A New Parametric Tropical Cyclone Tangential Wind Profile Model

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(Manuscript received 13 April 2012, in final form 12 October 2012)

ABSTRACT

A new parametric tropical cyclone (TC) wind profile model is presented for depicting representative surface pressure profiles corresponding to multiple-maxima wind profiles that exhibit single-, dual-, and triple-maximum concentric-eyewall wind peaks associated with the primary (inner), secondary (first outer), and tertiary (second outer) complete rings of enhanced radar reflectivity. One profile employs five key parameters: tangential velocity maximum, radius of the maximum, and three different shape velocity parameters related to the shape of the profile. After tailoring the model for TC applications, a gradient wind is computed from a cyclostrophic wind formulated in terms of the cyclostrophic Rossby number. A pressure, via cyclostrophic balance, was partitioned into separate pressure components that corresponded to multiple-maxima cyclostrophic wind profiles in order to quantitatively evaluate the significant fluctuations in central pressure deficits. The model TC intensity in terms of varying growth, size, and decay velocity profiles was analyzed in relation to changing each of five key parameters. Analytical results show that the first shape velocity parameter, changing a sharply to broadly peaked wind profile, increases the TC intensity and size by producing the corresponding central pressure fall. An increase (decrease) in the second (third) shape velocity parameter yields the pressure rise (fall) by decreasing (increasing) the inner (outer) wind profile inside (outside) the radius of the maximum. When a single-maximum tangential wind profile evolves to multiple-maxima tangential wind profiles during an eye replacement cycle, the pressure falls and rises are sensitively fluctuated.

1. Introduction

Over the years, a variety of parametric wind profile models have been developed to depict representative tangential wind and/or pressure profiles within a tropical cyclone. Originally introduced to tropical meteorology by Depperman (1947), a Rankine (Rankine 1882) vortex model was used in numerous studies to approximate the inner core of solid-body rotation of the tropical cyclone (TC). Outside the core, the tangential wind decreased, with some values of about 0.6 ± 0.1 being inversely proportional to radius from the TC center. The velocity distributions were found to give a good approximation to the tangential wind profiles of TCS (Hughes 1952; Riehl 1954, 1963; Malkus and Riehl 1960, among others). Although the modified Rankine model had some observational support (Gray and Shea 1973), the model did not allow computation of a relation between the maximum wind and minimum sea level pressure because the radial integral of the gradient wind acceleration was not bounded at a large radius (Willoughby and Rahn 2004).

Schloemer (1954) proposed an alternative model in which a TC surface pressure field was assumed to follow a modified rectangular hyperbolic function with...
radius, and the gradient-balance wind was calculated from the radial variation of hurricane pressure. Myers (1957) adopted the Schloemer model to derive a pressure–wind relation in which the maximum balanced wind was proportional to the square root of the pressure difference between large radius and the TC center. Adopting the Schloemer model, Holland (1980) introduced an additional parameter, now commonly referred to as the Holland $B$ parameter, into an inverse exponential function of radius to represent a parametric pressure field. The field in turn must be differentiated with respect to radius to obtain a gradient wind. This parameter was critical in allowing the pressure profile, and thus the controlling width of the wind profile spanning the maximum, to accommodate the gamut of velocity shapes typically found in mature TCs. The Holland model, however, suffers from two major shortcomings, as noted by Willoughby and Rahn (2004): (i) the areas of strong inner winds in the eyewall and of nearly calm winds at the storm center were unrealistically wide compared to observations, and (ii) the outer winds outside the eyewall decayed too rapidly with increasing radial distance from the center. Lack of extensive reconnaissance aircraft datasets prior to 1980s apparently contributed to the shortcomings of the Holland model. Since then, modern technologies produce better datasets that offer much potential for increased knowledge about detailed TC wind profiles including multiple-maxima concentric eyewall tangential winds. As a result, the datasets led Holland et al. (2010) to improve the original Holland (1980) model by (i) allowing the wind equation exponent to vary in order to fit outer wind observations, and (ii) adding a secondary wind profile to the preexisting primary wind profile. Their discussion on secondary wind maxima, however, was superficial because it was unclear how variability of outer wind (i.e., secondary wind) profiles was related to changes in TC intensity, strength, and/or size during an eye replacement cycle.

Two shortcomings of the Holland (1980) model led Willoughby et al. (2006, hereafter WDR) to develop a different parametric model that was designed specifically to fit wind observations and has been extensively tested on reconnaissance aircraft data. The WDR model outperformed the Holland model by increasing a number of parameters to characterize and obtain an optimum fit to complex observed wind profiles. One profile was a piecewise, continuous approximation defined by three different functions in different parts of the TC wind structure. Inside the eye, the first function used a power of radius to control the shape of the inner profile. Within the eyewall, the second function used a smooth, radially varying polynomial ramp function to replace the unrealistic discontinuity (i.e., cusp) in the infinitesimal radial thickness of the Rankine tangential velocity maximum with a smooth polynomial transition joining the inner and outer profiles. Outside the eyewall, the third function used a dual exponential to control two different decay rates of the outer profile: (i) the rapid wind decay close to the eyewall and (ii) the slow wind decay farther away from the eyewall. Using reconnaissance aircraft data, the radial profiles of tangential wind and geopotential height were shown to be able to fit well the observed profiles of flight-level tangential wind and geopotential height. Additionally, in his unpublished work with the WDR model, Willoughby successfully developed multiple-maxima wind profiles that resembled to those in Hurricane Allen (1980).

The Holland $B$ parameter cannot model all TCs at all times because this limited parameter ranges from 0.75 to 2.5, as noted by Willoughby and Rahn (2004), WDR, Vickery and Wadhera (2008), and Vickery et al. (2009a,b) in their discussion of parametric characteristics of the Holland model. The Holland $B$ formulation is so rigid that it is difficult to have control over the shape of the different regions of the radial distribution of the gradient wind. The difficulty of the model provides motivation to apply the Wood and White (2011) parametric tangential wind profile model (developed for high Rossby number vortices such as thunderstorm mesocyclones and tornadoes) to a TC and determine whether the model is capable of achieving an optimum fit to realistic TC pressure and multiple-maxima, concentric eyewall tangential wind profiles with and without the eyewall replacement cycle (ERC).

The objective of this study is to extend the existing parametric tangential wind profile model of Wood and White (2011) to tailor the model for TC applications. In section 2, the following five key model parameters controlling the radial profile of tangential wind are described: maximum velocity, radius of the maximum, and three shape velocity parameters that independently control different portions of the profile. Physical interpretations of each shape velocity parameter are described. The description follows the computation of a model gradient wind from a cyclostrophic wind approximation formulated in terms of a cyclostrophic Rossby number. Section 3 discusses partitioning of the radial profile of model TC gradient and cyclostrophic winds into as many as three wind maxima. The total surface pressures are partitioned into separate components of pressures that correspond to triple wind maxima. Additionally, the partitioned central surface pressure deficits corresponding to three peak tangential wind profiles are derived. Mathematical formulas for gradient and cyclostrophic vorticity for multiple wind maxima are developed and discussed in section 4.
Section 5 both uses a TC wind structural variability [similar to the Merrill (1984) concept] and a TC emulator to present a discussion on the manner in which a model TC intensifies, strengthens, and changes size by examining the behaviors of central surface pressure deficits and radial profiles of cyclostrophic and gradient winds, vorticity, and surface pressure with and without an ERC. Finally, conclusions and future work are presented in Section 6.

Our parametric TC tangential wind profile model is a part of our research to obtain an optimum fit to radial profiles of aircraft flight-level data. If the parametric model successfully replicates the general aspects of observed profiles of wind and pressure (or geopotential height) in TCs, then we will eventually apply the model to azimuthal variations of high-resolution Doppler velocity signatures of vortices for various applications such as developing statistics of vortex wind profiles for use in enhancing the National Weather Service warnings.

2. Parametric modeling of TC gradient–cyclostrophic winds

a. Wood–White parametric wind model

The parametric tangential wind profile $V_{WW}$ formulated by Wood and White (2011) for inviscid, axisymmetric flow is given by

$$V_{WW}(r; m) = V_X f(r; k, h, l)$$

and

$$f(r; k, h, l) = \frac{\eta^k k^l}{(\eta - k + k \rho^h)^l}, \quad 1 \leq k < \eta, \quad \lambda > 0,$$

where $V_{WW}$ is the tangential velocity as a function of radius $r$, and $\rho = r/R_X$ is a dimensionless radius. The Wood–White profile employs a model vector of five key parameters: $m = [V_X, R_X, k, h, l]^T$, where $V_X$ is maximum tangential velocity; $R_X$ radius of the maximum; and $k$, $h$, and $l$ represent different shape velocity parameters that are related to different shapes of the velocity profile. Note that $k$ and $\eta$ are the same parameters “$k$” and “$n$” as initially developed by Wood and White (2011).

The parametric model (1) assumes a circular wind flow pattern and does not adequately depict the actual surface boundary layer winds. Surface boundary layer winds including wind reduction factor, storm translation effects, and the degree of overwater wind axisymmetry and asymmetry (e.g., Kepert 2001, 2006a,b) could be added to the model for use in storm surge models, models of wind-driven seas, and other oceanic responses to TCs.

b. Physical interpretations

Figure 1 shows how each shape velocity parameter $(k, \eta, \lambda)$ influences the shape of a normalized velocity $V^*$ as a function of (a) the growth parameter $k$, (b) the decay parameter $\eta$, and (c) the size parameter $\lambda$; $V_X^* = 1$ is the normalized tangential velocity peak at $\rho = 1$.

Fig. 1. Varying radial profiles of normalized tangential velocity $V^*$ as a function of (a) the growth parameter $k$, (b) the decay parameter $\eta$, and (c) the size parameter $\lambda$; $V_X^* = 1$ is the normalized tangential velocity peak at $\rho = 1$. 

The $\eta$ parameter may be thought of the “decay” parameter since it controls the size of the decaying outer velocity profile (Fig. 1b) producing a transition to bowl-to-U-shaped (nonlinear) profiles with increasing width of zero tangential velocity at the vortex center.

The $\eta$ parameter may be thought of the “decay” parameter since it controls the size of the decaying outer velocity profile (Fig. 1b). The parameter is given as $\eta = \kappa - \mu$, where $\mu < 0$ is the power-law exponent that
governs the outer velocity profile in the Rankine vortex model (see the appendix). The higher the \( h \) value, the more rapidly the tangential velocity decreases with \( r \) beyond the radius of maximum wind (RMW).

The \( l \) parameter may be thought of the “size” parameter since it controls the radial width of the velocity profile in the annular zone of the maximum. When \( l = 1 \), a broadly peaked profile results (Fig. 1c). As \( l \to 0 \), the profile transitions to a sharply peaked profile that resembles the idealized Rankine velocity profile. Both \( \kappa \) and \( \eta \) alone cannot allow adjustment of the sharply profile at \( \rho = 1 \). Although the \( \lambda \) parameter may determine the size of both inner and outer circulations, the \( \eta \) parameter plays an important role in determining the size of the only outer circulation in the Merrill (1984) and Weatherford and Gray (1988b) sense.

The radial profile families of normalized tangential velocity as functions of \( \kappa, \eta, \) and \( \lambda \) are schematically presented in Fig. 2. In each panel of the figure, three varying values of \( \eta = \kappa + \mu \) with \( \mu = -0.1, -1.0, -2.0 \)
are presented for each selected values of $\kappa$ and $\lambda$. As one progresses from the top to the bottom panels, $\kappa$ and $\eta$ remain unchanged with decreasing $\lambda$. Consequently, 1) a broadly peaked profile undergoes a change to a sharply peaked profile and 2) the three radial profiles merge together to form the same profile inside $\rho = 1$. When $\lambda$ is constant, $\eta$ changes with increasing $\kappa$ and the curvature of the profile progressively changes such that the V-shaped profile (left column of Fig. 2) transitions into the bowl-shaped profile (center column) and eventually into the U-shaped profile (right column).

Users/analysts interested in applying this parametric model to some actual data or simulations are encouraged to experiment with each of adjustable shape velocity parameters ($\kappa$, $\eta$, $\lambda$). One’s ability to fine-tune any parameters that represent physically meaningful aspects of the wind profile is much like adjusting the Holland $B$ parameter to obtain the desired results. Once they have gained familiarity with adjustable parameters, the users/analysts are able to reproduce actual or simulated TC wind profiles for a wide range of applications (e.g., hurricane risk model or model initialization for wind specification).

c. Gradient versus cyclostrophic wind approximations

The tangential wind in (1) is inappropriately scaled for TC tangential wind owing to the absence of the Coriolis parameter, which is not important to the scaling argument for a tornado. To tailor the Wood–White parametric wind model for TC applications, we follow Willoughby (1990b, 1995, 2011) and use the fact that (1) can be converted to the gradient $V_G$ and cyclostrophic $V_C$ winds in terms of the cyclostrophic Rossby number [$R_C = V_C/(fR)$] such that

$$V_G = \left(\frac{1}{2}\left(\frac{1}{R_C} \pm \sqrt{1 + \frac{1}{4R_C^2}}\right)^2\right)^2, \quad R_C \neq 0,$$  \hspace{1cm} (3)

where $f$ is the Coriolis parameter. Note that (3) is much simpler than gradient balance because of the following reasons. If the wind represented by (1) is assumed to be the gradient wind, then the inward radial integral from large radius of the Coriolis term $fV_G$ (in gradient balance) becomes infinite (Willoughby 1995). The difficulty of the infinite pressure difference (e.g., $P_e - P_s$, where $P_e$ is the central pressure at the storm center, and $P_s$ the environmental pressure at radial infinity) arises because it is the pressure, not the wind, that is specified at the beginning. This problem can be avoided by first computing $V_C$ (which will be described in detail in section 3), and then calculating both $V_G$ in (3) and the pressure from cyclostrophic balance, which will be also discussed in that section.

For the high Rossby number gradient wind approximation, (3) may be rewritten equivalently\(^1\) as

$$V_G = \frac{V_C}{\frac{1}{2}\left(\frac{1}{R_C} \pm \sqrt{1 + \frac{1}{4R_C^2}}\right)} = \frac{V_C}{1 + \frac{1}{2R_C}}. \hspace{1cm} (4)$$

Here we have chosen the positive root (nonanomalous flow) and recognized that $1/8R_C^2 \ll 1$. Now, we substitute for $R_C$ in (4) as follows:

$$V_G = \frac{V_C}{1 + \frac{1}{2V_C}fr} = V_C - \frac{fr}{2}. \hspace{1cm} (5)$$

Scale analysis of (5) indicates that this approximation is valid only when $R_C > 10$. Equation (5) is used to derive a parametric vorticity formula in section 4. In (3)–(5), $V_C$ is expressed as, with the aid of $\phi$ in (1),

$$V_C = (V_X\gamma)\phi, \hspace{1cm} (6)$$

where the product of $V_X$ and $\gamma$ is defined as the scaled cyclostrophic tangential wind maximum, and $\gamma$ is a dimensionless variable computed from

$$\gamma = \frac{1}{2}\left\{\frac{1}{R_{CX}} + \frac{1}{\sqrt{1 + \frac{1}{4R_{CX}^2}}}\right\}, \quad \text{for} \quad R_{CX} \leq 10, \hspace{1cm} (7a)$$

and

$$\gamma \approx \left(1 + \frac{1}{2R_{CX}}\right), \quad \text{for} \quad R_{CX} > 10, \hspace{1cm} (7b)$$

where (7b) is obtained in a manner analogous to the development of the denominator in (4). Here, $R_{CX} = V_X/(fR_X)$ is the local cyclostrophic Rossby number based upon the observed RMW (or $R_3$), the observed maximum wind $V_X$, and the Coriolis parameter $f$ evaluated at the TC’s center position. It is used to solve

\(^1\) Using the binomial series for the square root term and ignoring higher-order terms, it can be shown that $(1 \pm x)^n = 1 \pm nx \pm \frac{a(a-1)x^2}{2!} \pm \ldots \approx 1 \pm ax$. 

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the gradient–wind relation for the cyclostrophic wind that gives the approximately correct (i.e., the observed $V_X$) gradient wind at the RMW when the gradient–wind relation is applied to the cyclostrophic wind profile. As a practical matter, it is almost as good to use $V_C = V_G + f r/2$ [see (5)] to adjust both the cyclostrophic wind value and the radius of maximum cyclostrophic wind. In other words, the radius of maximum cyclostrophic wind and the value of the maximum cyclostrophic wind are adjusted by differentiating $V_C$ with respect to $r$ and setting the result to zero (i.e., $\partial V_C/\partial r = 0$).

According to Willoughby (1990b) and Willoughby and Rahn (2004), the typical local cyclostrophic Rossby numbers $R_{CX}$ range from 10 to 100 in TCs. As an example, Fig. 3 illustrates how the different radial profiles of normalized model tangential wind vary as a function of $R_{CX}$. The plots of three radial profiles of normalized model tangential wind $V_{WW}^*(=V_{WW}/V_X)$, $V_{G}^*(=V_G/V_X)$, and $V_{C}^*(=V_C/V_X)$ are compared for selected values of local cyclostrophic Rossby number $R_{CX}$. The model parameters that produced the $V_{WW}^*$ profile are $\kappa = 1.5$, $\eta = 2.0$, and $\lambda = 0.1$.

![Fig. 3. Normalized model tangential wind profiles of $V_{WW}^*$ (solid curve), $V_{G}^*$ (short-dashed curve), and $V_{C}^*$ (long-dashed curve) as functions of the normalized radial distance ($\rho = r/R_X$) for selected values of local cyclostrophic Rossby number $R_{CX}$. The model parameters that produced the $V_{WW}^*$ profile are $\kappa = 1.5$, $\eta = 2.0$, and $\lambda = 0.1$.](image-url)

FIG. 3. Normalized model tangential wind profiles of $V_{WW}^*$ (solid curve), $V_{G}^*$ (short-dashed curve), and $V_{C}^*$ (long-dashed curve) as functions of the normalized radial distance ($\rho = r/R_X$) for selected values of local cyclostrophic Rossby number $R_{CX}$. The model parameters that produced the $V_{WW}^*$ profile are $\kappa = 1.5$, $\eta = 2.0$, and $\lambda = 0.1$.

In (7a), for small $R_{CX}$, the maximum value of $V_{C}^*$ at $\rho = 1$ always is larger than the maximum $V_{G}^*$, which in turn is always greater than the $V_{WW}^*$ peak in normal (cyclonic) flow around a closed low pressure system. Keep in mind that this $\rho = 1$ can occur no matter how close to (or very far from)
the TC center is the ρ. Beyond ρ = 1, \( V_{C}^e \) increasingly deviates and rapidly decreases from \( V_{W}^e \) with the radius from the storm center. Gradient flow is a good approximation to the wind outside the vortex core, but it is strictly valid only when radial accelerations of the radial wind are small.

When \( R_{CX} \) becomes large in (7b), \( \gamma \to 1 \), indicating that two radial profiles of \( V_{C}^e \) and \( V_{G}^e \) approach the \( V_{WW}^e \) profile and nearly merge together in each panel (Fig. 3) to form one radial profile for large \( \rho \). At the same time, the peak values of \( V_{C}^e \) and \( V_{G}^e \), however, diverge beyond \( \rho = 1 \) for all the \( R_{CX} \) values. The radial profiles of cyclostrophic and gradient winds (Figs. 3a,b) favorably resemble those of Willoughby (1995, 2011).

### 3. Partitioned pressure–wind profiles

**a. Partitioned wind profiles for multiple-maxima eyewall tangential wind profiles**

Based on the seminal work of Willoughby et al. (1982), intense symmetric TCs have been observed to exhibit multiple, well-defined wind maxima during ERCs. Two (three) concentric eyewalls were defined by two (three) tangential wind peaks in each of the aircraft radial legs and by two (three) well-defined rings of enhanced radar reflectivity located at different radii from the centers of the TCs. An echo-free moat and a saddle-shaped wind profile were situated between each pair of concentric eyewalls. Good examples of the primary, secondary, and tertiary tangential wind maxima within concentric eyewalls were also provided in McNoldy (2004) and Sitkowski et al. (2011), which displayed flight-level tangential wind profiles observed by aircraft in Hurricanes Juliette (2001) and Frances (2004), respectively. According to Sitkowski et al. (2011), several TCs exhibiting multiple wind maxima show evidence of multiple ERCs. Figure 4a provides an example from Hurricane Frances (2004). The weakening primary wind maximum (identified by 1) was associated with a decaying primary (inner) concentric eyewall, while the intensifying secondary wind maximum (identified by 2) was associated with a developing secondary (first outer) concentric eyewall as an ERC neared its completion. The tertiary wind maximum (identified by 3) appeared to originate from spiral rainbands that ultimately arranged into a tertiary (second outer), well-defined concentric eyewall from which an additional ERC arose. There have been no documented cases of more than three well-defined tangential wind peaks in aircraft profiles of TCs.

Fig. 4. Radial profiles of (a) storm-relative tangential wind \((m \, s^{-1})\) and (b) surface pressure (hPa) in Hurricane Frances observed on 30 Aug 2004. The beginning and ending times (UTC) of aircraft flight-level collection data are indicated at the top-left and top-right corners, respectively, of (a). In (a), triple wind maxima, identified by 1, 2, and 3, are observed for each aircraft right leg [from Sitkowski et al. (2011)]. In (b), arrows indicate the possible locations of kink reflected in the pressure profile. The radial distance (km) is from the TC center located at 19.380°N, 59.352°W at 1812:49 UTC. (Data obtained courtesy of NOAA/AOML/HRD.)

In view of the above, the triple concentric eyewall structure can be considered as a triple vortex composed of the inner vortex, the first and second outer vortex configurations. We now show that the hypothesis on a three-vortex composite in TC wind profiles enables the description of complex tangential velocity distributions. By isolating the primary tangential wind profile from the secondary and tertiary tangential wind profiles, the total cyclostrophic wind \( V_{C} \) profile in (3) may be partitioned into the primary \( V_{P} \), secondary \( V_{S} \), and tertiary \( V_{T} \) tangential wind profiles:

\[
V_{C} = \sum_{i=1}^{3} V_{i} = V_{P} + V_{S} + V_{T},
\]

where the subscript \( i \) represents the number of individual wind profiles [i.e., \( i = 1 \) corresponding to \( P \) (primary), \( i = 2 \) to \( S \) (secondary), and \( i = 3 \) to \( T \) (tertiary)]. This subscript applies to at least one radial profile of tangential wind. The partitioned tangential wind components in (8), with the aid of (1) and (6), are expressed by
\[ V_i = (V_{iX} \gamma_i) \phi_i, \quad \gamma_i = \frac{1}{2} \left( \frac{1}{R_{iX}} + \frac{1}{2} \left[ 1 + \frac{1}{4 R_{iX}^2} \right] \right), \]

\[ R_{iX} = \frac{V_{iX}}{f R_{iX}}. \tag{9a} \]

\[ \phi_i = \frac{\eta_i^r \rho_i^r}{(\eta_i - \kappa_i + \kappa_i \rho_i^r \eta_i^r)^{1/2}}, \quad \text{and} \quad \rho_i^r = \frac{r}{R_{iX}}, \tag{9b} \]

where the subscript \( X \) refers to the radius at which the maximum tangential wind occurs. In (9a), the primary eyewall wind component \( V_P \) is the primary tangential wind in a basic-state vortex flow. The secondary \( V_S \) and tertiary \( V_T \) tangential wind perturbations added to the primary vortex are the perturbations that form a double- and/or triple-peaked tangential wind profile when they are imposed on the basic state (e.g., Schubert et al. 1980).

In the developing stage of a secondary eyewall, a broad region of nearly constant boundary layer tangential winds form underneath the new eyewall, followed by increased inflow and enhanced convergence in the boundary layer (Wu et al. 2012; Huang et al. 2012). Using (8), the cyclostrophic wind balance for an axisymmetric TC is employed to derive a single pressure profile from the multiple-maxima wind profiles, and is presented later in the subsequent subsection.

b. Cyclostrophic wind balance

The relationship between aircraft flight-level tangential winds and pressures in TCs is well represented using the gradient wind approximation (Willoughby 1990a,b, 1991, 2011). As mentioned previously in section 2c, avoiding the problem of the unbounded radial integral of \( f V_G \) as the outer limit of integration to compute the pressure profile approaches infinity allows one to facilitate the computation of the pressure from the cyclostrophic–wind relation. Cast in terms of geopotential height \( Z \) of an isobaric surface instead of surface pressure (e.g., Willoughby and Rahn 2004), the geopotential height is obtained by integrating the cyclostrophic relationship radially inward from an environmental geopotential height \( Z_E \). The result is given by

\[ Z(r) = Z_E + \frac{1}{g} \int_{r}^{\infty} \frac{V_G^2(r')}{r'} dr', \tag{10} \]

where \( Z_E \) is the environmental (undisturbed) geopotential height at infinitely far from the TC center, \( g \) is the acceleration of gravity, and \( r' \) is a dummy variable for the integration, which is done numerically. Equation (10) can be extrapolated from the aircraft flight level to the surface under the assumption of the mean temperature \( T = (T_{SS} + T_{FL})/2 \) of a tropical atmospheric layer between the flight level (subscript FL) and the sea surface (SS), where \( T_{SS} = 300 \text{ K} \) is the typical temperature at the sea surface and \( T_{FL} \) is the temperature at the flight level. Assuming that \( T \) is independent of \( r \) and substituting (10) into the hypsometric equation yields the surface pressure \( P_{SFC} \) within the TC region:

\[ P_{SFC} = \frac{P_F}{P_T} \exp \left[ \frac{1}{R_d T} \int_{r}^{\infty} \frac{V_G^2(r')}{r'} dr' \right], \tag{11} \]

where

\[ P_F = P_{FL} \exp \left( \frac{g Z_E}{R_d T} \right). \tag{12} \]

Here, \( P_{FL} \) is the pressure altitude at the flight level outside the TC region, \( R_d \) is the gas constant for dry air, and \( P_F \) is the asymptotic environmental surface pressure that is extrapolated from \( Z_E \) in the environmental tropical atmosphere. Equation (11) contains the pressure-adjusted flight-level tangential winds but is extrapolated to the pressure at the sea surface. In this study, the \( Z_E \) values of 850 and 700 hPa in the U.S. Standard Atmosphere, 1976 [(Committee on Extension to the Standard Atmosphere) COESA 1976] are, respectively, 1457 and 3012 m. Inward radial integration of (11) requires a known environmental pressure \( P_F \) at the sea surface and at a large radius where a zero asymptotic cyclostrophic wind occurs. Since \( P_F \) is chosen to be the mean value of 1014.3 hPa based on their climatological analyses of Knaff and Zehr (2007), the calculated values of \( T \), respectively, are found to be 278 and 282 K for the mean temperature of the standard 700- and 850-hPa layers at \( Z_E \). This mean surface pressure is only representative of the Atlantic basin. Furthermore, (11) involves an integral that is calculated using the trapezoidal rule (e.g., Press et al. 1992, 125–126).

c. Partitioned pressure profiles for multiple-maxima eyewall tangential wind profiles

In the cyclostrophic–wind relationship, a total surface pressure \( P_{TOT} \) corresponds to the radial profiles of the total (summed) cyclostrophic winds and is obtained by incorporating (8) into (11). As a consequence, \( P_{SFC} (=P_{TOT}) \) is partitioned into three different pressure components that correspond to the individual radial profiles of \( V_F, V_S, \) and \( V_T \), respectively,

\[ P_{SFC}(r) = \frac{P_F(r)P_S(r)P_T(r)}{P_F}, \tag{13} \]

where the partitioned surface pressure components are expressed by
\[ P_p(r) = P_E \exp \left\{ \frac{1}{R_d T} \int_{r}^{r'} \left[ \frac{V_p^2(r') + V_p(r')V_S(r') + V_p(r')V_T(r')}{r'} \right] dr' \right\}, \quad (14a) \]

\[ P_S(r) = P_E \exp \left\{ \frac{1}{R_d T} \int_{r}^{r'} \left[ \frac{V_S^2(r') + V_p(r')V_S(r') + V_S(r')V_T(r')}{r'} \right] dr' \right\}, \quad \text{and} \quad (14b) \]

\[ P_T(r) = P_E \exp \left\{ \frac{1}{R_d T} \int_{r}^{r'} \left[ \frac{V_T^2(r') + V_p(r')V_T(r') + V_T(r')V_S(r')}{r'} \right] dr' \right\}. \quad (14c) \]

where the partitioned radial surface pressure gradient components are given by

\[
\frac{\partial P_p(r)}{\partial r} = \frac{P_p(r)}{R_d T} \left[ \frac{V_p^2(r) + V_p(r) V_S(r) + V_p(r) V_T(r)}{r} \right], \quad (16a)
\]

\[
\frac{\partial P_S(r)}{\partial r} = \frac{P_S(r)}{R_d T} \left[ \frac{V_S^2(r) + V_p(r) V_S(r) + V_S(r) V_T(r)}{r} \right], \quad (16b)
\]

\[
\frac{\partial P_T(r)}{\partial r} = \frac{P_T(r)}{R_d T} \left[ \frac{V_T^2(r) + V_T(r) V_T(r) + V_T(r) V_S(r)}{r} \right]. \quad (16c)
\]

Here (16a)–(16c) reveal that the radial pressure gradient fluctuations are concentrated inside the radii of multiple-maxima eyewall tangential winds.

The total central surface pressure deficit \( \Delta P_{SFC} \) thus is obtained by integrating (15) radially inward from \( P_E \), yielding

\[ \Delta P_{SFC}(r) = \Delta P_{TOT}(r) = \Delta P_p(r) + \Delta P_s(r) + \Delta P_T(r), \quad (17) \]

where the partitioned central surface pressure deficit components are expressed by

\[
\Delta P_p(r) = \frac{1}{P_E} \int_{r}^{0} P_S(r') P_T(r') \frac{\partial P_p(r')}{\partial r'} dr', \quad (18a)
\]

\[
\Delta P_s(r) = \frac{1}{P_E} \int_{r}^{0} P_p(r') P_T(r') \frac{\partial P_s(r')}{\partial r'} dr', \quad \text{and} \quad (18b)
\]

\[
\Delta P_T(r) = \frac{1}{P_E} \int_{r}^{0} P_p(r') P_S(r') \frac{\partial P_T(r')}{\partial r'} dr'. \quad (18c)
\]
As discussed later in the subsequent section, (17)–(18) are useful in diagnosing and evaluating the significant fluctuations in partitioned central surface pressure deficits during the ERC.

4. Partitioned gradient and cyclostrophic vorticity for multiple-maxima eyewall tangential wind profiles

For axisymmetric flow, absolute vorticity $\xi_A$ is calculated as

$$\xi_A = \xi_R + f,$$  
(19)

where $\xi_R = \partial V/\partial r + V/r$ is the relative vorticity. The so-called gradient vorticity may be obtained by replacing the subscript $R$ by $G$ on the first right-hand side of (19), and is given by $\xi_G = \partial V_G/\partial r + V_G/r$. Substituting (5) into this vorticity gives

$$\xi_G = \xi_C - f,$$  
(20)

where $\xi_C (= \partial V_c/\partial r + V_c/r)$ is defined as the “cyclostrophic vorticity.” The formulation in (20) works well within a couple hundred kilometers of the TC center. After substituting (8) into the cyclostrophic vorticity, $\xi_C$ can easily be partitioned into three different vorticity components ($\xi_P$, $\xi_S$, $\xi_T$) that correspond to the individual radial profiles of $V_P$, $V_S$, and $V_T$, respectively, and are given by

$$\xi_C = \sum_{i=1}^{3} \xi_i = \xi_P + \xi_S + \xi_T.$$  
(21)

Following the vorticity derivation of Wood and White (2011), $\xi_P$, $\xi_S$, and $\xi_T$ are obtained by substituting (9) into (21):

$$\xi_i = \frac{\partial V_i}{\partial r} + \frac{V_i}{r} = \kappa_i \omega_i + \frac{\eta_i - \kappa_i + (\kappa_i - \eta_i)\rho^{\eta_i/\lambda_i}}{\eta_i - \kappa_i + \kappa_i\rho^{\eta_i/\lambda_i}} + \omega_i,$$  
(22)

where the partitioned curvature vorticity (angular velocity of rotation) $\omega_i$ component is given by

$$\omega_i = \frac{(V_i/k)\gamma_i}{R_iX} \left[ \frac{\lambda_i \kappa_i^{-1} \eta_i^{\kappa_i^{-1}}}{(\eta_i - \kappa_i + \kappa_i \rho^{\eta_i/\lambda_i})^2} \right].$$  
(23)

The reason why $\kappa_i \geq 1$ in (1)–(2) and also (23) is to avoid difficulty with a vorticity singularity at the vortex center if it is continued to $\rho = 0$ (Wood and White 2011). The exponent $\kappa_i - 1$ in $R_i^{\kappa_i^{-1}}$ in (23) reveals that if $0 < \kappa_i < 1$, the vorticity profile produces the singularity as $\rho \to 0$. The profile has the undesirable property that, $0 < \kappa_i < 1$, vorticity is infinite along the vortex axis. Difficulty with the singularity can be avoided by using $\kappa_i \approx 1$ only. Now, (19) becomes

$$\xi_A = \xi_G + f = \xi_C = \xi_P + \xi_S + \xi_T.$$  
(24)

Interestingly, $f$ does not appear after $\xi_C$ and also $\xi_P + \xi_S + \xi_T$; however, the Coriolis parameter in terms of $R_{CX}$ is included in (9).

Following the Wood and White approach, a critical point of partitioned vorticity peak $\xi_P$ represents a location of maximum vorticity value, and are given by

$$R_{\xi_P} = R_{CX} \left[ \frac{(\eta_i - \kappa_i)\alpha_i^{\kappa_i/\eta_i}}{\eta_i - \kappa_i \alpha_i} \right], \quad \eta_i - \kappa_i \alpha_i > 0.$$  
(25)

Here, $\alpha_i$ is a critical solution obtained with the quadratic formula derived by Wood and White, and is expressed by

$$\alpha_i = \kappa_i (2\kappa_i + \eta_i \lambda_i^{-1}) - \sqrt{[\kappa_i (2\kappa_i + \eta_i \lambda_i^{-1})]^2 - 4\kappa_i^2 (1 + \lambda_i^{-1})(\kappa_i^2 - 1)} > 0, \quad \lambda_i > 0.$$  
(26)

The reason for using (25) is to locate the radius where $\xi_{CX}$ is associated with partitioned local low pressure, as will be presented in this study.

5. TC simulation results

This section uses a TC simulator to provide what each input parameter ($V_X, R_X, \kappa, \eta, \lambda$) may be able to deduce about TC wind structure in terms of intensity, strength, and size changes (e.g., Merrill 1984). We performed parameter tests by varying at least one of the parameters while keeping other parameters unchanged. Finally, we presented a detailed discussion on the manner in which a model TC intensifies, strengthens, and changes in size. Our approach was similar to that of Knaff et al. (2011) who investigated the effects of fine-tuning their different parameters on the model TC intensity, strength, and size. Merrill (1984) pioneered the use of TC wind structural variability in terms of intensity, strength, and size. Intensity is measured by minimum sea level pressure or maximum
azimuthal wind in the primary vortex core; outer core wind strength is a spatially averaged tangential wind speed over an annulus between 100 and 250 km from the TC center (Weatherford and Gray 1988a,b); and size is the axisymmetric extent of gale-force (17 m s$^{-1}$) wind or the average radius of the outermost closed isobar (ROCI). These are illustrated in Fig. 5 as changes from an initial tangential wind profile (dashed curve).

There is little confusion that arises concerning the term “core intensity” surrounding the RMW (Fig. 5). The term does not indicate clearly how variability in the inner wind profile and/or in the shape profile straddling the maximum is related to a change in intensity. The term could be rectified by replacing core intensity by inner core average winds. For instance, tangential wind distributions have U-, V- and bowl-shaped structures (e.g., Fig. 1a) in the inner core of intense, mature hurricanes, as seen in examples of the flight-level profiles of tangential wind (Willoughby et al. 1982 among others) as well as the radial profiles of numerically evolving tangential wind, vorticity, and surface pressure minimum (Schubert et al. 1999; Kossin and Schubert 2001; Hendricks et al. 2009). Some of the distributions in the primary eyewall exhibit the broadly profiles of tangential wind (e.g., Fig. 1c); others display the peaked profiles, noted by WDR. We use surface pressure minimum as a proxy for intensity because it is our goal, via our equation (17), to investigate how a change in the shape and distribution of tangential wind directly affects intensity, strength, and size. For example, a transition of one shape profile into a different shape profile might produce a change in intensity, strength, and/or size as one goes progressively either (i) from top to bottom of Fig. 2 or from left to right, or (ii) from the initial broadly peaked profile (dashed curve) to the final sharply peaked profile (solid curve) in Fig. 5.

From extensive aircraft measurements, Willoughby (1990a,b, 1991, 2011) found that the gradient wind balance was a good approximation to the azimuthally averaged tangential winds in most portions of an inner eyewall above a boundary layer where the strong radial inflows dominate. The National Oceanic and Atmospheric Administration/Atlantic Oceanographic and Meteorological Laboratory/Hurricane Research Division (NOAA/AOML/HRD) flight-level data archive (Willoughby and Rahn 2004) provided consistent wind and geopotential height observations in hurricanes as functions of azimuth and time at fixed pressures. By averaging over azimuth and allowing for linear variation of the wind and geopotential height at fixed radii with time, these data provided an improved depiction of the time-varying azimuthally symmetric structure at a single level over 4–6 h (Willoughby 1990a,b). The maximum of the mean wind and its radius determined in this fashion were consistent estimates of RMW and $V_X$. The profiles also have the advantage of being close to gradient balance except (as in the boundary layer under the eyewall) where radial mean accelerations were large. It is important to recognize that the maximum azimuthally averaged wind and its radius were different from the average maximum wind and the average radius of maximum wind (i.e., mature TCs were rarely always symmetric over their lifetimes).

The individual wind profiles that made up the mean varied from it in interesting ways. They were generally not in gradient balance and many (if not most) of the individual profile wind maxima were stronger than the azimuthal mean maximum, but at different radii. The mean-wind profile was generally broader than the profiles that composed it because these maxima fell at different radii. As reported in the TC literature, many (but not all) of the supergradient winds appeared as the results of asymmetric radial decelerations (Kepert 2001; Kepert and Wang 2001) when analyzed in this way. A key strength of this analysis was that the variations among the observed profiles can be treated as perturbations (nonnecessarily linear) on a well behaved, nearly balanced mean vortex.

Since translation of the TC and the degree of overwater wind asymmetry were not accounted for in the model, the model TC was assumed to be axisymmetric and stationary in a quiescent, tropical environment (i.e., no external atmospheric influences or ocean interaction). The cyclone was assumed to be positioned at 20°N latitude in this study. This latitude was close to the average latitude of 23.7°N based on their climatological analyses of Knaff and Zehr (2007). Note that this mean latitude is only representative of the Atlantic basin.

Table 1 lists the selected parameter values used for our 12 experiments. The main reason for presenting the
side-by-side panels in Figs. 6–11 is to compare the impact of changing one (or more) input parameter (e.g., $\kappa$) on the two different TCs’ radial profiles of $V_C$, $V_G$, $R_C$, $\zeta_A$, $\zeta_G$, $P_{SFC}$, and $\Delta P_{SFC}$. Our parametric model is capable of extending a wind profile out to a large radius at which the tangential wind decreases asymptotically to zero. In this study, the outer limit of 250 km was chosen because details in the portions of the wind and pressure structures near the TC center were displayed for ease of readability.

### a. Hurricanes with single-maximum eyewall tangential winds

The input parameters ($V_{PX}$, $R_{PX}$, $\kappa_P$, $\eta_P$, $\lambda_P$) are listed in Table 1 for initializing a single-maximum (primary) wind profile in a model mature hurricane. We set $V_{SX}$ and $V_{TX}$ equal to zero in (9) so that (8), (9), (13)–(18), and (24)–(25) were reduced to the following form:

$$V_C = V_P,$$

$$V_P = (V_{PX}\gamma_P)\phi_P, \quad \gamma_P = \frac{1}{2}\left(\frac{1}{R_{PCX}} + \frac{1}{2} + \frac{1}{4R_{PCX}^2}\right),$$

$$R_{PCX} = \frac{V_{PX}}{fR_{PX}},$$

$$\phi_P = \frac{\lambda_P \eta_P \kappa_P}{(\eta_P - \kappa_P + \kappa_P \rho_P \eta_P)^{\lambda_P}}, \quad \text{and} \quad \rho_P = \frac{r}{R_{PX}},$$

(27)

(28a)

(28b)

In (18), we set $P_S(r')/P_E = P_P(r'/P_E = 1$, thus giving (30). Using the available cyclostrophic wind in (27), the gradient wind readily is calculated from (3). Figure 6 shows the radial profiles of primary cyclostrophic $V_C$ and gradient $V_G$ winds, primary cyclostrophic $\zeta_C$ and gradient $\zeta_G$ vorticities, cyclostrophic Rossby number $R_C$, surface pressure $P_{SFC}$, and central surface pressure deficit $\Delta P_{SFC}$ in the model tropical storm and hurricane $A$.

A tropical storm illustrates a nondeveloping TC when the shape velocity parameters of $\kappa_P$, $\eta_P$, and $\lambda_P$ (Table 1) produce a nearly flat wind profile (Fig. 6a). Here $V_G$ (corresponding $\zeta_G$) approaches $V_C$ (corresponding $\zeta_C$) when $R_C$ is large near the TC center (Figs. 6a,b). When $R_C$ decreases with large radius, $V_G$ ($\zeta_G$) deviates from $V_C$ ($\zeta_C$) and more slowly decreases with radius than does $V_C$ ($\zeta_C$). The flat wind profile of $V_C$ is supported by the fairly uniform radial pressure gradient (Fig. 6c); the central surface pressure minimum is $-19$ hPa.

A change in $V_{PX}$, $R_{PX}$, $\kappa_P$, $\eta_P$, and $\lambda_P$ (Table 1) evolves the tropical storm into hurricane $A$. As expected, these TCs grow and strengthen during the transition from the
tropical storm to hurricane intensity as a comparison between the left and right panels of Fig. 6 shows. The central surface pressure minimum falls from 219 to 241 hPa; vorticity concentration, initially located at the tropical storm center, is progressively displaced toward \( R_{X} \), where the strongest gradient of the inner wind profile occurs in hurricane A. At the same time, the primary eyewall vorticity maximum \( \zeta_{X} \) increases. Note that \( R_{X} \) is calculated from (32), suggesting that \( \zeta_{X} \) is associated with the central surface pressure minimum. The annular pattern of vorticity is characteristic of a two-celled structure—an axial sinking motion surrounded by an annular convective-scale updraft along the inner edge of the primary eyewall that slopes radially outward with increasing altitude (Jorgensen 1984).

It is vital to point out that (3) is optimized to describe a relatively peaked eyewall wind maximum. Because of the way the cyclostrophic Rossby number \( R_{C} \) changes with radius (Fig. 6b), a relatively broad cyclostrophic wind profile with its maximum at 100 km can result in dislocation of the radius of maximum gradient wind toward the TC center. This maximum occurs at 85 km where \( \partial V_{G}/\partial r = 0 \), as indicated by an upward-pointing arrow adjacent to number represents the radius \( R_{X} \) where maximum vorticity \( \zeta_{X} \) occurs.
\(\partial R_C/\partial r\) changes the radial gradient enough to move the gradient wind maximum away from the radius where \(\partial V_C/\partial r = 0\) toward the TC center. When the \(V_C\) profile is sharply peaked, the displacement of the radius of the gradient wind peak disappears (Fig. 6d). Thus, a mathematical adjustment for the high Rossby number gradient wind approximation with its relatively broad profile is needed.

Figures 7–9 provide what the model parameters (\(\mathbf{m} = [V_X, R_X, \kappa, \eta, \lambda]^T\)) may deduce about the wind changes in a TC having only a single-maximum (primary) eyewall tangential wind. In each experiment, the parameters are used to construct each idealized \(V_C\) and \(V_G\) profiles and the resulting \(R_C\), \(\zeta_A\), \(\zeta_G\), \(P_{SFC}\), and \(\Delta P_{SFC}\) profiles. As shown in Table 1, hurricane A (Fig. 6d) is used as a reference hurricane with the constant parameters. This hurricane is compared with other experimental hurricanes (B–G) when at least one of five free parameters is varied (bold numbers). Comparison of one variable parameter to the one identical reference parameter aids in determining how variations in the shape, strength, and/or size of a selected wind profile are related to changes in TC intensity, as will be discussed in subsequent subsections.
1) VARIABLE \( V_X \)

Idealized tangential wind profiles are constructed with constant \( R_{PX}, \kappa_P, \eta_P, \) and \( \lambda_P \) but variable \( V_{PX} \) from hurricane A to major hurricane B (Fig. 7a). The TC experiences increasing intensity and size as \( V_{PX} \) increases from 45 to 60 m s\(^{-1}\) (indicated by bold numbers shown in Table 1) and its size of the outer circulation increases (as comparison between black and gray curves in the figure illustrates). The corresponding profiles of \( \xi_A, \xi_G, \) and \( R_C \) in hurricane B (Fig. 7b) are similar to those in hurricane A, except that the former profiles have increased in magnitude. The corresponding central surface pressure deficit (Fig. 7c) is deepening (\(-72 \) hPa) in hurricane B, compared to \(-41 \) hPa in hurricane A (Fig. 6f). This is in general agreement with the analyses of Weatherford and Gray (1988b), Croxford and Barnes (2002), and Knaff et al. (2011) who showed that an increase in eyewall tangential wind maximum was strongly correlated with the TC intensity (decreasing surface pressure minimum).

2) VARIABLE \( R_X \)

A change in \( R_{PX} \), when other parameters are held constant, produces a change in the wind structure from hurricane A to hurricane C (Table 1). The variable \( R_{PX} \) plays a role in modulating the radial profiles of \( V_C, V_G, R_C, \xi_A, \xi_G, \) and \( P_{SFC} \), as shown in the right panels of Fig. 7. Only the central surface pressure minimum \( \Delta P_{SFC} \) is unaffected (Fig. 7f). At RMW (\( =R_{PX} \) = 50 km, the \( V_G \) peak is about 1 m s\(^{-1}\) weaker than the \( V_C \) maximum or the ratio of \( V_G \) to \( V_C \) is \( 1/(1 + 0.5R_C^{-1}) = 0.97 \) [e.g., see (4)], which is always less than 1.0 for \( R_C > 10 \). The reason (i) why \( \Delta P_{SFC} = -43 \) hPa at the center of hurricane C (Fig. 7f) is slightly lower than \( \Delta P_{SFC} = -41 \) hPa at the center of hurricane A (Fig. 6f) and (ii) the \( V_C \) peak associated with hurricane C is slightly higher than that associated with hurricane A is owing to the product of \( V_{PX} \) and \( \gamma_P \) as calculated from (28a) and also Fig. 3b. Since the difference between these pressure deficits is insignificant, the deficit appears to be relatively insensitive to variations in \( R_{PX} \), although the variable \( R_{PX} \) appears to produce a size change of the outer wind profile. This finding was consistent with Holland (1980), Weatherford and Gray (1988b), and Knaff et al. (2011) who showed that the variable \( R_{PX} \) is independent of the surface pressure minimum. As a result, the variable may affect a size change but does not have an effect on the intensity of a TC.

3) VARIABLE \( \kappa \)

Numerical TC simulations revealed variability in the shape of inner profiles of tangential wind, vorticity, and surface pressure near the TC center (Schubert et al. 1999; Kossin and Schubert 2001; Hendricks et al. 2009). To mimic such a shape profile, we explore the role in shaping the inner wind profile by making \( \kappa_P \) variable, while keeping other model variables fixed in hurricanes A and D (Table 1). The initial \( \kappa_P \) and \( \eta_P = \kappa_P + 0.5 \) values of 1.5 and 2.0 associated with hurricane A were increased to 3.5 and 4.0 associated with hurricane D. The reason for changing \( \eta_P \) is to keep the outer profile unchanged. As comparison between hurricane A’s inner wind profile (gray curve in Fig. 8a) and that of hurricane D illustrates, a change in \( \kappa_P \) modulates the inner profiles of \( V_C, V_G, R_C, \xi_A, \) and \( \xi_G \) (Figs. 8a,b).

In hurricane D, the tangential wind distribution inside the RMW (Fig. 8a) exhibits the bowl-shaped profile; the corresponding vorticity profile (Fig. 8b) may be described as a thin annulus of strongly enhanced vorticity embedded in the primary eyewall with relatively weak vorticity in the eye and outside the eyewall. This profile results from barotropic instability near the RMW (Schubert et al. 1999). The simulated profiles of tangential wind (Fig. 8a) and vorticity (Fig. 8b) resemble the radial distributions of numerically evolving tangential wind and vorticity (Kossin and Schubert 2001, their Figs. 3 and 11; Hendricks et al. 2009, their Fig. 3) and the flight-level profiles of tangential wind and vorticity observed in Hurricanes Guillermo (1997) and Gilbert (2008) of Kossin and Schubert (2001, their Fig. 1).

Corresponding to the vorticity profile (Fig. 8b), the \( P_{SFC} \) profile inside about \( r = 20 \) km is flatter (black curve in Fig. 8c) than that in hurricane A (gray curve). Simultaneously, \( \Delta P_{SFC} \) rises from \(-41 \) hPa in hurricane A to \(-34 \) hPa in hurricane D, thus producing a decrease in intensity. This rise occurs when the quasi-V-shaped profile of tangential wind in hurricane A transitions into the bowl-shaped profile in hurricane D, as a result of increasing \( \kappa_P \). The shapes of the simulated pressure profile (Fig. 8c) corresponding to the changing tangential wind and corresponding vorticity profiles (Figs. 8a,b) bear resemblance to those of Schubert et al. (1999), Kossin and Schubert (2001), and Hendricks et al. (2009).

A transition from a U-shaped to a V-shaped profile of tangential wind inside the primary eyewall over a period of a few hours or less was documented by Kossin and Eastin (2001) who used aircraft flight-level data to investigate the kinematic and thermodynamic distributions within the eye and eyewall of Hurricane Diana (1984). The changes in the inner wind profile result in measurable and physically important differences in the eyewall vorticity (Fig. 8b) and central surface pressure (Fig. 8c).

Croxford and Barnes (2002, their Fig. 13a) observed the U-shaped profiles of storm-relative tangential wind...
in the primary eyewall of Hurricane Emily (1993), although the profiles remained nearly unchanged and the primary tangential wind maxima increased intensity from 0700 through 1000 to 1600 UTC. During this 9-h period, the RMW slowly contracted inward the TC center. Unfortunately, the radial profiles of corresponding surface pressure in this hurricane were not documented, even though Croxford and Barnes (2002) showed that increased tangential wind maximum decreased the central surface pressure minimum, as is consistent with Figs. 7a,c.

In view of the above, the variable $k_P$, while holding other variables constant, is shown to produce a change in the inner profiles of tangential wind, vorticity, and surface pressure, which in turn affects the intensity of a TC. At the same time, the outer profile appears to be relatively insensitive to variations in $k_P$ because this parameter is dominant near the TC center and does not significantly affect the outer profile as comparison between the black and gray curves in Figs. 8a–c shows.

4) VARIABLE $\eta$  
In the last experiment, the radial profiles were compared by varying $k_P$ from model hurricanes A to D, while other variables were held constant. We now investigate the role of $\eta_P$ in the behavior of the radial profiles of $V_C, V_G, R_C, \zeta_A, \zeta_G, P_{SFC}$, and $\Delta P_{SFC}$. Table 1 and the right panels of Fig. 8 illustrate a change in $\eta_P$ from hurricane A to E. The salient feature in Fig. 8d is that a decrease in $\eta_P$ increases a size of the outer circulation, which is responsible for changing the outer vorticity profiles (Fig. 8e) and deepening the $P_{SFC}$

![Image of diagrams showing Hurricane D and Hurricane E profiles for tangential winds, vorticity, and surface pressure.](image-url)
profile at the TC center (Fig. 8f). The central surface pressure deficit $\Delta P_{SFC}$ drops from $-41$ hPa associated with hurricane A to $-60$ hPa associated with hurricane E. While $\eta_P$ appears to be relatively insensitive to the inner wind profiles near the TC center, the size change caused by $\eta_P$ can affect an intensity of a TC. The $\eta_P$ parameter may be a useful and diagnostic parameter in estimating the size of an outer circulation in terms of the radii of 34- (18), 50- (26), and 64-kt (33 m s$^{-1}$) winds as well as the ROCI in four quadrants of the TC (e.g., Demuth et al. 2006).

5) VARIABLE $\lambda$

Now that we comprehend how the different $\kappa_P$ and $\eta_P$ variables, respectively, control the inner and outer profiles, we further explore the role of $\lambda_P$ in influencing the profiles. In an experimental hurricane F, $\lambda_P$ is increased to 1.0 from 0.2 associated with hurricane A (Table 1). As discussed previously, $\lambda_P$ primarily controls the radial width of the wind profile straddling the wind maximum (e.g., Figs. 1c and 2).

Figure 9 illustrates the role of $\lambda_P$ in sizing and shaping the overall wind profile in two different model hurricanes. When $\lambda_P = 1.0$, a horizontal distance of the profile at a given tangential wind level in hurricane F is wider than that in hurricane A (Fig. 9a), thus resulting in a broadly peaked profile. Such a variation in the width produces a change in the radial profiles of $R_C$, $\zeta_A$, and $\zeta_G$ by subtly displacing their maximum values at $\tilde{R}_{P_{SFC}} = 5$ km closer to the TC center in hurricane F (Fig. 9b) than $\tilde{R}_{P_{SFC}} = 13$ km closer to the RMW in hurricane A (Fig. 6e). Furthermore, an increase in $\lambda_P$ causes $P_{SFC}$ to
Table 2. Details of the tangential wind profile experiments including vortex parameters and results in terms of the TC wind profile, corresponding central (total) surface pressure minimum, and intensity changes. The arrow ↑ represents an increase in magnitude; the arrow ↓ represents a decrease in magnitude; NC denotes no change. The deepening D and filling F of the central (total) surface pressure minimum are indicated.

<table>
<thead>
<tr>
<th>Experimental TC</th>
<th>Parameter</th>
<th>Size</th>
<th>Surface pressure min</th>
<th>Intensity</th>
</tr>
</thead>
<tbody>
<tr>
<td>Major hurricane B</td>
<td>$V_X$</td>
<td>↑</td>
<td>$D$</td>
<td>↑</td>
</tr>
<tr>
<td>$V_X$</td>
<td>↓</td>
<td>$F$</td>
<td>↓</td>
<td></td>
</tr>
<tr>
<td>Hurricane C</td>
<td>$R_X$</td>
<td>↑</td>
<td>NC</td>
<td>NC</td>
</tr>
<tr>
<td>$R_X$</td>
<td>↓</td>
<td>NC</td>
<td>NC</td>
<td></td>
</tr>
<tr>
<td>Hurricane D</td>
<td>$\kappa$</td>
<td>↑</td>
<td>NC</td>
<td>$F$</td>
</tr>
<tr>
<td>$\eta$</td>
<td>↓</td>
<td>$D$</td>
<td>↑</td>
<td></td>
</tr>
<tr>
<td>Hurricane E</td>
<td>$\eta$</td>
<td>↑</td>
<td>$F$</td>
<td>↓</td>
</tr>
<tr>
<td>$\kappa$</td>
<td>↓</td>
<td>$D$</td>
<td>↑</td>
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</tr>
<tr>
<td>Hurricane F</td>
<td>$\lambda$</td>
<td>↑</td>
<td>$F$</td>
<td>↓</td>
</tr>
<tr>
<td>Hurricane G</td>
<td>$\lambda$</td>
<td>↓</td>
<td>$F$</td>
<td>↓</td>
</tr>
</tbody>
</table>

fall and $\Delta P_{SFC}$ to decrease (Fig. 9c), while the width of the central surface pressure minimum decreases. This wind structure may increase the strength of the TC.

Conversely, when $\lambda_P$ is varied from 0.2 to 0.01, the size of the inner and outer wind profiles at a given tangential wind level in Hurricane G is shorter than that in Hurricane A, thus producing a sharply peaked profile (Fig. 9d and Table 1). Just inside the RMW, the peak $R_C$, $\xi_A$, and $\zeta_G$ values (Fig. 9e) are displaced from $R_{\text{PE}} = 13$ km in Hurricane A (Fig. 5f) to $R_{\text{PE}} = 19$ km in Hurricane G (Fig. 9f). Just outside the RMW, $\xi_A$ and $\zeta_G$ then drop off abruptly in a short radial distance before decreasing gradually to zero with increasing radius (Fig. 9e). In addition, such a variation in $\lambda_P$ results in the $P_{\text{SFC}}$ rise at the eye (Fig. 9f).

Based on the simulation results in hurricanes F (G), an increase (a decrease) $\lambda_P$ tends to produce an increase (a decrease) in intensity, strength, and size when other variables are fixed. Such various shapes of tangential wind distributions encompassing the wind maxima are seen in many examples of the flight-level profiles of tangential wind in TCs. Furthermore, the advantage of $\lambda_P$ is that it is operating independently of $\kappa_P$ and $\eta_P$.

Table 2 is summarized to illustrate the details of the experimental hurricanes B–G including model parameters in terms of the tangential wind profile, corresponding total surface pressure minimum, intensity, and size changes.

b. Major hurricanes with dual-maximum eyewall tangential winds

In an intense, symmetric TC, an eyewall replacement cycle occurs when replacement of the inner eyewall by the outer eyewall coincides with a decrease in storm intensity (Willoughby et al. 1982; Willoughby 1990a; Black and Willoughby 1992). The cycle is often accompanied by significant fluctuation in maximum wind speed (i.e., initial weakening followed by reintensification) and central surface pressure. To determine how the TC wind structural changes are related to variations in intensity, strength, and size during which a single-maximum wind profile evolves to a dual-maximum wind profile, we set $V_{r_X}$ equal to zero in (9) so that (8), (9), (13)–(18), and (24)–(25) are reduced to the following:

$$V_C = V_P + V_S,$$

$$V_P = (V_{P_X} \gamma_P) \phi_P,$$

$$\gamma_P = \frac{1}{2} \left( \frac{1}{R_{PCX}} + 2 \sqrt{1 + \frac{1}{4 R_{PCX}^2}} \right),$$

$$R_{PCX} = \frac{V_{P_X}}{f R_{PX}},$$

$$\phi_P = \frac{\eta_P^\lambda_P \kappa_P}{(\eta_P - \kappa_P + \kappa_P \rho_P^\lambda_P)^s},$$

$$\rho_P = \frac{r}{R_{PX}}.$$  

$$V_S = (V_{S_X} \gamma_S) \phi_S,$$

$$\gamma_S = \frac{1}{2} \left( \frac{1}{R_{SCX}} + 2 \sqrt{1 + \frac{1}{4 R_{SCX}^2}} \right),$$

$$R_{SCX} = \frac{V_{S_X}}{f R_{SX}},$$

$$\phi_S = \frac{\eta_S^\lambda_S \kappa_S}{(\eta_S - \kappa_S + \kappa_S \rho_S^\lambda_S)^s},$$

$$\rho_S = \frac{r}{R_{SX}}.$$  

$$P_{SFC} = P_{TOT} = \frac{P_P P_S}{P_E},$$

$$\Delta P_{SFC} = \Delta P_{TOT} = \Delta P_P + \Delta P_S,$$

$$\xi_A = \xi_C = \xi_S,$$

$$R_{r_X} = R_{\text{PE}} \frac{(\eta_P - \kappa_P \rho_P^\lambda_P)^s}{(\eta_P - \kappa_P \rho_P^\lambda_P)^s},$$

$$R_{r_X} = R_{\text{PX}} \frac{\eta_S - \kappa_S \rho_S^\lambda_S}{\eta_S - \kappa_S \rho_S^\lambda_S},$$

where

$$P_P = P_E \exp \left\{ \frac{1}{R_d T} \int_0^{\infty} \left[ \frac{V_P^2(r') + V_P(r') V_S(r')}{r'} \right] dr' \right\}.$$  

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\[ P_S = P_E \exp \left\{ \frac{1}{R_d T} \int_0^r \frac{V_S^2(r') + V_S(r')V_P(r')}{r'} \, dr' \right\}, \]  
(41)

\[ \Delta P_P = \frac{1}{P_E} \int_0^r P_S(r') \frac{\partial P_P(r')}{\partial r'} \, dr', \]  
(42)

\[ \Delta P_S = \frac{1}{P_E} \int_0^r P_P(r') \frac{\partial P_S(r')}{\partial r'} \, dr'. \]  
(43)

Note that \( P_S(r')/P_E, P_S(r')/P_E < 1 \) because \( V_{PX}, V_{SX} \neq 0 \). Also, \( P_P(r')/P_E = 1 \) since \( V_{TX} = 0 \). In (33), we can combine dual profiles to produce realistic matches to observations with double wind maxima. The input parameters are listed in Table 1 for initializing dual-maximum (primary and secondary) wind profiles in model mature hurricanes H, I, and J. Note that some of the parameters are variable, while other parameters are held constant.

A new outer eyewall in response to convective heat release induces a new secondary tangential wind \( V_S \) perturbation, while the developing secondary eyewall migrates slowly inward (Willoughby et al. 1982). This may be illustrated in Fig. 10a. Addition of the \( V_S \) profile to the preexisting \( V_P \) profile in hurricane A (Fig. 6a) results in a significant change in the \( V_C \) and \( V_G \) profiles and also in a developing saddle-shaped wind minimum beyond 50 km from the TC center. We believe that a large area of the central secondary wind \( V_S \) calm occurs at the eye and underneath the primary eyewall region so that the calm does not modify the preexisting primary \( V_P \) profile inside about \( r = 50 \) km, where the zero \( V_S \) profile is flat. This profile requires that \( \kappa_S \) be large and hence \( \eta_S = \kappa_S + 1.0 \), and also \( \lambda_S \) be increased to 1.0 (Table 1). This is quasi-subjectively trying to construct the double profiles \( (V_P \) and \( V_S) \) to match an observed outer wind profile. Wu et al. (2012) and Huang et al. (2012) showed a secondary eyewall formation in association with the broadening of the secondary wind in Typhoon Sinlaku (2008).

In hurricane H, the developing secondary wind increases the strength (Fig. 10a) and intensity by decreasing the surface pressure minimum (Fig. 10c), even when the primary wind remains unchanged. It is notable that the \( P_P \) and \( P_{TOT} \) profiles corresponding to the evolving \( V_S \) profile are changed. The \( P_P \) profile (gray curve in Fig. 10c) in hurricane H, prior to the inception of the secondary eyewall, is identical to that (black curve in Fig. 6f) in hurricane A. The \( P_P \) profile (short dashed curve) has dropped because the nonzero \( V_S \) profile in the product of \( V_P \) and \( V_S \) [e.g., see (40)–(41)] partially contributes to \( P_P \) and \( P_S \), even when \( V_{PX} \) and \( R_{PX} \) remain unchanged in both hurricanes A and H. In this situation, the \( P_{TOT} \) drop is approximately \(-9 \) hPa at the TC center.

The TC continues to grow and strengthen during the transition from hurricane H to hurricane I, as dual inspection between Figs. 10a,d shows. As the strength continues to increase and the secondary eyewall slowly contracts inward (not shown), the radial profiles of \( P_P, P_S \), and \( P_{TOT} \) continue to fall significantly. Note that variations in \( V_{SX}, R_{SX}, \kappa_S, \eta_S, \) and \( \lambda_S \) (Table 1) undergo major changes in shaping the secondary wind profile (e.g., from the much broadly peaked profile of \( V_S \) in hurricane H to the sharply peaked profile with a prominent saddle-shaped wind minimum located between the primary and secondary eyewalls in hurricane I). Such variables produce dramatic surface pressure drops. It is notable that the \( P_P \) profile (gray curve in Figs. 10c,f) would have occurred without the presence of the \( P_S \) profile associated with the secondary eyewall.

Another important feature in Figs. 10c,f is that the product of \( P_P \) and \( P_S \) and the result divided by \( P_E \) in (36) are reflected in the “kink” (relatively depressed pressure) located at \( R_{55x} \) on the evolving \( P_{TOT} \) profile. As the secondary eyewall contracts and strengthens, the kink moves from \( R_{55x} = 132 \) to \( R_{55x} = 91 \) km. Concurrently, the secondary eyewall vorticity peak (\( \zeta_{55x} \)) has experienced a substantial increase (Figs. 10b,e).

The evolution of the wind profiles from hurricane I to hurricane J is illustrated in the top panels of Figs. 10d and 11a, respectively. When the inner eyewall collapses, the TC decreases intensity as the strength in the secondary eyewall continues to increase. The TC then increases intensity again as the former secondary eyewall transitions into the primary eyewall that continues to intensify. According to Willoughby et al. (1982), the total surface pressure \( (P_{TOT}) \) minimum increases when the \( P_P \) rise because the weakening of the inner eyewall exceeds the \( P_S \) fall due to the strengthening of the outer eyewall, as comparison between Figs. 10f and 11c illustrates. When the inner eyewall eventually dissipates, the \( V_P \) profile vanishes and the \( P_P \) profile returns to the \( P_E \) profile (not shown).

There is a subtle similarity between the simulated profiles of tangential wind and pressure in Figs. 10a,c,d,f and radial profiles of the flight-level tangential wind and 700-hPa geopotential height observed by aircraft in Hurricane Allen (1980) [Figs. 4 and 15 of Willoughby et al. (1982)]. There is also a resemblance between the simulated profiles and the flight-level (450-m altitude) tangential wind and surface pressure profiles from aircraft observations of Hurricane Hugo (1989) in Figs. 11a and 15 of Marks et al. (2008). The simulated profiles of
primary and secondary tangential wind and vorticity (Figs. 10d, e) resemble the flight-level profiles from aircraft observations of Hurricane Gilbert (1988) of Kossin et al. (2000, their Fig. 1b).

In summary, Fig. 12 presents evolution of a simple ERC. The radial profiles of $V_G$ and $P_{TOT}$ in hurricanes A, I, J, and H have been assimilated into the resultant profiles. According to Willoughby et al. (1982) and Willoughby (1988, 1995), a symmetric, mature TC first experiences increasing intensity when a well-defined primary (inner) eyewall exists. The TC then increases the strength when a secondary (outer) ring of convection in its developing stage forms around the primary eyewall. The TC further decreases intensity as the fully developed secondary eyewall contracts and strengthens when the primary eyewall weakens and eventually vanishes. Finally, the TC increases intensity again when the well-defined secondary eyewall remains while transitioning into a new primary (inner) eyewall. At the end of the ERC cycle, the TC would be larger, more intense, and stronger, but the timing of the changes would be out of phase.
In the left panels of Fig. 12, the white, thin curve is calculated from the gradient–wind relation, given by

\[ V_G = \frac{rf}{2} \pm \sqrt{\left(\frac{rf}{2}\right)^2 + r \alpha \frac{\partial P_{TOT}}{\partial r}}, \quad (44) \]

where \( \alpha = R_d T/P_{TOT} \) is the specific volume of air, and \( \partial P_{TOT}/\partial r \) is calculated from (15), via (14) and (16). The positive root here is chosen. In the right panels of Fig. 12, the \( P_{TOT} \) profile is indicated. This white curve (Figs. 12a–d) is superimposed on the black, thick curve that represents \( V_G \) in (3) for comparison. It is demonstrated that (44) coincides with (3) well. Provided that the radial profiles of the partitioned cyclostrophic wind components in (14) and (16) are known, (13) and (15) are more accurate than the limited Schloemer (1954) relation. Furthermore, the simulated wind and corresponding pressure profiles (Fig. 12) compare favorably with the flight-level tangential wind and surface pressure profiles from aircraft observations of Hurricane Wilma (2005) of Sitkowski et al. (2012).
c. Major hurricanes with triple-maximum eyewall tangential winds

Triple concentric eyewalls consisting of three complete rings of enhanced radar reflectivity with echo-free moats have been rarely documented. Typhoon June (1975) may be the first reported case of triple eyewalls observed by reconnaissance aircraft (Holliday 1976). McNoldy (2004) documented the radial profiles of the triple concentric eyewalls observed by aircraft in Hurricane Juliette (2001). Sitkowski et al. (2011) documented that Hurricane Frances (2004) (see Fig. 4) and Hurricane Ivan (2004) exhibited triple-maximum tangential winds during the ERCs.

To simulate a simple TC with triple-maximum tangential winds, a new radial profile of a tertiary tangential wind \( V_T \) was added to the profiles of preexisting \( V_P \) and \( V_S \) in hurricane J to construct the resultant profile in hurricane K (Table 1 and Fig. 11d). It was assumed that the tertiary concentric eyewall contained the tertiary tangential wind maximum \( V_{TX} \) with the same characteristic circulations and structures as the primary and secondary concentric eyewalls. Radial profiles of the partitioned surface pressures \( \Delta P_{TOT} \) are indicated.
and total central surface pressure deficit ($\Delta P_{\text{TOT}} = \Delta P_P + \Delta P_S + \Delta P_T$) in (17)–(18) corresponding to $V_P$, $V_S$, and $V_T$ are depicted in Figs. 11d–f. The corresponding profiles of $R_C$, $\xi_A$, $\xi_G$, $\xi_P$, $\xi_S$, and $\xi_T$ are also illustrated in the figure.

Concurrent with changes in the radial tangential wind distributions in major hurricane K, the $P_{\text{TOT}}$ profiles exhibit substantial changes, as scrutiny of Figs. 11c,f shows. The $P_{\text{TOT}}$ minimum has dropped from 963 to 953 hPa; the deficit $\Delta P_{\text{TOT}} = -51$ hPa has decreased to $-61$ hPa. This is a result of the developing $P_T$ profile (long–short dashed curve in Fig. 11f) that corresponds to the slowly intensifying $V_T$ profile. As a consequence, $\Delta P_T = -6$ hPa is produced. It is notable that $\Delta P_P = -19$ hPa and $\Delta P_S = -36$ hPa have decreased slightly since the nonzero $V_T$ profile partially contributes to $P_P$, $P_S$, and $P_T$ [e.g., see (14)].

The subtly depressed pressure (kink) at $R_{\text{SX}} = 187$ km from the TC center (Fig. 11f) is again associated with the developing tertiary eyewall vorticity $\xi_T$ maximum (Fig. 11e). This kink is analogous to the other kink associated with the well-pronounced secondary eyewall vorticity maximum $\xi_{\text{SX}}$ at $R_{\text{SX}} = 45$ km from the center (Figs. 11e,f).

The marked kinks reflected in the simulated surface pressure profiles (Fig. 11f) resemble those observed in Hurricane Frances (2004) (Fig. 4b), particularly at 35 km on the left side of the eye and 40 km on the right side. Based on our simulations, the slightness of the tertiary kinks in Frances’ pressure profile (located at 95 km on the left side and 115 km on the right side) may be explained by the weak but developing tertiary wind and vorticity maxima associated with the developing tertiary ring of radar reflectivity.

More research is needed on the climatological characteristics of tertiary wind profiles including the wind maxima in association with tertiary eyewalls. The question needed to be addressed is as follows: Do the tertiary eyewall structures associated with tertiary tangential wind maxima have the same characteristic circulation and structures as the primary and secondary concentric eyewalls? This is important because anticipating changes in TC intensity due to multiple ERCs is one of the most challenging aspects of TC forecasting.

6. Conclusions and future work

The new parametric TC wind profile model has been developed by tailoring the Wood and White (2011) parametric tangential wind profile for TC applications. The model avoids the problem of the unbounded radial integral of the Coriolis term in gradient wind balance as the outer limit of integration to calculate the pressure profile approaches infinity. A gradient wind $V_G$ is readily derived from a cyclostrophic wind $V_C$ formulated with the cyclostrophic Rossby number $R_C$, and pressure is simply computed from cyclostrophic balance, provided that radial profiles of the partitioned cyclostrophic wind components in the parametric model are known. When $R_C$ is large near the TC center, $V_G$ (corresponding $\xi_G$) approaches $V_C$ (corresponding $\xi_C$). When $R_C$ decreases with increasing radius from the center, $V_G$ ($\xi_G$) slowly drops off with radius and also increasingly deviates from $V_C$ ($\xi_C$) that decreases with radius. The partitioning scheme is a useful diagnostic technique that permits investigators to quantitatively describe and interpret sensitive fluctuations in central surface pressure deficits and cyclostrophic and gradient wind maxima due to evolving primary, secondary, and/or tertiary eyewall tangential winds during ERCs.

The TC simulations demonstrate that the parametric model is capable of reproducing the representative results of flight-level winds and pressures as measured or inferred by aircraft reconnaissance data. The main conclusions of this study are as follows:

1) The $V_X$ variable, as expected, is shown to be sensitive to intensity and size by increasing the $V_X$ magnitude, while other variables (i.e., $R_X$, $\kappa$, $\eta$, and $\lambda$) remain unchanged.

2) The $R_X$ variable, while other variables are constant, is shown to be relatively insensitive to intensity and size of the inner and outer wind profiles.

3) The $\kappa$ variable, termed as the growth parameter, is shown to be sensitive to intensity by controlling the linearity and nonlinearity of the inner tangential wind profile inside the RMW only, while other variables are fixed. When $\kappa = 1$, a V-shaped (linear) profile is produced and related to the inner core of solid-body rotation. When $\kappa$ increases from 1.0, the V-shaped profile transitions to bowl- to U-shaped (nonlinear) profiles, the central width of wind calm is increased at the TC center, and the surface pressure minimum rises. Additionally, vorticity concentration is displaced from the center to some radius where the strongest gradient of the inner profile occurs. The resultant vorticity profile exhibits a ring of strongly enhanced vorticity embedded in the primary eyewall with relatively weak vorticity in the eye and outside the eyewall. This profile satisfies the necessary condition of barotropic instability within the eyewall (Schubert et al. 1999).

4) The $\eta$ variable, termed as the decay parameter, is shown to be sensitive to intensity by controlling the size of the outer wind profile outside the RMW only, when other variables are held constant. Since $\eta$ is
A single wind profile employing five key parameters can be partitioned into as many as triple wind maxima. The definitions of \( \kappa, \eta, \) and \( \lambda \) are more intuitive than those used previously (e.g., the Holland \( B \) parameter) because the choice of \( \kappa, \eta, \) and \( \lambda \) appears to be much easier to understand than those used previously. It relates them to the properties of real-world wind profiles well.

For any realistic axially symmetric TC wind profile with a single maximum, it is possible to adjust the parameters \( (V_X, R_X, \kappa, \eta, \) and \( \lambda) \) to match observed maximum wind, radius of maximum wind, radius of gale-force winds, and pressure fall from a TC periphery to the storm center. As a practical matter, (i) the growth parameter \( \kappa \) determines the vorticity profile inside the eyewall (Schubert et al. 1999; Kossin et al. 2000; Kossin and Eastin 2001; Kossin and Schubert 2001; Hendricks et al. 2009); (ii) the decay parameter \( \eta \) determines the size of an outer circulation in the Merrill (1984) and Weatherford and Gray (1988b) sense; and (iii) the size parameter \( \lambda \) determines the radial width of the velocity profile straddling the maximum. Adjustable parameters \( \kappa, \eta, \) and \( \lambda \) play vital roles in influencing a surface pressure minimum so that fine-tuning them is much like adjusting the Holland \( B \) parameter to get the proper results. Combining multiple profiles can produce realistic matches to observations with multiple-maxima winds. Manageable complications include conversions among observed, gradient, and cyclostrophic winds and (sometimes) lack of uniqueness of the parameter values that define fits to multiple-maxima observed profiles. Straightforward solutions exist to the first problem, but future study is needed to define consistent sets of parameters that fit complicated observed profiles.

Our near-future work will include application of the parametric model to radial profiles of reconnaissance aircraft wind and pressure (or geopotential height) data. A fitting algorithm using the data to fit the wind and pressure profiles will involve minimizing a cost function. The results will enable us to evaluate the distributions of the fitted parameters and critically examine the fitted profile's realism in comparison with observed tangential wind and pressure/geopotential height profiles in varying stages of TCs.

Potential applications of the TC parametric model include analytical or numerical model initialization for wind specification, hurricane risk model (e.g., Vickery and Twisdale 1995), model of wind-driven sea and other oceanic response to TC (e.g., Phadke et al. 2003), storm-surge inundation (e.g., Jelesnianski 1966), climatology of intensity and wind structure and pressure changes associated with ERCs (e.g., Sitkowski et al. 2011), and others. Also, it is possible for the parametric model to construct a two-dimensional horizontal wind and pressure fields for such applications.

Acknowledgments. The lead author would like to thank Neal Dorst of NOAA/AOML/HRD for his assistance in accessing the flight-level aircraft data of Hurricane Frances (2004). Particular thanks are extended to Corey Potvin of NOAA/OAR/NSSL for providing editorial assistance in the earlier version of the paper. The authors appreciate the constructive comments and suggestions provided by Pat Harr of the Naval Postgraduate School, John Knaff of NOAA/NESDIS Regional and Mesoscale Meteorology Branch, and another anonymous reviewer.

APPENDIX

Determination of the Decay Parameter \( \eta \) in a Mature TC

Riehl (1963) applied the Rankine vortex (RV) to approximate the observed tangential winds in mature TCs. In the RV model, a tangential velocity \( V_{RV} \) increases linearly from a circulation center to a radius \( R_X \) where the velocity \( V_X \) attains its maximum. Beyond \( R_X \), the velocity is inversely proportional to distance. Mathematically, the model is given by

\[
\frac{V_{RV}}{V_X} = \left( \frac{r}{R_X} \right)^{-\mu},
\]

where \( \mu \) is the power-law exponent that governs the inner and outer velocity profiles. According to Riehl (1963), \( \mu = -0.5 \) for \( r > R_X \), that is, the velocity decreases inversely as the square root of radius outside the radius of maximum wind. Using (A1), the outer decay
parameter $\eta$ in (2) can easily be determined by taking the limit of (2) as $\lambda \to 0$. The normalized tangential velocity profile of the Wood and White (2011) model exactly coincides with that of the modified Rankine vortex and is given by

$$
\lim_{\lambda \to 0} \left( \frac{V_{WW}}{V_X} \right) \begin{cases} 
\left( \frac{r}{R_X} \right)^\kappa, & r \leq R_X, \\
\left( \frac{r}{R_X} \right)^{(\kappa-\eta)}, & r > R_X,
\end{cases}
$$

$1 \leq \kappa < \eta.

(A2)

The estimated values of $\kappa$ and $\eta$ easily are determined by setting (A1) equal to (A2) and then taking the natural logarithm of the result, which yields

$$
\kappa - \eta = \mu = \begin{cases} 
1.0, & r \leq R_X, \\
-1.0, & 0.5, & r > R_X.
\end{cases}
$$

(A3)

When $\kappa = \mu = 1.0$, for example, the tangential velocity increases linearly from a circulation center to $R_X$. When $\mu = -1.0$, $\eta = \kappa - \mu = \kappa + 1.0$ meaning the velocity is inversely proportional to distance $r$ beyond $R_X$ (potential flow). When $\mu = -0.5$, $\eta = \kappa - \mu = \kappa + 0.5$, indicating that beyond $R_X$, the tangential velocity decays more slowly than $r^{-1}$. In this study, $\mu = -0.5$ is used, which is close to some values of about $-0.6 \pm 0.1$. These outer velocity distributions were found to give a good approximation to the observed outer wind profiles of TCs (Hughes 1952; Riehl 1954, 1963; Malkus and Riehl 1960; Gray and Shea 1973). Tropical cyclones are often characterized by a relatively slow decrease of tangential wind outside the RMW, and hence by a corresponding cyclonic vorticity skirt (Mallen et al. 2005).

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