Near-Ground Rotation in Simulated Supercells: On the Robustness of the Baroclinic Mechanism*

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ABSTRACT

This study addresses the robustness of the baroclinic mechanism that facilitates the onset of surface rotation in supercells by using two idealized simulations with different microphysics parameterizations and by considering previous results. In particular, the importance of ambient crosswise vorticity relative to baroclinically generated vorticity in the development of near-ground cyclonic vorticity is analyzed. The storms were simulated using the CM1 model in a kinematic base state characterized by a straight-line hodograph. A trajectory analysis spanning about 30 min was performed for a large number of parcels that contribute to near-surface vertical-vorticity maxima. The vorticity along these trajectories was decomposed into barotropic and non-barotropic parts, where the barotropic vorticity represents the effects of the preexisting, substantially crosswise horizontal storm-relative vorticity. The nonbarotropic part represents the vorticity produced baroclinically within the storm. It was found that the imported barotropic vorticity attains a downward component near the surface, while the baroclinic vorticity points upward and dominates. This dominance of the baroclinic vorticity is independent of whether a single-moment or double-moment microphysics parameterization is used. A scaling argument is offered as explanation, predicting that the baroclinic vertical vorticity becomes increasingly dominant as downdraft strength increases.

1. Introduction

One of the outstanding questions in tornado research remains the origin of “seed” ground-level rotation¹ that horizontal convergence can act upon to realize a compact vortex (e.g., Davies-Jones et al. 2001). It has been established that the onset of this initial rotation is due to the rearrangement of initially horizontal vortex lines within downdrafts (Davies-Jones and Brooks 1993; Walko 1993; Wicker and Wilhelmson 1995; Adlerman

et al. 1999; Davies-Jones et al. 2001; Davies-Jones and Markowski 2013; Dahl et al. 2014; Markowski et al. 2014; Schenkman et al. 2014; Parker and Dahl 2015), assuming negligible preexisting vertical vorticity in the storm’s environment. In general terms, these horizontal vortex lines may either be generated within the storm by buoyant or frictional torques, or the horizontal vorticity may be imported into the downdraft from the environment. This study is part of an ongoing effort to explore which of these contributions dominate in what situation, which is crucial for a complete understanding of tornado dynamics. In an attempt to tackle this problem, Dahl et al. (2014) applied a vorticity decomposition technique to quantify the roles of storm-generated and imported vorticity in an idealized free-slip simulation of the Del City, Oklahoma, supercell. They found that the imported ambient vorticity (treated as “barotropic vorticity”) did not contribute much to the development of vertical-vorticity maxima at the lowest model level. Rather, the near-surface vertical vorticity originated primarily from horizontal buoyancy torques within the storm. This behavior is due to the orientation of the ambient vorticity vector relative to the storm-relative flow: if this barotropic vorticity is mostly streamwise it

¹ In this study, “ground level” rotation or “near ground” rotation refer to rotation of air about a vertical axis an arbitrarily small distance above the lower boundary.

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will tend to remain aligned with the velocity vectors. Hence, as the trajectories turn on the horizontal plane near the ground, the barotropic vorticity likewise turns horizontally and thus cannot contribute to *vertical* near-ground vorticity, as already anticipated by Davies-Jones and Brooks (1993) and Davies-Jones et al. (2001). However, if the ambient vorticity has a crosswise component, it is generally possible for this vorticity to contribute to, and perhaps dominate, the vertical vorticity at the downdraft base. Rotunno and Klemp (1985) simulated a supercell in an environment with crosswise vorticity. From their work [Fig. 12 in Rotunno and Klemp (1985)] it may be inferred that in their simulation the barotropic vertical vorticity is negative near the ground and that it is dominated by the positive baroclinic vertical vorticity.² However, Rotunno and Klemp (1985) and other studies demonstrating the dominance of the baroclinic mechanism (Davies-Jones and Brooks 1993; Wicker and Wilhelmson 1995; Adlerman et al. 1999) used a warm-rain Kessler microphysics parameterization, which has long been known to overestimate low-level baroclinity (e.g., Markowski 2002). The question is thus whether the prevalence of the baroclinic mechanism is merely an artifact of the microphysics parameterization.

The purpose of this study is to analyze the relative importance of the baroclinic and barotropic mechanism in the presence of crosswise vorticity employing the vorticity decomposition approach and to test the sensitivity of the results by using two different microphysics schemes. Also, a scaling argument is offered as explanation for the results.

This study is focused on the initial development of tornado-cyclone-scale vorticity maxima at the lowest model level. That is, only the seed near-ground rotation for possible tornadogenesis is addressed herein. Whether or not this vorticity is actually concentrated into a strong tornado-like vortex by vertical stretching is a separate problem not considered in this study [but it is discussed elsewhere, e.g., by Markowski and Richardson (2014)].

2. Methods

a. Experimental design

The goal is to produce simulations of a supercell that develops vertical vorticity ζ at the lowest model level while ingesting appreciable crosswise storm-relative

vorticity. The most straightforward way to accomplish this goal is to use a unidirectionally sheared base-state flow (e.g., Rotunno and Klemp 1985), as detailed below. The simulations were carried out with the Bryan cloud model, version 1 [CM1; Bryan and Fritsch (2002)], release 17. The forward trajectory calculations (see section 2c) within the model were modified, using a fourth-order Runge–Kutta time integration and Lagrange polynomials for the spatial interpolation to obtain the velocities at the parcels' locations (rather than the trilinear interpolation in the standard distribution of CM1). The horizontal model domain ($\sim 125 \times 125 \text{ km}^2$) has a grid spacing of 250 m. The vertical grid spacing varies from 100 m near the ground to 250 m at the domain top, which is at 20 km AGL. The lowest scalar model level is at 50 m AGL. A sponge layer is employed in the uppermost 6 km and the lateral boundary conditions are open while the lower and top boundaries are free slip, and the Coriolis parameter is set to zero. One of the simulations uses a single-moment Lin-type microphysics parameterization (Gilmore et al. 2004), in which the rain-intercept parameter was reduced to 10^6 m^{-4} to prevent overly cold outflow (Dawson et al. 2010). The other simulation utilizes the double-moment Morrison microphysics scheme (Morrison et al. 2009) using the “hail-like” graupel option.

Guided by Rotunno and Klemp (1985), the base state is given by a unidirectional wind profile, where the x component of the base-state flow, $\bar{u}$, increases linearly with height from -15 to $+15 \text{ m s}^{-1}$ within the lowest 7500 m AGL and remains constant above. The y component of the base-state flow is zero. The thermodynamic base state is given by the Weisman and Klemp (1982) analytical profile. The storm was initiated using convergence forcing as described by Loftus et al. (2008), using a minimum divergence of -10^{-3} s^{-1} applied in a 2000-m-deep layer in the center of the domain for 15 min, with the shape control parameters $\lambda_x = \lambda_y = 10^4$ (Loftus et al. 2008). Convergence forcing (rather than the “warm-bubble” initiation) was necessary to prevent the storm from evolving into a quasi-linear convective system. The splitting storms evolve in a practically symmetric fashion and develop into persistent, discrete supercells that each produce several compact and deep vortices in contact with the ground (with maximum vertical vorticity at the lowest model level in each case reaching about 0.1 s^{-1}) during the simulation period of 5400 s. In the remainder of this paper we will focus on the cyclonic, right-moving storms. In each simulation the grid was transformed to a frame stationary with respect to the right-moving cell by subtracting an average storm motion of $\mathbf{c} = (-4, -4) \text{ m s}^{-1}$ from the horizontal velocity vectors. This implies that the trajectory analysis (see section 2c) pertains approximately to the storm-relative

² The material circuit analyzed by Rotunno and Klemp (1985) was located in a regime of appreciable baroclinity at the initial time, such that the association of the initial circulation with the ambient contribution is somewhat uncertain. However, because the baroclinic production (see their Fig. 12) was positive at the initial time while the circulation was negative, it seems likely that the ambient circulation indeed was negative.

frame. After the split, both cells propagate symmetrically off the hodograph, so that the vorticity attains a storm-relative streamwise component. However, the alignment of the vorticity vector averaged over all analyzed parcels (see section 2c) still deviates by 64° to the right of the average storm-relative velocity vector at the time the trajectory analysis is started, so that this scenario is suitable for testing the direct effect of ambient crosswise vorticity on near-ground rotation.

b. Vorticity decomposition

In this study the vorticity separation technique described by Dahl et al. (2014) is used. In general terms, the total vorticity at a given time t may be decomposed into two parts: a barotropic part, which is due to the rearrangement of vorticity present at an arbitrary initial time, and a nonbarotropic part, which is due to *production* of vorticity by pressure (and frictional/diffusional) torques, and subsequent reconfiguration (e.g., Dahl et al. 2014 and the references therein). The barotropic vorticity may be determined by calculating the deformation gradient of a fluid volume along its trajectory, which is done by tracking the relative displacements of parcels within “Lagrangian stencils” (i.e., sets of six parcels that are each centered around the parcel of interest and initially aligned along the three Cartesian axes). Once the barotropic vorticity is determined, the nonbarotropic vorticity may simply be inferred from the difference between the known total vorticity and the barotropic vorticity along the trajectories of interest. Herein the barotropic vorticity represents the ambient vorticity, which characterizes the kinematic environment of the storm. The nonbarotropic vorticity then represents the storm-generated vorticity. This interpretation requires that the forward trajectories are launched far enough away from the storm such that the initial vorticity is not contaminated by baroclinic production in the storm’s far field. The procedure to obtain suitable trajectories is described next.

c. Parcel trajectories

The objective is to obtain highly accurate forward trajectories calculated on the large time step (2.0 s) within CM1 and to analyze those parcels that acquire positive vertical vorticity while descending through the lowest model level. This criterion is consistent with the notion that the initial vertical vorticity in supercells is generated in downdrafts (Davies-Jones 1982; Davies-Jones and Brooks 1993; Davies-Jones 2000; Davies-Jones and Markowski 2013; Dahl et al. 2014; Parker and Dahl 2015).

Dahl et al. (2012) suggested that forward trajectories near vorticity extrema are more accurate than backward trajectories. Moreover, forward trajectories can be calculated within CM1 on every large model time step

without the need to store such high-resolution output. The disadvantage of forward trajectories is that the initial locations of the trajectories ending up in a certain region of interest, are unknown. In contrast to the approach by Dahl et al. (2014), an iterative technique using backward trajectories was used to identify the source regions of relevant parcels. First, a dense cloud (~ 2020 parcels km^{-3}) of forward trajectories was seeded at 3000 s in a 3-km-deep, $20 \times 20 \text{ km}^2$ box surrounding the main downdraft cores that produce vertical vorticity at their bases. Only those trajectories were captured with $\zeta > 0.001 \text{ s}^{-1}$ and vertical velocity $w < -0.5 \text{ m s}^{-1}$ ($0.0 < \zeta < 0.0005 \text{ s}^{-1}$ in the double-moment run, as detailed below) as they descended through the 40–60 m AGL height interval centered at the lowest model level (50 m AGL). The time window within which the above kinematic criteria needed to be fulfilled, covered the period between 3200 and 3900 s. This period includes the development of several ζ maxima, rendering the analysis more general compared to just focusing on a single ζ maximum. Finally, the y component of the velocity was required to be less than zero, which was done to reduce the number of parcels swept toward the north [those parcels getting trapped along the rear-flank gust front (RFGF) are most likely to be relevant in tornadogenesis]. The parcels were not tracked below the lowest scalar model level at 50 m AGL because of uncertainties in the specification of the lower boundary condition for the horizontal winds (see Dahl et al. 2014). This was deemed an acceptable approach because the focus is on the downdraft production of potential seed vertical vorticity for tornado-like vortices.

Once parcels of interest were identified, backward trajectories were calculated for an interval of 30 min, using history files every 30 s and a second-order Runge–Kutta scheme with a time step of 2.0 s. This 30-min time interval was found to be necessary to ensure that the parcels are far enough away from the storm such that their initial vorticity is sufficiently unperturbed, as detailed below. These comparatively coarse backward trajectories merely served as guidance for where to seed the (~ 30 min) forward trajectories in a restart run. The initial positions at $t = 1200$ s of these long-history trajectories were located in the subdomain $[76, 96] \times [61, 71] \times [0.05, 3] \text{ km}^3$ (the origin of the coordinate system is at the southwest bottom corner of the domain), and forward integration was again done on the large time step within CM1. The initial distance between the parcels along the Cartesian axes was 50 m, yielding 4836 060 parcels. The output interval of the trajectory data is 10 s.

For the single-moment run, the same filter criteria as for the initial set of forward trajectories were applied, yielding 3481 parcels of interest. For the initial (barotropic)

FIG. 1. Shown are the 1695 trajectories at 3780 s that contribute to vertical vorticity at the lowest model level, color coded based on their initial altitude (see color bar). Shown are (top) the projection onto the (x, z) plane, (right) the projection onto the (y, z) plane, and (left, main panel) the horizontal projection. In the main panel, the solid black contour shows the 20-dBZ reflectivity and the dashed black contour shows the -3 m s^{-1} vertical velocity at 3129 m AGL. Vertical vorticity at the lowest model level is represented by red contours (positive values: solid, contoured for 0.01, 0.03, and 0.05 s^{-1} ; negative values: dashed, contoured for -0.05 , -0.03 , and -0.01 s^{-1}). Wind vectors at the lowest model levels are also shown, and the extent of the cold pool (-1-K potential temperature perturbation) is represented by the blue line.

vorticity to represent the ambient vorticity as accurately as possible, another filter was applied to keep only those parcels whose initial vertical vorticity was 0.0005 s^{-1} or less, and whose horizontal vorticity was perturbed by less than 10% from the base-state value, which yielded 1846 parcels. Finally, to obtain the barotropic vorticity, another restart run is necessary, this time including the Lagrangian stencils surrounding each of the 1846 parcels, analogous to the description by [Dahl et al. \(2014\)](#).

About 10% of the Lagrangian stencils became strongly and highly asymmetrically deformed, such that the barotropic vorticity could no longer be inferred accurately. This may in part be related to trajectory errors in the strongly divergent region near the ground beneath intense downdrafts [analogous to errors experienced by the backward trajectories analyzed by [Dahl et al. \(2012\)](#)]. The deformation of the stencils may be quantified by the magnitude of the material deformation gradient tensor. Relative to a Cartesian basis the deformation gradient is given by

$$F_{ij} = \frac{\partial x_i}{\partial a_j}, \quad (1)$$

where x_i are the spatial coordinates of the parcel at time t , a_j are the initial coordinates of the parcel, and the indices i and j are running from one to three. The magnitude of the deformation gradient is approximately given by

$$F \approx \left(\sum_{ij} \frac{\Delta x_i}{\Delta a_j} \frac{\Delta x_j}{\Delta a_i} \right)^{1/2}, \quad (2)$$

where Δx_i is the distance along the x_i axis of the parcels initially aligned along the x_j axis, and Δa_j is the initial parcel separation along the x_j axis, which is 2 m. Considering the distribution of F for all stencil volumes (not shown) and omitting the outliers ($F > q_3 + 1.5 \times \text{IQR}$, where q_3 is the 75th percentile of the distribution and IQR is the interquartile range) leaves only those parcels with a deformation magnitude of less than 90. This

criterion conveniently filters out those parcels belonging to a common stencil that strongly diverge at the base of a downdraft. If only these “well behaved” stencils ($F < 90$) are retained, a total of 1695 parcels remains.

For the double-moment simulation, the same initial conditions and filtering criteria of the trajectories in the downdraft were used, except that $0.0 < \zeta < 0.0005 \text{ s}^{-1}$ and $-0.3 < w < 0 \text{ m s}^{-1}$. These criteria were chosen to include those parcels with near-zero positive vertical vorticity in weak downdrafts (thereby also testing the robustness of the parcel-selection criteria), while at the same time keeping the overall number of identified parcels manageable.³ To identify those parcels that were sufficiently unperturbed initially, a 5% perturbation of the horizontal vorticity was admitted and the vertical-vorticity magnitude needed to be less than $5 \times 10^{-5} \text{ s}^{-1}$. These more stringent criteria compared to the single-moment run were used to reflect the smaller ζ threshold stipulated at the downdraft base. Using again the $F < 90$ criterion yielded 330 parcels of interest for the simulation with the double-moment microphysics scheme.

3. Results

a. Single-moment microphysics simulation

An overview of the storm including the analyzed trajectories is shown in Fig. 1. The trajectories, as in previous simulations (e.g., Adlerman et al. 1999; Dahl et al. 2012, 2014; Markowski and Richardson 2014; Parker and Dahl 2015), originate from the lowest few kilometers above the ground. This trajectory sample includes several parcels that start out near the ground, but then rise along the left-flank convergence boundary (Beck and Weiss 2013) and subsequently descend within the main downdraft north of the mesocyclone. The trajectories reach the lowest model level at different times [at the time shown, some of the parcels are already rising in the main updraft near $(x, y) = (63, 70) \text{ km}$ while other parcels are just approaching the lowest model level, e.g., near $(x, y) = (67, 76) \text{ km}$].

The result of the vorticity decomposition applied to all these trajectories is shown in Fig. 2, displaying the average over all 1695 trajectories of horizontal barotropic and nonbarotropic vorticity. To obtain the average, the trajectories were transformed to a common time frame defined with respect to the time when the parcels reach

FIG. 2. Horizontal projection of the average nonbarotropic (blue) and barotropic (red) vorticity vectors along the average trajectory ($n = 1695$ parcels) over a period of 800 s (~ 13 min) before the lowest model level (50 m AGL) is reached at t_0 . The vectors are plotted every 40 s.

the lowest model level. The initial barotropic vorticity of the parcels is determined by the base state and keeps pointing northward throughout the analysis period, but undergoes substantial horizontal stretching before reaching the base of the downdraft. The nonbarotropic vorticity is due primarily to southward horizontal buoyancy torques, consistent with Fig. 3, and subsequent horizontal stretching.⁴

The baroclinic and barotropic vorticity parts along the averaged trajectory in the (y, z) plane are shown in Fig. 4. While the barotropic vorticity attains a downward component during descent, the baroclinic vorticity vector is tilted upward while descending “tail first,” consistent with the “DJB93” process [after Davies-Jones and Brooks (1993); see also Davies-Jones (2000); Dahl et al. (2014); Markowski and Richardson (2014); Parker and Dahl (2015)]. However, in the classic DJB93 conceptual model the vertical vorticity develops in the (s, z) plane, where s is the direction of the trajectory. Herein, the baroclinically generated vorticity is tilted upward while it still has an appreciable crosswise horizontal component, but it subsequently becomes horizontally

³ Isolating thousands of trajectories out of a set of several million parcels is computationally quite expensive. Increasing the upper boundary of the ζ interval would have increased the number of parcels.

⁴ Since baroclinic vorticity production is rather inhomogeneous and unsteady, it is impossible to display the buoyancy field at a single time and height representative of the baroclinic vorticity generation for all parcels. However, the results for individual parcels were carefully checked and the inferred baroclinic vorticity is consistent with the horizontal buoyancy gradients within the storm. As in the Del City simulation (Dahl et al. 2014), the dominant vorticity production occurs in the lowest few 100 m above the ground within strong horizontal buoyancy gradients at the periphery of the main downdraft north of the mesocyclone. It thus seems justified to refer to the nonbarotropic vorticity as baroclinic vorticity.

FIG. 3. Shown is every 10th trajectory of the set containing 1695 parcels. The buoyancy field (including hydrometeor load) is shaded and the *direction* of the baroclinic vorticity production is shown by the arrows. The magnitude of baroclinic production is proportional to the horizontal buoyancy gradients: (top) at 1818 m AGL and 3000 s and (bottom) 265 m AGL and 4200 s.

aligned with the trajectory during its southward turn. The basic result is that the ambient vorticity contributes negatively to the ζ maxima but is overwhelmed by the baroclinic vertical vorticity, as also implied by Rotunno and Klemp (1985) for the wam-rain microphysics scheme.

The distributions of total, barotropic, and baroclinic vertical vorticity of all parcels as they are descending through 50 m AGL are shown for two deformation-magnitude thresholds in Fig. 5a ($F < 50$) and Fig. 5b ($F < 10$). For larger F (not shown), the distributions contain an increasing number of large vertical-vorticity magnitudes. This is not surprising because the initial vortex lines and the vortex lines that have been generated baroclinically are frozen into the fluid volumes. In these cases ($F > 90$) the mean and median values of the

barotropic ζ distributions also remain negative (and those of the baroclinic ζ distributions are positive). Figure 5b highlights that these results are not due to the small number of large vertical-vorticity magnitudes belonging to strongly deformed volumes and that the dominance of the baroclinic vertical vorticity does not depend on the threshold for maximum allowed deformation (the difference between the average baroclinic and barotropic vertical vorticity for several F thresholds, including no F filter at all, are all statistically significant based on a t test with a significance level of 5%).

Several additional tests were performed to evaluate the robustness of the results: (i) to assess the technique used to obtain the barotropic vorticity, this vorticity was calculated by numerically integrating the 3D vorticity equation but omitting baroclinic- and frictional-production terms

FIG. 4. Average baroclinic (blue) and barotropic (red) vorticity vectors in the (y, z) plane, plotted for the last 210 s (3.5 min) before the average trajectory ($n = 1695$) reaches the lowest model level (50 m AGL) at t_0 (vectors plotted every 10 s).

(this integration was performed using updates of the forcing function every 10 s and an integration step of 0.1 s); (ii) to assess the sensitivity of the results to errors in the initial orientation and magnitude of the vorticity (cf. the base-state value), the analysis was repeated using the assumption that the initial barotropic vorticity of each of the 3481 parcels was given precisely by the base-state vorticity $\bar{\eta} = 0.004 \text{ s}^{-1}$; and (iii) the same experiment was performed using a previous version of the CM1 model (implying a different set of trajectories and slightly reduced accuracy of the trajectory calculations). All these tests yield qualitatively (and for the

most part, quantitatively) the same results as those reported herein (see Table 1).

b. Double-moment microphysics simulation

Now that the dominance of the baroclinic vorticity within the near-ground vorticity maxima has been established for the single-moment run, we turn to the sensitivity of this result using a double-moment scheme. Overall, the simulation with the double-moment microphysics parameterization evolves similarly to the single-moment simulation. The 330 trajectories that contribute to near-ground vorticity maxima are shown in Fig. 6 and generally originate from lower altitudes than those in the single-moment run. Strikingly, in the double-moment simulation there are fewer downdrafts and less frequent downdraft surges, and thus fewer ζ extrema in the cold pool (Fig. 7). Moreover, the horizontal baroclinic vorticity production is weaker overall than in the single-moment run (Figs. 7b,d). However, as also shown in Figs. 7b and 7d the ζ maxima in each simulation emanate from the most intense downdraft cores and conspicuously emerge from concentrated regions of large horizontal buoyancy gradients (see also the animated version of Fig. 7, available in the online supplemental material). To better understand the relative importance of baroclinic and barotropic contributions in this case, the horizontal projection of the averaged two vorticity parts is shown in Fig. 8. The vorticity parts evolve qualitatively identical to those in the single-moment run, and again the barotropic negative vertical vorticity is overwhelmed by the cyclonic baroclinic contribution (Fig. 9). This experiment demonstrates that also with a double-moment microphysics scheme the baroclinic contribution dominates.

FIG. 5. Box-and-whisker plots of the total, baroclinic, and barotropic vertical-vorticity distribution of the parcels as they reach the lowest model level (50 m AGL). (a) Parcels with $F < 50$ and (b) parcels with $F < 10$. Values above $q_3 + 1.5 \text{ IQR}$ or below $q_1 - 1.5 \text{ IQR}$ are shown as red crosses (q_1 and q_3 are, respectively, the 25th and 75th percentile of the distribution and IQR is the interquartile range).

TABLE 1. Sample averages of barotropic and baroclinic vertical vorticity at the downdraft base for the single-moment simulation. Shown are the results for different methods, filter thresholds, and model runs. The method of obtaining the barotropic vorticity is either via Cauchy’s formula (the “Lagrangian stencil technique”) or RK2 integration. The filter based on initial perturbation is referred to as “IC filter” and is described in the text. The F filter pertains to the maximum allowed deformation magnitude, as also described in the text. The symbol η_0 represents the initial (southerly) vorticity of the parcels. CM1r16 refers to the previous release (r16) of the CM1 model.

Method	IC filter	F filter	N	Avg ζ_{BT} (s^{-1})	Avg ζ_{BC} (s^{-1})
Cauchy	Yes	$F < 90$	1659	−0.0019	0.0045
Cauchy	Yes	No	1846	−0.0024	0.0050
Cauchy	No	No	3378	−0.0029	0.0054
RK2	Yes	No	1846	−0.0024	0.0050
Cauchy	η_0 set to $0.004 s^{-1}$	No	3378	−0.0025	0.0051
RK2 CM1r16	Yes	No	783	−0.0007	0.0038

Summarizing the results so far, the warm-rain Kessler scheme (Rotunno and Klemp 1985), as well as the Lin-type single-moment scheme and the Morrison scheme presented herein, each exhibiting different outflow characteristics, favor the baroclinic mechanism. A cartoon of the general behavior of the vorticity in these simulations is shown in Fig. 10.

4. Discussion

a. Why does the baroclinic mechanism seem to dominate?

It is intriguing that the above results and a large number of previous studies analyzing observed storms

FIG. 6. This figure displays the 330 trajectories for the double-moment simulation at 4200 s that contribute to vertical vorticity at the lowest model level, color coded based on their initial altitude (see color bar; note the different scale compared to Fig. 1). Shown are (top) the projection onto the (x, z) plane, (right) the projection onto the (y, z) plane, and (left, main panel) the horizontal projection. In the main panel, the black contour shows the 20-dBZ reflectivity and the dashed black contour shows the $-3 m s^{-1}$ vertical velocity at 3129 m AGL. Vertical vorticity at the lowest model level is represented by red contours (positive values: solid, contoured for 0.01, 0.03, and $0.05 s^{-1}$; negative values: dashed, contoured for -0.05 , -0.03 , and $-0.01 s^{-1}$). Wind vectors at the lowest model levels are also shown, and the extent of the cold pool ($-1-K$ potential temperature perturbation) is represented by the blue line.

FIG. 7. A snapshot of the simulation using (a),(b) the single-moment microphysics scheme and (c),(d) the double-moment microphysics scheme. (a) Buoyancy (including hydrometeor load; shaded), vertical velocity (-2 m s^{-1} ; black contours) at 265 m AGL, and positive ζ at the lowest model level (contoured for 0.005 and 0.01 s^{-1}) at 4500 s . (b) As in (a), but that the shaded field is the magnitude of the buoyancy torque. (c),(d) As in (a),(b), but for the double-moment simulation and at 4710 s .

and idealized simulations consistently find that downdraft production of vertical vorticity near the ground is due primarily to the baroclinic mechanism.⁵ This implies that the barotropic mechanism is ineffective for a wide range of representations of cloud microphysics ranging from warm-rain (Rotunno and Klemp 1985; Davies-Jones and Brooks 1993; Wicker and Wilhelmson 1995; Adlerman et al. 1999) to Lin-type (Dahl et al. 2014 and this study) to double-moment (this study) to idealized heat sink

(Markowski and Richardson 2014; Parker and Dahl 2015) parameterizations, as well as observed cases (Markowski et al. 2008, 2012). It is, thus, tempting to speculate that there is a fundamental reason that leads to this dominance of baroclinic vorticity.⁶ The leading-order effect is most likely that tornadic environments tend to be dominated by streamwise ambient storm-relative vorticity, implying that in such cases the ambient vorticity does not contribute to ground-level ζ as discussed in section 1. But,

⁵ The author is aware of only one study that suggests that ambient vorticity is the dominant contributor to an intense near-ground vortex in a supercell (Mashiko et al. 2009). However, these authors calculated parcel histories of only about 5 min prior to the parcels entering the vortex, which makes it rather unlikely that the initial (barotropic) vorticity corresponded to the ambient vorticity.

⁶ The basic downdraft processes simulated by Parker and Dahl (2015) were not changed in important ways when surface friction was included in their simulations, implying that at least for the onset of near-ground rotation, surface friction is not the dominant contributor. This point will be addressed again at the end of this section.

FIG. 8. Horizontal projection of the two vorticity parts for the average trajectory in the double-moment simulation. The nonbarotropic (blue) and barotropic (red) vorticity vectors are plotted along the average trajectory ($n = 330$ parcels over a period of ~ 25 min) before the lowest model level (50 m AGL) is reached at t_0 . The vectors are plotted every 50 s.

why does the baroclinic vorticity also seem to dominate in cases where the ambient storm-relative vorticity has an appreciable crosswise component?

An answer might be provided by a simple scaling argument. In the absence of initial vertical vorticity at the ground, a downdraft is needed that locally depresses horizontal vortex lines, which results in vertical vorticity at the surface [see e.g., Fig. 10.3 in Markowski and Richardson (2010)]. The focus in the following argument will be on the reorientation of horizontal vorticity into the vertical in a negatively buoyant downdraft,⁷ because this determines the sign and magnitude of the two ζ parts delivered at the downdraft base (subsequent stretching will not change whether baroclinic or barotropic vertical vorticity dominates). The role of horizontal divergence on the vertical vorticity will thus not be considered. Under this condition, the vertical barotropic vorticity ζ_{BT} is given by [cf. Eq. (20) in Davies-Jones (1984)]

$$\zeta_{BT} = \omega_{BT}^h \frac{\partial h}{\partial s}, \quad (3)$$

where h is the height of the vortex line relative to some reference height, s is now parallel to the local horizontal vortex line, and ω_{BT}^h is the magnitude of the horizontal barotropic vorticity, which is assumed to have a nonzero component parallel to the downdraft gradient. The slope of the vortex line, $\partial h / \partial s$, needs to be nonzero at the ground for surface rotation, which requires ω_{BT}^h to have a crosswise component if the trajectories are smooth.

Now let T represent a typical time scale for parcels to move through downdrafts (and buoyancy extrema), H the characteristic vertical displacement of parcels from their initial height, and L the length scale characterizing the downdraft periphery (e.g., where $|\nabla_h w|$ is nonzero, w being the vertical velocity). The dependencies of the magnitude of the vertical barotropic vorticity may then be estimated by

$$\zeta_{BT} \sim \omega_{BT}^h H L^{-1}, \quad (4)$$

$$\sim \omega_{BT}^h W T L^{-1}, \quad (5)$$

where W is the characteristic downdraft velocity and the symbol $\sim$ means “varies with.” Since the vertical velocity arises from the time integral of the buoyancy B , we obtain

$$\zeta_{BT} \sim \omega_{BT}^h T^2 B L^{-1}, \quad (6)$$

⁷ Dynamically forced downdrafts, such as the occlusion downdraft (Klemp and Wilhelmson 1983; Markowski 2002) are approximately irrotational and, hence, cannot directly produce horizontal vorticity baroclinically. The argument herein pertains only to downdrafts produced by negative buoyancy.

FIG. 9. Average baroclinic (blue) and barotropic (red) vorticity vectors in the (y, z) plane for the double-moment simulation, plotted for the last 100 s before the average trajectory ($n = 330$) reaches the lowest model level (50 m AGL) at t_0 (vectors plotted every 10 s).

or

$$\zeta_{BT} \sim \omega_{BT}^h \omega_{BC}^h T, \quad (7)$$

where ω_{BC}^h is the horizontal baroclinic vorticity, which roughly scales with TBL^{-1} . Similarly, for the magnitude of the vertical baroclinic vorticity ζ_{BC} we get

$$\zeta_{BC} \sim \omega_{BC}^h \omega_{BC}^h T. \quad (8)$$

The downdraft-relative flow, which was found to be critical in the production of near-ground ζ by Parker and Dahl (2015) is indirectly included via the advective time scale T .⁸

If we assume a constant characteristic time scale, the implication is that ζ_{BC} increases as a quadratic function of the accumulated baroclinity (represented by ω_{BC}^h) while ζ_{BT} increases linearly (Fig. 11). Further, the downdraft strength may be assumed to scale with the baroclinic vorticity (i.e., stronger downdrafts are accompanied

by stronger horizontal downdraft gradients). This implies that the magnitude of baroclinic vertical vorticity delivered at the ground increases as a quadratic function of downdraft strength while the barotropic vertical-vorticity magnitude increases only linearly (Fig. 11). Even if T is allowed to increase with increasing baroclinic vorticity (i.e., if the assumption is made that the downdraft diameter also increases with downdraft strength, thus increasing the advective time scale T), the above argument remains qualitatively valid.

For a storm exhibiting a variety of downdraft intensities, the argument predicts that weaker downdrafts will tend to deliver less vertical vorticity at its base than stronger downdrafts. However, the important finding is that the barotropic vorticity dominates only within those weaker downdrafts. This corresponds to the region left of the vertical line in Fig. 11. While the stronger downdrafts more effectively tilt horizontal barotropic vorticity (ζ_{BT} increasing linearly with downdraft strength), the horizontal baroclinic vorticity is tilted even more effectively (ζ_{BC} increasing quadratically with downdraft strength; Fig. 11). Physically, this happens because the downdraft gradient not only is the agent that facilitates the tilting, but this gradient is a manifestation of the baroclinic vorticity itself. This may be the reason that in the double-moment simulation, while there are fewer intense downdrafts than in the single-moment run (Fig. 7), only those more intense downdrafts produce appreciable

⁸ An assumption implicit in this argument is that the orientation of the horizontal vorticity parts does not depend on the accumulated baroclinity ($\sim$ downdraft strength). The barotropic horizontal vorticity is assumed to be given by the base state and, hence, does not vary for a given storm. That the baroclinic vorticity does not change its horizontal orientation appreciably as downdraft intensity is increased was confirmed by idealized downdraft simulations such as those in Parker and Dahl (2015; not shown).

FIG. 10. Conceptual model of the vorticity evolution along a typical trajectory (black line) within the simulations analyzed herein. Red arrows represent barotropic vorticity and blue arrows represent baroclinic vorticity.

vertical vorticity, and that this vorticity mainly originates from baroclinic torques.⁹

Based on the above argument it may be speculated why strong surface ζ extrema dominated by barotropic vorticity seem to be rare: downward tilting of horizontal vortex lines (and downward advection of the resulting vertical vorticity) is most effective in strong downdrafts. Weaker downdrafts on the other hand, are ineffective at tilting horizontal vorticity into the vertical and transporting the vertical vorticity to the ground. However, the barotropic mechanism only dominates in this weak-downdraft regime. The resulting weak barotropic vertical surface vorticity may take too long to be stretched into an intense vortex within time scales that parcels typically spend in horizontally convergent flow. Put another way, the scaling argument predicts that as downdraft strength increases not only does the total vertical vorticity delivered at the downdraft base increase, but also that this vorticity is increasingly dominated by baroclinic vorticity.

Near-ground rotation in axisymmetric simulations (Markowski et al. 2003; Davies-Jones 2008; Parker 2012) is dominated by barotropic vortex-line reconfiguration, because the azimuthal baroclinic vorticity cannot be tilted into the vertical. However, the horizontal flow field in which the downdraft is embedded in axisymmetric simulations is not particularly representative of sheared, 3D convective storms. Thus, in the above

⁹ In the simulations discussed herein, there is a considerable amount of variation among the parcels regarding residence time within downdrafts and, hence, accumulated baroclinic vorticity, as well as horizontal deformation, which in turn yields a large range of horizontal barotropic vorticity magnitudes among the parcels, making it difficult to test the prediction of the above argument. Instead, one probably would have to perform highly controlled experiments that only vary downdraft strength/baroclinity [perhaps similar to those by Parker and Dahl (2015)].

FIG. 11. The curves describe the surface vertical-vorticity parts [baroclinic (blue) and barotropic (red)] at the downdraft base as a function of downdraft intensity based on the scaling argument. The vertical dashed line marks the downdraft intensity above which the baroclinic vertical vorticity dominates. The horizontal barotropic vorticity is assumed to be constant.

argument a more realistic setting was assumed with nonaxisymmetric horizontal flow through the downdraft, which was found to be the basic requirement for the onset of near-ground rotation by Davies-Jones (2000) and Parker and Dahl (2015).

b. The role of surface friction

In this study surface friction is neglected and the focus is strictly on the relative roles of ambient and storm-generated vorticity at the base of downdrafts. At this stage of vorticity acquisition, surface friction should play a rather small role as the horizontal-velocity profile of air reaching the surface during descent has not yet adjusted to surface friction [Letchford et al. 2002; a sample of measured wind profiles within thunderstorm outflows can be found in Gunter and Schroeder (2015)]. Parker and Dahl (2015) found no appreciable difference between their simulations with and without surface friction. This is in contrast to Schenkman et al. (2014), who did find that frictional torques at the base of downdrafts were the dominant source of horizontal vorticity for some of the parcels they analyzed. More research is needed to explain this discrepancy, but it is likely that the completion of tornadogenesis as well as tornado maintenance rely on processes beyond the barotropic and baroclinic mechanisms discussed herein.

The idealized base-state shear profile used herein could not be maintained in the presence of surface friction, which would alter the low-level ambient

vorticity. The orientation and perhaps the magnitude of the barotropic vorticity of near-ground parcels riding up the left-flank boundary would thus be expected to vary from the results presented herein. The explicit effect of surface friction in the context of vorticity decomposition is left for future research.

5. Conclusions

In this study the relative importance of ambient crosswise (barotropic) vorticity and storm-generated (baroclinic) vorticity in producing vertical-vorticity maxima at the base of downdrafts in supercells was investigated. The goal was to analyze how robust the baroclinic mechanism is. Two supercells in unidirectional shear were simulated, using a single-moment and a double-moment microphysics parameterization, respectively. A large number of forward trajectories that contribute to cyclonic vorticity at the base of downdrafts was analyzed for a time period of about 30 min and the vorticity was decomposed into barotropic and baroclinic parts. Independent of the microphysics parameterization, the barotropic vorticity remains weaker than the baroclinic vorticity and is tilted downward within downdrafts, while the baroclinic vorticity has a much larger magnitude and is tilted upward.

The observation based on this study and previous work that the dominance of the baroclinic mechanism seems rather insensitive to the microphysics parameterizations (and the shear profiles) may be related to the following factors: (i) in cases with streamwise ambient vorticity, the barotropic contribution to near-ground rotation is small because streamwise vorticity becomes horizontal along trajectories near the surface; and (ii) in cases with crosswise ambient vorticity, a scaling argument predicts that the baroclinic vertical vorticity becomes increasingly dominant as downdraft strength increases. That is, the imported barotropic vorticity tends to be overwhelmed by baroclinic vorticity except in the weakest downdrafts, which, however, do not yield much vertical vorticity altogether at their base. This mostly barotropic vorticity may be too weak to be concentrated effectively by horizontal convergence.

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