Sampling Error Damping Method for a Cloud-Resolving Model Using a Dual-Scale Neighboring Ensemble Approach

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ABSTRACT

In ensemble-based assimilation schemes for cloud-resolving models (CRMs), the precipitation-related variables have serious sampling errors. The purpose of the present study is to examine the sampling error properties and the forecast error characteristics of the operational CRM of the Japan Meteorological Agency (JMANHM) and to develop a sampling error damping method based on the CRM forecast error characteristics.

The CRM forecast error was analyzed for meteorological disturbance cases using 100-member ensemble forecasts of the JMANHM. The ensemble forecast perturbation correlation had a significant noise associated with the precipitation-related variables, because of sampling errors. The precipitation-related variables were likely to suffer this sampling error in most precipitating areas. An examination of the forecast error characteristics revealed that the CRM forecast error satisfied the assumption of the spectral localization, while the spatial localization with constant scales, or variable localization, were not applicable to the CRM.

A neighboring ensemble (NE) method was developed, which was based on the spectral localization that estimated the forecast error correlation at the target grid point, using ensemble members for neighboring grid points. To introduce this method into an ensemble-based variational assimilation scheme, the present study horizontally divided the NE forecast error into large-scale portions and deviations. As single observation assimilation experiments showed, this “dual-scale NE” method was more successful in damping the sampling error and generating plausible, deep vertical profile of precipitation analysis increments, compared to a simple spatial localization method or a variable localization method.

1. Introduction

Recently, many numerical weather prediction (NWP) centers have routinely used cloud-resolving models (CRMs) that explicitly forecast hydrometers for the improvement of precipitation forecasts (e.g., Saito et al. 2006). To make the most of the CRMs at short lead times, the assimilation of precipitation information (e.g., Makihara 2000, 63–111) is desirable (Aonashi and Eito 2011, hereafter referred to as AE). While precipitation data were confined to areas where ground-based observations have been made, recent satellite microwave imager (MWI) and radar data have enabled us to obtain global precipitation information (Kummerow et al. 1998). MWI brightness temperatures (TBs) and radar reflectivity are nonlinear and flow-dependent functions of various atmospheric and surface variables. Some studies

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proposed ensemble-based variational assimilation (EnVar) in order to address these problems.

AE pointed out that sampling errors have more serious effects on precipitation-related variables (precipitation rate, vertical wind speed, etc.) than other variables. This is because sample numbers of nonzero precipitation were much smaller than those of other variables for most grid points. AE also reported that the CRM precipitation-related variables have distinct forecast error features (narrow horizontal correlation scales, etc.), which differ from the other variables.

The sampling error is the common problem with ensemble-based assimilation schemes; hence, various types of sampling error damping methods have been developed. “Spatial localization” is the most widely used method, damping the sampling error in a distance by reducing forecast error covariance with increasing separation distance (Houtekamer and Mitchell 1998; Hamil et al. 2001; Lorenc 2003). Kang et al. (2011) developed a “variable localization” method that neglected the cross correlation between variables that were not physically related, and reduced the sampling error between these variables. Buehner and Charron (2007) proposed a “spectral localization” method that damped the sampling error in spectral space by reducing the forecast error correlation with increasing wave-number difference. Buehner (2012) and Buehner and Shlyaeva (2015) combined the spatial and spectral localization methods to use different spatial localization lengths, in terms of the spectral modes.

To damp the sampling error of CRM precipitation-related variables, we need to use a sampling error damping method whose hypothesis for the forecast error characteristics are satisfied with the forecast errors of these variables. We also need to increase the sample numbers in the forecast error covariance calculation.

Hence, the purpose of the present study is twofold:

1) to examine the sampling error properties and forecast error characteristics of the Japan Meteorological Agency (JMA) operational CRM and
2) to develop a CRM sampling error damping method based on the CRM forecast error characteristics.

To achieve the first aim, we performed 100-member ensemble forecasts of the JMA operational CRM, for various precipitation cases. Then, we analyzed the sampling error properties and spatial, intervariable, and spectral characteristics of the CRM ensemble forecast perturbations.

To achieve the second aim, we developed a sampling error damping method that was based on the spectral localization that estimated the forecast error correlation at the target grid point, using ensemble members for neighboring grid points. We performed single observation assimilation experiments in order to compare this method to conventional sampling error damping methods.

2. CRM ensemble forecast and the meteorological cases

a. CRM ensemble forecast

In the present study, we adopted the JMA Nonhydrostatic Model (JMANHM), which had 400 grid points in the x and y directions, with a horizontal resolution of 5 km and 38 levels in the vertical (Saito et al. 2006). This CRM predicted horizontal and vertical momentums as the dynamic variables, and pressure perturbations and potential temperature as the thermodynamic variables. This CRM also employed a bulk microphysical scheme in order to predict, explicitly, the mixing ratios of six hydrometers (water vapor, cloud water, cloud ice, rain, snow, and graupel) and the number concentrations of cloud ice, snow, and graupel (Ikawa and Saito 1991).

Similar to AE, we performed 100-member ensemble forecasts of the CRM, each of which started with perturbed initial values. To this end, we added random perturbations to the routine JMA regional analysis dataset (Ishikawa and Koizumi 2002). To make the perturbations, we employed a method of generating “three-dimensional random fields with prescribed covariance structure” (Houtekamer and Mitchell 1998). This method derived random fields from horizontal and vertical correlation scales. Following AE, we prescribed the horizontal scales as 100, 200, 300, 500, or 1000 km and the vertical scales as 3, 5, or 7 km. Then, we calculated the random perturbations from the combination of horizontal and vertical scales by this method. This enabled us to use the same initial perturbations for the ensemble forecasts for all meteorological cases. The boundary conditions for the forecasts were provided by hourly forecasts of the JMA regional spectral model (JMA 2002), starting with the above routine regional analysis. Note that boundary conditions were not perturbed in our ensemble forecasts.

b. The meteorological cases

We chose typical precipitation disturbances around Japan as the target cases. We executed the CRM ensemble forecasts for a typhoon case (initial time: 1500 UTC 9 June 2004), an extratropical low case (initial time: 2100 UTC 26 January 2003), and a baïu case (initial time: 0000 UTC 1 June 2004).

We employed the ensemble forecasts after 7 h of the initial time for the CRM forecast error analysis in order to spin up the model precipitation. Figure 1 shows the
mean of the ensemble forecasts of the precipitation rate (shades) and surface pressure (contours) at 7 h after the initial time. For the typhoon case (Fig. 1a), heavy convective precipitation areas were located to the north of the typhoon center, and around the typhoon spiral band. For the extratropical low case (Fig. 1b), heavy precipitation areas were extended from central Japan to the sea, south of western Japan, and surrounded by weak stratiform precipitation areas. For the baiu case (Fig. 1c), weak convective precipitation areas were extended from southern China to the sea, south of Japan.

3. Analysis of the CRM ensemble forecast perturbations

a. The sampling error of the CRM ensemble forecast

We derived the forecast differences between each ensemble member and the ensemble mean at 7 h after the initial time (hereafter referred to as the ensemble forecast perturbations), and calculated the correlation using the 100 samples of the ensemble forecast perturbations. In this calculation, we selected the horizontal ($u$, $v$) and vertical wind speeds ($w$), potential temperature (PT), ratio of total water content to the saturation specific humidity (RTW), precipitation rate (PR), and surface pressure ($P_s$) at the 38 height levels. Hereafter, we refer to this method as the conventional method.

First, we show in Fig. 2 the ensemble forecast perturbation correlation, generated by the conventional method. We found that there was significant noise related to the precipitation-related variables, because of the sampling error, in peripheries of precipitation areas (e.g., point A in Fig. 1a) where few members forecasted precipitation. On the other hand, the noise levels were low in heavy-rain cores (e.g., point B in Fig. 1a) where most members forecasted precipitation. Figures 2a and 2b illustrate the horizontal autocorrelation of $u$ and PR, respectively, at a height of 2880 m, between point A in Fig. 1a and surrounding grid points at every 25 km (five grids). The $u$ correlation smoothly decreased with the distance from point A, while the correlations were elongated in the east-northeast and west-southwest directions. On the other hand, the PR correlation was noisy, with high-correlation ($>0.5$) points scattered over a wide range ($>200$ km). Figures 2c and 2d illustrate the horizontal autocorrelation of $u$ and PR, respectively, at a height of 2880 m, at point B in Fig. 1a. The PR correlation had no high-correlation ($>0.4$) points except for the target point. The $u$ correlation had a narrower horizontal scale than that at point A.

Figure 3a shows the vertical cross correlation of various variables at point A. In this figure, parameter indexes (PI) represent PR (1–20), $w$ (21–40), RTW (41–60), PT (61–80), $u$ (81–100), and $v$ (101–120) at 20 height levels (20, 360, 710, 1180, 1460, 1770, 2110, 2480, 2880, 3310, 3770, 4260, 4780, 5330, 5910, 6520, 7160, 8530, 10020, and 12480 m), as well as $P_s$ (121). We found that PR had unrealistic, block-shaped, high-correlation patterns between most vertical levels, due to the sampling error. The cross correlation between PR and $w$ also had similar unrealistic, block-shaped, high-correlation patterns. On the other hand, other variables had plausible patterns, with small correlation coefficients, except for the near-diagonal elements. Figure 3b shows the vertical cross correlation of the variables at point B. The precipitation-related variables did not have unrealistic block-shaped patterns. Cross-correlation coefficients between different variables or different vertical levels were larger than those at point A.

The unrealistic vertical correlation can degrade the EnVar analyses, in particular, in MWI TB assimilation. This is because the EnVar determines the vertical profiles of the analysis increments, predominantly based on the forecast correlation, and also because MWI TBs are functions of the column-accumulated water subsistence.

To understand what portion of each precipitation area suffered from this sampling error, we introduced the mean distance between points whose horizontal ensemble forecast perturbation correlation with the target point was greater than 0.5, as the index of the sampling error (hereafter referred to as DIST). DIST becomes greater when high-correlation points exist in the distance, as shown in Fig. 2b.

Figure 4 illustrates the DIST of PR at every 25 km (five grids), at a height of 2880 m, as calculated by the conventional method for the target cases. This shows that DIST was larger than 80 km in most precipitation areas, with the exception of heavy precipitation cores, in the central parts (e.g., point B in Fig. 1a). It also shows that DIST tends to increase in the peripheries of the precipitation areas. This suggests that PR is likely to suffer from sampling errors in most precipitation areas.

b. Examination of the ensemble forecast perturbations

The sampling error damping methods, detailed in the introduction, are applicable to the CRM only when the presumptions of these methods (i.e., the forecast error characteristics) are satisfied with the CRM. Therefore, we checked the spatial, intervariable, and spectral characteristics of the ensemble forecast perturbation for the target cases.

1) SPATIAL CHARACTERISTICS

First, we checked the horizontal correlation of the ensemble forecast perturbations. We calculated the
horizontal moving average of the ensemble forecast perturbation correlation, over the whole CRM forecast domain, in order to reduce the sampling error of the conventional method. In this calculation, we employed the vertical wind speed, $w$, instead of PR. This is because $w$ had a high correlation with PR [the details of this are reported upon in section 3b(2)], and a nonzero correlation in precipitation-free areas (hereafter, we refer to PR and $w$ as the precipitation-related variables). Figure 5a shows the domain-averaged ensemble forecast

FIG. 1. The mean of the CRM ensemble forecasts at 7 h after the initial time. Shading and contours denote precipitation rates and surface pressure. Shown are results for (a) a typhoon (initial time: 1500 UTC 9 Jun 2004), (b) an extratropical low (initial time: 2100 UTC 26 Jan 2003), and (c) a baiu case (initial time: 0000 UTC 1 Jun 2004).
FIG. 2. (a) The horizontal autocorrelation of $u$ at a height of 2880 m between point A in Fig. 1a and surrounding points at every 25 km (five grids), calculated by using the CRM ensemble forecasts at 7 h after the initial time by the conventional method. (b) As in (a), but for PR. (c) As in (a), but at point B in Fig. 1a. (d) As in (b), but at point B in Fig. 1a.
perturbation correlation of $u$, RTW, and $w$ in the $x$ direction, at a height of 2880 m, for the typhoon case. The horizontal scale (with correlation coefficient $= 0.5$) is approximately 15 km for $w$, and larger than 100 km for other variables. This suggests that the precipitation-related variables had narrow horizontal forecast errors, equivalent to the effective resolution of the CRM, while wider forecast error modes (>100 km) were dominant for other variables.

The CRM forecast error can vary between precipitating and nonprecipitating areas, as pointed out by Michel et al. (2011). Considering this, we calculated the domain-averaged ensemble forecast perturbation correlation in the $x$ direction, classified with PR, for the target areas (Figs. 5b–d). These results suggest that the precipitation-related variables had deeper autocorrelation patterns, compared to the other variables. They also suggest that the PR forecast error, in particular, tended to be vertically coherent because the precipitating particles in the upper levels rapidly dropped down to the lower levels, and that the vertical scale of the forecast error correlation, for all variables, increased with the precipitation rate.

We obtained similar results for the spatial characteristics of ensemble forecast perturbations for the extratropical low and baiu cases. Thus, we concluded that the spatial scales of the CRM forecast error were very different between the precipitation-related variables and the other variables. This suggests that a simple spatial localization, using constant localization scales, is detrimental to the CRM.

2) INTERVARIABLE CORRELATION

We checked the cross correlation of the ensemble forecast perturbations between different variables. Figure 6a illustrates that the domain-averaged correlation had a negative correlation of around $-0.6$ between RTW and PT, and a positive correlation of around 0.2 between $w$ and RTW, at the same vertical levels. The ensemble forecast perturbation cross correlation,
however, changed substantially in terms of the precipitation rate. In precipitation-free areas (Fig. 6b), the intervariable cross correlation was negligible, except for that between RTW and PT, at the same vertical levels. In weak precipitation areas (Fig. 6c), the PR forecast error had a large positive correlation (>0.6) with the \( w \) forecast error at higher levels, while the intervariable cross correlation became small between RTW and PT. In heavy precipitation areas (Fig. 6d), the precipitation-related variables, and other variables, had significant forecast error cross correlations because dynamic and thermodynamic disturbances were induced in the areas associated with this heavy precipitation.

We obtained similar results for the intervariable correlation of the ensemble forecast perturbations for the extratropical low and baiu cases. Thus, we concluded that the intervariable characteristics of the CRM forecast errors depend heavily on the precipitation rate. We need to consider this dependency if we develop a variable localization scheme for the CRM.

3) SPECTRAL CHARACTERISTICS

To check the spectral characteristics of the ensemble forecast perturbations, first, we Fourier transformed local correlations of the ensemble forecast perturbations into wave space (hereafter referred to as the power

![Fig. 4. DIST of PR at a height of 2880 m calculated from the CRM ensemble forecasts (7 h after the initial time) by using the conventional method. Results are shown for (a) a typhoon (initial time: 1500 UTC 9 Jun 2004), (b) an extratropical low (initial time: 2100 UTC 26 Jan 2003), and (c) a baiu case (initial time: 0000 UTC 1 Jun 2004).](image)
Figure 7 shows the domain-averaged power spectrum of the ensemble forecast perturbation correlation of $w$ and $u$, at a height of 2880 m, for the typhoon case, as calculated by the conventional method (note that this figure emphasized the small amplitudes). The power spectrum for $w$ had a wide range of diagonal amplitudes, while the off-diagonal amplitudes were negligible. This pattern is close to that for the delta function and, therefore, is consistent with the narrow horizontal scale of the $w$ forecast error. On the other hand, the power spectrum of $u$ only had nonzero amplitudes for low wavenumbers ($<16$), including the off-diagonal modes. We think that these off-diagonal modes were mainly caused by horizontal variations of the forecast perturbation correlation, due to precipitation disturbances. It should be noted that, even for $u$, the power spectrum amplitudes were negligible for the off-diagonal modes with large wavenumber differences ($>5$).

The above results were valid for the ensemble forecast perturbation correlation for the target points classified with the precipitation rates, as well as for the other meteorological cases. Thus, we conclude that the CRM forecast error satisfies the assumption of the spectral localization: that the power spectrum amplitudes were negligible between modes with large wavenumber differences.

4. Dual-scale neighboring ensemble method
a. Neighboring ensemble method

Based on the spectral characteristics of the CRM forecast errors, we adopted the spectral localization, in the horizontal, in order to damp the sampling error. The spectral localization assumed that the power spectrum of the forecast error correlation between the physical variable with parameter index $p1$ and wavenumber $k1$, and that...
FIG. 6. The domain-averaged vertical cross correlation of forecast errors of PR, w, RTW, PT, u, and v at the 20 levels, as well as Ps, for the typhoon case, calculated from the CRM ensemble forecasts (7 h after the initial time) by using the conventional method. [In (a) and (b) the correlation with PR is masked.] The parameter indexes are the same as in Fig. 3a. (a) Results for the whole CRM region. Averages over points with PR (b) <0.01, (c) ~1–3, and (d) >10 mm h⁻¹.
with parameter index p2 and wavenumber i2, decreased with the wavenumber differences. Based on this assumption, we derived a spectral-localized power spectrum \( \tilde{C}(i_1, i_2; p_1, p_2) \) as follows:

\[
\tilde{C}(i_1, i_2; p_1, p_2) = \tilde{C}_e(i_1, i_2; p_1, p_2) \times \tilde{W}(i_1 - i_2),
\]

where \( \tilde{C}_e(i_1, i_2; p_1, p_2) \) is the power spectrum of the ensemble forecast perturbation correlation and \( \tilde{W}(i_1 - i_2) \) is the weight expressed as a function of the wavenumber difference, \((i_1 - i_2)\).

As Buehner and Charron (2007) discussed, Eq. (1) is equivalent to the following weighted moving average in the grid space:

\[
\text{Corr}(k_1, k_2; p_1, p_2) = \int \text{Corr}_e(k_1 + s, k_2 + s; p_1, p_2) W(s) \, ds,
\]

where \( \text{Corr}(k_1, k_2; p_1, p_2) \) is the forecast error correlation between \((k_1, p_1)\) and \((k_2, p_2)\), \( s \) is the movement from the target grid point \((k_1, k_2)\), and \( W(s) \) is the weight expressed as a function of the movement \( s \). In the present study, we chose the horizontal scale of the precipitation-related variables (15 km) as a scale of the movement \( D_s \), and adopted \( W(s) = 1/\sqrt{2\pi D_s} \exp[-(s/D_s)^2] \). The ensemble forecast perturbation correlation between \((k_1 + s, p_1)\) and \((k_2 + s, p_2)\) is \( \text{Corr}_e \), which is written as follows:

\[
\text{Corr}_e(k_1 + s, k_2 + s; p_1, p_2) = \frac{1}{N} \sum_{n=1}^{N} e_n(k_1 + s; p_1) e_n(k_2 + s; p_2),
\]

where \( N \) is the total ensemble number and \( e_n(k; p) \) is the forecast error of ensemble member \( n \) for the horizontal grid point \( k \) and the physical variable with the parameter index \( p \), which is normalized by the forecast error standard deviation at that point.

Using Eq. (2), we can increase the sample numbers in the forecast error correlation calculation, compared to the conventional method. We simplified this equation by setting \( W(s) \) to zero outside a neighboring block of the target points (hereafter referred to as the reduced-grid box). With this simplification, we can approximate Eq. (2) as the ensemble forecast perturbation correlation calculation using neighboring ensemble members within the reduced-grid box (hereafter referred to as the neighboring ensemble (NE)):

\[
\frac{1}{M} \sum_{m=1}^{M} W(s_m) \text{Corr}_e(k_1 + s_m, k_2 + s_m; p_1, p_2) = \frac{1}{NM} \sum_{m=1}^{M} \sum_{n=1}^{N} W(s_m) e_n(k_1 + s_m; p_1) e_n(k_2 + s_m; p_2) = \frac{1}{L} \sum_{l=1}^{L} e_{l1}(k_1; p_1) e_{l2}(k_2; p_2),
\]
where $M$ is the number of horizontal grid points within the reduce-grid box, $L = N \times M$,

$$
\varepsilon_l^i(k; p) = \sqrt{W(s_m)\varepsilon_n^i(k + s_m; p)} \quad \text{for} \quad l = (m - 1) \times N + n.
$$

(5)

Then, we can write the forecast error covariance using the above NE forecast perturbation correlation and standard deviation:

$$
P_{\text{NE}}^i(k_1, k_2; p_1, p_2) = \sigma(k_1; p_1)\text{Corr}(k_1, k_2; p_1, p_2)\sigma(k_2; p_2).
$$

(6)

Hereafter, we refer to this method as the NE method. As shown in Eq. (4), the NE method estimates the forecast error correlation at the target grid point using the ensemble forecast perturbations for grid points within the reduced-grid box. This increases the number of samples used for the forecast error correlation calculation, at the cost of a decrease in the spatial resolution of the correlation to that of the reduced-grid box. In the present study, we adopted a 5 × 5 grid as the reduced-grid box, which was comparable to the CRM effective spatial resolution.

b. Ensemble-based variational assimilation scheme

In this section, we briefly describe the EnVar scheme proposed by AE. From the observation $\mathbf{y}$, the mean $\mathbf{x'}$, and forecast perturbation covariance $\mathbf{P}_f$ of $N$-member ensemble forecasts $\mathbf{x}_f^i$, $n = 1, N$, AE derived the analysis of the ensemble mean $\mathbf{x}$ by minimizing the following three-dimensional cost function:

$$
J_a = \frac{1}{2}(\mathbf{x} - \mathbf{x'})^T(\mathbf{P}_f)^{-1}(\mathbf{x} - \mathbf{x'}) + \frac{1}{2}(\mathbf{y} - \mathbf{H}(\mathbf{x'}))^T\mathbf{R}^{-1}(\mathbf{y} - \mathbf{H}(\mathbf{x'})),
$$

(7)

where $\mathbf{H}$ and $\mathbf{R}$ denote the forward calculation operator and the observation error covariance. The localized forecast error covariance $\mathbf{P}_f^l$ is expressed as a Schur product (denoted by $\circ$) of $\mathbf{P}_f^i$ and a correlation matrix:

$$
\mathbf{S} : \mathbf{P}_f^l = \mathbf{P}_f^i \circ \mathbf{S}.
$$

(8)

AE performed horizontal spatial localization by adopting weights of the function of the horizontal distance for $\mathbf{S}$.

Following Lorenz (2003), AE assumed that the analysis increment $\mathbf{x} - \mathbf{x'}$ belonged to the subspace spanned by the ensemble forecast perturbations. Hence, the analysis increment can be written with ensemble forecast perturbations $\mathbf{e}_n^i = 1/\sqrt{N - 1}(\mathbf{x}_f^i - \mathbf{x'})$ and their amplitudes $\mathbf{a}_n$ (Wang, 2010) as follows:

$$
\mathbf{x} - \mathbf{x'} = \sum_{n=1}^{N} \mathbf{e}_n^i \circ \mathbf{a}_n = [\text{diag}(\mathbf{e}_1^i), \ldots, \text{diag}(\mathbf{e}_N^i)]\mathbf{a},
$$

(9)

where diag is an operator that turns a vector into a diagonal matrix and $\mathbf{a}$ is a vector formed by concatenating $N$ vectors $\mathbf{a}_n$, $n = 1, N$ (Wang, 2010).

Substituting Eq. (9) into Eq. (7), the cost function can be written in terms of $\mathbf{a}$:

$$
J_a = \frac{1}{2}\mathbf{a}^T\mathbf{A}^{-1}\mathbf{a} + \frac{1}{2}(\mathbf{y} - \mathbf{H}(\mathbf{a}))^T\mathbf{R}^{-1}(\mathbf{y} - \mathbf{H}(\mathbf{a})),
$$

(10)

where $\mathbf{A}$ is a correlation matrix for $\mathbf{a}$ and is expressed as a block diagonal matrix with $N$ identical submatrices $\mathbf{S}$. To calculate the optimal analysis, AE iteratively minimized Eq. (10).

c. Introducing the NE method to EnVar

In the present study, we introduced the NE method to the above EnVar scheme by adopting a horizontal correlation calculated from the NE forecast perturbations as the correlation matrix $\mathbf{S}$, and expressing the analysis increment subspace using the NE forecast perturbations.

To calculate $\mathbf{S}$, we developed a method for calculating the horizontal forecast perturbation correlation of the combination of different physical variables. Specifically, we adopted the singular value decomposition (SVD) eigenmodes of the vertical cross correlation of the NE forecast perturbations $\mathbf{C}_f'$ as the combination:

$$
\mathbf{C}_f' = \mathbf{U}\Gamma\mathbf{U}^T, \quad \mathbf{U} = (\mathbf{u}_1, \ldots, \mathbf{u}_p),
$$

(11)

where $\mathbf{u}$ is the eigenvector, $\Gamma$ is the eigenmatrix [a diagonal matrix whose diagonal elements $(\gamma_1, \ldots, \gamma_p)$ are the eigenvalues] of $\mathbf{C}_f'$, and $P$ is the total number of the physical variables at all vertical levels.

In this calculation, however, we need to consider that the precipitation-related variables had narrower horizontal forecast error scales than other variables. To address this, we horizontally divided the NE forecast perturbations, $\varepsilon_l^i(k; p)$, into large-scale portions, $\varepsilon_{lA}^i(k; p)$, and deviations, $\varepsilon_{ld}^i(k; p)$:

$$
\varepsilon_{lA}^i(k; p) = \Phi_A \varepsilon_l^i(k; p)
$$

$$
\varepsilon_{ld}^i(k; p) = \Phi_d \varepsilon_l^i(k; p),
$$

(12)
where $\Phi$ is the horizontal low-pass filter used to generate the 13 $\times$ 13 grid averages in the present study and $\Phi_d$ is the high-path filter used to generate the deviations. We chose this threshold value (13 grids $\sim$65 km) because it was an intermediate value between the horizontal error scales of the precipitation-related variables (15 km) and the other variables (100 km) described in section 3b. In the present study, we assumed that the precipitation-related variables had zero large-scale portions.

Then, we derived the SVD eigenmodes of the vertical cross correlation of the large-scale portions and the deviations. We adopted the horizontal correlation of the first eigenmodes as $S$ for both scales. Hereafter, we refer to this method as the dual-scale neighboring ensemble (DuNE) method.

If we assume that the analysis increment belongs to the subspace spanned by the $L$-member DuNE forecast perturbations, the EnVar computational load becomes prohibitive because the subspace dimension is too large. To avoid this, we developed an approximation method for reducing the dimension of the DuNE forecast subspace (Lermusiaux and Robinson 1999). In this method, we first calculated the SVD eigenmodes of the vertical cross covariance of the DuNE forecast perturbations $P_{DuNE}$:

$$P_{DuNE}^s = V^T \Lambda V = (v_1, \ldots, v_p),$$

where $v$ is the eigenvector and $\Lambda$ is the eigenvalue matrix $\Lambda = \begin{pmatrix} \lambda_1 & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \lambda_p \end{pmatrix}$.

Then, we approximated the ensemble forecast perturbations using the first–$N$th eigenvectors and eigenvalues:

$$e_n^l \sim \lambda_n v_n \quad n = 1, N.$$  

**d. Application of the DuNE method**

We applied the above DuNE method in order to calculate the forecast perturbation correlation of the large-scale portions, $\text{Corr}_A(k_1, k_2; p_1, p_2)$, and the deviations, $\text{Corr}_d(k_1, k_2; p_1, p_2)$. Then, we merged $\text{Corr}_A(k_1, k_2; p_1, p_2)$ and $\text{Corr}_d(k_1, k_2; p_1, p_2)$ into the forecast perturbation correlation of the physical variables in the following way:

$$\text{Corr}(k_1, k_2; p_1, p_2) = [\text{Corr}_A(k_1, k_2; p_1, p_2)\sigma_A(k_1; p_1)\sigma_A(k_2; p_2)$$

$$+ \text{Corr}_d(k_1, k_2; p_1, p_2)\sigma_d(k_1; p_1)\sigma_d(k_2; p_2)]/[\sigma_A(k_1; p_1)\sigma_A(k_2; p_2) + \sigma_d(k_1; p_1)\sigma_d(k_2; p_2)],$$

where $\sigma_A(k; p)$ and $\sigma_d(k; p)$ represent the large-scale and deviation modes of the NE forecast perturbation standard deviation, respectively.

First, we applied the DuNE method in order to estimate the forecast perturbation correlation at the grid points where the conventional method gave a large sampling error. Figures 8a and 8b illustrate the horizontal autocorrelation of $u$ and PR, respectively, at a height of 2880 m, at point A in Fig. 1, as calculated by the DuNE method. The PR correlation had no high-correlation (>0.2) points, except for at the target point. This indicates that the NE method significantly reduced the sampling error of the precipitation-related variables, as shown in Fig. 2b. The $u$ correlation pattern was similar to that shown in Fig. 2a. Figure 8c shows the vertical cross correlation of the variables at point A, as calculated by the DuNE method. We found that the PR had a positive correlation with the $w$ at higher levels. This pattern is close to that of the domain average over the weak precipitation areas (Fig. 6c).

This suggests that the DuNE method was successful in estimating a realistic vertical cross correlation between the precipitation-related variables. The vertical cross correlations between the other variables were similar to that shown in Fig. 3a.

Next, we calculated DIST in order to check how the DuNE method damped the sampling error. Figures 9a–c illustrate the DuNE-calculated DIST of the precipitation rates, at 2880 m, for the target cases. A comparison with Fig. 4 indicates that the DuNE method reduced DIST to lower than 40 km for the most points, except for in the northern parts of the extratropical low. Figure 9d illustrates the DuNE horizontal autocorrelation of PR, at a height of 2880 m, at point C in Fig. 9b, in the northern part of the extratropical low. Figure 9d shows a plausible horizontal correlation pattern that decreases gradually with distance from the peak at the target point. Hence, we concluded that the large DIST observed in this region was generated as a result of the authentic horizontal forecast error correlation of precipitation, not by the sampling error. The above results indicate that the DuNE method damped the horizontal sampling error noise for the precipitation-related variables, while still retaining the authentic horizontal forecast error correlation.
To validate the DuNE method, we assumed that the ensemble forecast perturbation calculated by the conventional method was close to the true value at grid points with a small sampling error, and compared the DuNE-calculated and ensemble forecast perturbation results at points with small DIST, such as point B in Fig. 1a. As a result, we found negligible differences in standard deviations between the DuNE-calculated and ensemble forecast perturbations (figure not shown). Both the conventional and DuNE methods yielded

![Fig. 8. (a) As in Fig. 2a, but calculated by using the DuNE method. (b) As in Fig. 2b, but for results calculated by using the DuNE method. (c) As Fig. 3a, but for results calculated by using the DuNE method.](image)
similar “delta function” patterns for the horizontal autocorrelation of PR (figure not shown) at point B. Figures 10a and 10b show the vertical cross correlations of PR, $w$, and RTW, at point B, as calculated by the conventional and DuNE methods, respectively. Figures 10a,b show that the DuNE-calculated correlation agreed well with the ensemble correlation, while the peak cross correlation decreased by approximately 0.1.
The above results indicate that quantitative changes were negligible between the DuNE-calculated and ensemble forecast perturbations.

5. Assimilation experiments

To compare the DuNE method with the conventional sampling error damping methods, we performed experiments assimilating single-observation data into the EnVar schemes using these methods.

a. Experimental procedures

In the experiments, we assimilated PR or $u$ at point A, at a height of 2880 m for the typhoon case (2200 UTC 9 June 2004). We made “the observation” of PR ($u$) by adding an increment of 1 mm h$^{-1}$ (1 m s$^{-1}$) to the mean of the ensemble forecasts after 7 h of the initial time (1500 UTC 9 June 2004). We also prescribed the STD of the observation error as 2.7 mm h$^{-1}$ (2.7 m s$^{-1}$). We developed EnVar schemes using a spatial localization method, a variable localization method, and the DuNE method. Hereafter, we refer to these schemes as SL, VL, and DN, respectively. In SL, we approximated each element of the correlation matrix $S$, $s(k1, k2)$, as a Gauss function of distances:

$$s(k1, k2) = \exp[-(d_h/D_h)^2 - (d_v/D_v)^2],$$  \hspace{1cm} (16)

where $d_h$ and $d_v$ are the horizontal and vertical distances between points k1 and k2, respectively, and $D_h$ and $D_v$ are the horizontal and vertical localization lengths, respectively. We prescribed the localization lengths as the domain average of the forecast error scales of $u$ for the typhoon case ($D_h = 250$ km, $D_v = 1$ km). In VL, we set horizontally narrow, vertically wide localization lengths ($D_h = 50$ km, $D_v = 6$ km) for precipitation-related variables, neglecting forecast error correlation between these variables and others whose localization lengths were the same as SL. DN employed the DuNE method described in section 4c. DN expressed the analysis increment subspace using the NE forecast perturbations, while SL and VL assumed the analysis increment subspace spanned by the ensemble forecast perturbations.

b. Results

Table 1 shows analysis increments and STDs of the ensemble forecast perturbations of PR and $u$ at point A at a height of 2880 m, as calculated by SL, VL, and DN. This indicates that DN made a larger PR analysis increment and a larger ensemble forecast perturbation.
FIG. 11. Analysis increments calculated from a single observation at point A at a height of 2880 m. (Increments were normalized by the analysis increments at point A at a height of 2880 m.) (a) Horizontal distribution of the normalized analysis increments at a height of 2880 m, generated by SL. Shading (contours) denotes PR (u) analysis increments calculated from the PR (u) single observations. (b) Vertical profiles of the normalized analysis increments of PR (red broken line) and u (black solid line) at point A, generated by SL. (c) As in (a), but for results generated by VL. (d) As in (b), but for result generated by VL. (e) As in (a), but for results generated by DN. (f) As in (b), but for results generated by DN.
STD than SL and VL, while $u$ analysis increments were similar. This is because the conventional methods generated noisy patterns of PR ensemble forecast perturbation STDs due to the sampling error.

Figure 11a illustrates the horizontal analysis increments at a height of 2880 m, generated by SL. Shading (contours) denotes PR ($u$) analysis increments calculated from the PR ($u$) single observation. Both increments were normalized by the analysis increments at point A at a height of 2880 m. Figure 11a shows that the sampling error caused noisy PR analysis increments over a wide range. On the other hand, the $u$ analysis increments followed a smooth pattern. Figure 11b illustrates vertical profiles of the normalized analysis increments of PR (red broken line) and $u$ (black solid line) at point A, generated by SL. The vertical profile of the PR analysis increments was similar to that of $u$, and much shallower than the vertical forecast error correlation of the precipitation-related variables shown in sections 3 and 4. The results indicate that the simple spatial localization using constant localization lengths was not appropriate for the CRM.

Figures 11c and 11d illustrate normalized analysis increments of PR and $u$, generated by VL. The narrow horizontal localization in VL eliminated the erroneous pattern of the PR analysis increments that appeared in SL. While the wide vertical localization in VL generated a deeper profile of the PR analysis increments, it also made an unrealistic upper-level peak at around 7 km because of the sampling error. VL generated horizontal and vertical $u$ analysis increments similar to those of SL.

Figures 11e and 11f illustrate normalized analysis increments of PR and $u$, generated by DN. In the

| TABLE 1. Analysis increments and STDs of the ensemble forecast perturbations of PR and $u$ at point A at a height of 2880 m, calculated by SL, VL, and DN. |
|------------------------|------------------------|------------------------|------------------------|
| Analysis increment (PR, mm h$^{-1}$) | Analysis increment ($u$, m s$^{-1}$) | Ensemble forecast perturbations STD (PR, mm h$^{-1}$) | Ensemble forecast perturbations STD ($u$, m s$^{-1}$) |
| SL | 0.151 | 0.391 | 1.176 | 2.356 |
| VL | 0.152 | 0.391 | 1.176 | 2.356 |
| DN | 0.413 | 0.387 | 2.267 | 2.363 |
horizontal, DN confined the PR analysis increments within 50 km of point A. In the vertical, DN generated a deep profile of the PR analysis increments, and damped the upper-level peak that appeared in VL. We think that this vertical profile is plausible because it is consistent with the vertical forecast error correlation of the precipitation-related variables.

Based on the above results, we consider the DuNE method to have an advantage over the conventional sampling error damping methods in terms of damping the sampling error and generating the deep vertical profile for the PR analysis increments. The conventional sampling error damping methods could not generate plausible vertical profiles because they needed narrow vertical localization lengths in order to damp the sampling error of the precipitation-related variables.

6. Summary and discussion

a. Summary

The purpose of the present study was twofold:
1) to examine the sampling error properties and the forecast error characteristics of the JMA operational CRM and
2) to develop the CRM sampling error damping method based on the CRM forecast error characteristics.

To achieve the first aim, we analyzed the CRM ensemble forecast perturbations for precipitation disturbance cases, using the 100-member CRM ensemble forecasts. We found that significant noise was present in the horizontal and vertical correlations and the STDs of ensemble forecast perturbation for the precipitation-related variables, due to the sampling error, in areas where other variables had plausible correlations. We also found that the precipitation-related variables were likely to suffer from the effects of this sampling error in most precipitation areas, with the exception of heavy precipitation cores in the central areas. An examination of the forecast error characteristics revealed the following:

1) the horizontal forecast error autocorrelation patterns were narrow for the precipitation-related variables (~15 km), while they changed (160–40 km), in terms of the precipitation rate, for other variables;
2) the forecast error cross correlation between CRM variables changed substantially in terms of the precipitation rate; and
3) when we transformed the horizontal forecast error autocorrelation into wave space, the correlation matrix had negligible values for the off-diagonal elements with large wavenumber differences.

To achieve the second aim we first developed the NE method, which is based on the assumption of the spectral localization. The NE method estimated the forecast error correlation at target grid points using the ensemble forecast perturbations for grid points within a reduced-grid box. To introduce this method into the EnVar scheme, we horizontally divided the NE forecast perturbations into large-scale portions and deviations (DuNE method). As the single-observation assimilation experiments showed, the DuNE method was more successful in damping the sampling error and generating plausible, deep vertical profiles for the PR analysis increments, compared to the simple spatial localization method or the variable localization method.

b. Discussion

1) FEATURES, APPLICABILITY, AND LIMITATION OF THE DUNE METHOD

The main characteristic of the DuNE method is that it obtains a large sample number (here 2500) by using the NE within a reduced-grid box. The assimilation experiment results indicated that this contributed to a dampening of the sampling error and generated deep vertical analysis increments for the precipitation-related variables simultaneously. On the other hand, the conventional sampling error damping methods needed narrow vertical localization lengths in order to damp the sampling error of these variables. It should also be noted that the DuNE method performed no spatial localization in the vertical.

The DuNE method is similar to the “spatial/spectral covariance localization” (Buehner 2012) and “scale-dependent covariance localization” (Buehner and Shlyaeva 2015) methods in terms of its use of the spectral localization and scale separation of ensemble forecast perturbations. The DuNE method, however, has simpler procedures than those methods, as it uses the NE forecast perturbations instead of transforming the forecast error into the spectral space. Additionally, the DuNE method only used two scales for the CRM forecast error: the large scale and the deviation that was dominated by the precipitation-related variables, based on the CRM forecast error analysis.

Whether the DuNE method is applicable to other CRMs depends on whether the CRM satisfies the forecast error characteristics required (the spectral characteristics, difference in horizontal forecast error scales between precipitation-related and other variables, and significant intervariable cross correlation in precipitation areas). We checked the CRM forecast errors detailed in previous studies and found that many of these CRMs seemed to satisfy the above conditions. For
example, Pannekoucke et al. (2014) filtered the ensemble forecast perturbation of the Application de la Recherche à l’Opérationnel à Meso-Echelle (AROME-France) model with the use of a simple spatial averaging method (a simplified NE method) for the background error covariance modeling. Michel et al. (2011) reported the narrow horizontal forecast error correlation scales of their CRM precipitation-related variables and intervariable cross correlation in rainy areas. Therefore, we inferred that the DuNE method is applicable to many CRMs; needless to say, it is imperative to examine the forecast error characteristics of the target CRM beforehand.

Sampling errors are more serious for precipitation-related variables, compared to other variables, as not all ensemble members have precipitation at the target point. To address this, in the present study, we employed the DuNE method and obtained 2500 samples at each reduced-grid box. In reality, however, large-scale displacement errors can occur for precipitation forecasts, as AE pointed out. In such cases, we cannot expect to retrieve a large enough number of precipitating samples in the observed precipitation areas by using the DuNE method alone. To overcome this limitation, we need to combine the DuNE method with other methods, such as the displacement error correction method proposed by AE, when we construct operational data assimilation systems.

2) FUTURE DIRECTIONS

In the present study, we applied the DuNE method to a selection of precipitation disturbance cases, in order to check its performance. So far, the results seem to be very encouraging. We executed observing system simulation experiments (OSSEs) to incorporate MWI TBs into the EnVar using the DuNE method (Aonashi et al. 2014). Based on the OSSE results, we are also planning to conduct future assimilation experiments using real observation data.

Conversely, further examination of the NE forecast perturbations is required, in particular, over land, because the NE forecast perturbations may be influenced by topographic inhomogeneity within the reduced-grid boxes. For this reason, we are planning to employ a 1000-member ensemble forecast (Kunii 2014) that will enable us to analyze the forecast error at each grid point, using sampling numbers similar to those of the DuNE method detailed in the present study. We also would like to use this 1000-member ensemble forecast in order to study the variations of the horizontal forecast error scales of the stratiform precipitation outlined in section 4d.

We are also planning to improve the DuNE method by allowing cross correlation between the large scale and the deviation, similar to Buehner and Shlyaeva (2015). As they pointed out, this can avoid the loss of information on ensemble forecast error inhomogeneity.

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