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ABSTRACT

Conditional nonlinear optimal perturbation (CNOP), which is a natural extension of the linear singular vector into the nonlinear regime, is proposed in this study for the determination of sensitive areas in adaptive observations for tropical cyclone prediction. Three tropical cyclone cases, Mindulle (2004), Meari (2004), and Matsa (2005), are investigated. Using the metrics of kinetic and dry energies, CNOPs and the first singular vectors (FSVs) are obtained over a 24-h optimization interval. Their spatial structures, their energies, and their nonlinear evolutions as well as the induced humidity changes are compared. A series of sensitivity experiments are designed to find out what benefit can be obtained by reductions of CNOP-type errors versus FSV-type errors. It is found that the structures of CNOPs may differ much from those of FSVs depending on the constraint, metric, and the basic state. The CNOP-type errors have larger impact on the forecasts in the verification area as well as the tropical cyclones than the FSV-types errors. The results of sensitivity experiments indicate that reductions of CNOP-type errors in the initial states provide more benefits than reductions of FSV-type errors. These results suggest that it is worthwhile to use CNOP as a method to identify the sensitive areas in adaptive observation for tropical cyclone prediction.

1. Introduction

Typhoons or hurricanes originating from tropical cyclones are some of the most destructive disasters and cause great loss of lives and property each year throughout subtropical regions of the world. In addition to studies of their structures, intensities, background circulations, and their relationships to climate events (Chan 1995, 2000), much effort has been put into improving their prediction. Based on predictability studies of tropical cyclones, it was realized that the improvement of tropical cyclone track and intensity forecasting depends on an accurate initial analysis and the assimilation of observations (Riehl et al. 1956; Simmons et al. 1995; Froude et al. 2007). Consequently, significant efforts have been devoted to improve tropical cyclone prediction through supplemental observations in data-sparse areas.

Many organizations have conducted experiments to gather observations in and around tropical cyclones, for example, the synoptic flow experiments of the National Oceanic and Atmospheric Administration (NOAA) Hurricane Research Division (HRD) between 1982 and 1996, and the World Meteorology Organization (WMO) Special Experiment Concerning Typhoon Recurvature.
and Unusual Motion in 1990 (SPECTRUM-90). Studies showed that extensive observations obtained in the general region around the cyclones did not conclusively improve forecasts more than observations only obtained in particular regions (Franklin and DeMaria 1992; Aberson 2002). Naturally the researchers have thought of using adaptive observations for tropical cyclones.

The adaptive observation strategy, which places observations in specific regions according to weather events such as tropical cyclones, is viewed as an effective way to improve the skill of numerical weather prediction. Many studies have shown that adaptive observations can, on average, reduce analysis errors more than the same number of nonadaptive observation, and fewer adaptive observations can reduce the error by the same amount as a larger number of nonadaptive observations (Morss et al. 2001; Kim et al. 2004). This idea has been implemented in a set of field experiments, including the Fronts and Atlantic Storm-Track Experiment (FASTEX; Snyder 1996; Joly et al. 1997), the North Pacific Experiment (NORPEX; Langland et al. 1999a), Dropwindsonde Observations for Typhoon Surveillance near the Taiwan Region (DOTSTAR; Wu et al. 2005), Winter Storm Reconnaissance (WSR; Szunyogh et al. 2000, 2002), and the North Atlantic Observing System Research and Predictability Experiment Regional Campaign (NATReC; Petersen et al. 2006). Results from these field experiments showed that the forecast skills are improved by assimilation of targeted observations (Langland et al. 1999b; Langland 2005; Buizza et al. 2007).

A key issue in adaptive observation is the determination of the sensitive areas where additional observations are expected to yield a better forecast than observations taken in other regions. Currently, several strategies have been applied to identify the sensitive areas. One strategy is based on the adjoint technique, such as singular vectors (SVs; Palmer et al. 1998), adjoint sensitivities (Ansell and Mass 2006), and the adjoint-derived sensitivity steering vector (ADSSV; Wu et al. 2007). The adjoint of the forward tangent propagator of the numerical model is required for their calculation, so they are also named adjoint-based sensitivities (Kim et al. 2004). Another strategy is ensemble based, for example, the ensemble transform (Bishop and Toth 1999), the ensemble Kalman filter (Hamill and Snyder 2002), and the ensemble transform Kalman filter (Bishop et al. 2001). In the sensitive area determination, other methodologies are also suggested, for example, the quasi-inverse linear method (Pu et al. 1997), and the breedinglike method mentioned in the study of Lorenz and Emanuel (1998).

Most of the methodologies mentioned above have been tested in field experiments. The results are promising (Bergot et al. 1999; Langland et al. 1999a; Szunyogh et al. 2000; Toth et al. 2002). However, it is difficult to distinguish which of the methods is best. The reason is that usually two or more strategies are used simultaneously in the same field experiment and the targets they define are often mixed, or different approaches are adopted for different weather processes (Bergot 1999).

In particular, determining which approach in tropical cyclone adaptive observations is optimal in view of the improvement of forecast skill is an interesting issue in this research area (Wu et al. 2007).

Even though SV has the limitation of linear approximation, it has been adopted in the study of atmospheric predictability and adaptive observations with the energy norm (Bergot et al. 1999; Reynolds and Rosmond 2003), since the spectra of the dominant SVs with respect to energy metric is consistent with the spectra of estimates of the analysis error variance (Palmer et al. 1998).

In FASTEX and NORPEX, sensitive areas for special observations were defined by the initial-time structure of leading SVs using a norm based on the perturbation energy. Langland et al. (1999a) and Gelaro et al. (1999) found a 10% 2-day mean error reduction during NORPEX and a 22%–38% error reduction in FASTEX. This suggests that the strategy of SVs for targeted observing is feasible despite the limited success. In the recent work of Buizza et al. (2007), both data-injection and data-denial experiments for a very large number of cases (315 cases, selected to cover both summer and winter) were examined. Their conclusion is that observations taken in the SV-target areas are more valuable than observations taken in random areas, and the impact of the observations is different depending on the region, on the season, and on the baseline observing system. This may be related to the basic-state sensitivity of SVs (Puri et al. 2001; Buizza et al. 1997; Barkmeijer et al. 2003).

Since SV is based on linear approximation, the initial perturbation must be sufficiently small and the time period must not be too long when it is used. Because of these deficiencies of SV and because the motions of the atmosphere and oceans are dominated by complicated nonlinear systems, Barkmeijer (1996) modified the technique for linearly fastest-growing perturbations and attempted to construct fast-growing perturbations for the nonlinear regime. However, he recognized that this technique did not always result in the nonlinearly fastest-growing perturbation. A similar procedure was described in Oortwijn and Barkmeijer (1995) and Oortwijn (1998). Mu and Duan (2003) proposed a new method called conditional nonlinear optimal perturbation (CNOP), which acquires the maximum value of the cost function.
Based on physical problems. This method has been used in some research fields, such as ENSO predictability (Duan et al. 2004; Duan and Mu 2006; Mu et al. 2007b), the sensitivity and the passive variability of ocean thermohaline circulation (THC; Mu et al. 2004; Sun et al. 2005), the double-gyre ocean circulation (Terwisscha van Scheltinga and Dijkstra 2008), the nonlinear behavior of baroclinic unstable flows (Rivière et al. 2008), as well as the ensemble forecasting (Mu and Jiang 2008). Primarily, it has been used in identifying the sensitive area for heavy rainfall (Mu et al. 2007a). The results suggest that the forecast results benefit more from the reductions of the CNOP-type initial errors than the reductions of the (first singular vector) FSV-type errors. This indicates that it is feasible to use CNOP for the determination of sensitive areas in adaptive observation.

In this paper, we utilize the fifth-generation Pennsylvania State University–National Center for Atmospheric Research (PSU–NCAR) Mesoscale Model (MM5) and its adjoint model to study the applicability of conditional nonlinear optimal perturbation to tropical cyclone–targeted observation. Norms of kinetic energy and dry energy are used. The initial moist perturbations are set to zero, but the final changes on specific humidity due to the initial errors of other variables are inspected. Additionally, the benefits from the reductions of initial uncertainties are examined.

The structure of this paper is as follows. Section 2 provides an introduction to CNOP and FSV, and section 3 introduces the model and the experiment design. Section 4 gives a comparison of CNOPs and FSVs characterized by different metrics. Section 5 describes the differences between the developments of CNOPs and FSVs, including the subsequent specific humidity changes. Section 6 presents the results of sensitivity experiments. A brief summary and discussion are given in the final section.

2. The method: CNOP and FSV

Suppose we have the following model:

\[
\begin{align*}
\frac{\partial X}{\partial t} + F(X) &= 0, \\
X_{t=0} &= X_0
\end{align*}
\]  
(1)

where \(X\) is the state vector of the model with initial value \(X_0\), and \(F\) is a nonlinear partial differential operator. The solution of (1) can be expressed in discrete form:

\[
X_t = M(X_0),
\]  
(2)

where \(M\) is a nonlinear propagator, and \(X_t\) is the value of \(X\) at time \(t\).

To measure the development of \(X_0\), appropriate norms must be chosen. In discrete form, this is equivalent to choosing symmetric positive definite matrices \(C_1\) and \(C_2\). An initial perturbation \(\delta X_0\) of vector \(X_0\) is called CNOP if and only if

\[
J(\delta X_0) = \max_{\delta X_0, \delta X_0 = \beta} J(\delta X_0),
\]  
(3)

where

\[
J(\delta X_0) = [PM(X_0 + \delta X_0) - PM(X_0)]^T \times C_2 [PM(X_0 + \delta X_0) - PM(X_0)].
\]  
(4)

and \(\delta X_0^T C_1 \delta X_0 \leq \beta\) is a constraint condition of initial perturbations with the presumed positive constant \(\beta\) representing the magnitude of the initial uncertainty. The first guess of the initial perturbation \(\delta X_0\), which is usually taken as the difference between the model outputs at 2 times, should be adjusted to satisfy the constraint condition \(\delta X_0^T C_1 \delta X_0 \leq \beta\). Here \(P\) is a local projection operator and takes value 1 (0) within (without) the targeted region. Note that the norms used in the cost function and the initial constraint condition may be the same, depending on the physical problem. It is clear that the CNOPs depend on the choice of \(\beta, P, C_1,\) and \(C_2\). Some sensitivity studies for \(\beta, C_1,\) and \(C_2\) are presented in following sections.

CNOP is the global maximum of cost function \(J\). There exists a possibility that \(J\) attains its local maximum in a small neighborhood of a particular point in phase space. Such an initial perturbation is called the local CNOP (Mu and Zhang 2006). In some physical problems the CNOP and the local CNOP possess clear physical meanings. For example, Duan et al. (2004) revealed that the CNOP (local CNOP) acquired on the climatological background state is most likely to evolve into an El Niño (La Niña) event and acts as the optimal precursor for El Niño (La Niña).

Suppose that the initial perturbation \(\delta X_0\) is sufficiently small and the integration time interval is moderate, then the development of \(\delta X_0\) in discrete form can be approximated by

\[
\delta X_t = L(\delta X_0),
\]  
(5)

where \(L\) is the forward tangent propagator. The term \(\delta X_t\) is the linear development of \(\delta X_0\) at time \(t\). According to Barkmeijer et al. (2003), the first singular vector \(\sigma_1\) of \(L\) satisfies the following (with respect to the norms \(C_1\) and \(C_2\):

\[
\sigma_1^2 = \max_{(\delta X_0^T C_1 (\delta X_0) > 0)} \frac{[L(\delta X_0)]^T C_2 [L(\delta X_0)]}{(\delta X_0)^T C_1 (\delta X_0)}.
\]  
(6)

Additionally, if \(v_1\) is the first singular vector of \(L\), then
\[(C_1)^{-1}(LC_2L)v_1 = \sigma_1^2 v_1, \quad (7)\]

where superscript \(-1\) denotes the inverse of the matrix.

In this paper, we will numerically solve the following nonlinear optimization problem to obtain the FSV:

\[J(\delta X_0^*) = \max_{\delta X_0^*} J(\delta X_0), \quad (8)\]

where

\[J(\delta X_0) = [PL(\delta X_0)]^T C_2 [PL(\delta X_0)]. \quad (9)\]

Thus, both CNOP and FSV can be obtained by using the same optimization algorithm to facilitate comparison, and for further simplicity \(C_1 = C_2 = C\). The definition of FSV in (8) and (9) can also be found in Ehrendorfer and Errico (1995).

If \(C_{ke}\) represents the metric of kinetic energy and \(C_{te}\) the total dry energy, then in a continuous expression

\[(\delta X_0)^T C_{ke} (\delta X_0) = \frac{1}{D} \int_D \int_0^1 (u'^2 + v'^2) \sigma \, du \, dD, \quad (10)\]

\[(\delta X_0)^T C_{te} (\delta X_0) = \frac{1}{D} \int_D \int_0^1 \left( \frac{c_p}{T_r} T'^2 + \frac{R_a T_r}{p_r} \left( \frac{p_r}{p_r} \right)^2 \right) \sigma \, du \, dD, \quad (11)\]

where \(c_p\) and \(R_a\) are the specific heat at constant pressure and the gas constant of dry air, respectively (with numerical values of 1005.7 J kg\(^{-1}\) K\(^{-1}\) and 287.04 J kg\(^{-1}\) K\(^{-1}\)). The reference parameters are the following: \(T_r = 270\) K and \(p_r = 1000\) hPa. Here \(u', v', T', p_r\), which are components of the state vector, are the perturbed zonal and meridional wind components, temperature, and surface pressure, respectively. The integration extends over the full domain \(D\) and the vertical direction \(\sigma\).

In this study, the calculations are done for \(C = C_{ke}\) and \(C = C_{te}\). It is noted that when \(C_{te}\) is concerned, \(\delta X_0\) is only composed of \(u', v'\); namely, initial changes are only allowed for \(u\) and \(v\). Similarly, when \(C_{te}\) is used, initial changes are only made on \(u, v, T,\) and \(p_r\). It is pointed out that specific humidity, which neither appears explicitly in the cost function nor in the initial perturbations, would also vary with time since the water vapor equation is included in the model. This is also discussed in section 5.

3. Experiment design

a. The model and the optimization algorithm

Our study utilizes a component of MM5 (Dudhia 1993), which includes the nonlinear MM5, its tangent linear model (TLM), and corresponding adjoint model (Zou et al. 1997). The following physical parameterizations are used: dry convective adjustment, grid-resolved large-scale precipitation, the high-resolution PBL scheme, and the Kuo cumulus parameterization scheme. The initial and boundary conditions are from the National Centers for Environment Prediction (NCEP) Global Forecasting Systems (GFS) global reanalysis (1° × 1°) interpolated to the MM5 grids. The horizontal resolution is 60 km, and the vertical range is divided into 11 sigma levels, with the top pressure at 100 hPa. The model domains, which change with cases, are shown in the corresponding figures, where verification areas are marked by boxes.

The optimization algorithm employed is the spectral projected gradient 2 (SPG2) (Birgin et al. 2001), which calculates the minimum value of a function of several variables subject to box or ball constraints. During the calculation, the cost function implemented is \(J_t(\delta X_0) = -J(\delta X_0)\), with the same initial constraint condition \(\delta X_0^T C_1 \delta X_0 \leq \beta\). The gradient of the cost function with respect to the initial perturbation is needed for the SPG2 algorithm, and the adjoint model of MM5 is used to calculate the gradient efficiently. Here we emphasize that according to the theory of nonlinear optimization, the adjoint model is applicable for calculating the gradient of the cost function in spite of its linearity, which is similar to the case of four-dimensional variational data assimilation. Generally speaking, a numerical algorithm only yields a local maximal or minimal point. To obtain CNOP, we have tried as many initial perturbations as possible. If they all converge to one maximal point, the maximal point will be regarded as CNOP. If there are several local maximal points, we compare their values of cost function, and choose the largest one as CNOP, others as local CNOPs.

b. Description of cases

Case A is Northwest Pacific Tropical Cyclone Matsa that occurred in August 2005, which caused great loss in China. The ability of MM5 to simulate this case accurately is checked based on a 36-h simulation initialized at 1200 UTC 4 August 2005. Figure 1a shows that the model storm (dashed) moves along the observed track (solid), but slightly faster than as is observed with an error less than 100 km, so it is feasible to use the MM5 model in this study. The model domain shown in Fig. 1a covers 55 latitude × 55 longitude grids.

Case B is Tropical Cyclone Meari (2004) also in the Northwest Pacific Ocean. Figure 1b shows the 36-h simulation initialized at 1200 UTC 25 September 2004. The model storm track (dashed) runs parallel and
south of the observed track (solid). The errors are also acceptable. The model domain is $51 \times 55$ grids (Fig. 1b).

Case C is Tropical Cyclone Mindulle, which also occurred in 2004 and in the Northwest Pacific Ocean. Its 36-h simulation is displayed in Fig. 1c. The initialization time is 1200 UTC 27 June 2004. The simulated track (dashed) and the observed track (solid) are similar. The model domain size is $41 \times 51$ grids (Fig. 1c).

The track data are from the Chinese Meteorology Administration (CMA). The optimization time interval is 24 h for all three cases with Matsa initialized at 0000 UTC 5 August 2005, Meari at 0000 UTC 26 September 2004, and Mindulle at 0000 UTC 29 June 2004.

c. Choice of constraint values

As mentioned in section 3a, an initial constraint condition $\delta X|C_1, \delta X_0 \leq \beta$ is employed during the optimization. It is clear that the constraint condition is not only determined by the metric $C_1$ but also by the value $\beta$. Hence, the choice of $\beta$ is of importance. To choose an appropriate $\beta$, a series of experiments with different values should be carried out. In this paper, we follow the rules that when the magnitudes of temperature and wind components of CNOP are comparable with the counterparts of current analysis errors, the constraint value $\beta$ is regarded as acceptable.

The results of the Matsa case with $\beta = 0.0003 \text{ J kg}^{-1}$ and $\beta = 0.03 \text{ J kg}^{-1}$ characterized by total dry energy metric are presented in Fig. 2. The optimization time interval is 24 h.

Figure 2 displays the temperature and wind field on the $\sigma = 0.7$ level (approximately 800 hPa). The pattern of CNOP is similar to that of FSV with $\beta = 0.0003 \text{ J kg}^{-1}$ (Figs. 2a,b). This indicates that the linear assumption is tenable with such a constraint and within this time period. When increasing the constraint to $\beta = 0.03 \text{ J kg}^{-1}$, the pattern of the CNOP becomes much different from that of FSV (Figs. 2c,d) as well as that of CNOP with $\beta = 0.0003 \text{ J kg}^{-1}$, while the FSV pattern remains the same and the FSVs only differ in magnitude. The studies of other levels come to a unanimous result (figures not shown). In terms of the magnitudes, the maximum of the temperature and wind on the $\sigma = 0.7$ level are about 0.1 K and 0.15 m s$^{-1}$, respectively, with $\beta = 0.0003 \text{ J kg}^{-1}$, while 1 K and 1.5 m s$^{-1}$, respectively, with $\beta = 0.03 \text{ J kg}^{-1}$. Apparently, the latter is closer to the current analysis errors. Moreover, the results with respect to kinetic energy with constraint $\beta = 0.03 \text{ J kg}^{-1}$ are also acceptable (with the maximum about 2 m s$^{-1}$, Fig. 3).

Therefore, the constraint 0.03 J kg$^{-1}$ is used in the Matsa case. Similarly, we chose 0.03 J kg$^{-1}$ for both the Meari and Mindulle cases.

It is worthwhile to mention that other constraint values are also studied (figures not shown), the patterns and magnitudes of the CNOP vary gradually with the change of the constraint value, while the FSV patterns stay the same and the magnitude increases. This shows that the FSV pattern is independent of the constraint value and it only represents the fastest linear development direction, while the CNOP can better depict the development information of initial errors.

4. Comparisons of CNOPs and FSVs

In this section, the patterns of CNOPs and FSVs of a 24-h optimization time interval are studied. As mentioned in section 2, the kinetic energy (KE) and total dry energy (TE) are chosen as metrics. The constraint is the same for the KE metric and the TE metric for convenience of comparisons, although this may result in the magnitudes of CNOP (FSV) using the KE norm being a little larger than that of CNOP (FSV) using the TE norm. Subsequently, the CNOP calculated using the KE norm is marked as KE-CNOP for simplicity. Similarly,
we use notations KE-FSV, TE-CNOP, and TE-FSV, respectively.

The patterns of the wind components of KE-CNOP are similar to those of TE-CNOP (Figs. 3a and 2c), with differences existing mainly in the magnitude. Such similarities are also found in the results for KE-FSV and TE-FSV (Figs. 3b and 2d). Additionally, similarities are found in all three cases (figures with respect to the KE norm for the other two cases are not shown). This is consistent with the result of Palmer et al. (1998). Therefore, in the remainder of this section, we describe the results using the TE norm, and the “TE-” is omitted in TE-CNOP and TE-FSV.

**a. Horizontal structures**

We only present the patterns of CNOP or FSV on the level $\sigma = 0.7$, since the results of comparisons on other levels are similar.

The Matsa case is studied first. The structure of CNOP in this case differs greatly from that of FSV, although both are localized. The wind component of CNOP at the northeast side of the cyclone center exhibits an anticyclonic circle and maximizes at its southwest side where the maximum temperature is located (Fig. 2c). Both the maximum wind and the minimum temperature of FSV appear at the southeast side of the cyclone center and...
correspond in space. For CNOP, there is also a minimum in temperature with local maximum winds at the southeast side of the cyclone center.

The study of Meari comes to a little different result. In this case, the structure of CNOP is similar to that of FSV. From patterns presented on the level $\sigma = 0.7$ (Figs. 4a,b), both the wind components of CNOP and FSV present cyclonic (anticyclonic) circulations at the eastern (western) side of the cyclone center with the maximum located in the northeast, while the temperature components of both CNOP and FSV present an alternating pattern in the northeast to the cyclone center. For CNOP, there are local maxima and minima in temperature located near the cyclone centers, as can be seen in Fig. 4a.

The result for Mindulle is consistent with that of Matsa. That is, there exist considerable differences between the structures of CNOP and FSV. The wind pattern of CNOP presents a wavelike pattern as anticyclone and cyclone interlaced from around the cyclone center to the far outside with clear temperature structures in the northeast (Fig. 4c), while the FSV wind pattern seems more localized near the cyclone center and with alternating circulations only to the northwest of the cyclone center (Fig. 4d). Both the temperatures of CNOP and FSV are spatially consistent with their wind fields. This relation is more apparent in FSV.

Examination of these three cyclones reveals that for CNOP the maximum values of wind and temperature are located at the northeast side of the current cyclone centers. The following possible explanation is proposed. Kitade (1981) suggested that the Jacobian term $[-\mathbf{J}(\psi, \nabla^2\psi)]$, where $\psi$ is the streamfunction, $\mathbf{J}$ is the horizontal Jacobian operator, and $\nabla^2$ is the Laplacian] would produce northwestward movement of the vortex, so it is important in the vortex motion. The investigations of these three cases found that the maximum wind and temperature of CNOP could result in larger nonlinear terms than those induced in FSV (Figs. 5a,b, figures of other cases not shown). This supports the view that CNOP may be more important in tropical cyclone prediction. Corbosiero and Molinari (2003) pointed out that heavy convective activity would impact the tropical cyclone motion, which might suggest another candidate interpretation. Figures 5c,d show that regions of the wind and temperature maximum or minimum of CNOP and FSV, where strong convergence (divergence) are observed at low levels, are contained inside the areas of the strong convergence (divergence) of basic states (figures of other cases not shown). Thus, those areas indicated by CNOP and FSV may be sensible.

b. Vertical structures

Figure 6 shows the vertical distributions of temperature along lines AB (Fig. 2c), CD (Fig. 4a), and EF (Fig. 4c) for the Matsa, Meari, and Mindulle cases, respectively. It is clear that both the CNOPs and FSVs present distinct baroclinic structures. As for Matsa, the slope of CNOP is greater than that of FSV from about 850 hPa to the top while it is a little smaller below 850 hPa. As for Meari, the slopes of CNOP and FSV are comparable.
from the surface to the top with that of CNOP a little larger than that of FSV. As for Mindulle, the CNOPs are more westward tilted than FSVs, which appear to be almost vertical.

These baroclinic structures of CNOP and FSV may partially account for their potential for rapid amplification, as indicated by Ehrendorfer and Errico (1995). Since the slopes of the CNOPs are almost always greater than those of FSVs, it may provide an interpretation for the faster development of CNOP than FSV as indicated in the subsequent sections.

As mentioned in section 2, the local CNOPs may be found when there exists a local maximum in the cost function. During the calculations of the above three cases, we did find local CNOPs for some cases. For example, a local CNOP for Matsa is found with respect to the TE metric, and it shows a pattern opposite to that of CNOP and resembles the pattern of −FSV (figure not shown) to some extent. For comparison, we will examine the developments of the local CNOP and the −FSV in the next section.

In summary, this section demonstrates that the structures of CNOPs may differ greatly from those of FSVs depending on the cases. The wind components of the CNOPs consistently show up as semicircular features within which the temperatures centers are located. For FSV, on the other hand, there is no obvious rule for the patterns found in the three cases as in the CNOP, but the FSVs show more localized structures than the CNOPs. The investigation of vertical structures shows that the CNOPs and FSVs exhibit westward-tilting structures. The CNOPs are tilted more than FSVs, and this partly explains the CNOPs’ larger developments.

FIG. 4. As in Fig. 2, but for the (a),(b) Meari and (c),(d) Mindulle cases with total dry energy norm.
5. The developments of CNOPs and FSVs

In view of prediction uncertainties, it is of interest to investigate the growth of CNOPs and FSVs over the optimization interval. In this section we will study the energy evolution of CNOPs and FSVs as well as the nonlinear development of their structures by the final time. As mentioned in section 2, although there is no initial perturbation of specific humidity when using kinetic or dry energy as the metrics, the perturbations of other variables may also result in changes of the specific humidity at the final time. In the following the wind, temperature, and specific humidity at the final time will be investigated together.

a. Structures

The results of the Matsa case is addressed first. Figures 7a,b show that the patterns of the wind components after the nonlinear development of the KE-CNOPs are similar to those of TE-CNOPs. Figures 7d,e demonstrate a
FIG. 6. The vertical distributions of temperature (K) for CNOP and FSV along (a),(b) line AB in Fig. 2c; (c),(d) line CD in Fig. 4a; and (e),(f) line EF in Fig. 5a.
similarity between the results of KE-FSV and TE-FSV as well.

The temperature changes due to the initial KE-CNOP and TE-CNOP are similar in the verification area as both present a positive center to the northeast side of the verification area, where the specific humidity reaches its minimum (Figs. 7a,b). The temperature changes caused by the KE-FSV are similar to that caused by TE-FSV while the specific humidity changes caused by the KE-FSV and TE-FSV differ greatly in the verification area (Figs. 7d,e). The KE-FSV results in a prominent negative center of specific humidity in the verification area, while there are no evident changes of specific humidity in the results of the TE-FSV.

Fig. 7. For the Matsa case, the final-time changes of specific humidity ($Q$, shaded), temperature (contour), and wind (vector) that develop from initial perturbations of (a) KE-CNOP, (b) TE-CNOP, (c) TE-local CNOP, (d) KE-FSV, (e) TE-FSV, and (f) TE-FSV. The boxes indicate the verification area. The filled circle indicates the position of the current cyclone.
The changes caused by the local CNOP and FSV with respect to TE norm are also examined. Figures 7c,b show that the results of TE-local CNOP differ much from those of TE-CNOP. Also, the development of FSV differs from the development of FSV (Figs. 7f,e), which means that initializing the model with the negative of the FSV does not produce a mirror image solution at the end of the forecast interval, as would be expected for a linear system, but not for the nonlinear model.

In spite of the above-mentioned differences, there also exist some similarities in these results. First, the locations of centers of specific humidity almost coincide with centers of temperature. Second, the CNOP-type perturbations would result in larger specific humidity changes than the corresponding FSV types. Third, large changes of all these variables have occurred in the verification area and the site where the cyclone is currently located. This means that all the perturbations checked here would heavily affect the verification area and cyclogenesis. In other words, CNOP or FSV type errors of initial states are important since they may yield bad forecasts.

For the cases of Meari and Mindulle, the results of KE-CNOPs (figures not shown) resemble those of TE-CNOPs, which are similar to the case of Matsa. Here, we mainly present the results of TE-CNOP, and the “TE-” is omitted in TE-CNOP and TE-FSV later in this section.

Figures 8a,b show that for Meari the development of CNOPs are similar to the development of FSVs. The similarity even exists in the specific humidity field. This may result from the similarity of their initial structures (Figs. 4a,b). In this case, only the wind contains a maximum in the verification area, in which there are also obvious changes of temperature and specific humidity. This suggests that the initial errors considered would have considerable effects on the wind prediction in the verification area and the cyclone, while the temperature and humidity prediction would be relatively less affected.

The results of Mindulle shown in Figs. 8c,d illustrate that a considerable difference exists between the developed CNOP and the developed FSV, though there are prominent wind, temperature, and specific humidity changes in the verification area for both two. As mentioned above, this implies that both CNOP- and FSV-type errors would heavily affect the verification area forecast, but the impacts are different as CNOP (FSV) would result in an overall lower (higher) temperature prediction in the verification area.

The explorations of these two cases also demonstrate that the locations of the centers of specific humidity always coincide with the centers of temperature and the CNOP-type errors would result in larger specific humidity changes than the FSV types.

At the optimization time the westward tilt of CNOP and FSV structures for all three cases are not as obvious as that at the initial time (figures not shown).

b. Energy evolution

Although both CNOP- and FSV-type errors would affect the targeting forecast, the former are more important as they would result in a worse forecast. This is not only reflected by the humidity changes but also by the energy increments.

Figure 9 shows the total dry energy evolutions of CNOPs and FSVs in the verification area during the optimization time interval. It is clear that for Matsa the energy of CNOP develops fastest of all and the local CNOP a little slower, while the energies of FSV and FSV develop slowly. At the final time, the energy of CNOP is about 3 times that of FSV. For Meari, because of the similarities between the CNOP and FSV, the energy evolutions also present similarities except in the final several hours with small differences. Mindulle shows quite different characteristics. In this case, the total dry energy of FSV evolves faster than that of CNOP over the first 12 h, and then decreases, while the total dry energy of CNOP increases slowly first, experiences a small fluctuation, then increases quickly so that at the final time it is nearly 3 times that of the energy of FSV. Here we point out that this does not necessarily imply the intensification of the tropical cyclone. For example, if the tropical cyclone decays, it would cause large perturbation energies. Examinations of the kinetic energy evolution yield similar results (figures not shown).

In the above investigation, it is seen that for all three cases the energy evolution of the CNOP is larger than that of the FSVs at the final time no matter how it changes during the processes. This implies that for all three cases the CNOP-type errors would cause a worse prediction than the FSV type at the final time.

6. Sensitivity experiments

In the previous sections, the patterns of the CNOPs were shown to differ from those of FSVs in some cases, and that the nonlinear development of the CNOPs was seen to be larger than that of FSVs at the final time. In addition, the changes of specific humidity at the final time were more heavily affected by the CNOPs than by FSVs. Therefore, eliminating the CNOP-type errors in the analysis seems important. The goal of this section is to estimate what benefit can be obtained by reductions of the CNOP-type errors versus the FSV-type errors. A series of sensitivity experiments is designed to investigate this question.
First we define the following:

\[
J_1(\delta X_0) = [PM(X_0 + \delta X_0) - PM(X_0)]^T \times C[PM(X_0 + \delta X_0) - PM(X_0)]
\]

(12)

\[
J_2(\delta X_0) = [PM(X_0 + c\delta X_0) - PM(X_0)]^T \times C[PM(X_0 + c\delta X_0) - PM(X_0)]
\]

(13)

where \(c\) is a constant less than 1, and \(\delta X_0\) represents the CNOPs or the FSVs. Here \(J_1\) is the prediction error caused by the CNOPs or FSVs, while \(J_2\) evaluates the forecast errors caused by the reductions of the CNOPs or FSVs, so the benefits from the reductions of the CNOPs or FSVs can be obtained by the ratio defined as follows:

\[
\frac{J_1(\delta X_0) - J_2(\delta X_0)}{J_1(\delta X_0)}. \tag{14}
\]
Table 1 shows the sensitivity experiment results for the Matsa case. For both KE and TE metrics, the benefits from the reductions of the CNOPs are larger than those from reductions of the FSVs. For example, setting \( c \) to 0.75 makes an about 49.4% improvement of the verification area forecast for CNOP in the measure of dry energy norm, while for FSV it results in only a 15.3% improvement. Further setting \( c \) to 0.25, the benefits increase to 86.4% for CNOP versus 46.5% for FSV. The results in terms of the kinetic energy norm are similar.

In sections 4 and 5 it is shown that for the Meari case, the patterns of the CNOPs and FSVs are similar both at the initial and final times. Their energy developments are also consistent except for during the final few hours. Similarly, amplitude reductions of CNOPs and FSVs produce little differences in the results (Table 2). The benefits gained from reducing the CNOPs are comparable to or slightly larger than those gained from reducing the FSVs. The results for Mindulle are more similar to those for Matsa (Table 3). The differences between the benefits from the amplitude reductions of the CNOPs and FSVs are more remarkable; for example, 84.8% versus 25.1% improvements are obtained for CNOP versus FSV with the coefficient \( c \) of 0.25 in the measure of dry energy norm. In this case, setting \( c \) to 0.75 makes a relative small benefit compared to the other two cases. Note that in this case for FSV when \( c \) was set to 0.75, no benefits are gained, on the contrary, the forecast becomes worse. This phenomenon is possible since the errors develop nonlinearly.

7. Summary and discussion

Singular vectors have been applied extensively for adaptive observations of tropical cyclones, yet they have limitations in terms of their linear approximation. Conditional nonlinear optimal perturbation (CNOP), which is a natural extension of linear singular vectors into the nonlinear regime, is proposed to identify sensitive areas in targeting strategies for tropical cyclones forecasts. The first SV (FSV) is also taken into account and compared with the CNOPs.

CNOPs and FSVs are calculated by the spectral projected gradient 2 (SPG2) optimization algorithm for three tropical cyclone cases with a 24-h optimization time interval. Kinetic energy (KE) and total dry energy (TE) are chosen as the metrics. The same metric is used in the constraint and cost function. The constraint value is specified according to the magnitude of the current analysis error. The spatial structures, their energies, and their nonlinear evolutions of the CNOPs as well as their
inducing humidity changes are compared with those of the FSVs.

The results indicate that the pattern of the wind component of the CNOPs characterized by the KE norm is similar to that characterized by the TE norm and only differs in the magnitude. This is found in all three cases, and it is found for the FSV results as well.

For the Matsa and Mindulle cases, the structures of the CNOPs as well as their evolutions at the final time differ greatly from those of the FSVs. In terms of magnitude, the CNOPs are comparable to the FSVs at the initial time, whereas the developments of the CNOPs are much larger than those of the FSVs. In addition, the changes in specific humidity caused by the CNOPs are more discernible than those caused by the FSVs. However, for the case of Meari, there is little difference between the structures of the CNOPs and FSVs, and this is also the conclusion reached for the developed CNOPs and FSVs. The similarity of the CNOPs and FSVs means that linear approximation is acceptable in this case.

Finally, a sensitivity experiment is designed to find out to what extent an improvement in forecast errors can be obtained by reducing the amplitude of CNOP- or FSV-type errors. All three cases lead to a consistent conclusion that reducing the CNOP-type errors provides more benefit than reducing FSV-type errors, although the benefits from CNOP in the Meari case are not as great as the other two cases. In addition, there is a possibility that the reduction of FSV-type errors could lead to a worse prediction in certain cases.

In summary, the structures of the CNOPs may differ greatly from those of the FSVs depending on the constraint, metric, and basic state. The verification area as well as the tropical cyclone forecasts are more heavily affected by CNOP-type errors than by FSV-type errors. The reductions of CNOP-type errors provide more benefit than those of FSV-type errors. This suggests that it is worthwhile to further study the applicability of the CNOP method to identify sensitive areas in the adaptive observation.

This study makes the first step toward the application of CNOP to adaptive observation of tropical cyclones. There are quite a few issues that require further explorations. For example, how to determine the sensitive area according to the information offered by the CNOPs. In the applications of SV to adaptive observation, usually, several leading SVs are considered simultaneously in the determinations of the sensitive area, so it is worthwhile to study how to use CNOP, as well as local CNOP and other SVs together for the same purpose. Two other issues are the coarse resolution in this paper and how the results shown in this paper are affected by resolution, which will be examined in the future fine-resolution simulations.

The CNOP is expected to reveal the information in the initial analysis error that has maximum impact on tropical cyclone forecasts. It is confessed that for SVs there are more sensible metrics such as background error covariance (Gelaro et al. 2002) and more meaningful cost functions such as those used by Henderson et al. (2005) and Hoffmann et al. (2006; Hoffmann 2006). This suggests that it is of importance to choose an appropriate constraint condition such as the covariance of the analysis error, and construct a cost function that takes account of track, intensity, and even the economical effects. There would be something different if $c$ is taken as other metrics; however, we anticipate that the main conclusions of this paper will not be changed since the main differences between the CNOP and the FSV are caused by nonlinear processes. Of course, all these aspects need to be investigated in the future work.

It is well known that moist physical processes are of importance in some weather events, for example, Rosenthal (1978) and Schade and Emanuel (1999) have shown that moist processes are crucial in tropical cyclones. Ehrendorfer et al. (1999), Barkmeijer et al.

### Table 1. The benefits (%) obtained from the reductions of CNOP or FSV with KE and TE, respectively, in sensitivity experiments for Matsa. The values in the table are given by Eq. (14) in text.

<table>
<thead>
<tr>
<th>Coef $c$</th>
<th>KE CNOP</th>
<th>FSV</th>
<th>TE CNOP</th>
<th>FSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>91.3%</td>
<td>63.3%</td>
<td>86.4%</td>
<td>46.5%</td>
</tr>
<tr>
<td>0.50</td>
<td>69.5%</td>
<td>38.3%</td>
<td>69.9%</td>
<td>26.3%</td>
</tr>
<tr>
<td>0.75</td>
<td>42.3%</td>
<td>17.8%</td>
<td>49.4%</td>
<td>15.3%</td>
</tr>
</tbody>
</table>

### Table 2. As in Table 1, but for the case of Meari.

<table>
<thead>
<tr>
<th>Coef $c$</th>
<th>KE CNOP</th>
<th>FSV</th>
<th>TE CNOP</th>
<th>FSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>94.2%</td>
<td>93.9%</td>
<td>94.8%</td>
<td>92.1%</td>
</tr>
<tr>
<td>0.50</td>
<td>74.7%</td>
<td>74.6%</td>
<td>75.0%</td>
<td>67.3%</td>
</tr>
<tr>
<td>0.75</td>
<td>46.2%</td>
<td>43.2%</td>
<td>42.0%</td>
<td>35.7%</td>
</tr>
</tbody>
</table>

### Table 3. As in Table 1, but for the case of Mindulle.

<table>
<thead>
<tr>
<th>Coef $c$</th>
<th>KE CNOP</th>
<th>FSV</th>
<th>TE CNOP</th>
<th>FSV</th>
</tr>
</thead>
<tbody>
<tr>
<td>0.25</td>
<td>91.6%</td>
<td>47.8%</td>
<td>84.8%</td>
<td>25.1%</td>
</tr>
<tr>
<td>0.50</td>
<td>62.2%</td>
<td>27.3%</td>
<td>53.8%</td>
<td>7.5%</td>
</tr>
<tr>
<td>0.75</td>
<td>26.6%</td>
<td>15.3%</td>
<td>24.1%</td>
<td>−5.3%</td>
</tr>
</tbody>
</table>
(2001), and Errico et al. (2004) investigated the moist SV. Particularly Hoskins and Coutinhol (2005) studied the moist SV and related them to cyclone predictability. In this paper we limited the study to moist changes resulting from the initial uncertainties in wind, temperature, and pressure, rather than initial moisture error. Considering the impact of accounting for moisture in the above work related to moist SVs, it would appear worthwhile to extend the current study to the moist CNOPs and investigate the role of initial moisture errors in future work.

Furthermore, it is of interest to compare CNOP with other methods, such as adjoint sensitivities and the ensemble transform Kalman filter (ETKF). In addition, in order to advance the success of operational adaptive observation of tropical cyclones, it is crucial to investigate the efficiency of optimization algorithms for solving CNOPs since we are required to determine the targeted regions in advance.

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REFERENCES


