A New Assessment of the Predictability of Tropical Cyclone Tracks

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(Manuscript received 14 September 2010, in final form 13 April 2011)

ABSTRACT

The inherent predictability of tropical cyclone tracks has received much attention since the 1980s. It is still an issue because of the recent improvement of track forecasts by numerical models. The aim of this study is to assess this predictability limit globally using an approach devised by Lorenz on several up-to-date numerical models. The differences between forecasts valid at the same instant are considered to be error values; the doubling time of these small errors leads to an estimated upper bound on predictability. This method is here applied on cyclone position forecasts obtained from three different global operational models (from ECMWF, Météo-France, and the Met Office) over the main tropical cyclone basins in the world and during three recent cyclone seasons (2006–09).

The resulting estimates of predictability largely exceed the values that are commonly accepted in the literature. The doubling time of small errors is found between 30 and 50 h. An important consequence is that cyclone track forecasts have not reached their predictability limit yet. It is argued that the previous methods for computing the predictability of tropical cyclone tracks did not constrain the environment and the structure of the cyclones initially. But the Lorenz method could still underestimate the inherent predictability of tropical cyclone tracks. The sensitivity of the predictability estimates to the model characteristics is discussed. In particular, the use of wind bogus is suggested to avoid serial correlations between successive forecasts and to accelerate error growth.

1. Introduction

For several decades, forecast errors in tropical cyclone tracks have been regularly decreasing (Avila et al. 2006). Nowadays, cyclone forecasts heavily rely on numerical weather prediction. The rapid increasing amount of observations, especially from satellites, as well as the improvement of numerical models and of their assimilation schemes are certainly the main reasons for this error decrease. Nevertheless, there should be a nonzero limit for tropical cyclone track error. The existence of a natural, inherent predictability bound for every atmospheric process is due to the nonperiodic property of the atmosphere (Lorenz 1963).

Several studies have been assessing the inherent predictability of tropical cyclone tracks in different basins. A common measure of predictability is the doubling time of small position errors. Fraedrich and Leslie (1989) applied a nonlinear system analysis to a climatology of tracks around Australia, followed by Aberson (1998) and Aberson and Sampson (2003) in other basins. The divergence of cyclone positions leads to an estimate of the doubling time of small errors of roughly 15 h around Australia, 10 h in the northwest Pacific, and 40 h in the North Atlantic. These values contradict (Aberson and Sampson 2003) a previous climatology-persistence study (Pike and Neumann 1987), showing that the northwest Pacific is the basin where tropical cyclone tracks are the most predictable. Using simplified numerical models in the Pacific and the Atlantic basins, Leslie et al. (1998) showed that the inherent predictability of cyclone tracks in every basin was 40% down to the error using of these models. The values they obtained in the Atlantic were consistent with the results of Fraedrich and Leslie (1989), but the estimates were only slightly dependent on the basin. As a consequence, past results are not fully consistent. These approaches relied either on climatology, on statistical models, or on simplified numerical models. They estimated a mean predictability per basin. Because
of the remarkable improvement of operational models for cyclone track prediction in the very recent years, it is relevant to assess the inherent global predictability with other methods based on up-to-date numerical models. Moreover, some global models, like the one from the European Centre for Medium-Range Weather Forecasts (ECMWFF), have such improved that one can wonder if they have not reached the predictability limit for tropical cyclone tracks (Fiorino 2009); the ECMWF forecast error roughly equals the inherent error values by Leslie et al. (1998). Considering the inconsistency between previous studies, the suspicion that a nonperfect numerical model may reach the estimated predictability limit brings back the inherent predictability of tropical cyclone tracks to a present research issue.

Lorenz (1982, hereafter L82) designed a statistical method to compute a lower bound and an upper bound on the predictability of weather patterns. The input data were 100 days of 500-hPa geopotential height forecasts from the ECMWF operational model, from 0 day up to 10-day forecast terms. An obvious lower bound on predictability is the forecast error of an up-to-date numerical model. An upper bound is given by the doubling time of the growth of small errors, where error values are obtained as the difference between numerical forecasts. Measuring error growth as the evolution of the difference between model states has been classic since the first studies on predictability (Charney et al. 1966; Smagorinsky 1969; L82).

L82 argued that the lower and the upper bounds on predictability are getting closer as the numerical models improve. Obviously, the lower bound increases with model upgrading. As for the upper bound, L82 reviewed several articles showing that the doubling time of small errors decreases as the model simulates more processes with more accuracy and particularly the small-scale processes. The estimated upper bound on predictability should decrease as the model becomes more complex, and this was verified by Simmons et al. (1995). Consistently, the natural predictability limit is enclosed between the lower and upper bounds estimated by a numerical model, and these bounds should get closer as models better represent the atmosphere.

The L82 method is expected to be promising to assess the predictability bounds on tropical cyclone tracks. It has been used to determine the predictability of different atmospheric features. Bengtsson and Hodges (2006) updated and extended L82 by applying the same method to extratropical 500-hPa height fields and to wind fields in the tropics, using the ECMWF model. Bengtsson et al. (2005) and Froude et al. (2007) applied the L82 method to cyclone tracks. They studied the predictability of extratropical cyclone tracks using the ECMWF global model, aiming to infer how much future observation and model improvements could reduce forecast error.

An estimate of the predictability of cyclone tracks would be critical information to know what improvement may be expected from further research. It would provide objective information about what forecast error reduction may be expected from future improvements in the forecast model and in the assimilation of observations. Although the numerical models that are used for cyclone prediction are the same as operational numerical weather prediction systems, some specific developments are dedicated to cyclone modeling. One is the bogus technique (Heming et al. 1995), which consists in forcing the assimilation of pseudo-observations (wind or pressure) into the model initial state. The pseudo-observations are deduced from an idealized cyclone structure observed by satellite imagery. These specific techniques generally improve cyclone forecasts (Heming 2009).

Section 2 recalls the general principle of the L82 method and presents the data. Section 3 shows the results for three numerical models. Before concluding, a discussion is provided in section 4 about the difference with the previous predictability estimates and about the validity of the L82 method for cyclone tracks.

2. Methodology and data

L82 computed the mean distance between atmospheric states that consist of forecasts from different base times that are valid at the same instant. The distance used by L82 to study synoptic-scale weather patterns was the rms of the spectral components of 500-hPa geopotential height. For cyclone position, the mean distance $E_{j,k}$ between $l$ hours and $k$ hours forecasts is given by the formula:

$$E_{j,k} = N_{j,k}^{-1} \sum_{i=1}^{N_{j,k}} d(M_{ij}, M_{ik}), \quad (1)$$

where $M_{ij}$ is the cyclone position in the $i$ hours model forecast valid at instant $i$, $N_{j,k}$ is the number of forecasts, and $d(.,.)$ is the distance along the earth’s great circle (Bengtsson et al. 2005). Cyclone positions in the forecasts are computed using a simple and robust tracking algorithm (appendix A). The time step $\Delta t$ is 12 h.

Like in many predictability studies (Charney et al. 1966; L82; Bengtsson et al. 2005), the mean distance $E_{j,k}$ is assumed to represent a mean error value. L82 analyzed the time evolution of the error values $E_{j,k}$ to characterize the error amplification due to the processes that the model is able to represent. L82 considered that the error doubling time computed from the $E_{j,k}$ values may
be an upper bound on inherent predictability because an imperfect model reproduces uncompletely the processes of the real atmosphere.

The dataset consists of numerical forecasts from three up-to-date global operational models (Table 1): the Integrated Forecast System (IFS) from the ECMWF, the Unified Model (UM) from the Met Office, and the Action de Recherche Petite Echelle Grande Echelle (ARPEGE) model from Météo-France. The following shall be noted:

- IFS has the highest horizontal and vertical resolutions on the 2006–09 period,
- UM and ARPEGE have similar resolutions over the period, and
- UM is the only model that assimilates wind bogus pseudo-observations (Heming et al. 1995).

The dataset covers the tropical cyclones in every cyclone basin except for the north Indian Ocean over the October 2006–October 2009 period. These basins are the ones for which the Regional Specialized Meteorological Centers (RSMC) or other warning centers issue and broadcast cyclone forecasts. RSMC analyses will serve as reference in the present study.

For the purpose of comparing the statistics of the three different models, an homogeneous dataset (Tsuyuki et al. 2002) is built, in which a forecast is taken into account if and only if the tropical cyclone is present in the reference and is detected (using the tracking algorithm of appendix A) in every model forecasts. For each model, an inhomogeneous dataset is merely obtained as the cyclones that are present in the reference and that are detected in the model. The homogeneous dataset is required to ensure that the different results between the models are not due to different sampling. However, the homogenization procedure may favor one model among the others and may skew the results. The inhomogeneous dataset shall confirm the results obtained with the homogeneous data.

3. Results

For each model, the $E_{j,k}$ values are connected like in L82 (Fig. 1). The solid bold curve $\{E_{0,1}, E_{0,2}, E_{0,3}, \ldots\}$, called the total error, is the forecast error with regard to the model analysis, which estimates the lower bound on predictability. The dashed bold curve represents the forecast error relative to the observed cyclone position (RSMC analysis). The other curves are defined as $C_p = \{E_{0,p}, E_{0,1+p}, E_{j+1,1+p}, \ldots\}$. They represent the amplification of the error by the model, starting from different initial error values $\{E_{0,p}\}$.

The $C_p$ curves (Fig. 1) increase as expected. They do not cross one another and they are well ordered, in the sense that $C_p$ is higher than $C_q$ for $p > q$, and that every $C_p$ is lower than the total error. The number of forecasts for the homogeneous dataset $N_{j,k}$ is higher than 600 for all $j$ and $k$ and reach the value of 1250 for $j = 0, k = 1$. The well-ordered $C_p$ curves and the size of the samples guarantee the reliability of the statistics. The $C_p$ curves give an upper bound on the predictability of tropical cyclone tracks. The difference between the total error (lower bound on predictability) and the $C_p$ curves (upper bound on predictability) is the maximum error reduction that may be expected from future model improvements.

The bold curves show that IFS performs as the best model, followed by UM and then by ARPEGE. At 48- and 72-h forecast terms, the total mean error of the IFS model is 160 and 225 km, respectively. The inherent predictability estimated by Leslie et al. (1998) is approximately 150 km at 48 h and 220 km at 72 h. The IFS performance is therefore very close to the estimated predictability limit, consistently with observations by Fiorino (2009). This leaves two options: either IFS is a perfect model for cyclone tracks, or the existing predictability bound estimates are not upper bounds on predictability. It may also be noticed that the total error growth of IFS (solid bold curve) yields approximately a 36-h time period for the errors to double (between the 12-h term and the 48-h term, the mean error grows from 85 to 170 km). This value is much higher than the inherent doubling time of small position errors found in the literature (15 h). But this difference could be due to the fact that the IFS errors cannot be considered to be small. The very slow initial growth of the IFS error from the RSMC analysis (dashed bold curve) is similar to what has been observed on IFS 500-hPa height fields (Bengtsson and Hodges...
The estimation of the doubling time of small errors requires some further analysis of the $C_p$ curves. Figure 2 represents the cloud points of the estimated error growth $Y = (E_{j+1,k+1} - E_{jk};$ local derivative of the $C_p$ curves) as a function of the estimated error value $X = (E_{jk} + E_{j+1,k+1})/2$ (local value of the $C_p$ curves), for every model and for the homogeneous dataset. The doubling time of error $\tau$ associated with each of these dots can be written as

$$\tau = \ln(2) \frac{E}{dE/dt} = \ln(2) \frac{X}{Y}. \quad (2)$$

Similar points $(X, Y)$ for the total error (solid bold curve of Fig. 1) are plotted as crosses in Fig. 2. The crosses stand apart from the dots and suggest that future model improvement is possible (L82). The same conclusions apply to the inhomogeneous dataset (Fig. 3).

A difficulty for assessing the doubling time of small errors from Fig. 2 is that there is no point for $X$ below 60 km, which means that extrapolation is needed toward small error values. L82 proposed and justified the parabolic relationship $dE/dt = aE - bE^2$, or equivalently $Y = aX - bX^2$. This equation is consistent with the observation by L82 that the dots lay along a curve that is quasi-linear with a slight saturation for the highest error values. It is also supported by theoretical considerations (Lorenz 1969). The saturation term $-bX^2$ means that the error growth rate decreases as the error increases due to nonlinear processes (Smagorinsky 1969), up to a point where the error is so large that it equals the difference between random states. This parabolic hypothesis seems to apply well to the present results. Figures 2 and 3 show a tendency for $Y$ to increase when $X$ increases and saturation occurs for large $X$ values, consistently with the parabolic assumption. The points are spread along the ascending part of a parabola, contrary to L82, where they were lying along its descending part. The cloud points $(X, Y)$ are fitted to the parabola $Y = aX - bX^2$ that minimizes the quadratic difference between the points and the parabola (see appendix B). The doubling time of small errors $\tau_0$ is obtained by the following equation:

$$\tau_0 = \lim_{E \to 0} \left( \ln(2) \frac{E}{dE/dt} \right) = \ln(2)/a. \quad (3)$$

The estimated $\tau_0$ values for each model are shown in Table 2. The estimates are consistent between the homogeneous and the inhomogeneous data, with $\tau_0 \sim 50$ h for IFS and ARPEGE, and $\tau_0 \sim 30$ h for UM. For larger error values, the doubling time $\tau$ decreases slowly; it lies between 50 and 60 h for errors around 200 km. These
$t_0$ largely exceed the previous predictability estimates ($\sim 15$ h).

A rigorous comparison with previous studies is done using a plot of error growth against error values (Fig. 4, similar to Fig. 2). The curve associated with the value
Table 2. Initial doubling time $\tau_0$ for the cyclone position forecasts from the three models on the homogeneous data (H) and on the inhomogeneous data (NH). Values are obtained by a least squares fitting of the points $(X, Y)$ to a parabola $Y = aX - bX^2$.

<table>
<thead>
<tr>
<th>Model</th>
<th>IFS</th>
<th>UM</th>
<th>ARPEGE</th>
</tr>
</thead>
<tbody>
<tr>
<td>Doubling time $\tau_0$ (H)</td>
<td>44 h</td>
<td>31 h</td>
<td>48 h</td>
</tr>
<tr>
<td>Doubling time $\tau_0$ (NH)</td>
<td>42 h</td>
<td>32 h</td>
<td>46 h</td>
</tr>
</tbody>
</table>

$\tau_0 = 15$ h and the dots issued from Leslie et al. (1998) are plotted together with the predictability estimates by UM and IFS. Dots from Leslie et al. (1998) are obtained by computing the $(X, Y)$ values on the inherent predictability error evolutions in the Atlantic, northwest Pacific, and southwest Pacific estimated by the barotropic and baroclinic models (see caption of Fig. 4 for more details). Figure 4 confirms the results of Leslie et al. (1998): the curve $\tau_0 = 15$ h fits well the dots for the smallest error values (around 60 km). For the higher values, the doubling time $\tau$ decreases rapidly and converge toward the UM estimates. For every error value, the dots issued from Leslie et al. (1998) lay largely over the curve representing the IFS estimate ($\tau_0 = 42$ h).

As a conclusion, the doubling time of intrinsic error estimated by IFS (and also ARPEGE) for all error values are higher than the estimates provided by previous studies, particularly the one by Leslie et al. (1998). For small error values (below 60 km), the estimates from IFS, ARPEGE, and UM rely on a justified parabolic extrapolation that takes into account the nonlinear saturation for large errors. Although such an extrapolation guarantees that $\tau < \tau_0$, it could not be excluded to find larger values of $\tau_0$ if data were available below 60 km. The present study still shows significant higher doubling times of errors than the previous ones. Some possible reasons for these differences and for the difference between models (IFS and ARPEGE against UM) are discussed in the next section.

4. Discussion of the results

a. On the difference between these results and previous predictability estimates

A definition of inherent predictability is given by Lorenz (1969) as the doubling time of the difference between two initial similar atmospheric states, measured using a relevant metric. For instance, L82 investigated the predictability of synoptic-scale weather patterns by measuring the rms differences in geopotential fields at 500 hPa. For tropical cyclone tracks, it is commonly accepted that the motion of tropical cyclones is largely driven by its large-scale environment (Chan and Gray 1982). The cyclone structure and intensity also have a significant influence on its future track (Chan 2005). The definition of inherent predictability for cyclone tracks should take into account these external influences. Predictability estimates could thus be obtained as the doubling time of the difference between two atmospheric states for which

- the cyclone positions are infinitely close; and
- the difference between their environment and their cyclone structure, which may be called the surrounding flow of the cyclone center, is infinitely small.

With regard to this definition, the methods used by previous studies will be carefully examined.

1) THE METHOD OF FRAEDRICH AND LESLIE (1989)

Fraedrich and Leslie (1989) applied a nonlinear system analysis to the cyclone tracks gathered in different zones around Australia at the same initial point. The doubling time of the divergence of cyclone tracks from this initial point was taken as the inherent predictability of cyclone tracks. Cyclones with very different surrounding flows may be gathered inside the same sample. Hence, a significant part of track divergence may be due to the differences in the initial environmental forcings. Although Fraedrich and Leslie (1989) assumed that regional differentiation accounts for much of the external or climatic forcing, the surrounding flows of the different cyclones are not controlled to be close. It is thus very likely that this method (also used by Aberson 1998;
Aberson and Sampson (2003) underestimate the doubling time of small errors.

2) THE METHOD OF LESLIE ET AL. (1998)

The main purpose of Leslie et al. (1998) was to assess the difference between practical predictability (i.e., error by an up-to-date numerical model) and inherent predictability. Their approach relied on the use of a numerical model, either barotropic or baroclinic. The error associated with inherent predictability was obtained by assuming that the model was perfect. The tracks forecasted by this perfect model were relocated every 24 h at the observed position. Such a relocation guarantees that position error is small, but errors in the surrounding flow may be large. Moreover, the perfect-model assumption may lead to an underestimation of the doubling time of error.

Leslie et al. (1998) and Fraedrich and Leslie (1989) found similar doubling time estimates of small errors, which are likely to be underestimations of inherent predictability regarding the above arguments.

3) THE L82 METHOD

The L82 method is presently applied to cyclone positions obtained from current operational models. A consequence is that the surrounding flow of the cyclone is predicted by the model, and is not obtained as the natural evolution of the atmosphere. The difference of cyclone position between two model forecasts may originate from three sources:

- the intrinsic growth of the difference of cyclone position in a perfectly predicted surrounding flow,
- the cumulating effect of the growth of the difference of the cyclone surrounding flow,
- the cumulating effect of the modeling error of the cyclone surrounding flow.

It follows that the result of the L82 method is not strictly consistent with the definition of predictability given previously: the third source of error may yield an artificial underestimation of the error doubling time. This undesirable error is still lower than the supplementary error sources generated by previous methods. Indeed, forecasting the evolution of the surrounding flow using a present numerical model is expected to generate a lower error on cyclone position than selecting randomly initial surrounding flows. This explains why the L82 method yields higher error doubling time than previous studies.

The predictability bound on tropical cyclone tracks given by the L82 method is not strictly an upper bound. With future improvement of numerical models, the doubling time of position errors shall undergo two contradictory evolutions. On one hand, it shall decrease following the argument of L82: as model become more complex, the doubling time of the difference between model states shall decrease. On the other hand, it shall increase as a consequence of progress in modeling the surrounding flow of cyclones. This flaw of the L82 method applied to cyclone positions is still expected to have a lower impact on the predictability bound than the ones from previous studies.

b. On the difference between estimates from UM and the other two models

The predictability bounds estimated by UM ($\tau_0 \sim 30$ h) and IFS and ARPEGE ($\tau_0 \sim 50$ h) are markedly different. Since L82 relies on the use of forecasts starting from different base times, the serial correlation between successive track forecasts (Aberson and Sampson 2003) should be addressed. Aberson and DeMaria (1994, their appendix B) proposed a diagnosis for measuring such serial correlations. Table 3 shows the separation time at different lead time for the three models, computed on the homogeneous forecast sample. For UM, the separation time (around 13 h) is close to the true one (12 h), which means that the UM track forecasts starting from successive instants are mostly uncorrelated. For IFS and ARPEGE, the separation time is around 16 h, which is associated to a moderate correlation compared to the 30-h value obtained by Aberson and DeMaria (1994) with a barotropic model in the Atlantic. The serial correlations of IFS and ARPEGE may yield an overestimation of inherent predictability.

The explanation for the different predictability bounds should come from differences between the models. The assimilation of wind pseudo-observations to constrain a cyclone in the analysis is specific to UM and it is known to play a major role in cyclone numerical forecasts (Heming 2009), and it may explain a part of the different predictability estimates between UM and the other models. The first expected effect of wind bogus is to relocate a cyclone from a wrong predicted position to its observed location. Among the data that could constrain cyclone position, such as scatterometer winds or satellite
radiances, the bogus pseudo-observations are known to have the most important impact (Montrotty et al. 2008). The UM wind bogus is thus probably responsible for avoiding the serial correlations between track forecasts.

Wind bogus may also have some negative impacts on the representation of the cyclone surrounding flow. Pseudo-observations are deduced from the sum of an idealized vortex superimposed on a uniform environmental wind at several isobaric levels in the middle and low troposphere (Heming et al. 1995). This idealized representation introduces some error with respect to the real structure of the cyclone. Moreover, the pseudo-observations are forced to be assimilated and the Gaussian linear assumptions on which variational data assimilation relies (Talagrand and Courtier 1987) are not expected to treat correctly large errors such as those induced by the displacement of an intense vortex. Although wind bogus reduces the initial error position, these additional errors in the surrounding flow may accelerate the growth of the predicted position error. The error doubling time computed by UM may therefore be an underestimation of predictability.

The IFS and ARPEGE values ($t_0 \sim 50$ h) are overestimations due to serial correlations. The UM wind bogus may lead a too low doubling time ($t_0 \sim 30$ h) due to an artificial acceleration of error with time. As a consequence, the inherent predictability of tropical cyclone tracks should reasonably be enclosed within the estimates from the three models UM, ARPEGE, and IFS. Although it has not been demonstrated that wind bogus is the sole reason for elucidating the differences between UM and the other models, it is likely that this scheme explains a large part of it.

5. Conclusions

The main result of the present study is that the inherent predictability of tropical cyclone tracks was largely underestimated in past studies. A new estimation using the L82 method applied to several up-to-date numerical models leads to a doubling time of small position errors between 30 and 50 h. The doubling time of larger errors (around 200 km) increases slowly with error. Future improvement of numerical models may have contradictory effects on this estimate, leading to a small decrease or increase. The L82 approach applied to future numerical models could lead to even higher doubling times of errors.

One of the reasons why the inherent predictability of tropical cyclone tracks has been underestimated so far is that cyclone position has been treated independently from its surrounding flow. Tropical cyclone motion is highly dependent on its environment and on its structure, and the study of position error growth should ensure that environmental errors are small. The previous predictability studies treated the environmental influence on a cyclone track as if it would remain unpredictable forever (i.e., numerical models would never be able to model it). The L82 method applied to cyclone tracks takes partially into account the environmental influence. A more precise estimate of the inherent predictability of tropical cyclones tracks would require some other new methods. It is necessary to separate the error that is due to the intrinsic position error growth in an evolving environment and the impact of the error of the surrounding flow.

More generally, there is a present trend in predictability studies to deal with meteorological objects or coherent structures (Casati et al. 2008; Plu et al. 2008; Hewson and Titley 2010), such as tropical or extratropical cyclones, vortices or convective cells, rather than integral fields. The present article shows that cyclone track predictability estimates may depend on how their environment is taken into account. The transfer of verification techniques from meteorological fields to structures should be done cautiously.

Acknowledgments. The Met Office and the ECMWF are warmly thanked for authorizing to analyze the data from their operational global models. The author is indebted to Jean-Marie Willemet, Rémy Lee-Ah-Siem, and Ghislain Faure for the development of the cyclone tracking algorithm and data homogenization. This article gained more clarity after the suggestions of two anonymous reviewers and after a careful proofreading by Marie-Dominique Leroux.

APPENDIX A

The Tracking Algorithm

The cyclone tracking algorithm used in the present study is applied to mean sea level pressure fields, and is computed on a latitude–longitude grid. For every cyclone, the operational RSMC analysis is used as a reference. For each model, the closest relative minimum to the operational RSMC analysis inside a $12^\circ \times 12^\circ$ square is taken as the first position of the cyclone. Starting from this initial position in the analysis, successive cyclone positions are sought in the forecasts every 6 h, at a maximum distance of 480 km from the previous position. This distance means that the cyclone track speed cannot be higher than 80 km h$^{-1}$. When two cyclones are present in the same basin and when the distance between them is below 1000 km, it has been verified that the tracking algorithm does not mix both trajectories.
APPENDIX B

Formulas for Fitting a Parabola to a Cloud Point

Let \( \{(x_i, y_i), i = 1, \ldots, n\} \) be a set of points, and the moments \( \mu_{jk} = n^{-1} \sum_{i=1}^{n} x_i^j y_i^k \). The parameters of the parabola \( Y = ax - bx^2 \) that best fits the cloud points are obtained after minimizing the function along \( a \) and \( b \):

\[
\sum_{i=1}^{n} \left( y_i - (ax_i - bx_i^2) \right)^2,
\]

which yields

\[
b = \mu_{4,0}(\mu_{5,0} - \mu_{2,1}), \quad \text{and} \quad a = (\mu_{5,0} \mu_{2,1} - \mu_{4,0} \mu_{1,1})/\left(\mu_{5,0}^2 - \mu_{4,0}^2 \mu_{2,0}\right).
\]

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