Reply

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1. Introduction

We appreciate Prof. Schultz's (Schultz 2007) careful examination of our multiple-Doppler radar study of a front (Stonitsch and Markowski 2007, hereafter SM). We agree with Schultz that none of his concerns affect any of the conclusions summarized in section 6 of SM (114–115) (in the original version of Schultz's comments, Schultz acknowledged that he "did not find fault in any of the principal conclusions advanced by SM in their impressive analysis").

The issues raised by Schultz, in the order he presents them, are as follows:

1) The scalar frontogenetical forcing may not necessarily reflect changes in the strength of a front owing to the fact that it ideally should be evaluated in a "quasi Lagrangian" reference frame.
2) The contribution of entrainment of warm prefrontal air into the frontal zone, and the subsequent mixing, to the horizontally differential diabatic heating effect was not compared with the contribution from a horizontally differential surface sensible heat flux.
3) A possible separation of the wind shift from the region of largest horizontal density gradient was not discussed.
4) The motion of fronts does not depend on their horizontal density gradients.

Each of the above issues is addressed below.

2. Evaluation of the scalar frontogenetical function

Schultz objects to the statement by SM (p. 109) regarding the observed weakening of the virtual potential temperature gradient across the front during deployment 2 (1910–2033 UTC 3 June 2002): "Throughout deployment 2, positive, albeit weak, frontogenetic forcing was observed along the front . . . One must therefore conclude that the weakening baroclinity was brought about by frontolytic differential diabatic heating . . ."

Schultz's point about the interpretation of the frontogenetical function has been made by practically countless others (e.g., Miller 1948; Sanders 1955, 1999; Keyser et al. 1988; Bluestein 1993, p. 248; Roebber et al. 1994; Ostdiek and Blumen 1995; Davies 1999). The frontogenetical function, $d|\nabla_p\theta_v|/dt$, defines the rate of change of the magnitude of the horizontal virtual potential temperature gradient following a parcel's motion, that is,

$$\frac{d|\nabla_p\theta_v|}{dt} = \frac{\partial|\nabla_p\theta_v|}{\partial t} + \mathbf{v}\cdot\nabla_p|\nabla_p\theta_v|. \qquad (1)$$

One must evaluate $\partial|\nabla_p\theta_v|/\partial t$ in a reference frame moving with the front (and presumably within the frontal zone) to assess changes in the strength of a front. In the reference frame of a front moving with velocity $\mathbf{c}$,

$$\left(\frac{\partial|\nabla_p\theta_v|}{\partial t}\right)_{\mathrm{fr}} = \frac{d|\nabla_p\theta_v|}{dt} - (\mathbf{v}-\mathbf{c})\cdot\nabla_p|\nabla_p\theta_v|, \qquad (2)$$

where the subscript fr indicates front-relative reference frame, $\mathbf{v}-\mathbf{c}$ is the front-relative wind, and $-(\mathbf{v}-\mathbf{c})\cdot\nabla_p|\nabla_p\theta_v|$ is advection of the virtual potential temperature gradient by the front-relative wind. Bluestein (1993, p. 248) concedes that "in practice, it is difficult to work in a quasi-Lagrangian frame because a front's motion vector usually varies as a function of location along the front." Schultz also notes that "[f]ew people have evaluated such a formulation in observed fronts." *To our knowledge, no one has evaluated such a formulation in observed fronts.*

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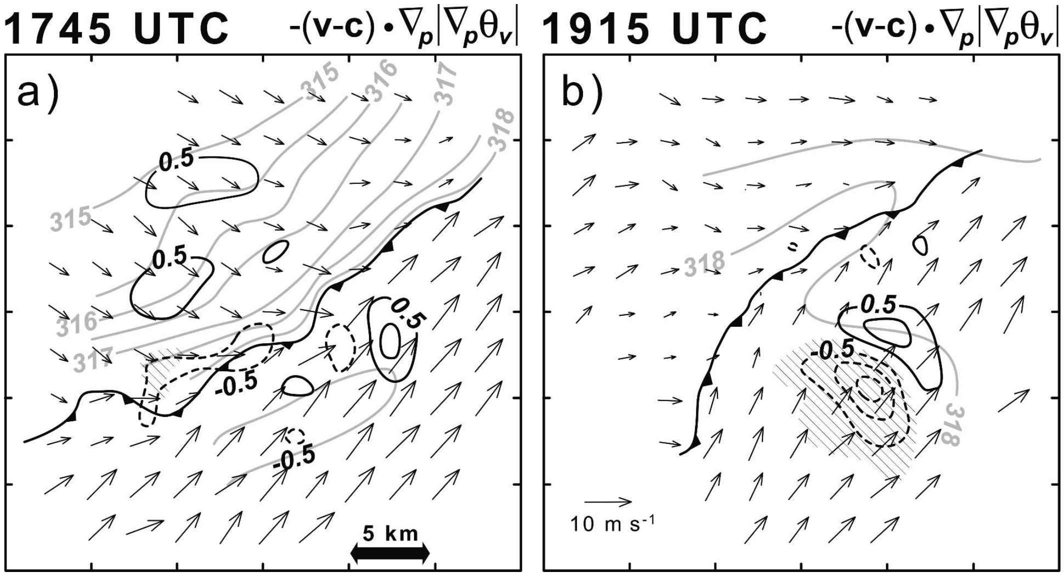

FIG. 1. (a) Quasi-horizontal advection of the quasi-horizontal potential temperature gradient by the front-relative wind [$-(\mathbf{v} - \mathbf{c}) \cdot \nabla_p |\nabla_p \theta_v|$; black contours every 0.5 K km^{-1} h^{-1}, with dashed contours indicating negative values and the zero contour suppressed] and mobile mesonet-derived θ_v field (gray contours every 0.5 K) at 1745 UTC 3 Jun 2002. The θ_v field is identical to that shown in Fig. 12b of SM. Dual-Doppler-derived horizontal wind vectors at the lowest grid level are shown at every 20th grid point and are identical to those appearing in Figs. 10b and 14b of SM. Hatching indicates regions where $\text{sgn}[(\partial |\nabla_p \theta_v| / \partial t)_{\text{fr}}] \neq \text{sgn}(d|\nabla_p \theta_v|/dt)$. The objectively analyzed front location also is indicated. (b) Same as in (a) but for 1915 UTC 3 Jun 2002. The θ_v field is that which appears in Fig. 12c of SM, and the wind vector field is identical to the wind vector fields in Figs. 10c and 14c of SM.

One might expect that $-(\mathbf{v} - \mathbf{c}) \cdot \nabla_p |\nabla_p \theta_v|$ would tend to vanish within frontal zones, at least for fairly well-behaved θ_v fields, because a frontal zone, by definition, is a region where $|\nabla_p \theta_v|$ is a relative maximum, thus $\nabla_p |\nabla_p \theta_v|$ would tend to be a relative minimum. In such cases, if $d|\nabla_p \theta_v|/dt$ is evaluated within a frontal zone, then it ought to be a good measure of whether a front is strengthening or weakening. It is likely for this reason that so many other investigators also have evaluated $d|\nabla_p \theta_v|/dt$ in their assessments of changes in the intensity of fronts (e.g., Newton 1954; Ballentine 1980; Keshishian and Bosart 1987; Doyle and Warner 1993; Roebber et al. 1994; Bryan and Fritsch 2000a,b, among others). Quoting Roebber et al. (1994, p. 31),

> [A]lthough the Lagrangian form of [the frontogenetical function] suggests the need for a complete trajectory analysis of air parcels . . . diagnostics presented in an Eulerian framework have proven quite useful for physically describing frontogenetical processes and their associated vertical circulations . . . *[the] advection of the temperature gradient is frequently small so that the real difference between Eulerian and Lagrangian calculations is not significant* (italics added).

On the other hand, fronts often do not have "well behaved" θ_v fields; for example, fronts often are viewed as discontinuities in the $\nabla_p \theta_v$ field (Bluestein 1993, 245–248), and strong cold fronts occasionally are observed to have a near discontinuity in their θ_v fields (e.g., Young and Johnson 1984; Shapiro 1984; Shapiro et al. 1985). When θ_v or $\nabla_p \theta_v$ is not easily differentiable, then it is more difficult to generalize what net effect $-(\mathbf{v} - \mathbf{c}) \cdot \nabla_p |\nabla_p \theta_v|$ might have on $(\partial |\nabla_p \theta_v| / \partial t)_{\text{fr}}$. For example, in cases of intense fronts like those documented by Young and Johnson (1984), Shapiro (1984), and Shapiro et al. (1985), $\nabla_p |\nabla_p \theta_v|$ might approach infinity at the front, but the front-relative wind would vanish as one neared the front.

In Fig. 1 we present fields of $-(\mathbf{v} - \mathbf{c}) \cdot \nabla_p |\nabla_p \theta_v|$ at 1745 and 1915 UTC 3 June 2002, derived from the dual-Doppler wind syntheses and mobile mesonet analyses of SM. We have chosen these two times because 1745 UTC (deployment 1) is the time at which the largest magnitudes of $-(\mathbf{v} - \mathbf{c}) \cdot \nabla_p |\nabla_p \theta_v|$ were observed within the frontal zone, and 1915 UTC (deployment 2) is within the time period for which Schultz has doubts that the weakening of the front could be accounted for by $d|\nabla_p \theta_v| dt$. As noted before, it is difficult to accommodate differential front motion like that which was observed (Fig. 5 of SM); an average $\mathbf{c}$ of 1 m s^{-1} from 310° is assumed in Fig. 1.

It is clear from Fig. 1 that within virtually all of the

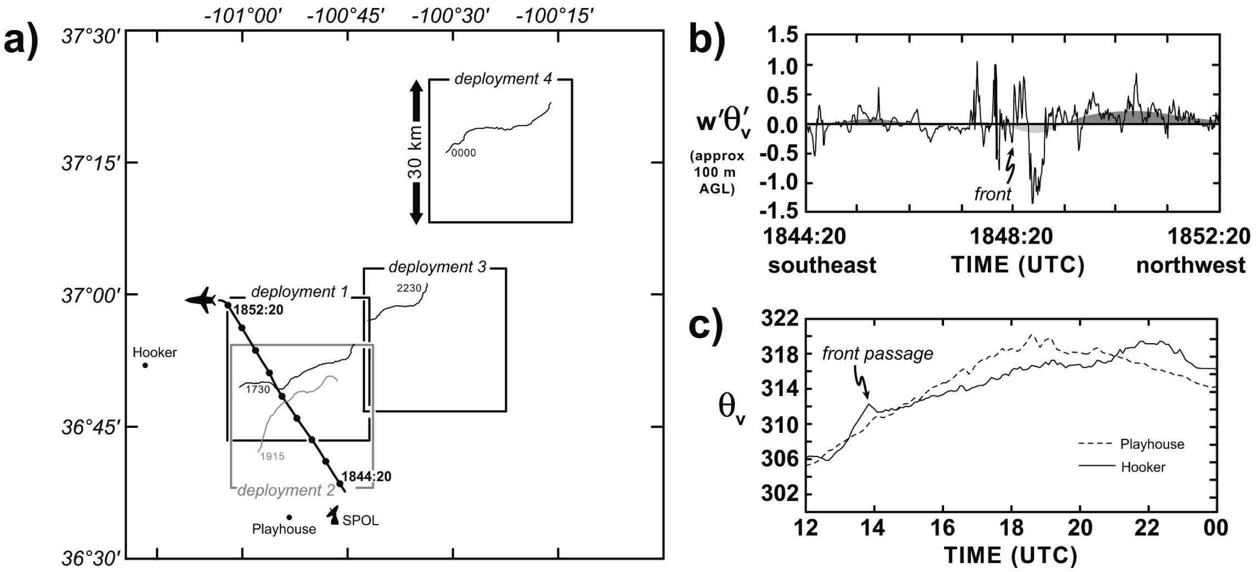

FIG. 2. (a) Dual-Doppler wind synthesis domains for deployments 1–4 (also see Fig. 5 in SM), with the 1844:20–1852:20 UTC University of Wyoming King Air transect overlaid. Filled circles appear along the flight track at 1-min intervals. Isochrones of the front at select times also are indicated (one for each deployment), with the time (UTC) indicated beside each isochrone. (b) Profile of $w'\theta_v'$ at approximately 100 m AGL observed by the King Air (black line), along with a spatially averaged profile ($\overline{w'\theta_v'}$; shaded). Units on the ordinate are °C m s^{-1}. (c) Time series of θ_v from 1200 to 0000 UTC 3–4 June 2002 at the Hooker and Playhouse surface observing sites [their locations are indicated in (a)].

frontal zone (with the minor exception of a small area at 1745 UTC indicated by hatching in Fig. 1a), *an evaluation of* $d|\nabla_p\theta_v|/dt$ within the frontal zone leads to the correct assessment of the sign of the changes in the intensity of the front (note that the contour interval used in Fig. 1 is half the interval used to contour $d|\nabla_p\theta_v|/dt$ in Fig. 14 of SM), neglecting diabatic effects. A larger area (but still representing only a small fraction of the analysis domain) within which an assessment of $d|\nabla_p\theta_v|/dt$ would lead to an incorrect assessment of the sign of $(\partial|\nabla_p\theta_v|/\partial t)_{\text{fr}}$ is evident at 1915 UTC (Fig. 1b), but it is not within the frontal zone.

Schultz also states, "[E]ven weakening fronts may exhibit positive frontogenesis due to deformation and convergence as air parcels in the warm sector approach the virtual potential temperature gradient associated with the front." But there must be air parcels exhibiting frontolysis following their motion *somewhere in the frontal zone* if the front is weakening, exotic temperature distributions aside, otherwise air would be approaching the front from *both sides* (this is the only way that parcels approaching the front from both sides can be experiencing positive frontogenesis following their motion). We do not comprehend how a front can be weakening if airstreams are approaching the front from both sides, barring nonconservation processes, which Schultz has not invoked.

3. Diabatic heating effects

SM concluded that diabatic processes must have led to the weakening of the front in the 1915–2030 UTC period, based on the observations of positive $d|\nabla_p\theta_v|dt$ during that time period (refer to the quote in the first paragraph of section 2). Although Schultz's original comments (D. M. Schultz 2007, personal communication) expressed skepticism, buoyancy flux data from the University of Wyoming King Air[1] are consistent with this mechanism proposed by SM.[2] The King Air arrived in the deployment area at approximately 1844 UTC. Fluxes computed along an approximately 50-km-long transect (Fig. 2a) roughly 100 m above ground level, spanning the 1844–1852 UTC period, indicate a horizontal gradient of buoyancy flux, $\overline{w'\theta_v'}$, with larger values on the cold side of the front (Fig. 2b). Although one would need to know the horizontal gradient of the vertical buoyancy flux divergence to assess frontogenetical forcing, the horizontal gradient apparent in Fig. 2b is consistent with the effect being frontolytic, assuming the typical, approximately linear decrease of buoyancy

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[1] LeMone et al. (2007) recently summarized the flux data obtained during the International H_2O Project (IHOP_2002).

[2] These data were first presented in our original reply (P. M. Markowski and J. R. Stonitsch 2007, unpublished manuscript).

flux from the surface to the top of the boundary layer. Actually, the frontolytic nature of this diabatic effect would be *underestimated* by consideration of only the buoyancy flux data (as opposed to considering both the horizontal gradient in the buoyancy flux and horizontal gradient in boundary layer depth) because of a deeper boundary layer on the warm side of the front (see Figs. 3 and 6 of SM).

Schultz (2007) presents several new objections. The first is that

> SM have not computed whether the time and space scales for the sensible heat flux and resultant turbulent mixing would be adequate to explain the observed frontolysis. Therefore, quantitative evidence for this mechanism is lacking in this case.[3]

Our response to this new criticism is simply that an evaluation of the relative roles of multiple possible diabatic processes in contributing to frontolysis (e.g., gradients in surface sensible heat flux versus entrainment of prefrontal air into the frontal zone, to be discussed below) was never a goal of SM, nor is it clear how one would ever do this. Another new criticism put forth by Schultz is that SM have omitted entrainment within the frontal zone, citing prior studies that have found that such an effect can be important in the presence of a thermally insulated lower boundary (i.e., surface heat fluxes are zero). We cannot easily extract this contribution from the buoyancy flux data shown in Fig. 2b, nor is it clear how the mechanism to which Schultz refers would have to be modified in the presence of strong surface fluxes. Furthermore, an exploration of all of the processes that might affect the buoyancy flux was simply never a goal of SM nor our original reply (P. M. Markowski and J. R. Stonitsch 2007, unpublished manuscript). Most importantly, the mixing process to which Schultz (2007) refers *is* a "frontolytic differential diabatic heating" process anyway (the reader is again referred to the quote in the first paragraph of section 2).

Schultz's claim about the number of studies that have indicated "entrainment and the resulting mixing to be a significant, if not the most significant, frontolytic process affecting the strength of fronts (Sanders 1955, 1999; Blumen and Piper 1999; Piper and Lundquist 2004)" is exaggerated. Sanders' (1955) only remark on the subject of mixing appeared near the end of his manuscript (551–552): "Presumably the eddy effects are frontolytical, but it is not possible to investigate the matter quantitatively at the present time." Regarding Sanders (1999), he wrote (p. 2401),

> Since the (frontal) zone was in an approximately steady state during the period examined (2200–0400 UTC), the inflowing air parcels ahead and behind must have been cooled and heated diabatically, respectively, to maintain a balance. Intense lateral mixing appears to have been the mechanism for these diabatic changes.

It is not evident that Sanders considered diabatic effects other than lateral mixing (e.g., horizontal gradients of sensible heat flux) during the time period to which his statements apply; thus, it is specious for Schultz to contend that Sanders "found" that lateral mixing was one of the most significant effects. More importantly, it is far from clear whether Sanders's "lateral mixing" is at all what Schultz describes in his reference to the work of Keyser and Anthes (1982) and Xu and Gu (2002). Regarding relatively recent work on turbulent dissipation in frontal zones cited by Schultz (Blumen and Piper 1999; Piper and Lundquist 2004), it is impossible for us to compare the contribution of mixing with other frontolytic effects, as frontogenetical forcing was not evaluated in these studies. Moreover, it is not easy to quantify the impact on the turbulence statistics of entrainment of warm prefrontal air into the frontal zone.

We maintain that frontolytic differential diabatic heating effects, in the form of a differential buoyancy flux (Fig. 2b), contributed to the weakening of the front during the 1745–2030 UTC time period.

4. Separation between the wind shift and the virtual potential temperature gradient

Schultz objects to the lack of attention given to a possible separation between the wind shift and virtual potential temperature gradient: "During 1915–2145 UTC, air with virtual potential temperature less than 317 K, formerly found in the analysis domain, is no longer found within the analysis domain, although the wind shift is." Schultz argues that this observation is suggestive of a separation between the wind shift and the virtual potential temperature gradient.

The evidence of a separation between the wind shift and virtual potential temperature gradient is weak for this case. A larger-scale surface analysis at 2030 UTC (Fig. 3) does not clearly indicate such a separation, although there is a significant data void in the region north of the dual-Doppler domain. The reason for the lack of virtual potential temperature readings less than 317 K after 1915 UTC is simply that there was ~4 K of

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[3] From the context (Schultz later proposes that the contribution to $\overline{w'\theta_v'}$ from the entrainment of prefrontal air is the dominant frontolytic diabatic heating process), we assume that Schultz's reference to "sensible heat flux" really refers to the contribution to $\overline{w'\theta_v'}$ that is a result of the ground being subjected to a net radiation surplus.

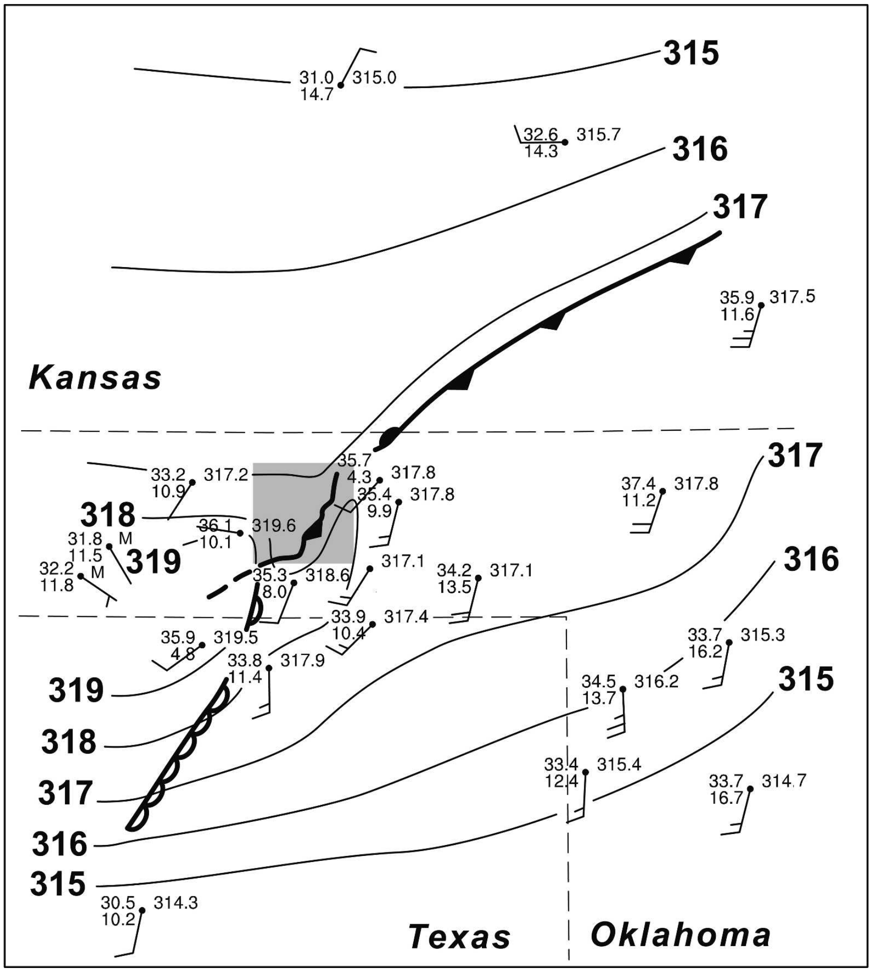

FIG. 3. Surface analysis at 2030 UTC 3 Jun 2002. Temperature (°C), dewpoint temperature (°C), virtual potential temperature (K), wind speed (half barb: 2.5 m s^{-1}; full barb: 5 m s^{-1}), and wind direction are plotted in the station models. Virtual potential temperature is analyzed at 1-K intervals. The front and dryline positions also have been analyzed. The shaded rectangular region is dual-Doppler wind synthesis region during deployment 2 (1910–2033 UTC). The analysis in this region has been guided by the higher resolution data in SM (e.g., see their Fig. 12d).

diurnal warming between 1600 and 1900 UTC at the surface observing stations in the area (e.g., Fig. 2c).

Regardless of whether such a separation developed, lack of evidence aside, this subject was well beyond the scope of the present study. Our focus was on the finecale kinematic attributes of the front and possible implications for convection initiation. The subject of convection initiation was one of the foci of the IHOP_2002 (Weckwerth et al. 2004).

5. The motion of fronts

Schultz takes issue with SM's statement (p. 109) that "... the motion of the front slowed throughout deployment 2, perhaps in response to the weakening density gradient across the front." Schultz insinuates that the horizontal density gradient exerts little or no influence on the motion of a front: "[A] casual survey of classic cold-frontal literature indicates that the speed *and direction* of motion of cold fronts are not related to the cross-frontal density difference ..." (italics in original).

Contrary to Schultz's arguments, density gradients most certainly *do* affect the movement of fronts. The motion of fronts is highly dependent upon the front-normal isallobaric gradient. For example, pressure rises (falls) on the cold (warm) side of a cold front cause the low pressure trough associated with the front to move toward the warm air. Because the density gradient is largest on the cool side of a front, the front movement tends to be highly correlated with the wind direction, and thus, density advection, on the cold side of a front. Cold fronts have cold advection (which contributes to pressure rises) on their cold sides and warm fronts have warm advection (which contributes to pressure falls) on their cold sides. The magnitude of the density gradient is crucial—for a given wind field, if the density gradient weakens, then the density advection is reduced, as is the isallobaric gradient and speed of the front. For further details, we recommend the discussion and illustration presented by Bluestein (1993, p. 259, 268).

In earlier versions of the correspondence, Schultz asserted that

> the slowing down of the front during deployment 2 is likely caused by the reversal of the flow in response to the change in the pressure gradient . . . Density-current dynamics, as implied by SM, likely have little to do with the frontal motion in this case.[4]

We did not previously argue—nowhere in SM does "density current" appear—and do not wish to argue now that this particular front should be regarded as a density current. But if one was to apply density current theory properly, one would have to account for changes in the headwind or tailwind influencing the front [e.g., Eq. (3) in Smith and Reeder (1988)]. Although it is not clear how one would most appropriately do this, even the "flow reversal" hypothesis advanced by Schultz, in principle, could be accommodated crudely by density current theory by varying the strength of the headwind or tailwind. Smith and Reeder (1988, p. 1941) submit that one of the biggest difficulties in applying density current theory is not the fact that horizontal density gradients are unimportant, but rather "the formulae are often difficult to apply because of uncertainties in determining from observations appropriate values for the variables involved."

6. Final remarks

It is worth repeating that none of the issues raised by Schultz affects the conclusions appearing in section 6 of SM (114–115):

- Vertical vortices were strongest when the horizontal wind shear across the front was largest, suggesting that shearing instability played a role in the genesis and/or intensification of the vortices; however, in contrast to many previous studies, the spacing between vortices was highly irregular.
- When vertical vortices along the front were relatively weak, slabularity increased with the strength of the thermally direct frontal circulation (which closely responded to changes in the front-normal density gradient, as expected); therefore, during these times, slabularity was a good measure of the relative contribution to the vertical motion field from the frontal circulation compared with the contribution from boundary layer convection.
- When vertical vortices were relatively strong, it was difficult to generalize the relationship between slabularity and the strength of the thermally direct frontal circulation because of the ways by which strong vortices may influence the vertical velocity field, and thus slabularity.

We close by again expressing our thanks for the careful attention given to our article by Prof. Schultz.

Acknowledgments. Stimulating discussions with Drs. Peter Bannon, George Bryan, Mike Fritsch, Yvette Richardson, and George Young on these topics are gratefully acknowledged.

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[4] A portion of this quotation ("[t]he slowing down of the front during deployment 2 is likely caused by the reversal of the flow in response to the change in the pressure gradient") is retained by Schultz (2007). It is beyond the scope of the original work and our reply herein to evaluate the range of processes and their attendant range of scales that might have contributed to changes in the pressure gradient in this case. It goes without saying that horizontal density advection is one way by which the pressure field can be modified in a hydrostatic atmosphere.

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