Implications of Regime Transitions for Mountain-Wave-Breaking Predictability

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ABSTRACT

A suite of high-resolution two-dimensional ensemble simulations are used to investigate the predictability of mountain waves, wave breaking, and downslope windstorms. For relatively low hills and mountains, perturbation growth is weak and ensemble spread is small. Gravity waves and wave breaking associated with higher mountains exhibit rapid perturbation growth and large ensemble variance. Near the regime boundary between mountain waves and wave breaking, a bimodal response is apparent with large ensemble variance. Several ensemble members exhibit a trapped wave response and others reveal a hydraulic jump and large-amplitude breaking in the stratosphere. The bimodality of the wave response brings into question the appropriateness of commonly used ensemble statistics, such as the ensemble mean, in these situations. Small uncertainties in the initial state within observational error limits result in significant ensemble spread in the strength of the downslope wind speed, wave breaking, and wave momentum flux. These results indicate that the theoretical transition across the regime boundary for gravity wave breaking can be interpreted as a finite-width or blurred transition zone from a practical predictability standpoint.

1. Introduction

As stably stratified air flows over a topographic obstacle, gravity waves are generated and propagate away from the mountain. Vertically propagating mountain waves may amplify, overturn, and break, due to factors such as the decrease of atmospheric density with altitude, nonlinearity, and vertical gradients of the ambient winds and stability, all of which influence the wave amplitude. Wave breaking is thought to be a threshold phenomenon occurring when the wave grows beyond a critical amplitude (e.g., Lindzen 1988; Bacmeister and Schoeberl 1989). To attempt to quantify this threshold characteristic of mountain waves, regime diagrams have been proposed that depict a sharp delineation between the gravity wave and wave-breaking regimes as a function of the nondimensional mountain height, \( \frac{N h_m}{U} \), (e.g., Smith 1989; Smith and Grönås 1993; Ólafsson and Bougeault 1996) where \( N \) is the Brunt–Väisälä frequency, \( h_m \) is the mountain height, and \( U \) is the incident wind velocity. Similarly, Peltier and Clark (1979) describe downslope windstorms that form beneath wave-breaking layers as a bifurcation phenomenon governed by the occurrence of breaking. Mountain-wave breaking remains an active area of research because of the profound multiscale influences on the atmosphere that include: clear-air turbulence (Clark et al. 2000); downslope windstorms (Klemp and Lilly 1975); stratospheric vertical mixing of water vapor, aerosols, and chemical constituents (Dörnbrack and Dürbeck 1998); potential vorticity generation (Schär and Durran 1997); and the significance of orographic drag on the large-scale circulation (Bretherton 1969).

Although numerical models have been able to successfully simulate gravity wave generation, evolution (e.g., Smith and Grönås 1993; Schär and Durran 1997) and breakdown (Clark and Peltier 1977; Bacmeister and Schoeberl 1989), basic questions remain regarding the nature of the predictability of topographically forced phenomena such as mountain waves, downslope windstorms, and wave breaking. Lorenz (1969) hypothesized that perturbation growth rates increase as the horizontal scale of the phenomenon decreases. This effectively limits the intrinsic predictability, which cannot be lengthened by reducing the initial error beyond a...
certain point. As for topographically forced phenomena, it remains an open question whether the predictability limit is just a few hours or days. Anthes et al. (1985) hypothesized that mesoscale phenomena forced by the lower boundary attain enhanced predictability; however, their results were subsequently found to be overly optimistic because of lateral boundary conditions, numerical dissipation, and adjustment processes (Errico and Baumhefner 1987; Vukicevic and Errico 1990). Nuss and Miller (2001) found that mesoscale predictions of wind and precipitation associated with landfalling fronts are extremely sensitive to small changes in the flow pattern relative to the orography, as deduced through simulations made with small modifications to the topography orientation. In addition, a number of studies suggest that rapid growth occurs at small scales, particularly when moist processes are important (Ehrendorfer et al. 1999; Zhang et al. 2002, 2003, 2007). Doyle et al. (2007) applied a high-resolution nonhydrostatic tangent linear and adjoint modeling system to explore the predictability limits of mountain waves. They found that as the mountain height is increased into the wave-breaking regime, perturbation growth becomes extremely rapid. For example, perturbation growth can exceed 50 m s\(^{-1}\) in a 30-min period associated with wave breaking. Using an initial state from the 11 January 1972 downslope windstorm that occurred along the Colorado Front Range (Klemp and Lilly 1975; Lilly 1978), Doyle et al. (2007) found that perturbation growth is most nonlinear (and therefore, the assumption of tangent linear perturbation growth least appropriate) near the wave-breaking threshold, rather than within the wave-breaking regime forced by even higher terrain. Their study motivates further exploration of gravity wave predictability and perturbation growth in these regimes within a fully nonlinear context, that is, using nonlinear ensembles.

Results of an intercomparison of 11 different nonhydrostatic model simulations of the 11 January 1972 downslope windstorm is described in Doyle et al. (2000). Although upper-level wave breaking was predicted by all of the models in comparable locations, there were a number of significant differences among the simulations including the nature of the breaking and the lower-tropospheric wave structure, which was characterized by a hydraulic jump in most of the models and large-amplitude waves in several of the simulations. In the Doyle et al. (2000) intercomparison, all numerical models used an identical initial state essentially exploring the influence of model error on mountain-wave predictability. It follows that a remaining question is the degree to which mountain-wave predictions are sensitive to the initial conditions. In this study, we perform high-resolution ensembles of two-dimensional numerical simulations of the 11 January 1972 downslope windstorm in order to further investigate the sensitivity of downslope windstorm and mountain-wave predictions to the initial state. An extreme downslope windstorm and multiple regions of complex upper-level wave breaking develop in this case, and represent a challenging mesoscale predictability scenario. The overall goal of this study is to explore the predictability and characteristics of the numerically simulated downslope windstorm and wave-breaking characteristics, and the influence of uncertainties in the initial state. The model description is presented in section 2. The ensemble simulation results are described in section 3, followed by the conclusions in section 4.

2. Numerical model description and experimental design

The numerical simulations in this study are performed using the atmospheric portion of the Coupled Ocean–Atmosphere Mesoscale Prediction System (COAMPS; Hodur 1997), which is based on a finite-difference approximation to the fully compressible, nonhydrostatic equations and uses a terrain-following vertical coordinate transformation. In this application, the finite-difference schemes are of second-order accuracy in time and space with the exception of horizontal advection, which is fourth-order accurate. A time-splitting technique with a semi-implicit formulation for the vertical acoustic modes is used to efficiently integrate the compressible equations (Klemp and Wilhelmson 1978). A prognostic equation for the turbulence kinetic energy (TKE) budget is used to represent the planetary boundary layer and free-atmospheric turbulent mixing and diffusion (Hodur 1997; Doyle and Durran 2007). These idealized simulations are dry with a free-slip lower-boundary condition. A radiation condition proposed by Orlanski (1976) is used for the lateral boundaries with the exception that the Doppler-shifted phase speed \((u \pm c)\) is specified and temporally invariant at each boundary (Pearson 1974; Durran et al. 1993). The mitigation of reflected waves from the upper boundary is accomplished through a radiation condition formulated following the Durran (1999) approximation to the Klemp and Durran (1983) and Bougeault (1983) techniques.

The model is applied in a two-dimensional mode with a horizontal grid increment of 1 km and a vertical grid spacing of 200 m up to an altitude of 25 km (125 vertical levels). The topography for the 2D simulations are specified using a witch of Agnesi profile:
where $h_m$ is the mountain height and $a$ is the mountain half-width. In the control simulation, we use $a = 10$ km and $h_m = 2$ km, which are typical values for the Colorado Front Range. A series of ensemble simulations are conducted with various values for $h_m$: 100, 1000, 1500, 1750, and 2000 m. Although portions of the continental divide upstream of Boulder can be considered quasi-two dimensional, the numerical model is applied in a two-dimensional mode in this study to facilitate efficient, high-resolution ensembles of wave-breaking predictions that are straightforward to analyze. The horizontal boundaries are located $10a$ (100 km) upstream and $12a$ (120 km) downstream of the mountain. The forecasts are conducted to a 4-h integration time. The results at the 4-h simulation time are characterized by variations on relatively slow time scales with respect to predictions of the windstorm and upper-level wave breaking.

In this study we use an initial state corresponding to the 11 January 1972 windstorm along the Colorado Front Range. This windstorm has been analyzed from both observational (Lilly and Zipser 1972; Lilly 1978) and numerical (Klemp and Lilly 1975, 1978; Peltier and Clark 1979) perspectives because of the extreme nature of the event and rare direct in situ measurements of the mountain-wave structure. The analysis of research aircraft flight data are presented in Klemp and Lilly (1975, 1978). The analysis reveals a large-amplitude wave that extends over 300 hPa in the upper troposphere and is nearly coincident with a deep layer of weak or near-reversed cross-mountain flow, which is likely a manifestation of mountain-wave amplification and breaking. Lilly (1978) concluded that the 60 m s$^{-1}$ downslope winds along the lee slope are consistent with a deep layer between 200 and 700 hPa upstream of the mountains passing through a 260-hPa layer just above the surface in the lee. The initial conditions for the simulations conducted in this study are horizontally homogeneous and based upon the upstream 1200 UTC 11 January 1972 Grand Junction, Colorado, sounding shown in Fig. 1 and identical to that used by Doyle et al. (2000, 2007). The sounding contains several distinct shear layers, strong low-level cross-mountain winds ($\sim 15$ m s$^{-1}$), and a well-defined stable layer above the mountain crest between 3 and 5 km, characteristics favorable for downslope windstorms (e.g., Brinkman 1974; Durran 1986). The cross-mountain wind speed approaches zero near 21 km indicative of the presence of a critical level, which inhibits wave propagation above this level and mitigates possible wave reflection from the top boundary.

A 20-member ensemble initialized with small perturbations to the sounding described above is performed. Random perturbations of a maximum $\pm 1$ K and $\pm 1$ m s$^{-1}$ are made to each temperature and wind speed data point in the sounding for each member, as indicated in Fig. 1 (each level is perturbed independently, so there are no vertical correlations imposed). The sounding perturbations are within the limits of expected radiosonde errors. In this method, the perturbations are horizontally homogeneous and will not sweep through the domain during the integration. The homogeneous perturbations have the additional conceptual advantage that they can be interpreted as nearly indistinguishable profiles that may be used as input for a gravity wave drag parameterization, such as that used in GCMs. The perturbation temperature field at the 3-h simulation time for one of the ensemble members, computed as the difference between the ensemble member and the control simulation, is shown in Fig. 2a. The perturbation field indicates that the largest growth takes place in the lower stratosphere associated with the wave breaking, which will be discussed in more detail in the following section. For comparison, a second type of perturbation method is explored using ran-
dom perturbations that are uncorrelated in space and time and applied to the initial state and upstream boundary. The maximum amplitude of the random perturbations are $\pm 10 K$ and $\pm 1 m s^{-1}$ and are temporally invariant at the upstream boundary. These uncorrelated random perturbations yield a similar structure as the homogenous method, as illustrated for the 3-h potential temperature perturbation field shown in Fig. 2b. Finally, a third method using random spatially uncorrelated perturbations of a maximum 10 m to the terrain height produces a similar pattern, although the final-time perturbation magnitude is reduced by approximately an order of magnitude (not shown). In this study, we used the homogeneous perturbations to initialize the ensemble based on the advantages discussed above, although the three methods appear to yield broadly similar results.

3. Results

a. Control simulation

The potential temperature and cross-mountain wind speed for the control simulation ($h_m = 2000 m$) at 3- and 4-h integration times are shown in Fig. 3. In the stratosphere, two deep regions of wave breaking are apparent, near approximately 10–15 km and approximately 16–20 km, with a vertical wavelength of approximately 3–4 km. The model-predicted turbulent kinetic energy indicates layers of stratospheric wave breaking with an upstream vertical tilt. At both simulation times, undulating secondary waves analogous to trapped waves or an undular bore are present in the stratosphere along strongly stratified layers and are positioned beneath the respective wave-breaking layers. A strong leeside windstorm is present near the surface, positioned beneath a layer of tropospheric wave breaking that extends from 5 to 9 km. The lee-slope wind speed maximum is 75 and 85 m s$^{-1}$ at the 3- and 4-h integration times, respectively. A well-defined internal hydraulic jump, as evident in the potential temperature and velocity fields, is positioned near the base of the lee slope at the 3-h integration time and the jump propagates an additional 20 km downstream in the following hour of the simulation. In general, aside from the location of the hydraulic jump, the basic characteristics of the flow field are quite similar at 3 and 4 h. The control simulation has many characteristics in common with those of the 11 nonhydrostatic models presented in the Doyle et al. (2000) intercomparison study.

The control simulation reproduces the salient characteristics of the gravity wave-breaking event analyzed by Klemp and Lilly (1975) and Lilly (1978) reasonably well, including the wind speed maximum in the lee, the hydraulic jump, the midtropospheric cross-mountain wind component minimum in the lee, and the weak stability layer associated with the midlevel gravity wave steepening and breaking. However, the simulated hydraulic jump extends to a greater altitude and has a steeper slope than indicated by the aircraft observations. Also, the observational analysis does not support the model-simulated downward tropopause depression associated with the breaking superpositioned over the low-level hydraulic jump. These differences between the simulation and observational analyses may arise because of the simplicity of the model architecture, such
as the inviscid and two-dimensional configuration, as well as the simplified model initial state and topography representation. Simulations conducted with surface friction by Doyle et al. (2000) using the identical sounding considered here indicate that the leeside downslope windstorm is diminished and the hydraulic jump movement is restricted to a position closer to the mountain. They also found that three dimensionality acts to decrease the overall upper-level wave-breaking activity, in part because the flow is able to split around or pass over the topography, in contrast to two-dimensional flow that is either blocked or is forced over the obstacle. It should be noted that these simplifications were also present in the multimodel intercomparison discussed in Doyle et al. (2000). Despite these assumptions, the overall characterization of the flow is quite realistic.

### b. Ensemble simulations

Several suites of ensemble simulations are conducted with varying mountain heights of 100, 500, 1000, 1500, 1750, and 2000 m. The motivation for performing these ensemble simulations with differing mountain heights is to explore aspects of gravity wave and windstorm predictability in the various regimes of response. We refer to the following regimes as the linear, small-amplitude wave, large-amplitude wave, bifurcation, and wave-breaking regimes corresponding to $h_m/\Delta H = 100, 1000, 1500, 1750,$ and 2000 m, respectively. Linear wave theory has been previously addressed in the literature (e.g., Smith 1980, 1989) and the rationale for regime diagrams describing mountain waves and topographic flows is discussed in Smith (1989), Smith and Grønås (1993), and Ólafsson and Bougeault (1996). The simulations conducted with $h_m = 1750$ m suggest that the mountain waves are near the cusp of breaking, hence we refer to this ensemble as the bifurcation regime.

The ensemble simulations using $h_m = 100$ m exhibit little variance after 4 h (not shown) with the gravity wave perturbation magnitudes and locations being very similar. The final time perturbation magnitudes are similar to the initial state perturbations with little growth exhibited. In this linear regime, the wave characteristics appear to be quite predictable. Although the
wave amplitudes are larger in the $h_m = 500$-m ensemble, the ensemble variance remains small with respect to the windstorm and wave characteristics.

As the mountain height is increased, the ensemble variance increases substantially. A summary of the results from the ensembles performed with $h_m = 1000$, 1500, 1750, and 2000 m at the 4-h simulation time is shown in Fig. 4. In the small-amplitude wave regime, little spread is apparent in the isentropes (Fig. 4a), and the variance of potential temperature (Fig. 4b) and cross-mountain wind speed (Fig. 4c) is less than 25 K$^2$ and 25 m$^2$ s$^{-2}$, respectively. The ensemble shows small variations in the positions and amplitudes of the lee-wave crests and troughs. As the mountain height is increased to 1500 m, the ensemble spread increases as apparent in the isentropic spaghetti diagram (Fig. 4d). The wave troughs and crests are no longer coincident with regard to position or amplitude. The 320-K isentropes indicate at least one member attains a near-neutral lapse rate, presumably associated with the onset of wave breaking. The ensemble potential temperature variance (Fig. 4e) is relatively small and exhibits modest maxima in the vicinity of vertically propagating waves in the stratosphere and several very weak localized maxima associated with the lee waves. The $u$-wind component variance (Fig. 4f) exhibits a local maximum along the lee slope associated with variability in the downslope windstorm and a broad region of large variance aloft.

The ensemble performed using a mountain height of 1750 m exhibits the largest variability of all the simulations conducted. The 304-K isentrope (Fig. 4g) suggests that approximately one-half of the ensemble members are in a large-amplitude trapped wave state and the remaining members produce a propagating hydraulic jump associated with the downslope windstorm. Since the distribution of the low-level flow is basically bimodal with some members in a lee-wave state and others in a hydraulic jump state, the perturbation growth appears to be nonlinear and the ensemble mean does not provide a proper characterization of the flow. For example, the mean isentropes in the lower troposphere (Figs. 4g,h) exhibit neither a trapped wave nor hydraulic jump feature; rather, a blend of the two states exists. In this bifurcation regime, large variances are apparent in the region of the first trapped wave and hydraulic jump (Fig. 4h), as well as along the lee slopes associated with the downslope windstorm (Fig. 4i). In the midtroposphere, large variability exists in the wave-breaking region positioned above the lee slope in the 5–9-km layer (see control simulation in Fig. 3b), particularly apparent in the spread of the 320-K isentrope. The largest ensemble $u$-component variance is present in the upper troposphere above and just downstream of the hydraulic jump or initial lee wave (Fig. 4i) associated with gravity wave breaking and in regions of lower-stratospheric wave breaking, coincident with the $\theta$-variance maxima (Fig. 4h). In contrast, the transition between gravity wave regimes associated with flow over smaller mountains is not subject to a well-defined threshold response. Thus, the transitions for these regimes occur more gradually and are consistent with relatively small ensemble spread (e.g., Figs. 4a–f).

All of the members in the ensemble that used a 2000-m mountain height contain a low-level internal hydraulic jump signature in the lee (Fig. 4j). Surprisingly, there is little variance among the members in the hydraulic jump position and amplitude, as apparent in the 304-K isentrope, although there does appear to be a bimodal distribution in the height of the 320-K isentrope in the well-mixed wave-breaking region. A local $u$-variance maximum (Fig. 4i) is present at the surface along the lee slope near the hydraulic jump, although it is reduced by 2–3 times relative to the bifurcation regime ensemble (Fig. 4i). Aloft, the variance in the wave-breaking regions in the upper troposphere and lower stratosphere are reduced as well (Figs. 4k,l) relative to the $h_m = 1750$-m ensemble, in spite of the wave amplitudes being larger in the 2000-m mountain ensemble. For example, the 356- and 400-K mean ensemble isentropes exhibit larger amplitudes and more vigorous wave breaking in the $h_m = 2000$-m ensemble (Fig. 4j) than in the 1750-m mountain ensemble (Fig. 4g).

Skillful forecasts of topographically forced flows, such as downslope winds, have been previously speculated to be achievable with reasonably accurate depiction of the large-scale conditions (e.g., Mass et al. 2002). In the ensemble simulations performed in this study, small deviations in the large-scale conditions are applied such that the perturbations are within conventional measurement errors. Very large ensemble variance in the maximum wind speed is apparent in the various gravity wave regimes. A summary of the variation of the maximum leeside wind speed at the lowest model level, 100 m, for each ensemble member as a function of the mountain height at the 4-h simulation time is shown in Fig. 5. Relatively little range in the maximum wind speed values is apparent for the ensembles that used a low mountain height, namely, the $h_m = 100$, 500, and 1000 m. The range of the maximum leeside wind speed is largest for the $h_m = 1500$- and 1750-m ensembles, approximately 30 and 25 m s$^{-1}$, respectively. The ensemble that used the highest mountain, 2000 m, attains the largest mean maximum wind speed, $\sim$85 m s$^{-1}$, yet this ensemble exhibits a range
Fig. 4. Ensemble spaghetti diagrams of potential temperature (K) for the 304-, 320-, 336-, and 400-K contours for different mountain heights corresponding to $h_m = (a) 1000$ m, (d) 1500 m, (g) 1750 m and (j) 2000 m for the 4-h simulation time. The ensemble mean is shown by the dashed line, which is yellow for 304 K and black for the other mean isentropes. The ensemble mean potential temperature (contours every 8 K) and variance (color shading interval 25 K$^2$) are shown for $h_m = (b) 1000$ m, (e) 1500 m, (h) 1750 m, and (k) 2000 m for the 4-h simulation time. The corresponding ensemble mean $u$ (contours every 8 m s$^{-1}$) and variance (color shading interval 25 m$^2$ s$^{-2}$) are shown for $h_m = (c) 1000$ m, (f) 1500 m, (i) 1750 m, and (l) 2000 m.
among the members of only 11 m s\(^{-1}\). An additional ensemble simulation conducted with \(h_m = 2250\) m indicates a 15 m s\(^{-1}\) spread of the leeside wind speed maximum, which is significantly less than the range attained for the \(h_m = 1500\) - and 1750-m ensembles. The fact that the leeside wind speeds are greatest in the \(h_m = 2000\) - and 2250-m ensembles and yet exhibit significantly less spread than the \(h_m = 1500\) - and 1750-m ensembles, provides further justification for the notion of predictability degradation near regime boundaries. These results indicate how relatively small uncertainties in the large-scale conditions result in significant uncertainty in the downslope wind speed for the large-amplitude wave and bifurcation regimes.

c. Wave momentum flux

The representation of the vertical flux of horizontal momentum due to orographic effects has been shown to be important for the large-scale general circulation (e.g., Palmer et al. 1986; Lott 1995; Kim et al. 2003). The momentum flux is also a proxy for the wave activity. The momentum flux is computed for each ensemble member as follows:

\[
M_x = \rho \int_{-\infty}^{\infty} u'w' \, dx, \tag{2}
\]

where the primes are deviations from the two-dimensional domain average and \(w\) is the vertical wind component. Vertical profiles for each ensemble member corresponding to the \(h_m = 1500\), 1750-, and 2000-m mountain heights are shown in Fig. 6. The profiles are generally negative with the largest negative values near the surface. The larger near-surface negative values for the larger mountain heights are consistent with the expectation of a net drag that the topography imparts on the westerly flow. Relatively little variation is apparent in the momentum flux for the 1500-m mountain ensemble simulation above 5 km. The relatively large range in near-surface momentum flux for the \(h_m = 1500\)-case is consistent with the large range of near-surface wind speed maxima shown in Fig. 6. Small positive momentum fluxes may arise in this case because of the presence of trapped mountain waves that extend to the boundary and the finite domain used for the calculations. The 1750- and 2000-m mountain height ensembles exhibit substantial variance between members, particularly in the 3–12-km layer. The momentum flux spread is nearly equivalent for these two ensembles, which suggests a comparable uncertainty in this quantity. Relatively small perturbations in the mean state (e.g., Fig. 1) lead to a large spread in the momentum flux profile (Fig. 6). Some of the variance in the momentum flux profiles may arise because of differences in phase as well as amplitude of the waves. Variations in the momentum flux may also occur because of differences in the evolution of the mean flow, as discussed in Chen et al. (2005). The large wave drag spread arising from nearly identical large-scale conditions, or what could be viewed as nearly equivalent mean states in a GCM grid cell, motivates the application of stochastic parameterization approaches (e.g., Palmer 2001) to gravity wave drag representation in large-scale weather and climate models.

d. Nonlinearity

The degree to which the ensemble simulations are nonlinear can be explored through the calculation of the relative nonlinearity index \(\Theta\) (Gilmour et al. 2001) defined as

\[
\Theta = \frac{\|\delta^+ + \delta^-\|}{0.5(\|\delta^+\| + \|\delta^-\|)}, \tag{3}
\]

where \(\delta^+\) represents an ensemble member initialized with a set of perturbations and \(\delta^-\) denotes the use of the same set of perturbations with the opposite sign. For perturbations that are the same size, a spatial correlation between the positive and negative perturbations of \(-1\) would correspond to \(\Theta = 0\), a correlation of \(0\) would correspond to \(\Theta = 1.41\), and a correlation of \(1\) would correspond to \(\Theta = 2\). For relatively linear growth, one would expect \(\Theta \sim 0\).

The relative nonlinearity index is computed for the linear, large-amplitude wave, bifurcation, and wave-
breaking regimes and the temporal evolution for representative members is shown in Fig. 7a. We use positive and negative perturbations corresponding to the first ensemble member to illustrate the utility of the relative nonlinearity index in this context. The perturbation growth for the 100-m high mountain regime is relatively slow and predominantly linear. The wave-breaking regime and the two members shown for the bifurcation regime show a similar evolution of the nonlinearity index up to approximately 3 h. After the 3-h simulation time, the hydraulic jump member performed with $h_m = 1750$ m exhibited a further increase beyond 1.4 in the nonlinearity index consistent with positively correlated fields. Doyle et al. (2007) used tangent linear and adjoint models to explore nonlinearity issues using the identical sounding. They found that as the mountain height is increased, the tangent linear approximation generally becomes less accurate. Similar to the results from the ensemble simulations performed in this study, the strongest nonlinearity was found to occur for flows very near the wave-breaking threshold, rather than fully within the wave-breaking regime forced by higher terrain.

The ensemble spread of the $u$-wind perturbations based on the mean-square difference between each ensemble member and the control simulation is shown in Fig. 7b for the 4-h simulation time. Little spread is apparent for the relatively low-terrain ensembles in contrast to the large spread in the ensembles using high mountains. The spread is a maximum for the ensembles performed with $h_m = 1750$ m with two clusters apparent, consistent with the notion of a bifurcation in the wave response. A group of five ensemble members are clustered with relatively small perturbations of less than 8 $\text{m s}^{-1}$ corresponding to the trapped wave members. A second cluster is apparent with perturbations of 10–13 $\text{m s}^{-1}$ that are composed of members with responses similar to internal hydraulic jumps.

e. Sensitivity to critical levels

An additional ensemble simulation was performed to explore the sensitivity to the mean state, specifically the imposition of a mean-state critical level. Here we specify a reduction in the mean state wind velocity approaching 0 at an altitude of 10 km (Fig. 1). As mountain waves approach a layer of zero wind velocity or critical level, the vertical wavelength approaches zero and wave action accumulates beneath the critical layer leading to wave breaking and energy absorption (Dörnbrack et al. 1995; Grubišić and Smolarkiewicz 1997). Partial or complete reflection of gravity waves may occur at a critical level when the Richardson number, $R_i = N^2/(\frac{\partial u}{\partial z})^2$, is less than 2 (e.g., Breeding 1971; Wang and Lin 1999). As a result of the wave breaking, energy absorption, and reflection at the critical level, a significant reduction in the wave propagation into the lower stratosphere occurs in the case considered.

The ensemble mean and variance for potential temperature and $u$-wind component are shown in Fig. 8 for the simulation conducted with a mean state critical level at 10 km using $h_m = 2000$ m. The potential temperature ensemble mean indicates a train of large-
amplitude lee waves that extend downstream from the mountain (Fig. 8a) in contrast to the hydraulic jump apparent in the control ensemble performed with $h_m/H_1 = 2000$ m (Fig. 4k). The potential temperature and wind velocity (Fig. 8b) variance is larger in the lower troposphere than in the $h_m/H_1 = 2000$ m control ensemble (Figs. 4k,l). The spread in the maximum wind speed at the surface is 13 m s$^{-1}$ and similar to the control ensemble (Fig. 5). However, the $u$-variance maximum near the surface is similar to that of the variance maximum in the bifurcation ensemble (Fig. 4i), which was larger than any of the other simulations. The variance is considerably reduced in the lower stratosphere as a result of the critical level absorption present near the tropopause. Thus, it appears that the critical level is enhancing predictability at higher altitudes above the critical level, but decreasing it near the surface.

4. Conclusions

In this study, we have explored the predictability of mountain waves, wave breaking, and downslope windstorms using a high-resolution nonhydrostatic model.
applied in a two-dimensional configuration. Ensemble simulations are performed using an initial state derived from an upstream sounding for the well-known 11 January 1972 Boulder, Colorado, windstorm event. Random, horizontally homogeneous perturbations to the wind and temperature fields are combined with the upstream sounding to initialize the ensemble. Other perturbation methods such as nonhomogenous random perturbations and random perturbations to the terrain result in similar patterns of perturbation growth.

The control simulation, conducted with a mountain height of 2000 m, exhibits a strong leeside windstorm of 85 m s\(^{-1}\) that is present with a deep internal hydraulic jump downstream of the wind speed maximum. Wave breaking is apparent above the mountain lee slopes in the midtroposphere and in numerous layers in the stratosphere that spread upstream and downstream of the mountain, vertically bounded by strongly stratified layers that contain secondary gravity waves. Overall the control simulation contains many of the characteristics observed by the research aircraft and simulated in a previous multimodel intercomparison exercise (Doyle et al. 2000).

A series of ensembles are conducted for flow over an idealized two-dimensional mountain of heights that vary from 100 to 2000 m. Mountain waves exhibit multiple regimes with regard to perturbation growth and sensitivity to the initial state. In the linear regime, which is valid at the small mountain height limit, the simulations exhibit little ensemble spread and the growth rate of the perturbations is small. The perturbations also retain their linear properties throughout the simulation. As the mountain height is increased, small- and moderate-sized mountain waves are excited in the wave regime that exhibits trapped wave characteristics in the lower troposphere and an absence of wave breaking in the lower stratosphere. The spread in the maximum leeside wind speed at the lowest model level varies by less than 5 m s\(^{-1}\) in the linear and trapped wave regimes for mountain heights of 100, 500, and 1000 m.

At the regime boundary between mountain waves and low-level wave breaking, the ensemble variance is the largest. In this regime, a bifurcation behavior of the ensemble is apparent with about one-half of the members exhibiting trapped wave characteristics in the low levels and the other half revealing a hydraulic jump and large-amplitude breaking in the lower stratosphere. The ensemble spread in this bifurcation regime is substantially larger than in the mountain-wave or wave-breaking regimes as a result of this bimodal response. The ensemble spread of the maximum leeside wind speed at the lowest model layer is over 25 m s\(^{-1}\) for the mountain-wave and bifurcation regimes \((h_m = 1500\) and 1750 m), which is considerable larger than the 11 m s\(^{-1}\) spread for the \(h_m = 2000\)-m ensemble, which attained the strongest downslope winds. Representative hydraulic jump members selected from the bifurcation and wave-breaking regimes appear to exhibit the greatest nonlinearity. The wave momentum flux exhibits large spread for the bifurcation and wave-breaking regimes, which has implications for gravity wave drag parameterization. Relatively small uncertainties in the initial state result in significant spread in the strength of the downslope wind speed, wave breaking, as well as wave momentum flux. The transition between gravity wave regimes associated with flow over smaller mountains is not subject to a distinct threshold response and exhibits less ensemble spread, suggestive of generally greater predictability.

The simplifying assumptions of two dimensionality and free-slip lower boundary considered here may limit the generality of the results. Future studies of mountain waves and topographic circulations should be conducted with no-slip boundary conditions and more realistic boundary layers to better understand the predictability implications. Additionally, the differences in error growth for two- and three-dimensional topographic circulations need to be considered.

Quantifying mesoscale predictability continues to be a major challenge. The results of this study suggest that the predictability of wave breaking, downslope windstorms, and mountain-wave-induced turbulence are limited by initial condition sensitivity, particularly for cases when the simulated states are close to regime boundaries, such as at the wave-breaking threshold. Relatively small perturbations within observational error limits, can lead to rapid perturbation growth, and suggests that predictability may be indeed limited for strongly forced topographic flows, even when the synoptic scale is well observed. The bimodality of the mountain-wave response exhibited in the bifurcation regime brings into question the appropriateness of commonly computed ensemble statistics, such as the ensemble mean. This may be especially problematic for the mesoscale, which encompasses many phenomena that have threshold characteristics such as convection, terrain flows, and mesoscale instabilities. The transition across regimes boundaries for various mesoscale threshold phenomena that are distinct from a theoretical perspective may be manifested as a finite-width or blurred transition zone from a practical predictability standpoint. Thus, prediction of wave breaking and associated turbulence, which are of significance for the aviation community, may only be achievable through probabilistic approaches.
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