x_0 + \frac{\delta}{\cos y_0} \frac{u(x_0, y_0)}{\sqrt{\upsilon(x_0, y_0)^2 + u(x_0, y_0)^2}} \quad \text{and}$$

$$y_1 = y_0 + \delta \frac{\upsilon(x_0, y_0)}{\sqrt{\upsilon(x_0, y_0)^2 + u(x_0, y_0)^2}}.$$

Here δ has the same units as y. The factor $\cos y_0$ compensates for the decreasing zonal distance between grid points near the poles. We determine $(x_2, y_2), (x_3, y_3), \ldots, (x_N, y_N)$ iteratively according to the same formula with shifted indices, and determine $(x_{-1}, y_{-1}), (x_{-2}, y_{-2}), \ldots, (x_{-N}, y_{-N})$ similarly, working backward:

$$x_{-j-1} = x_{-j} - \frac{\delta}{\cos y_{-j}} \frac{u(x_{-j}, y_{-j})}{\sqrt{\upsilon(x_{-j}, y_{-j})^2 + u(x_{-j}, y_{-j})^2}}$$

and

$$y_{-j-1} = y_{-j} - \delta \frac{\upsilon(x_{-j}, y_{-j})}{\sqrt{\upsilon(x_{-j}, y_{-j})^2 + u(x_{-j}, y_{-j})^2}}.$$

We choose $N\delta$ to be roughly the length of a latitude circle.

2) We now have points with coordinates (x_j, y_j), $j = -N, \ldots, +N$. We interpolate the atmospheric variable of interest ϕ onto each such point (x_j, y_j). We then localize $\phi(x_j, y_j)$ using a Gaussian filter function centered at $j = 0$:

$$\overline{\phi}(x_j, y_j) = \phi(x_j, y_j) \exp\left(-\alpha^2 \frac{j^2}{N^2}\right).$$

We choose $1/\alpha$ to be roughly the length of the wave packet we wish to analyze, as a fraction of the length of the latitude circle.

3) Taking $s = j\delta$, with $\phi(s)$ given by $\overline{\phi}(x_j, y_j)$ for $|s| \leq$

δN, we input this new definition of $\phi(s)$ into the algorithm described in ZEA03 to obtain the amplitude $A(x_j, y_j)$. We estimate the wave packet envelope at (x_0, y_0) by the amplitude $A(x_0, y_0)$ at the center point.

We repeat the above three steps for every grid point.

The packet envelope in Fig. 1d is obtained using the same input variable $\phi(x, y)$ as in Fig. 1c [recall that $\phi(x, y)$ is the difference between the meridional wind component in the two forecasts]. We use $\delta = 1.75°$, $N = 90$, and $\alpha = 4$. The basic flow (u, v) was defined by a 20-day average of the wind field, shown by solid black contour lines in Fig. 1b. The C++/MATLAB Mex code for the technique is available from the authors.

Figure 1d illustrates that the new method does substantially better in tracking the alternating trail of positive and negative velocity difference contours (Fig. 1b), and, in addition, does not show the small additional spurious peak that results from application of the method of ZEA03.

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REFERENCES

Bloomfield, P., 2000: *Fourier Analysis of Time Series: An Introduction.* 2d ed. Wiley-Interscience, 261 pp.

Chang, E. K. M., 2000: Wave packets and life cycles of troughs in the upper troposphere: Examples from the Southern Hemisphere summer season of 1984/85. *Mon. Wea. Rev.,* **128,** 25–50.

——, and D. B. Yu, 1999: Characteristics of wave packets in the upper troposphere. Part I: Northern Hemisphere winter. *J. Atmos. Sci.,* **56,** 1708–1728.

Gabor, D., 1946: Theory of communication. *J. IEEE,* **93,** 429–457.

Persson, A., 2000: Synoptic-dynamic diagnosis of medium range weather forecast systems. *Proc. Seminars on Diagnosis of Models and Data Assimilation Systems,* Reading, United Kingdom, ECMWF, 123–137.

Szunyogh, I., Z. Toth, R. E. Morss, S. J. Majumdar, B. J. Etherton, and C. H. Bishop, 2000: The effect of targeted dropsonde observations during the 1999 Winter Storm Reconnaissance Program. *Mon. Wea. Rev.,* **128,** 3520–3537.

——, ——, A. V. Zimin, S. J. Majumdar, and A. Persson, 2002: On the propagation of the effect of targeted observations: The 2000 Winter Storm Reconnaissance Program. *Mon. Wea. Rev.,* **130,** 1144–1165.

Zimin, A. V., I. Szunyogh, D. J. Patil, B. R. Hunt, and E. Ott, 2003: Extracting envelopes of Rossby wave packets. *Mon. Wea. Rev.,* **131,** 1011–1017.