Representing Richardson Number Hysteresis in the NWP Boundary Layer

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ABSTRACT

Turbulence in the planetary boundary layer (PBL) transports heat, momentum, and moisture in eddies that are not resolvable by current NWP systems. Numerical models typically parameterize this process using vertical diffusion operators whose coefficients depend on the intensity of the expected turbulence. The PBL scheme employed in this study uses a one-and-a-half-order closure based on a predictive equation for the turbulent kinetic energy (TKE). For a stably stratified fluid, the growth and decay of TKE is largely controlled by the dynamic stability of the flow as represented by the Richardson number. Although the existence of a critical Richardson number that uniquely separates turbulent and laminar regimes is predicted by linear theory and perturbation analysis, observational evidence and total energy arguments suggest that its value is highly uncertain. This can be explained in part by the apparent presence of turbulence regime-dependent critical values, a property known as Richardson number hysteresis. In this study, a parameterization of Richardson number hysteresis is proposed. The impact of including this effect is evaluated in systems of increasing complexity: a single-column model, a forecast case study, and a full assimilation cycle. It is shown that accounting for a hysteretic loop in the TKE equation improves guidance for a canonical freezing rain event by reducing the diffusive elimination of the warm nose aloft, thus improving the model’s representation of PBL profiles. Systematic enhancements in predictive skill further suggest that representing Richardson number hysteresis in PBL schemes using higher-order closures has the potential to yield important and physically relevant improvements in guidance quality.

1. Introduction

The planetary boundary layer (PBL, acronyms defined in Table 1 for reference) is the portion of the lower troposphere that feels the influence of the earth’s surface on hourly time scales (Stull 1988). The layer’s rapid response to changes in surface properties is primarily a result of turbulent eddies that transport heat, momentum, and constituents (including water vapor) throughout its depth. These eddies are often generated by the interaction of the flow with the surface itself, either through buoyancy generated by heating at the lower boundary or by mechanical mixing resulting from surface drag (Kaimal et al. 1976; Caughey et al. 1979). Correctly parameterizing turbulent transports in the PBL is crucial to NWP since these primarily subgrid-scale motions largely determine PBL profiles of heat, moisture, and winds (Clarke 1970; Beljaars and Viterbo 1998). They thus have a direct impact on the forecasting of important near-surface quantities (Holtslag et al. 2013).

The separation of the momentum equation into mean and turbulent components using Reynolds decomposition yields a predictive equation for turbulent kinetic energy (TKE) that can be used to describe the turbulent eddies of the PBL. The dominant source terms in this equation relate to buoyancy and shear, contributions that are combined by Richardson (1920) to introduce the flux Richardson number $R_f$. The gradient form of this dimensionless quantity can be used to describe the evolution of eddies in the PBL (e.g., Klipp and Mahr 2004; Mauritsen and Svensson 2007; Sun et al. 2012), and has long been applied in studies of clear-air turbulence [e.g., Petterssen and Swinbank (1947), with early results summarized by Reiter and Lester (1967)].

In many of these investigations, a value of the gradient Richardson number (Ri) that separates turbulent from laminar flow regimes is sought; however, there remains...
disagreement between TKE balance (Richardson 1920), linear perturbation analysis (Taylor 1931; Miles 1961; Howard 1961), and observational estimates of this quantity [summaries are presented by Zilitinkevich and Baklanov (2002) and Grachev et al. (2013)]. This uncertainty is amplified by the difficulty of computing Ri from realistic profiles given the strong scale dependence of its definition (Reiter and Lester 1968; Van Gasel and Pelegri 2004; Balsley et al. 2008). A further explanation for the disparate estimates of this critical Richardson number (Ri,) is proposed by Woods (1969), who observes different Ri values for the forward (laminar to turbulent) and reverse (turbulent to laminar) transitions. This “Richardson number hysteresis” (hereafter Ri hysteresis) will be discussed more thoroughly in section 2b since it is fundamental to the current study.

The representation of unresolved turbulent mixing in numerical models requires the formulation of closure assumptions that relate the diffusivity of PBL eddies to the model’s state variables. Cuxart et al. (2006) provide an overview of modern PBL parameterizations, many of which use a TKE closure (Mellor and Yamada 1982); however, such higher-order closures are not the norm in operational NWP. The PBL scheme used for this investigation (Mailhot and Benoit 1998) is an exception, employing a prognostic TKE equation whose primary source term is a function of Ri under stably stratified conditions. Despite this dependence of TKE evolution on the dynamic stability of the flow, the original formulation of the scheme does not attempt to account for turbulence-regime-dependent Ri, values. To the best of our knowledge, the impact of Ri hysteresis has not been investigated within the context of any PBL parameterization for numerical models. In this investigation, we show that including a representation of this effect appears to be of benefit to NWP guidance quality.

This study begins with an overview of the datasets used to evaluate the model results and a description of the adopted PBL scheme in section 2. An expanded discussion of Ri hysteresis and its influence on TKE evolution is presented at the end of that section. In sections 3 and 4, the impact of the inclusion of this effect is evaluated within single-column and case study contexts, respectively. Thereafter, the results of a forecast sequence and a data assimilation cycle using the modified PBL parameterization are presented in section 5. The study concludes with a discussion of the findings in section 6.

2. Data and methods

This study uses analysis, surface, and radiosonde data to evaluate guidance quality. Pressure-level fields from the Interim European Centre for Medium-Range Weather Forecasts Re-Analysis (ERA-Interim; Dee et al. 2011) are employed to provide an independent representation of the atmospheric state. The T255 resolution (~79 km) of ERA-Interim is coarser than the forecast model studied here, leading to a potential overestimate of forecast errors at very fine scales. For this reason, reanalysis grids are used primarily to assess gross error structures, while surface and radiosonde data are used for quantitative guidance evaluation.

The configuration of the Global Environmental Multiscale (GEM; Côté et al. 1998) model employed throughout this study follows that of the Global Deterministic Prediction System (GDPS), run operationally at the Canadian Meteorological Centre (Bélair et al. 2009; Charron et al. 2012). The fully compressible primitive equations are solved using a two-time-level temporal scheme in Cartesian coordinates with 0.225° and 0.35° discretization in the meridional and zonal directions, respectively. Although GEM is capable of operating in a nonhydrostatic mode (Yeh et al. 2002), the GDPS employs the hydrostatic approximation because of its relatively coarse horizontal resolution. In the vertical, Charney and Phillips (1953) staggering is adopted, in which thermodynamic and momentum variables are offset by half levels (Arakawa and Konor 1996; Girard et al. 2014). An 80-level hybrid terrain-following log-hydrostatic-pressure coordinate relaxes to an isobaric surface at the 0.1-hPa model lid, with 10 levels below 850 mb (1 mb = 1 hPa) over low-lying terrain and the lowest thermodynamic level at ~20 m.

The physical parameterization package available in GEM is described by Mailhot et al. (1998). In the GDPS configuration, the model employs a Sundqvist (1978) gridscale precipitation scheme and a convective
parameterization based on Kain and Fritsch (1990, 1992). The effects of radiation are computed following the correlated-k technique (Li and Barker 2005), while the Interaction Soil–Biosphere–Atmosphere scheme predicts the evolution of land surface properties (Noilhan and Planton 1989; Bélair et al. 2003a,b). The McFarlane (1987) orographic gravity wave drag scheme reduces momentum in the upper atmosphere, with the effects of nonorographic gravity waves parameterized following Hines (1997a,b). At lower levels, Zdra et al. (2003) blocking acts to transfer momentum from the surface to the overlying column.

a. Description of the PBL scheme

The PBL parameterization used in the GDPS is described by Mailhot and Benoit (1982), Benoit et al. (1989), and Bélair et al. (1999). A brief summary of the elements of the PBL scheme most relevant to the current study is provided here. Height coordinates are used to develop the equations although they are transformed to σ coordinates for implementation in GEM.

Tendencies on the model’s state variables (temperature, moisture, and horizontal vector wind, denoted generally as \( \psi \)) are computed based on the convergence of turbulent fluxes into model layers:

\[
\frac{\partial \psi}{\partial t} = - \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \omega \psi' \right),
\]

where \( \rho \) is air density and \( \omega \) is vertical motion. Throughout this study, primes denote turbulent (unresolved) quantities, and overbars denote mean values over the grid cell. An assumption throughout the development of PBL schemes for NWP is that the distinction between the resolved and the unresolved scales of motion lines up with the spectral gap between large- and small-scale motions (Panofsky and Van der Hoven 1955; Fiedler and Panofsky 1970; Dalaunder and Sidi 1987). The degree to which this distinction is applicable depends on the resolution of the model, and even the existence of a spectral gap has been questioned (Nicholls 1985; Williams et al. 1996; Tjernström 2005).

Applying K theory allows for a recasting of Eq. (1) as

\[
\frac{\partial \psi}{\partial t} = \frac{1}{\rho} \frac{\partial}{\partial z} \left[ \rho K_x \left( \frac{\partial \psi}{\partial z} - \gamma \right) \right],
\]

where \( \gamma \) represents a countergradient mixing term and the \( x \) subscripts denote appropriate diffusion coefficients \( (K_x) \) for the variable in question (\( K_w \) for winds and \( K_H \) for heat and moisture). The countergradient term accounts for large eddy transports, and will not be considered further here since stably stratified boundary layers are of primary interest in this study. The diffusion coefficients are related to TKE in the one-and-a-half-order closure scheme (Mellor and Yamada 1974) employed by the GDPS:

\[
K_x = c \lambda \sqrt{E}.
\]

Here, \( E = 1/2(u'^2 + v'^2 + w'^2) \) is based on the three turbulent wind components, \( c = 0.516 \), and \( \lambda \) is the mixing length (see the appendix for details). The latter depends on the value of the gradient Richardson number, which is computed as

\[
R_i = \frac{g}{\theta_v} \frac{\partial \theta_v}{\partial z}.
\]

Terms I and II constitute the analytic part of the TKE tendency equation, while term III must be solved implicitly. The focus of this study is on the analytic component, for which the coefficients are defined as

\[
B = c \lambda \left( 1 - \frac{R_i}{P_i} \right) \frac{\partial \mathbf{V}^2}{\partial z} \quad \text{and} \quad C = 0.14 \frac{\lambda_{\text{diss}}}{\lambda_{\text{diss}}}.
\]

Since \( \lambda_{\text{diss}} \) is a positive-definite function of \( \lambda \), \( C > 0 \) and term II of Eq. (5) constantly depletes any TKE that exists (Kolmogorov 1941). Of more direct relevance to this investigation, the \( B \) coefficient of term I [Eq. (5)] can have either sign depending on the value of the flux Richardson number \( (R_f) \) in Eq. (6):

\[
R_f = \frac{R_i}{P_i} \quad \text{where} \quad \text{Pr} = \frac{K_M}{K_H}
\]
is the Prandtl number. In the PBL scheme considered here, \( \text{Pr} = 1 \) when \( \text{Ri} > 0 \), such that \( \text{Ri} = \text{Ri} \) for the stably stratified profiles of relevance to this study. Zilitinkevich et al. (2008) use total energy arguments to suggest that in fact \( \text{Pr} \rightarrow \infty \) at large \( \text{Ri} \): the sensitivity of the GEM PBL scheme to the \( \text{Pr} \)--\( \text{Ri} \) relationship will be investigated in a future study.

\textbf{b. Turbulence regimes and Ri hysteresis}

The formula for the TKE growth coefficient [Eq. (6)] suggests that \( \text{Ri} = 1 \) is of particular significance, since it distinguishes between regimes in which the dominant term \( \text{I} \) of Eq. (5) acts to create or destroy TKE. This appears as the point at which \( B = 0 \) in the \( \text{Ri} \)--\( B \) relationship for fixed values of \( \lambda \) and vertical shear as shown in Fig. 1a. Defining the critical Richardson number as \( \text{Ric} = 1 \) (Richardson 1920), supercritical regimes are associated with depletion of TKE by buoyant suppression \((B < 0)\), while subcritical values of \( \text{Ri} \) result in the mechanical generation of TKE as turbulent activity increases \((B > 0)\). A negative \( \text{Ri} \) is indicative of free convection (Priestley 1955), which leads to \( B \rightarrow 0 \) and to rapid increases in both TKE and the diffusion coefficients through Eq. (3).

From a physical perspective, \( \text{Ric} \) separates the turbulent (subcritical) regime from the laminar (supercritical) regime. However, the \( \text{Ri} \)--\( B \) relationship (Fig. 1a)
demonstrates that TKE evolution is highly sensitive to small subcritical departures from \( R_{ic} \). The rapid growth of TKE in the turbulent regime triggers enhanced vertical diffusion as the PBL scheme attempts to represent the resolved-scale effects of boundary layer eddies. Given this large response to the model’s estimate of \( R_{ic} \), careful treatment of the transition between the regimes is required.

The TKE balance-based \( R_{ic} = 1 \) is consistent with \( R_{fc} = 1 \) under the adopted assumption that \( Pr = 1 \) (Taylor 1914), and also with the nonlinear analysis of Abarbanel et al. (1984). However, both observations and linear theory suggest that much lower values are required for TKE generation. A review of early observational estimates of \( R_{ic} \) is provided by Oke (1970), wherein evidence is presented to support \( R_{ic} = 0.1 \). This value is even lower than the hydrodynamic estimate of \( R_{ic} = 0.25 \) suggested by linear perturbation theory (Taylor 1931; Miles 1961; Howard 1961). The influence of \( Pr \), the impact of the dissipative term [term III in Eq. (5); discussed by Gage (1971)], nonlinearity, and the time-dependent nature of the background state (Majda and Shefter 1998) combine to explain some of the apparent inconsistencies in these estimates.

More recently, Canuto and Minotti (1993) introduced the concept of turbulent potential energy, leading to the total energy formulation that is used by Zilitinkevich et al. (2008) to argue that no \( R_{ic} \) exists and that an unbounded increase of \( Pr \) in strongly stable atmospheres naturally limits \( R_f \lesssim 0.2 \). Zilitinkevich et al. (2008) also define “strong” (\( Ri < 0.1 \)) and “weak” (\( Ri > 1 \)) mixing regimes, with a transition zone between these, respectively, fully turbulent and wave-dominated states. They suggest that this regime nomenclature be used in the place of the traditional “turbulent” and “laminar,” since the latter does not account for the observed existence of turbulence in large-\( Ri \) states. Despite this potential weakness, the traditional regime names will be

### Table 2. Experiment names used throughout the text.

<table>
<thead>
<tr>
<th>Name</th>
<th>( R_{ic\text{ min}} )</th>
<th>( R_{ic\text{ max}} )</th>
<th>Description</th>
<th>Figure</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTL</td>
<td>1</td>
<td>1</td>
<td>Control</td>
<td>Fig. 1a</td>
</tr>
<tr>
<td>HYST1</td>
<td>0.25</td>
<td>1</td>
<td>Single-loop hysteresis</td>
<td>Fig. 1b</td>
</tr>
<tr>
<td>HYST2</td>
<td>0.25</td>
<td>2</td>
<td>Double-loop hysteresis</td>
<td>Fig. 1c</td>
</tr>
<tr>
<td>HYST2(_{\text{min}})</td>
<td>0.5</td>
<td>2</td>
<td>Reduced double-loop hysteresis</td>
<td></td>
</tr>
<tr>
<td>MIXLEN</td>
<td>1</td>
<td>1</td>
<td>Mixing length decreased (Blackadar 1962)</td>
<td></td>
</tr>
<tr>
<td>SMOOTH</td>
<td>0.22</td>
<td>1</td>
<td>Smooth single-loop hysteresis</td>
<td>Fig. 1d</td>
</tr>
</tbody>
</table>

![Potential Temperature at 0000 UTC 02 Jul 2006 (12h)](image1.png)

![TKE at 0000 UTC 02 Jul 2006 (12h)](image2.png)

![Ri at 0000 UTC 02 Jul 2006 (12h)](image3.png)

**Fig. 2.** Profiles from the 12-h (local midnight) state of the SCM of (a) potential temperature, (b) TKE, and (c) Ri from the surface to 2500 m. CTL is shown with black lines, while the integration including the effect of Ri hysteresis (HYST2) is shown with gray lines as indicated in the legends. Units are labeled along the abscissa of each panel. The PBL sublayers described in the text are identified in (a) for reference.
employed in this TKE-based study for consistency with the earlier investigations upon which it is founded. However, the important concept of a transition zone in Ri space is a key element of the current study.

Early investigations of the complexity of the transition between turbulent and laminar states are presented by Woods (1969) and Businger (1969). Fluorescent dye experiments performed by Woods (1969) at the thermocline show that the regimes persist across a subregion of Ri space ($0.25 < \text{Ri} < 1$) that spans values similar to the Zilitinkevich et al. (2008) transition zone. Woods (1969) demonstrates that an initially laminar flow remains laminar for decreasing stability until $\text{Ri} \approx 0.25$ while an initially turbulent flow remains turbulent until $\text{Ri} \approx 1$ [Fig. 5 in Woods (1969) shows this diagrammatically]. The existence of different Ri, for turbulent and laminar initial states is further described by Canuto (2002), who also suggests that gravity wave sources can push Ri, for the reverse transition well beyond unity. A physical explanation for the asymmetry in turbulent energy growth in the PBL over the range $0.25 < \text{Ri} < 1$ is provided by Businger (1969), who used a TKE budget analysis to suggest that the TKE generated in the transition zone is available only to the finite-amplitude perturbations associated with preexisting turbulence. Although Ri hysteresis is sometimes qualitatively invoked to explain the behavior of systems in the turbulent–laminar transition zone (Toorman 2002; Andreas 2002; Grachev et al. 2013), no systematic study of the influence of this effect has been undertaken within an NWP context.

c. Implementation of Ri hysteresis in the PBL scheme

The implementation of Ri hysteresis in the PBL scheme is a relatively simple procedure that begins with the creation of a turbulence state (turbulent or laminar), defined for each grid point. This assignment takes place within an Eulerian framework for consistency with the

FIG. 3. As in Fig. 2, but for the lowest 500 m only. (a) Temperature, (b) Ri, (c) wind speed, and (d) wind direction. Dashed lines in (a), (c), and (d) represent Cabauw observations, with temperature values in (a) available only up to 200 m.
The end points of the hysteretic region are defined as $R_{i_{c_{\min}}}$ and $R_{i_{c_{\max}}}$ for the forward and reverse transitions, respectively. The physical nature of the flow can therefore be determined based on the current $R_i$ and the previous regime: turbulent if $R_i < R_{i_{c_{\min}}}$, laminar if $R_i > R_{i_{c_{\max}}}$, and persistent for intermediate $R_i$ values. The turbulence state is then updated accordingly. If a turbulence regime is not provided at model initialization, assignment is based on $R_{i_{c_{\min}}}$ in order to limit rapid TKE growth in the turbulence transition zone during model spinup. Additionally, $R_i$ hysteresis is not applied at the first prognostic level, where it is assumed that unresolved surface roughness elements shed eddies that are capable of triggering turbulent onset for any $R_i < 1$.

A set of modified $R_i-B$ relationships that describe different possible representations of $R_i$ hysteresis are shown in Figs. 1b,d, with the names of the related experiments summarized in Table 2. The simplified mixing-length estimate of Blackadar (1962) and the Delage and Girard (1992) stability functions are used to generate these analytic prototypes: other formulations would slightly alter the shapes of the $R_i-B$ relationships, but not the interpretations thereof. In its simplest form, the parameterization of $R_i$ hysteresis involves the creation of a subcritical branch for $R_{i_{c_{\min}}}<R_i<1$ in which the $B$ is not allowed to become positive for laminar flows (HYST1; Fig. 1b). This takes the form of a modification to $R_f$ following

$$R_f^* = \begin{cases} R_f, & \text{if } R_i < R_{i_{c_{\min}}} \text{ or } R_i > 1 \\ \max(R_f,1), & \text{if } R_i > R_{i_{c_{\min}}} \text{ and laminar} \end{cases},$$

which adjusts the $R_i-B$ relationship in Eq. (6). According to the physical interpretation of Eq. (5), this means that the forward transition does not occur until $R_i$ falls below $R_{i_{c_{\min}}}$ (0.25 in Fig. 1b).

The presence of abrupt changes in $B$ across $R_{i_{c_{\min}}}$ in the HYST1 implementation is not expected to lead to oscillatory behavior because of the intrinsic damping nature of hysteretic loops: a parcel is no longer on a locally discontinuous branch once it changes its turbulence state. Notwithstanding this expected damping, the impact of smoothed transitions in the $R_i-B$ relationships is investigated in a SMOOTH implementation (Fig. 1c). A heuristic modification to $R_f$,

$$R_f^* = R_{i_{c_{\min}}} + (R_f - R_{i_{c_{\min}}})^\alpha(1 - R_{i_{c_{\min}}})^{(1-\alpha)},$$

affects the $R_i-B$ in the same manner as Eq. (10). The impact of this adjustment for $R_{i_{c_{\min}}} = 0.22$ and $\alpha = 0.05$ (Fig. 1c) is shown to be minimal compared to the simpler HYST1 representation.

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Unpublished tests have shown that the use of the Lagrangian derivative on the left-hand side of Eq. (5) does not significantly alter the results in the current PBL scheme.
The implementation of Ri hysteresis in both HYST1 and SMOOTH is expected to result in a direct reduction of TKE, since the growth parameter $B$ is either reduced (laminar branch) or unchanged (turbulent branch). This systematic reduction in the activity of PBL eddies is potentially undesirable, particularly given the evidence presented for high-Ri turbulence in the studies cited in section 2b. The introduction of a second, supercritical hysteretic loop is implied by $Ri_{c}:\text{max}$, as shown in Fig. 1d (HYST2). This yields a slightly more complicated adjustment to the Ri–$B$ relationship of the form

$$R_f^* = \begin{cases} 
R_f, & \text{if} \ Ri < Ri_{c_{\text{min}}} \text{ or } Ri > Ri_{c_{\text{max}}} \\
\max(R_f, 1), & \text{if} \ Ri > Ri_{c_{\text{min}}} \text{ and laminar} \\
\min(R_f, 1), & \text{if} \ Ri < Ri_{c_{\text{max}}} \text{ and turbulent}
\end{cases}$$

(12)

In this case TKE in an initially turbulent flow resists buoyant suppression until $Ri > Ri_{c_{\text{max}}}$, in a manner symmetric to the subcritical hysteretic loop implemented in HYST1. The selection of $Ri_{c_{\text{max}}}:2$ is justified given the demonstration by Majda and Shefter (1998) that turbulence can persist at arbitrarily high Ri values in time-varying flows. Indeed, Kondo et al. (1978) show that turbulent motions essentially cease only for $Ri > 2$.

An additional set of experiments based on HYST2 in which $Ri_{c_{\text{min}}}$ is increased to 0.5 (HYST2$_{\text{min}}$; Table 2) is undertaken to assess the sensitivity of the results to the extent of the subcritical hysteretic loop: the analogous comparison for the supercritical loop may be made between HYST1 and HYST2.

The bulk of the analysis presented in the remainder of this study focuses on the HYST2 implementation of Ri hysteresis shown in Fig. 1d, since it is the configuration that has been selected for use in the operational GDPS.
The other representations of this process are used primarily to assess the sensitivity of the parameterization to modest changes in its implementation.

3. Column model results

The application of Ri hysteresis within a simplified framework allows for the development of a qualitative understanding of its impact on the evolution of PBL profiles. A single-column model (SCM) configured to run the third Global Energy and Water Cycle Experiment Atmospheric Boundary Layer Study (GABLS-3; Holtslag 2006; Baas et al. 2010; Bosveld et al. 2014) case is employed for this component of the investigation, because its state consists of a moderately stable nocturnal PBL with sharp diurnal transitions between stable and unstable profiles. The 24-h integration commences at local noon for a point near the Cabauw mast in the Netherlands (Van Ulden and Wieringa 1996), and uses prescribed atmospheric forcings to ensure that the influence of the synoptic-scale flow is properly represented within the SCM context. The SCM is run with 10-m vertical grid spacing and a 45-s time step to ensure that the solution has converged with respect to discretization errors. To the extent that these errors are random, the SCM results are expected to be representative of those produced by GEM when the latter are averaged over a sufficiently large area. To simplify the interpretation of the results, and for consistency with the discussion presented in section 2a, only the analytic component of the TKE equation is considered in these simulations [terms I and II of Eq. (5)] and cloud processes are deactivated. The results of the integrations are qualitatively similar when TKE diffusion is activated, although the differences are smoothed vertically by the additional mixing.

By midnight (12 h into the SCM integration), a stable nocturnal boundary layer has developed beneath a residual layer (RL) that extends to almost 2000 m (Fig. 2a). Turbulent kinetic energy in the RL decays more slowly in HYST2 than in CTL (Fig. 2b) because buoyant suppression is inactive for turbulent parcels with $R_i < 2$. This leads to a better-mixed RL whose lapse rate is closer to dry adiabatic, and to a general reduction of Ri in the profile (Fig. 2c).

The inversion that characterizes the nocturnal boundary layer is stronger and closer to observations with Ri hysteresis active (Fig. 3a), implying greater decoupling from the surface and an accelerated low-level jet (LLJ; Figs. 3c,d). The enhanced shear leads to a sharp decrease in Ri at the top of the nocturnal inversion, as shown in Fig. 3b. In CTL, turbulent mixing increases as soon as Ri drops below unity, leading to increased coupling and reduced LLJ strength throughout the overnight hours (Fig. 4). However, in HYST2 the flow remains laminar and the quasi-steady solution is able to support stronger shears without breaking down into turbulent eddies. The strength of the LLJ is still limited, since further shear increase would lead to Ri < 0.25 and the transition to a turbulent regime that would diffuse the jet core. Both the direction and speed of the LLJ are better predicted in HYST2, although the strength of the LLJ remains underrepresented, as noted for the majority of models participating in the GABLS-3 project (Bosveld et al. 2014).

At sunrise (0500 local time, 17 h into the integration), near-surface stability is reduced in both simulations as insolation exceeds 100 W m$^{-2}$ (Fig. 5a). As for the midnight profiles, the remaining nocturnal boundary layer inversion is deeper and stronger in HYST2, with
near-zero TKE between 100 and 300 m indicative of a shutdown of turbulent mixing (Fig. 5b). By this time, a profile of warm advection forcing is active, as prescribed for GABLS-3 simulations (dashed line in Fig. 5c). With Ri < 1 at all levels except at the nose of the inversion in CTL (Fig. 5d), turbulent eddies mix the supplied heat away from the temperature maximum (Fig. 5c). Conversely, the maintenance of laminar flow between 80 and 350 m in HYST2 (dashed line in Fig. 5d) reduces the redistribution of the advectively supplied

![Fig. 7. Analyses from ERA-Interim for (left) 0000 and (right) 1200 UTC 22 Mar 2007. (a),(b) Dynamic tropopause potential temperature (shaded in K, as shown along the grayscale bar), winds, and layer-mean 925–850-hPa relative vorticity (black contours at 5 × 10⁻² s⁻¹ intervals starting at 5 × 10⁻² s⁻¹). (c),(d) The 850-hPa temperature (plotted in °C as shown along the grayscale bar) and winds. The (e),(f) 2-m air temperature (plotted as for the row above) and sea level pressure (black contours at 4-hPa intervals). Surface fronts as analyzed by the Hydrometeorological Prediction Center for the features of interest are plotted in (c)–(f) for reference. All winds in this study are plotted with short barbs, long barbs, and pennants representing 5-, 10-, and 50-kt values, respectively.]
heat, thus increasing the inversion strength in the simulation. This reduction of mixing across the top of the inversion is shown in the following section to be an important attribute of Ri hysteresis, particularly under conditions of warm advection such as those imposed here.

The influence of Ri hysteresis on the timing of turbulence regime transitions can be seen in the TKE time series for the 450-m level, near the base of the RL (Fig. 6). The evening and morning transitions are slightly delayed, as are pulses of intermittent mixing (Mahrt 1998; Sun et al. 2012) near midnight and before dawn. These nocturnal mixing events last longer with Ri hysteresis active, as TKE decays more slowly following each mixing maximum. The flow at this level becomes laminar only briefly in HYST2, consistent with the TKE and Ri profiles shown in Fig. 2. Both the delay and

FIG. 8. (a) Surface air temperatures (solid lines at 4°C intervals with dashing below 0°C) and 10-m winds from ERA-Interim for 0000 UTC 22 Mar 2007, along with analyzed snow depths from the Canadian Meteorological Centre (Brasnett 1999) in cm, as indicated along the grayscale bar. Surface fronts are plotted as in Fig. 7. A line tracing the cross section shown in (b) appears as a heavy solid curve in (a). (b) The corresponding cross-frontal section of temperature (plotted as in (a)), Ri (shading as indicated along the grayscale bar), and section-parallel winds. Snow cover of >1 cm is shown with a heavy solid line on the orography in (b), and the vertical component of the wind field in the cross section is amplified by a factor of 100 for readability.
damping influences are expected consequences of hysteretic loops in general (Krasnosel’skii and Pokrovskii 1989). Indeed, the inclusion of Ri hysteresis has the effect of damping occasionally occurring numerical oscillations between the PBL and surface schemes in GEM; however, this secondary benefit of the parameterization of Ri hysteresis is not the main focus of this study.

4. Forecast case study

Operational forecasters with Environment Canada regularly report on the performance of GEM-based guidance systems so that model developers can address issues that adversely affect the forecast process. One of the primary failure modes noted by these meteorologists is that of the warm episode (hereafter italicized when used to denote this class of model error), in which surface air temperature (SAT; taken at the standard 2-m level) errors can exceed 5 K in short-range (day 1) predictions. This error has a direct impact on both temperature and precipitation-type forecasts, the latter because it appears primarily near the freezing point at the surface.

In this section, the impact of Ri hysteresis on a canonical warm episode error that occurred during a freezing rain event is investigated. This case (22 March 2007) is chosen because of the large amplitude of the guidance error rather than the quantity of freezing rain accumulation; however, it effectively demonstrates how poor numerical prediction of the PBL state can have a serious negative impact on forecasting for potential high-impact events. Moreover, the sensitivities shown here are representative of other warm episode cases studied during the development of the parameterization. Two overlapping regions of interest are considered in detail: the area with an elevated freezing-rain potential, and the area over which guidance SAT errors are a maximum.

a. Case description

The flow configuration during the 22 March 2007 event described here is not usual, consistent with the prevalence of the error mode associated with it. This section documents the development of a split surface front that creates a broad region with a PBL whose turbulence properties lie within the Zilitinkevich et al. (2008) transition zone. The formation mechanisms and relevance of these features must be identified and understood in order to develop a full understanding of the impact of Ri hysteresis on the numerical guidance.

The evolution of the synoptic-scale flow during this event is shown in Fig. 7. A region of reduced potential temperatures on the dynamic tropopause [defined as the 2-PVU (potential vorticity unit) surface, where
1 PVU = $10^{-6}$ m$^2$ K kg$^{-1}$ s$^{-1}$] is indicative of an upper-level trough located over central North America at 0000 UTC 22 March 2007 (Fig. 7a). Over the next 12 h, the trough acquires a neutral tilt as it progresses into the Great Lakes region (Fig. 7b). A local maximum in relative vorticity near Hudson Bay is the lower-level reflection of this developing baroclinically tilted system. The strip of high values of cyclonic vorticity over the midwestern United States is related to strong lateral shear between the northwesterly flow behind the trough axis and the strong southwesterly flow ahead of the developing cold front (Figs. 7c,d). The advection of warm air by the 40–50-kt (1 kt = 0.5144 m s$^{-1}$) southwesterly winds at 850 mb helps to build a broad thermal ridge over the northeastern United States and southeastern Canada over the 12-h period of interest.

The central sea level pressure of the developing cyclone south of Hudson Bay drops from 999 to 991 mb over the period, but eastward movement of the center toward a broad 1040-mb anticyclone over the western North Atlantic leads to rapidly intensifying southwesterly geostrophic winds (Figs. 7e,f). This flow is channeled up the Ohio River valley, resulting in strong lower-level warm advection and the poleward displacement of subfreezing SAT isotherms toward James Bay. Warmer SATs, however, remain relatively stationary along the snow line near the Great Lakes (Fig. 8a). Differential surface heating across the thermal gradient enhances the split lower-level frontal structure in which a pair of baroclinic zones are separated by a broad region of near-freezing SATs.

The region between the surface fronts is characterized by weak horizontal thermal gradients, enhanced static stability, and strong low-level vertical wind shear (Fig. 8b). The latter is particularly relevant, since it effectively reduces Ri despite the stable thermal profile. This vertical structure is delicately balanced as discussed in section 3, since an increase in the strength of the LLJ...
could trigger the growth of PBL eddies and result in strong vertical mixing that would reduce the strength of the warm nose and limit the freezing rain potential. It is the large size of the area in this configuration that makes this kind of case a challenging problem from both the NWP and public forecasting perspectives.

The region with an elevated freezing-rain potential lies between the surface fronts and is shown in Fig. 9 to move eastward with the cyclone between 0000 and 1200 UTC 22 March. In this study, the potential freezing-rain region is assessed as the area over which the 850-hPa warm-nose strength exceeds 4 K and SATs are <4°C. Although no soundings are available at the height of the event (0600 UTC 22 March), Fig. 10 shows a composite sounding for stations located within the potential freezing rain region at 1200 UTC 22 March [Fig. 9c; Albany, New York (ALB); Caribou, Maine (CAR); and Gray, Maine (GYX)]. The composite profile contains a 10-K warm nose at 900 mb, with subfreezing near-surface temperatures. The mean ERA-Interim sounding accurately depicts the free-tropospheric profile but fails to capture the full strength of the lower-level inversion, with a 6-K warm nose peaking at 850 mb. The difference in inversion amplitude likely arises from a combination of representativeness (area average versus three sounding points) and analysis errors, and is indicative of the errors that should be expected from any integration conducted at a global model resolution.

b. Guidance errors

Short-range NWP guidance errors for SAT in this case reach almost 10°C, peaking in the forecast initialized at 0000 UTC 22 March 2007. This large departure from the typical error level of 1–2°C is one of the primary reasons that this case is considered a warm episode archetype. The 12-h forecast SAT errors with respect to both observations and ERA-Interim are shown in Fig. 11a. Throughout the region between the split surface fronts, predicted SATs are too warm. Since this area covers the metropolitan centers of Ottawa and Montreal, the poor quality of the numerical guidance in this case had a large impact on the forecast process, and contributed to the late issuance of freezing rain warnings across Ontario and Quebec.

Errors are not restricted to SAT in this case as demonstrated by the 12-h forecast profiles in both the

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2 The selection of this threshold takes into account the fact that local effects may result in subfreezing SATs on unresolved scales. These localized cold areas expand the potential freezing rain region despite gridbox-mean temperatures that are slightly above freezing.
FIG. 12. Mean profiles from CTL in the (a),(b) potential freezing rain and (c),(d) maximum SAT error regions after 12 h of integration, valid 1200 UTC 22 Mar 2007. The regions are shown in Figs. 9c and 11a, respectively. (left) The CTL profiles (gray) are compared with ERA-Interim (black). (right) The departure of CTL temperature from ERA-Interim is shown for each region.
freezing rain and maximum error regions (Fig. 12). The amplitude of the warm nose is underestimated in the forecast, as is the strength of the southwesterly LLJ in the freezing-rain area. This vertical error structure, characterized by warm errors near the surface and cold errors aloft, is common in other, less extreme warm episode cases documented by forecasters. From a physical perspective, this error profile is consistent with overactive mixing in the PBL, which leads to the homogenization of the lower-level profiles despite strong synoptic-scale warm advection aloft.

The impact of the temperature error profile on precipitation type is evident in Fig. 13a. Although freezing rain is predicted across the region, the areal extent of the event is underestimated by the guidance. The probability of detection (POD) in the operational configuration (CTL) for even very light accumulations (0.5 mm between 0000 and 1200 UTC 22 March 2007) is only 37%, a fact that is reflected in the late issuance of warnings noted above.

The strength of vertical diffusion in GEM’s PBL scheme is related to TKE and the $\lambda$ through Eqs. (2) and
Since both of these quantities are themselves dependent on Ri, a comparison of Ri between the forecast and ERA-Interim is shown in Fig. 14 in the 950–850-hPa layer over which the excessive mixing in the forecast is maximized. Values of Ri over Ontario and Quebec, Canada, range from 0.25 to 1 in the 6-h forecast, lower than ERA-Interim estimates of 1–2. These depressed values lie within the Zilitinkevich et al. (2008) turbulence transition zone and may therefore be subject to the effects of Ri hysteresis.

To assess the impact of this process on short-range guidance for this warm episode, a rerun of the operational GDPS integration (CTL) initialized at 0000 UTC 22 March 2007 is undertaken using the HYST2 configuration. This integration represents an incomplete estimate of the impact of Ri hysteresis since the simple forecast-only experimental design does not account for accumulated errors that affect the analysis component. 

**Fig. 15.** As in Fig. 12, but for HYST2 (gray).
of the system. The impact of this process in a full assimilation cycle will be evaluated in section 5.

Near-surface and lower-tropospheric guidance errors in HYST2 are shown in Figs. 11b and 15, respectively. Errors in the short-range SAT predictions are reduced to <5 K, with lower temperatures forecast across the region of interest. Profiles from HYST2 within both the maximum error and potential freezing rain regions show an improvement that is consistent with a reduction of over-mixing. The mean freezing rain profile in Fig. 15a contains an 8°C warm nose, with a structure that is similar to that of the ERA-Interim analysis. This result is indicative of the maintenance of a laminar regime between the two surface baroclinic zones where Ri_{c,min} < Ri < 1, which results in the suppression of TKE growth across the region.

The improved lower-tropospheric thermal structure leads to improved freezing rain guidance, as shown in Fig. 13b. The area over which freezing rain is predicted expands beyond that of CTL to include a larger fraction of the stations that report such accumulations. This leads to an increase in the POD to 56% in HYST2, and to a near doubling of the equitable threat score (ETS) as shown in Table 3.

The reduction in subgrid-scale turbulence, and thus vertical diffusion within the PBL scheme, is evident in the prognostic TKE estimated by the model (Figs. 16a,b). Throughout the region dominated by southwesterly flow, lower-level TKE values in HYST2 are approximately half those of CTL. This results in reduced mixing lengths, which further act to decrease the diffusion coefficients in this area through Eq. (3) (Figs. 16c,d). Outside the warm sector, the activity of the PBL scheme remains essentially unchanged, as Ri values spend relatively little time in the turbulent transition zone.

The sensitivity of these results to the form of Ri hysteresis adopted in the integration is summarized in Tables 3 and 4. The configurations employed in these tests follow the strategies shown in Fig. 1, as described in Table 2. The single-sided implementation (HYST1) performs as well as HYST2 from both the freezing rain and SAT perspectives. This suggests that case study results are relatively insensitive to the value of Ri_{c,max}. Adjustments to Ri_{c,min}, however, have a substantial impact on guidance quality. The HYST2_{min} integration does not sufficiently suppress turbulent mixing, resulting in a reduced POD of freezing precipitation and SAT error indicators that lie approximately midway between CTL and HYST2. The effect of the SMOOTH Ri–B relationship is relatively small in terms of SAT; however, the freezing rain ETS for this integration remains below 0.3. It appears that the influence of this configuration on the sensitive Ri_{c,min} value results in a deterioration of predictive skill for precipitation type.

### Table 3. Predictive skill for freezing rain accumulations between 0000 and 1200 UTC 22 Mar 2007 in the integrations identified in the first column. The region shown in Fig. 13 is used for these calculations (53 stations reporting precipitation type), with a prediction threshold of 0.5-mm accumulation matched against freezing rain reports in the hourly observations. Acronyms used in the first row are probability of detection (POD), false alarm ratio (FAR), and equitable threat score (ETS).

<table>
<thead>
<tr>
<th>Integration</th>
<th>Ri_{c,min}</th>
<th>Ri_{c,max}</th>
<th>POD</th>
<th>FAR</th>
<th>ETS</th>
</tr>
</thead>
<tbody>
<tr>
<td>CTL</td>
<td>1</td>
<td>1</td>
<td>0.37</td>
<td>0.09</td>
<td>0.2</td>
</tr>
<tr>
<td>HYST1</td>
<td>0.25</td>
<td>1</td>
<td>0.56</td>
<td>0.06</td>
<td>0.35</td>
</tr>
<tr>
<td>HYST2</td>
<td>0.25</td>
<td>2</td>
<td>0.56</td>
<td>0.06</td>
<td>0.35</td>
</tr>
<tr>
<td>HYST2_{min}</td>
<td>0.5</td>
<td>2</td>
<td>0.44</td>
<td>0.08</td>
<td>0.25</td>
</tr>
<tr>
<td>MIXLEN</td>
<td>1</td>
<td>1</td>
<td>0.67</td>
<td>0.1</td>
<td>0.42</td>
</tr>
<tr>
<td>SMOOTH</td>
<td>0.22</td>
<td>1</td>
<td>0.48</td>
<td>0.07</td>
<td>0.28</td>
</tr>
</tbody>
</table>

Alternative strategies for reducing vertical diffusion within the PBL can of course be devised. From Eq. (3) it is clear that a reduction of the mixing length will directly impact the diffusion coefficients. The results of an experiment in which λ is decreased through the use of the Blackadar (1962) formulation, hereafter referred to as MIXLEN, are also summarized in Tables 3 and 4. While this approach yields the desired outcome of reduced vertical mixing, as illustrated by the high POD and cooled SATs, it increases both the freezing rain false alarm ratio (FAR) and SAT errors across the remainder of the continent. The SAT errors in the 12-h MIXLEN forecast shown in Table 4 exceed those of all other configurations. This result is consistent with those of other unpublished experiments that affect diffusion coefficients across the domain rather than targeting the region characterized by Ri values within the turbulent transition zone.

### 5. Systematic impacts of Ri hysteresis

The assessment of the impact of including Ri hysteresis in the PBL scheme presented in the preceding section promotes a physical understanding of the effect, but is necessarily incomplete. First, it deals only with a specific warm episode, albeit one that had a large impact on forecasts over a broad region. Although other cases were studied during the development of the parameterization and found to exhibit similar sensitivities (not shown), the representativeness of the results cannot be fully assessed with the experimental design used in section 4. Instead, a set of model integrations spanning a longer period is required. Second, the demonstrated impact of Ri hysteresis represents a lower bound given that the same analysis was used for the initialization of all integrations. A full assimilation cycle is required to determine the overall impact of this process on NWP guidance, since the modification to PBL mixing is present.
in both the trial (background) and forecast components of the cycle.

a. Impact in forecast sequences

To address the representativeness issues associated with the case study approach, a set of 2-month forecast sequences are run. The sequences discussed here cover February–March 2011, the period chosen for current “winter” GDPS development. The impact of Ri hysteresis on results from the “summer” period is small given the dominance of unstable PBLs over the continents, and they are therefore not shown. Forecast initializations are separated by 36-h intervals for independence, such that each sequence is composed of 44 integrations. In addition to allowing for a robust estimate of the impact of Ri hysteresis on guidance quality, the forecast sequences are computationally inexpensive enough to allow for some sensitivity testing of the alternative Ri–B relationships described in Fig. 1 and Table 2.

The results of the forecast sequence using the HYST2 implementation are shown in Fig. 17. Errors relative to Northern Hemisphere radiosonde observations after 24 h of integration are reduced throughout the lower troposphere for most variables, most notably for the standard deviation (SD) of temperature in the PBL. Near-surface cooling is suggested in the temperature

FIG. 16. Relevant PBL quantities after 6 h of the (left) CTL and (right) HYST2 integrations, valid 0600 UTC 22 Mar. (a),(b) Layer-mean surface–850-hPa TKE (m² s⁻² as shown along the grayscale bars) and λ (contoured at 30-m intervals). (c),(d) The resulting KH, shaded as shown along the grayscale bars. Analyzed surface fronts are plotted as in Fig. 7 for reference.
and height biases, a result that is consistent with the reduced vertical diffusion and enhanced lower-level stability identified in the case studies (sections 3 and 4c). The significance of the differences shown in Fig. 17 diminishes with forecast lead time as variability increases, such that the SD scores are statistically neutral in the forecast sequence at lead times beyond 72 h.

The results of forecast sequences using alternate Ri hysteresis configurations (Fig. 1 and Table 2) are summarized in Table 5 for the surface to 500-mb layer. Despite the apparently modest size of the error reductions, the long time period and large volume of evaluation data (approximately $10^5$ observations per variable) allows for the detection of statistically significant differences. The HYST1 configuration yields the largest improvements in wind biases and temperature errors, with HYST2 and SMOOTH demonstrating slightly smaller gains relative to CTL. The HYST2$_{\text{min}}$ sequence shows the smallest improvements, consistent with the noted sensitivity of the configurations to the critical value of the forward transition to turbulence.

b. Impact in an assimilation cycle

The impact of Ri hysteresis in a full data assimilation system is evaluated using a 2-month cycle of the GDPS covering the February–March 2011 period as for the forecast sequences (the summer cycle shows limited sensitivity for the same reason discussed in the previous section). Hysteresis is active in the nonlinear model throughout the four-dimensional variational assimilation cycle, but does not affect the tangent linear or adjoint formulations because of their adoption of a direct algebraic relationship between the model state and diffusion coefficients in the PBL scheme (Laroche et al. 2002).

The HYST2 implementation of Ri hysteresis was chosen for these cycles because preliminary results suggested that a worsening in the model’s SAT cold bias was related to the direct reduction of TKE in HYST1. However, in light of the results presented in the previous section and the introduction of an improved TKE initialization (also included in the HYST2 cycle), additional effort will be made in the near future to identify the optimal value of $\text{Ri}_{\text{c max}}$ in the GDPS.

The results of a forecast sequence initialized with the analyses generated by this assimilation cycle are shown in Figs. 18 and 19. At the 24-h forecast range (Fig. 18), the SD response to Ri hysteresis is similar to that noted in the forecast-only sequence: general improvements are observed throughout the lower troposphere for all variables. Unlike the forecast-only sequences however, in which the impact Ri hysteresis decays with lead time relative to the underlying error growth, the introduction of this effect within the cycle leads to significant improvements even at medium range (Fig. 19). Improvements in the temperature profile are concentrated in the PBL, but improvements in upper-tropospheric heights and winds are also evident. A preliminary evaluation of these results suggests that the changes in near-surface stability affected by this parameterization interact with the model’s orographic blocking scheme to influence the large-scale circulation (Zadra et al. 2003); however, a thorough examination of this interaction will be the subject of a future study.

6. Discussion

The PBL scheme in the Canadian operational global model (GDPS) employs a one-and-a-half-order closure that is based on the prediction of unresolved TKE. Given this quantity and the atmospheric profile, a set of vertical diffusion coefficients for heat, momentum, and constituents is determined. Both the time evolution of TKE and the relationships between the state variables and these coefficients are highly dependent on Ri. In this study, the relationship between Ri and the predictive TKE equation is modified to represent the effects of Ri hysteresis (Woods 1969). Instead of a single transition point between laminar and turbulent regimes at $\text{Ri}_{\text{c max}}$, the

<table>
<thead>
<tr>
<th>Integration</th>
<th>$\text{Ri}_{\text{c min}}$</th>
<th>$\text{Ri}_{\text{c max}}$</th>
<th>SAT error region</th>
<th>Continent</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td></td>
<td>Obs</td>
<td>ERA-Interim</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td>Bias SD</td>
<td></td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>CTL</td>
<td>1</td>
<td>1</td>
<td>5.9 2.2</td>
<td>5.4 1.3</td>
</tr>
<tr>
<td>HYST1</td>
<td>0.25</td>
<td>1</td>
<td>4.3 2.1</td>
<td>4.1 1.1</td>
</tr>
<tr>
<td>HYST2</td>
<td>0.25</td>
<td>2</td>
<td>4.3 2.1</td>
<td>4.1 1.1</td>
</tr>
<tr>
<td>HYST2$_{\text{min}}$</td>
<td>0.5</td>
<td>2</td>
<td>5.1 2.1</td>
<td>4.9 1.2</td>
</tr>
<tr>
<td>MIXLEN</td>
<td>1</td>
<td>1</td>
<td>3.3 2.1</td>
<td>3.0 1.4</td>
</tr>
<tr>
<td>SMOOTH</td>
<td>0.22</td>
<td>1</td>
<td>4.4 2.1</td>
<td>4.2 1.2</td>
</tr>
</tbody>
</table>
hysteresis implies that flows within a transition zone
($Ri_{\text{c,min}} < Ri < Ri_{\text{c,max}}$) are not in a state that is uniquely
defined by $Ri$: additional information about the current
state is required. Flows that are initially laminar remain
so until $Ri$ falls below $Ri_{\text{c,min}}$; conversely, turbulence
persists across the transition zone until $Ri < Ri_{\text{c,max}}$, as
shown in Fig. 1.

The impact of parameterized $Ri$ hysteresis on an SCM
integration of the GABLS-3 case is to enhance the LLJ
and to strengthen the nocturnal inversion that develops
in the presence of warm advection aloft. Both of these
outcomes are favorable given the tendency of models
participating in the project to underestimate the magni-
tude of these features (Bosveld et al. 2014). Furthermore,
Table 5. Summary of error changes in the forecast sequences identified in the first column with respect to CTL. Errors are computed as absolute bias and SD differences based on Northern Hemisphere radiosondes over the surface–500-hPa layer after 24 h of integration. Improvements are associated with error reductions (negative values) as in Fig. 17, with differences that are statistically significant at the 99% level shown in boldface.

<table>
<thead>
<tr>
<th>Integration</th>
<th>$R_i$,min</th>
<th>$R_i$,max</th>
<th>Wind speed (kt)</th>
<th>Height (m)</th>
<th>Temperature (K)</th>
<th>Dewpoint depression (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>Absolute bias</td>
<td>SD</td>
<td>Absolute bias</td>
<td>SD</td>
<td>Absolute bias</td>
<td>SD</td>
</tr>
<tr>
<td>HYST1</td>
<td>0.25</td>
<td>1</td>
<td>-0.05</td>
<td>-0.02</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>HYST2</td>
<td>0.25</td>
<td>2</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
<tr>
<td>HYST3</td>
<td>0.5</td>
<td>2</td>
<td>-0.007</td>
<td>-0.006</td>
<td>-0.03</td>
<td>-0.03</td>
</tr>
<tr>
<td>SMOOTH</td>
<td>0.22</td>
<td>1</td>
<td>-0.04</td>
<td>-0.02</td>
<td>-0.1</td>
<td>-0.1</td>
</tr>
</tbody>
</table>

A damping effect is evident in the SCM results, consistent with the properties of hysteretic loops in general.

The parameterization of $R_i$ hysteresis in GDPS forecasts for an archetypal warm episode that occurred in March 2007 demonstrates that the inclusion of this process can have an important impact on numerical guidance. A developing cyclone over Hudson Bay leads to strong warm advection up the Ohio River valley. The associated lower-level baroclinic zone splits to create a broad region across eastern Canada and New England that is characterized by a strong inversion, LLJ, and near-freezing temperatures at the surface. The $R_i$ values in this area lie within the turbulence transition zone (Zilitinkevich et al. 2008). Excessive vertical mixing by the operational PBL scheme is shown to be responsible for guidance errors approaching 10°C at short (6–12 h) lead times. The inclusion of $R_i$ hysteresis yields large improvements in temperature guidance throughout the PBL by suppressing the generation of TKE within this region.

The results of 2-month forecast sequences and data assimilation cycles show that the positive impact of including a parameterization of $R_i$ hysteresis is not limited to warm episode cases, nor is it restricted to the PBL. Although summer experiments show relatively little impact due to the prevalence of unstable boundary layers, the winter integrations demonstrate that temperature, height, and wind guidance are significantly improved in both the PBL and in the upper troposphere. The latter suggests that the correct representation of this effect may be both physically meaningful and relevant to PBL schemes that employ higher-order closures. The former presents the opportunity for a refinement of the underlying physical process upon which it is based, a pattern of behavior consistent with the systematic guidance improvements identified in this study.

With a trend toward the introduction of PBL schemes with higher-order closures in NWP, the results presented here suggest that a representation of $R_i$ hysteresis may become increasingly relevant in operational systems. Because this process is only relevant for $R_i$ values close to the turbulent transition zone, it plays an important role in the near-neutral conditions that challenge existing PBL schemes. These issues can have serious detrimental impacts on the quality of the numerical guidance for potentially high-impact forecasts such as those issued for the freezing rain event described in this study. To assess the more general applicability of a parameterization of $R_i$ hysteresis, investigations into alternate PBL formulations and under the total energy framework are required. We hope that this work will serve as the foundation for such further studies that will provide additional guidance to developers and users of higher-order PBL schemes regarding the potential importance of accounting for $R_i$ hysteresis during the estimation of turbulent mixing in the lower atmosphere.

Acknowledgments. ECMWF ERA-Interim data used in this study have been obtained from the ECMWF...
data server. Archived Environment Canada (EC) and National Weather Service text forecast data were downloaded from Iowa State University, while sounding data were obtained from the University of Wyoming. Michel Roch and Michel Desgagné of EC were instrumental in running the forecast sequences and assimilation cycle presented in this study. Discussions with Claude Girard and Martin Charron were very useful during the development of the parameterization.

APPENDIX

Definition of the Mixing Length

The implementation of Ri hysteresis described in section 2c does not involve any direct modification to the mixing-length (λ) component of the closure. However, since the mixing coefficients depend on λ through Eq. (3), a description of its definition in the GDPS is
worthwhile. The adopted formulation is a blended version of Blackadar (1962) and Bougeault and Lacarrère (1989) estimates, wherein the Blackadar (1962) component for a neutral atmosphere ($\lambda_n$) is represented by

$$\lambda_n = \min[k(z + z_\tau), \lambda_e].$$

(A1)

Here, $k = 0.4$ is the von Kármán constant, $z$ is the height above ground, $z_\tau$ is the aerodynamic roughness length, and $\lambda_e = 200 \text{ m}$ is the maximum value for $\lambda_n$, asymptotic in the original formulation and truncated here. The Blackadar-based mixing length is then

$$\lambda_{\text{Black}} = \frac{\lambda_n}{\phi}. \quad \text{(A2)}$$

which is dependent on the Delage and Girard (1992) stability functions:

$$\phi = \begin{cases} 
(1 - 40 \text{Ri})^{-1/6}, & \text{for } \text{Ri} < 0 \text{ (static instability)} \\
1 + 12 \text{Ri}, & \text{for } \text{Ri} \geq 0 \text{ (static stability)}
\end{cases}. \quad \text{(A3)}$$

The Bougeault and Lacarrère (1989) mixing length ($\lambda_{\text{Boug}}$) is directly computed based on the minimum vertical extent
of the buoyant displacements of a parcel whose kinetic energy is initially equal to that of the layer from which it departs [Eq. (11) in Bougeault and Lacarrère (1989)].

\[
\lambda^* = \begin{cases} 
\lambda_{\text{Black}} + \frac{z}{z_m} (\lambda_{\text{Boug}} - \lambda_{\text{Black}}), & \text{for } z < z_m = 500 \text{ m}, \\
\lambda_{\text{Boug}}, & \text{for } z \geq z_m.
\end{cases}
\]

Blending of the two estimates for \( \lambda \) in the GDPS follows an empirically derived two-step strategy that begins with a near-surface adjustment,

\[
\lambda = \begin{cases} 
\lambda^*, & \text{for } p \geq p_l = 550 \text{ hPa}, \\
\lambda_{\text{Black}} + \frac{p_l - p_h}{p_l - p_h} (\lambda^* - \lambda_{\text{Black}}), & \text{for } p_h < p < p_l, \\
\lambda_{\text{Black}}, & \text{for } p \leq p_h = 450 \text{ hPa}.
\end{cases}
\]

The value of \( \lambda \) computed by employing this technique is used with the prognostic TKE in Eq. (3) to yield an estimate of the turbulent diffusion coefficients (\( K_i \)) in integrations both with and without the Ri hysteresis.

REFERENCES


