Evaluating Implementations of the Immersed Boundary Method in the Weather Research and Forecasting Model

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ABSTRACT

The terrain-following coordinate system used by many atmospheric models can cause numerical instabilities due to discretization errors as resolved terrain slopes increase and the grid becomes highly skewed. The immersed boundary (IB) method, which does not require the grid to conform to the terrain, has been shown to alleviate these errors, and has been used successfully for high-resolution atmospheric simulations over steep terrain, including vertical building surfaces. Since many previous applications of IB methods to atmospheric models have used very fine grid resolution (5 m or less), the present study seeks to evaluate IB method performance over a range of grid resolutions and aspect ratios. Two classes of IB algorithms, velocity reconstruction and shear stress reconstruction, are tested within the common framework of the Weather Research and Forecasting (WRF) Model. Performance is evaluated in two test cases, one with flat terrain and the other with the topography of Askervein Hill, both under neutrally stratified conditions. WRF-IB results are compared to similarity theory, observations, and native WRF results. Despite sensitivity to the location at which the IB intersects the model grid, the velocity reconstruction IB method shows consistent performance when used with a hybrid RANS/LES surface scheme. The shear stress reconstruction IB method is not sensitive to the grid intersection, but is less consistent and near-surface velocity errors can occur at coarse resolutions. This study represents an initial investigation of IB method variability across grid resolutions in WRF. Future work will focus on improving IB method performance at intermediate to coarse resolutions.

1. Introduction

a. Background

As computing power increases, atmospheric models can be used at correspondingly finer resolutions. While increased resolution allows for improved representation of small-scale physical processes, errors can occur when steep terrain slopes are resolved. Many atmospheric models, including the commonly used Weather Research

and Forecasting (WRF) Model (Skamarock et al. 2008), employ terrain-following vertical coordinates, originally proposed by Gal-Chen and Somerville (1975). However, models that use terrain-following coordinates are prone to errors over steep slopes where grid cells can become overly skewed (e.g., Janjić 1977; Mahrer 1984; Schär et al. 2002; Klemp et al. 2003; Zängl 2002, 2003; Zängl et al. 2004; Lundquist et al. 2008, 2010b; Klemp 2011). Terrain slope-related errors reduce simulation accuracy and may prevent atmospheric models from running in regions of steep or complex terrain, thus limiting the study of flow interaction with important topographic features such as canyons and mountains.

Alternatives to terrain-following coordinates have been used to alleviate errors related to the terrain slope,

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allowing atmospheric models to work for arbitrarily complex terrain. One such alternative that has gained traction in the literature is the immersed boundary (IB) method, which was first used by [Peskin \(1972\)](#) to simulate cardiac mechanics and blood flow, and has since been applied to a variety of computational fluid dynamics applications [see the review of [Mittal and Iaccarino \(2005\)](#), and references therein]. When the IB method is used, the vertical coordinate is not required to conform to the terrain. Rather, the terrain is represented by an immersed surface that intersects the model grid arbitrarily. Boundary conditions are then applied along the immersed surface by modifying the governing equations in the boundary region.

IB methods have recently been used to study environmental flows ([Tseng and Ferziger 2003](#); [Senocak et al. 2004](#); [Chester et al. 2007](#); [Jafari et al. 2011](#); [Lundquist et al. 2010a, 2012](#); [Diebold et al. 2013](#); [Ma and Liu 2017](#); [DeLeon et al. 2018](#); [Bao et al. 2016, 2018](#); [Arthur et al. 2018](#); [Wiersema et al. 2018, 2020](#)); however, several of these studies ([Tseng and Ferziger 2003](#); [Lundquist et al. 2010a, 2012](#); [Arthur et al. 2018](#)) employed a ghost-cell approach with a no-slip bottom boundary condition for the velocity that has limited applicability to realistic atmospheric boundary layer (ABL) simulations. Due to the presence of unresolved surface roughness elements, ABL simulations typically parameterize the turbulent momentum flux (also known as the turbulent stress) at the surface using Monin–Obukhov similarity theory (MOST; [Monin and Obukhov 1954](#)). Although MOST is strictly valid for flow over flat and homogeneous terrain, it is widely applied in atmospheric models over a variety of terrain surfaces due to the lack of an accepted alternative.

To enable more realistic ABL simulation, IB methods have been adapted to include surface stress parameterizations, focusing on similarity theory. The first simulations of this kind were performed by [Senocak et al. \(2004\)](#), who studied neutrally stratified ABL flow over flat terrain with geostrophic forcing. IB methods based on similarity theory have since been extended to more general three-dimensional implementations, following two approaches. In the first, building on the work of [Senocak et al. \(2004\)](#), the velocity just above the terrain surface is reconstructed to fit a profile given by similarity theory ([Bao et al. 2018](#); [DeLeon et al. 2018](#)). In the second, the shear stress in the vicinity of the immersed surface is reconstructed using similarity theory ([Chester et al. 2007](#); [Diebold et al. 2013](#); [Ma and Liu 2017](#)). These approaches will be referred to here as velocity reconstruction and shear stress reconstruction (hereafter VR-IB and SR-IB) methods, respectively, and are discussed in further detail in [section 2](#).

Both of these methods have been validated and performed well for ABL large-eddy simulation (LES) cases with very fine grid resolution. For example, [Diebold et al. \(2013\)](#) and [Ma and Liu \(2017\)](#) used SR-IB to simulate flow over Bolund Hill, the site of a field and model intercomparison study ([Berg et al. 2011](#); [Bechmann et al. 2011](#)), with horizontal grid spacings of 2 m or less and vertical grid spacings of 1 m or less. [Bao et al. \(2018\)](#) used VR-IB to simulate flow over Bolund Hill at similar resolution, while [DeLeon et al. \(2018\)](#) used VR-IB with a slightly coarser horizontal resolution of 2–4 m. These studies have also used VR-IB to simulate flow over Askervein Hill, the site of a field campaign ([Taylor and Teunissen 1987](#)) with larger-scale topography than Bolund Hill. [Bao et al. \(2018\)](#) used a horizontal grid spacing of 5 m and a vertical grid spacing of 1 m near the surface, while [DeLeon et al. \(2018\)](#) used grid spacing ranging from roughly 10–25 m in the horizontal and 4–8 m in the vertical for this case.

Testing of IB methods for ABL applications at resolutions coarser than those in the aforementioned studies is limited. Although [Bao et al. \(2018\)](#) demonstrated good performance of VR-IB for flat terrain with 30 m horizontal grid spacing, such coarse resolution was not used for a case with more complex terrain. Additionally, [Phillips et al. \(2017\)](#) used VR-IB to study flow over complex terrain with horizontal and vertical grid spacings of approximately 30 and 4 m, respectively, for the purpose of improving the rating of power lines. However, there is evidence of poorer performance when coarser grids are used with IB methods (see discussion in [Chow et al. 2019](#)). For example, [Bao et al. \(2016\)](#) compared both the VR- and SR-IB methods to a standard WRF simulation over an idealized valley using 90 m horizontal grid spacing and found significant disagreement. To demonstrate this increased disagreement as resolution coarsens, results from simulations of flow over an idealized hill using both native WRF and the IB algorithms with 100 m horizontal grid spacing are shown in [Fig. 1](#). Large differences in predicted velocity profiles are seen between native WRF and the IB methods, especially in the lee of the hill, as will be discussed further below ([section 2d](#)).

Although the VR- and SR-IB methods have both demonstrated great promise for ABL applications, further testing is needed at the coarser resolutions used in many atmospheric studies. This testing is especially necessary if IB methods are to be used in the ABL “gray zone” or terra incognita, with horizontal grid spacings of roughly 10 m to 1 km (depending on the flow conditions), where neither LES nor planetary boundary layer (PBL) turbulence models are appropriate (see [Wyngaard 2004](#)). Resolved terrain slopes can still be large enough to

FIG. 1. Time-averaged x -direction velocity $\langle u_1 \rangle$ profiles for LES simulations of flow over a three-dimensional hill. Four cases are shown using native WRF, VR-IB, VR-IB with hybrid RANS/LES, and SR-IB boundary conditions. Profiles are shown along an x - z cross section through the terrain peak at $y = 3$ km for the (a) windward and (b) leeward sides of the hill. Note that the profile at the hill peak is shown in both (a) and (b).

degrade model performance when using terrain-following coordinates within the gray zone; it is therefore useful to understand the performance of IB methods at such resolutions.

One means of improving model performance at coarse-LES or gray-zone resolution is the hybrid Reynolds averaged Navier–Stokes (RANS)/LES approach of Senocak et al. (2007). In this approach, Prandtl’s mixing length model is applied near the surface, where the model grid is too coarse to adequately resolve turbulence in the LES sense, and then blended with an LES subgrid model aloft. The hybrid RANS/LES scheme has been used to improve VR-IB performance (Senocak et al. 2004; Phillips et al. 2017; DeLeon et al. 2018), but has not yet

been tested in WRF or with SR-IB, and will therefore be explored further in this work.

Because previous IB method studies have been performed using a variety of models, it is unclear if differences in performance are attributable to differences in models, IB method implementations, or both. Here, the VR- and SR-IB methods are tested in WRF, which provides a common platform for IB method implementation. As a widely used model for ABL applications, native WRF also provides a performance baseline against which the IB method can be compared for mild terrain slopes. In this work, the accuracy of the VR- and SR-IB methods is assessed at resolutions coarser than those in most previous studies (i.e., $\Delta x \geq 25$ m), where

the sensitivities of the methods become more evident. Several important model features are examined, including the location at which the immersed surface intersects the model grid, the grid resolution and aspect ratio, and the near-surface turbulence model. Sensitivity studies are first performed for an idealized neutral atmospheric boundary layer. The methods are then tested in the Askervein Hill case, for which model performance is also compared to observations.

b. Model framework

The VR- and SR-IB methods used here have been added into WRF version 3.6.1 within the framework of Lundquist et al. (2010a, 2012), with an overarching goal of incorporating IB methods into WRF's existing multiscale modeling system. Lundquist et al. (2010a) originally implemented the IB method into WRF in two dimensions with a Dirichlet no-slip boundary condition for the velocity, as well as coupling to WRF's MM5 surface layer module, Noah land surface module, RRTM longwave radiation module, and Dudhia shortwave radiation module. Lundquist et al. (2012) extended the implementation to three dimensions, performing large-eddy simulations using WRF's Smagorinsky turbulence model. Arthur et al. (2018) also coupled the IB method to WRF's topographic shading algorithm in order to simulate thermally driven flows over the complex terrain of Granite Mountain, Utah, during the MATERHORN field campaign (Fernando et al. 2015). It should be noted that Arthur et al. (2018) used a Dirichlet no-slip boundary condition and the Granite Mountain simulation was therefore presented as a semi-idealized demonstration case.

Bao et al. (2016) first implemented both the VR- and SR-IB methods in WRF and tested them under idealized conditions, while Bao et al. (2018) performed additional validation of VR-IB in WRF over Askervein Hill and Bolund Hill. Further development of the WRF-IB framework has included the implementation of a vertical grid refinement capability for nested domains (Daniels et al. 2016; Mirocha and Lundquist 2017), which is necessary for performing multidomain simulations spanning a large range of resolutions. Wiersema et al. (2018, 2020) developed a method for nesting IB domains within domains using the native terrain-following coordinate. They performed a multiscale simulation of flow around resolved buildings with vertical surfaces in downtown Oklahoma City using VR-IB, with nested domains ranging in horizontal grid spacing from roughly 6 km to 2 m. Model results were compared to observations from the Joint Urban 2003 field campaign (Allwine and Flaherty 2006).

2. Overview of surface boundary conditions

a. Standard implementation using terrain-following coordinates

Following MOST, atmospheric models typically parameterize the turbulent stress at the surface as

$$\tau_{i3,S} = -\rho C_D u_i(z_r) V_{\text{horz}}(z_r), \quad (1)$$

where S denotes the surface, ρ is the air density, u_i is the i th velocity component ($i = 1, 2$), $V_{\text{horz}} = [(u_1)^2 + (u_2)^2]^{1/2}$ is the horizontal wind speed, and z_r is a reference height near the surface. The exchange coefficient for momentum is defined as

$$C_D = \left[\frac{\kappa}{\ln\left(\frac{z_r - h}{z_0}\right) - \Psi\left(\frac{z - h}{L}\right)} \right]^2, \quad (2)$$

where $\kappa = 0.4$ is the von Kármán constant, h is the terrain height, z_0 is the surface roughness height, Ψ is the stability function, and L is the Obukhov length. This work focuses on cases with neutral stability, for which $\Psi = 0$ and Eqs. (1) and (2) and are commonly known as the logarithmic (or “log”) law. The standard WRF Model uses Arakawa C grid staggering with terrain-following coordinates, with $\tau_{i3,S}$ defined explicitly at a grid point on the terrain surface (Fig. 2a) to be used in the surface boundary condition for the velocity. In WRF's standard terrain-following, pressure-based coordinate system, the reference height $z_r = z(k = 1)$ is the height of the first velocity grid point, located one-half grid level above the surface. For this reason, the value of z_r can vary slightly as a function of x and y due to variation in the terrain or pressure fluctuations (even over flat terrain).

Vertical velocity u_3 is also defined explicitly on the terrain surface of WRF's staggered grid. Its value is therefore set to ensure no flow through the boundary using a kinematic condition,

$$u_{3,S} = u_{i,S} \frac{\partial h}{\partial x_i}, \quad (3)$$

where $i = 1, 2$ with summation over repeated indices and $u_{i,S}$ is extrapolated to the surface using a second-order Lagrange polynomial.

When a nonconforming grid is used in WRF with an IB method, grid points do not typically fall on the terrain surface, preventing use of this standard implementation of the surface boundary condition. The boundary conditions for velocity and/or turbulent stress must therefore be specified by other means.

FIG. 2. Two-dimensional (x - z) depiction of the surface boundary condition for (a) native WRF, (b) VR-IB, and (c) SR-IB. The solid black line denotes the terrain surface. Solid and dashed gray lines denote the staggered grid, with cell interfaces shown as solid lines and half-levels shown as dashed. Blue circles denote reconstruction points (RPs) or extrapolation points (EPs) at which a value must be specified by the given boundary condition. Blue squares denote interpolation points (IPs) that are used in the boundary condition calculation. Solid blue lines denote the surface-normal direction. In (c), the dotted blue line denotes the distance above the boundary d_r at which the surface-tangential velocity U_{IP}^t is calculated.

b. Immersed boundary method with logarithmic-law velocity reconstruction

The present VR-IB implementation follows the concepts presented in Senocak et al. (2004), DeLeon et al. (2018), and Bao et al. (2018). When VR-IB is used, a logarithmic velocity profile is assumed for the surface-tangential component of the velocity,

$$U^t = \frac{u_*}{\kappa} \ln\left(\frac{d_r}{z_0}\right), \quad (4)$$

where u_* is the friction velocity and d_r is the surface-normal distance to some reference point. As depicted in Fig. 2b, velocity reconstruction points (RPs) are chosen as the first velocity grid points above the boundary. The velocity is then found at an interpolation point (IP), which is located at the point of intersection between a surface-normal line drawn through the RP and the face of an adjacent computational cell that is not intersected by the IB. To calculate the velocity at the IP, interpolation is performed using the inverse distance weighted (IDW) scheme proposed by Franke (1982) and used with the IB method in Gao et al. (2007), Lundquist et al. (2010a, 2012), and Bao et al. (2018). Values at surrounding computational nodes within the fluid domain are used in the interpolation. A point coincident with the IB with $u_i = 0$ is also included in the IDW interpolation. This was found to improve performance for cases of flow over topography by yielding a more accurate velocity from the interpolation scheme.

Once the velocity at the IP is obtained, it is decomposed into components that are tangential and normal to the immersed surface. The tangential velocity at the RP is then calculated based on Eq. (4) using

$$U_{RP}^t = U_{IP}^t \frac{\ln\left(\frac{d_{RP}}{z_0}\right)}{\ln\left(\frac{d_{IP}}{z_0}\right)}, \quad (5)$$

where d_{RP} and d_{IP} are the surface-normal distances to the RP and IP, respectively. Equation (5) assumes that both the RP and IP lie within the logarithmic region of the flow and that u_* is constant within this region. The surface-normal velocity (U^n) at the RPs is calculated to ensure no flow through the IB with

$$U_{RP}^n = U_{IP}^n \frac{d_{RP}}{d_{IP}}. \quad (6)$$

After U_{RP}^t and U_{RP}^n are found, they are rotated into the grid coordinate to be used in the model solution. Following Bao et al. (2018), when the RP is within a surface-normal distance z_0 of the boundary, the tangential velocity U_{RP}^t is set to 0. Note that in other studies (e.g., Senocak et al. 2004; DeLeon et al. 2018) the bottom boundary is placed at the height of the roughness length z_0 .

Even though the velocity is reconstructed above the boundary, velocities beneath the boundary can still be used in the calculation of the resolved strain-rate tensor

$S_{ij} = (1/2)[(\partial u_i/\partial x_j) + (\partial u_j/\partial x_i)]$, which is used in the calculation of the eddy viscosity,

$$\nu_i = l_\Delta^2 |S_{ij}|, \quad (7)$$

where $l_\Delta = c_s \Delta$ with $c_s = 0.18$ as the Smagorinsky coefficient and $\Delta = (\Delta x \Delta y \Delta z)^{1/3}$ as the grid length scale. Since this can inadvertently affect the near-surface value of the eddy viscosity, the eddy viscosity should also be reconstructed using

$$\nu_{i,RP} = \nu_{i,IP} \frac{d_{RP}}{d_{IP}}, \quad (8)$$

which [as in Eq. (6) for the surface-normal velocity U^m] assumes a linear eddy viscosity profile in the surface-normal direction and $\nu_i = 0 \text{ m}^2 \text{ s}^{-1}$ at the surface (Senocak et al. 2004; DeLeon et al. 2018). While Bao et al. (2018) did not apply eddy-viscosity reconstruction in their original WRF implementation of VR-IB, the present VR-IB implementation applies this reconstruction.

A hybrid RANS/LES scheme proposed by Senocak et al. (2007) has been used with VR-IB (Senocak et al. 2004; Phillips et al. 2017; DeLeon et al. 2018), and is therefore included as an option here as well. The hybrid scheme works by blending the grid length scale in Eq. (7) with the Prandtl mixing length near the surface, assuming the near-surface grid is too coarse to adequately resolve turbulence for LES. Thus, l_Δ^2 in Eq. (7) is replaced by

$$l_{\text{mix}}^2 = \left[1 - \exp\left(-\frac{z-h}{h_B}\right) \right]^2 (c_s \Delta)^2 + \exp\left(-\frac{z-h}{h_B}\right) [\kappa(z-h)]^2. \quad (9)$$

The blending height h_B must be chosen, and it is recommended that $h_B/2\Delta > 1$ (Senocak et al. 2007).

c. Immersed boundary method with logarithmic-law shear stress reconstruction

The SR-IB method is implemented here following Chester et al. (2007) and Ma and Liu (2017). When SR-IB is used, the model stresses are reconstructed at the first grid points above the IB and are additionally extrapolated to the first grid points beneath the IB (see Fig. 2c). All velocities are set to zero beneath the terrain, but unlike in VR-IB, velocity values are not modified above the terrain.

Because different components of the stress tensor τ_{ij} are calculated at different locations on WRF's staggered grid, the stress reconstruction/extrapolation procedure

must be performed individually for each i, j pair. First, a reconstruction region is specified within $d_r = 1.2\Delta z$ above the surface. For each grid point within this region at which the stress is calculated (denoted $\tau_{ij,RP}$), a stress tensor $\tau_{ij,S}$ is constructed in a surface-rotated coordinate system. The surface-tangential stresses $\tau_{i3,S}$ are estimated following Eq. (1) using the surface-tangential velocities $u_{i,IP}^t$ at a surface-normal distance d_r from the boundary,

$$\tau_{i3,S} = -\rho \left[\frac{\kappa}{\ln\left(\frac{d_r}{z_0}\right)} \right]^2 u_{i,IP}^t U_{IP}^t, \quad (10)$$

where $U_{IP}^t = [(u_{i,IP}^t)^2]^{1/2}$ and $i = 1, 2$. Additionally, as $\tau_{ij,S}$ is constructed, $\tau_{31,S} = \tau_{13,S}$ and $\tau_{32,S} = \tau_{23,S}$, while all other i, j components are assumed to be zero (see Chester et al. 2007). The stress value at the reconstruction point $\tau_{ij,RP}$ is found by rotating $\tau_{ij,S}$ into the grid coordinate.

Next, the stress is extrapolated to the first grid points beneath the terrain $\tau_{ij,EP}$. For this procedure, the stress is found at three interpolation points denoted $\tau_{ij,IPm}$, with $m = 1, 2, 3$, at $d_{Em} = 1.1m\Delta z$ away from the extrapolation point in the surface-normal direction. If the first interpolation point is beneath the terrain, then d_{Em} is extended slightly as in Ma and Liu (2017). A second-order Lagrange polynomial is then used to extrapolate the stress,

$$\tau_{ij,EP} = 3\tau_{ij,IP1} - 3\tau_{ij,IP2} + \tau_{ij,IP3}. \quad (11)$$

Note that the values of d_r and d_{Em} are chosen following the WRF-based study of Ma and Liu (2017), and differ slightly from the values used by Chester et al. (2007). Additionally, while Chester et al. (2007) and Ma and Liu (2017) used a linear extrapolation function that requires only two interpolation points (i.e., $\tau_{ij,EP} = 2\tau_{ij,IP1} - \tau_{ij,IP2}$), a quadratic function was used here.

d. Illustrative comparison of immersed boundary method algorithms

To illustrate the differences between IB method implementations, example simulations of flow over an idealized hill are conducted using a horizontal grid spacing of 100 m, which is larger than that used in previous IB method studies. Several simulations are completed using WRF with a terrain-following grid, VR-IB with a nonconforming grid, and SR-IB with a nonconforming grid. VR-IB is also used in combination with the hybrid RANS/LES scheme of Senocak et al. (2007), as in DeLeon et al. (2018). At this resolution, the IB algorithm and the near-surface turbulence closure both affect the solution, especially in the lee of the hill (Fig. 1).

TABLE 1. Simulation details for idealized-hill and flat-terrain cases, including the domain size L , the number of grid points N , and the grid spacing. The vertical grid spacing Δz is constant up to $z_{\Delta z}$, above which it is stretched by a factor r until $\Delta z \approx 100$ m, above which it remains constant. Two numbers are reported for L_z and N_z (terrain-following and IB), which are larger for IB cases to allow for two grid cells beneath the terrain. Rayleigh damping is applied over the top z_{damp} of the domain.

Case	(L_x, L_y, L_z) (km)	(N_x, N_y, N_z)	$\Delta x = \Delta y$ (m)	Δz (m)	$z_{\Delta z}$ (m)	r	z_{damp} (km)
Idealized hill	(6, 6, 4/4.004)	(60, 60, 75/77)	100	20	450	1.05	2
Flat terrain	(3.2, 3.2, 1.5/1.516)	(100, 100, 70/72)	32	8	100	1.05	0.5

Following Lundquist et al. (2012), the cases are run in a $6 \text{ km} \times 6 \text{ km}$ horizontally periodic domain with $\Delta x = \Delta y = 100$ m grid spacing (see Table 1 for additional simulation details). The terrain is given by a two-dimensional Agnesi function,

$$h(x, y) = \frac{h_p}{1 + (x/L_h)^2 + (y/L_h)^2}, \quad (12)$$

where $h_p = 350$ m is the peak height and $L_h = 800$ m is the horizontal length scale of the hill. This results in a maximum terrain slope of roughly 15° , which is gentle enough that slope-related errors are relatively small and solutions on terrain-following and nonconforming grids can be compared directly. The flow is forced with a geostrophic velocity $u_1^g = 10 \text{ m s}^{-1}$ and the Coriolis parameter $f = 10^{-4} \text{ s}^{-1}$. WRF's Smagorinsky turbulence closure is used with a surface roughness $z_0 = 0.1$ m. Simulations are integrated over a period of 2 days and results are averaged over the second day (denoted by $\langle \cdot \rangle$, with output every 15 min), during which a quasi-steady state is reached. This time averaging minimizes the effect of both instantaneous turbulent fluctuations and inertial oscillations, thus allowing for direct comparison between simulations.

Both the VR- and SR-IB methods show differences relative to the native WRF solution. Variation in the flow profiles on the windward side of the hill is concentrated near the surface, and depends in part on where the IB intersects the model grid. For example, smaller variation is seen in the fourth and fifth profiles from the left in Fig. 1a because the first velocity grid point is nearly the same distance above the surface for native WRF (terrain-following) and IB (nonconforming) cases. Larger flow variation is found on the leeward side of the hill, where a logarithmic velocity profile is not necessarily expected. With the hybrid RANS/LES scheme, VR-IB performance more closely matches native WRF, although some variation is still evident near the surface. Additional testing of the hybrid scheme, including its use with WRF and SR-IB, is presented below.

Differences between native WRF and the IB methods are also caused by the way in which the log law is applied on the model grid. For native WRF, the log law is

applied vertically along grid lines, while for the IB methods it is applied in a surface normal direction (see Fig. 2). However, for the relatively gentle slopes considered in this study, differences related to this issue are expected to be small. Note that when using idealized Dirichlet no-slip boundary conditions and a constant eddy viscosity, Lundquist et al. (2012) found nearly exact agreement between a native WRF simulation with a terrain-following grid and a WRF simulation with a nonconforming grid using an IB method. This validated the IB implementation, though not the enforcement of a logarithmic velocity profile, which is necessary for modeling surface momentum fluxes in simulations of atmospheric flows. Thus, differences here are expected to be due to the different surface stress implementations in the IB methods compared to native WRF.

Despite the strong performance of IB methods in previous studies at high resolution (including those in WRF: Ma and Liu 2017; Bao et al. 2018), these preliminary results at coarser resolution show that different implementations of IB methods in WRF can give different flow solutions. To determine the causes of these differences, this study investigates IB method sensitivity to the location at which the immersed surface intersects the model grid, the grid resolution and aspect ratio, and the near-surface turbulence model. In what follows, additional test cases will be used to systematically evaluate and understand these model sensitivities. Analysis will focus first on neutral ABL cases in section 3, followed by cases over Askervein Hill in section 4.

3. Performance of immersed boundary method algorithms in a neutral atmospheric boundary layer

Although IB methods are intended to be used over complex terrain, test cases over flat terrain can provide a baseline understanding of IB method performance. In particular, modeled flow profiles can be compared to the theoretical log-law profiles on which surface turbulence parameterizations are based. Moreover, large-eddy simulations using Smagorinsky or similar subgrid-scale models have demonstrated sensitivity to grid aspect ratio $\alpha = \Delta x / \Delta z$. This sensitivity is not unique to WRF and

has been found in other finite-difference codes as well (see discussions in Sullivan et al. 1994; Chow et al. 2005; Brasseur and Wei 2010; Mirocha et al. 2010; Ercolani et al. 2017; Arthur et al. 2019). Because IB methods implemented into WRF are not immune to these issues, IB method sensitivity to different grid configurations will be explored. In this section, IB method performance is evaluated for a range of cases with varying IB location, grid resolution/aspect ratio, and near-surface turbulence scheme.

Simulations are completed in a domain with flat terrain, periodic lateral boundary conditions, a geostrophic forcing velocity $u_1^g = 10 \text{ m s}^{-1}$ with the Coriolis parameter $f = 10^{-4} \text{ s}^{-1}$, a boundary layer height $H = 1 \text{ km}$, and small initial near-surface velocity perturbations to trigger turbulence. The Smagorinsky turbulence closure is used with a surface roughness $z_0 = 0.1 \text{ m}$, and the hybrid RANS/LES scheme is used in some cases. Cases that do not use the hybrid RANS/LES scheme are referred to as “LES-only.” Additional simulation details are shown in Table 1. The simulations are integrated over a period of 2 days and results are averaged over the second day (denoted by $\langle \cdot \rangle$, with output every 15 min). Results are also averaged horizontally (x - y , denoted by an overbar).

a. Varying immersed boundary location

A primary reason for the variation in near-surface flow profiles in Fig. 1 is the location at which the IB intersects the model grid. Recall that MOST depends on z_r , a reference height above the surface that is taken to be the height of the first velocity grid point. When terrain-following coordinates are used, the first velocity grid point is a similar distance above the surface for all (x, y) such that $(z_r - h)$ in Eq. (2) is roughly constant. However, the same is not true for the IB method simulations because the IB intersects the grid arbitrarily and differently for each (x, y) location. For VR-IB, this effectively changes d_r in Eq. (4). For SR-IB, d_r , and thus the distance from the boundary at which $\tau_{i3,S}$ is estimated in Eq. (10), remain constant. However, the locations of $\tau_{ij,RP}$ and $\tau_{ij,EP}$ change relative to the point at which $\tau_{i3,S}$ is estimated.

To examine the sensitivity of the VR- and SR-IB methods to the location of the IB, several tests are completed with the same model grid but varying the vertical location of a flat-terrain surface. As depicted in Fig. 3, cases are run with flat terrain located at heights of $h = -3.95, -2, 0, 2$, and 3.85 m , thus spanning the range between the cell-centered vertical grid levels at $z \approx -4 \text{ m}$ and $z \approx 4 \text{ m}$ where the horizontal velocities u_1 and u_2 are calculated. Due to WRF’s pressure-based vertical coordinate, vertical grid levels do not have exact spacing. As a result, the terrain at $h = 0 \text{ m}$ is just above

the vertical grid line at $z = -0.0004 \text{ m}$. The value of $h = 3.85 \text{ m}$ is chosen such that d_{RP} is slightly greater than z_0 in the VR-IB case, thus preventing U_{RP}' from being set explicitly to 0.

The variability in the predicted velocity profiles relative to the log law also depends on the surface friction velocity $\langle \bar{u}_* \rangle_S$. These values were calculated using the most reasonable method available, and are presented in Table 2. In general,

$$u_* = \left\{ \frac{[(\tau_{13})^2 + (\tau_{23})^2]^{1/2}}{\rho} \right\}^{1/2}. \quad (13)$$

For native WRF, $\langle \bar{u}_* \rangle_S$ is calculated (as in the code) using Eqs. (1) and (2) with z_r as the height of the first velocity grid point above the surface. Although Eqs. (1) and (2) are not used explicitly in the code for the VR-IB boundary condition, the near-surface velocity is set based on the log law. Therefore, Eqs. (1) and (2) can be used to estimate $\langle \bar{u}_* \rangle_S$, again with z_r as the height of the first velocity grid point above the surface. For the SR-IB boundary condition, the velocity is not set explicitly near the surface, so Eqs. (1) and (2) do not provide a good estimate of $\langle \bar{u}_* \rangle_S$. A better estimate is made by interpolating the full $\langle \bar{u}_* \rangle$ profile to the surface. In this case, the stress in Eq. (13) includes both resolved and subgrid-scale (SGS) components, $\tau_{i3,\text{Tot}} = \tau_{i3,\text{Res}} + \tau_{i3,\text{SGS}}$, where $\tau_{i3,\text{Res}} = \rho u_i' u_3'$ and $\tau_{i3,\text{SGS}} = -2\rho \nu_t S_{i3}$, with $i = 1, 2$ (see, e.g., Arthur et al. 2019).

The VR-IB method with the LES-only surface treatment displays strong sensitivity to the location of the terrain relative to the model grid, as shown in the average velocity profiles in Fig. 4a. Note that the different profiles have been adjusted vertically so that the terrain is aligned. The VR-IB profile closely matches the native WRF profile when $h = 0 \text{ m}$ and the flat terrain is nearly coincident with w points, meaning that the IB and terrain-following grids are nearly identical. However, as h increases and the terrain surface moves closer to the first velocity grid point, a larger amount of drag is applied to the flow. This behavior was evident in the hill test case, for example in profiles 1 and 8 in Fig. 1a. The opposite is also true in that less drag is applied as the terrain surface moves farther away from the first velocity grid point (this behavior is less evident in Fig. 1). When compared to the theoretical log-law velocity profile

$$\frac{\langle \bar{V}_{\text{horz}} \rangle}{\langle \bar{u}_* \rangle_S} = \kappa \log \left(\frac{z - h}{z_0} \right), \quad (14)$$

for $z_0 = 0.1 \text{ m}$ (see Fig. 4b), the VR-IB LES-only solution generally overestimates the velocity in the surface layer, as does native WRF.

FIG. 3. Two-dimensional (x - z) depiction of the WRF grid and IB for flat-terrain IB method cases. The colored lines denote different flat terrain heights with colors corresponding to those used in Figs. 4 and 5. Left-side tick marks show the height of vertical grid lines, while right-side tick marks denote the WRF variables calculated at the given height.

VR-IB displays especially poor performance for the $h = 3.85$ m case (i.e., when d_{RP} is small) because too much drag is applied to the flow, as shown by the dotted purple line in Figs. 4a and 4c. Although the flow still conforms to the log law near the surface in Fig. 4c, the surface friction velocity is substantially underestimated (see Table 2 caption). This issue can be alleviated by moving the reconstruction point up one grid level when $d_{RP} < f_{tol}\Delta z$, where f_{tol} is a user-specified tolerance factor. This functionality has been added to the present implementation of VR-IB with $f_{tol} = 0.1$, and will be used in all remaining VR-IB simulations [overriding the implementation of Bao et al. (2018), which set the velocity to 0 within z_0 of the boundary]. A similar modification was used by Tseng and Ferziger (2003) in their IB method implementation. Improved performance is demonstrated in Figs. 4a and 4c by the solid purple line, which now roughly overlaps the red line corresponding to the $h = -3.95$ m solution.

Near-surface turbulence models may be used to augment the SGS scheme for both native WRF and IB method

boundary conditions. Here, the hybrid RANS/LES scheme presented above [Eq. (9)] is tested with a blending height $h_B = 48$ m. Notably, the hybrid scheme reduces the sensitivity of the VR-IB method to the location of the first velocity grid point above the surface compared to the LES-only case. When the hybrid scheme is used, VR-IB results more closely match both the WRF solution (see Fig. 4b) and the log law (see Fig. 4d). A narrower range of $\langle \bar{u}_* \rangle_S$ values is also found (0.39–0.47 vs 0.35–0.48, see Table 2).

However, because d_{RP} in Eq. (5) is a function of h , some sensitivity to h is an unavoidable feature of VR-IB. For example, even when the hybrid scheme is used, the method still applies too much drag to the flow when the reconstruction point is very close to the surface (i.e., when d_{RP} is small, compare dotted purple lines in Figs. 4a and 4c with those in Figs. 4b and 4d). Thus, moving the reconstruction point up one grid level when it falls within a specified distance from the surface again provides a performance improvement (see solid purple line in Figs. 4b and 4d) as in the LES-only case.

TABLE 2. Surface friction velocity $\langle \bar{u}_* \rangle_S$ (in m s^{-1}) for flat-terrain cases with varying terrain height h . Note that the updated VR-IB algorithm is used such that the reconstruction point has been moved up one grid level for the $h = 3.85$ m case. Without this update, $\langle \bar{u}_* \rangle_S = 0.19 \text{ m s}^{-1}$ for the LES-only case and 0.25 m s^{-1} for the hybrid RANS/LES case.

h (m)	WRF	VR-IB	SR-IB	WRF	VR-IB	SR-IB
	LES only	LES only	LES only	RANS/LES	RANS/LES	RANS/LES
−3.95	—	0.48	0.43	—	0.47	0.43
−2	—	0.46	0.44	—	0.45	0.44
0	0.43	0.42	0.46	0.42	0.43	0.44
2	—	0.35	0.46	—	0.39	0.45
3.85	—	0.47	0.47	—	0.46	0.45

FIG. 4. Planar- and time-averaged velocity profiles $\langle \bar{V}_{horz} \rangle$ for flat-terrain cases with varying terrain height h using the VR-IB method. The color corresponds to h as in Fig. 3 for the (a),(c) LES-only and (b),(d) hybrid RANS/LES cases. In (c),(d), velocities are normalized by the surface friction velocity $\langle \bar{u}_* \rangle_s$ and compared to the theoretical log-law profile. The dotted purple line in each panel corresponds to the VR-IB solution for $h = 3.85$ m without moving the reconstruction point up one grid level.

Compared with the VR-IB method, the SR-IB method shows substantially less sensitivity to the terrain height, as the average velocity profiles in Figs. 5a and 5b roughly overlap for each case. The SR-IB results also more closely follow the native WRF solution and the log law. However, near-surface variability related to the interpolation/extrapolation of the near-surface shear stress is evident. It should be noted that the native WRF and VR-IB solutions all match the log law exactly at the first grid point above the surface because the boundary condition sets this condition explicitly. However, this is not the case for SR-IB because neither the velocity nor the near-surface stress gradient are set explicitly.

Only a small change in the predicted velocity with a small improvement in log-law agreement is seen when the hybrid RANS/LES scheme is used with WRF and SR-IB (see Figs. 5b,d and Table 2). This is because both methods set the surface (or near-surface) stress explicitly without using the eddy viscosity. Conversely, the performance of VR-IB depends directly on accurate calculation of the near-surface stress gradient, which depends in turn on accurate calculation of the near-surface eddy viscosity.

b. Varying grid resolution and aspect ratio

Because previous IB method studies have generally used fine resolution, there is a need to better understand

FIG. 5. As in Fig. 4, but using the SR-IB method.

IB method performance at the coarser resolutions used by many ABL LES studies (e.g., Talbot et al. 2012; Nottrott et al. 2014; Mazzaro et al. 2017; Phillips et al. 2017; Muñoz-Esparza et al. 2018). Additionally, WRF and other finite-difference codes have a known dependence on the grid aspect ratio $\alpha = \Delta x / \Delta z$ (Mirocha et al. 2010; Ercolani et al. 2017; Arthur et al. 2019), so IB method performance is also evaluated over a range of α values. Here, the VR- and SR-IB methods are tested with the grid spacings shown in Table 3, which give aspect ratio values 1, 2, 4, 8, and 16. Results with LES-only surface treatment are presented first, followed by a subset of results using the hybrid RANS/LES scheme. These cases have similar setups to those discussed above (see Table 1), but with different grid spacing. That is, the domain size (L_x, L_y, L_z) remains the same, but ($\Delta x, \Delta y, \Delta z$) and thus (N_x, N_y, N_z) differ. Note that WRF's

aspect ratio dependence is expected to change with the surface roughness z_0 , the terrain slope, and other factors (Mirocha et al. 2010). Thus, the results presented here for flat terrain with $z_0 = 0.1$ m should not be considered universal.

For the LES-only cases, there is substantial variability in the flow solution depending on the grid resolution and aspect ratio, as seen in Fig. 6, which shows averaged velocity profiles for each case in Table 3 grouped by α . This is true both for native WRF and for the IB method implementations. Similar to previous studies using native WRF over flat terrain with $z_0 = 0.1$ m (e.g., Mirocha et al. 2010), a grid aspect ratio $\alpha = 4$ provides the best overall performance, with minimal differences between native WRF, the IB methods, and the theoretical log-law profile.

As the aspect ratio decreases to $\alpha = 2$ and 1, VR-IB outperforms both native WRF and SR-IB relative to the

TABLE 3. Grid aspect ratio α values for flat-terrain cases with different grid spacing. The case marked with a dash is not included.

		Δx (m)				
Δz (m)		16	32	64	128	
	8	2	4	8	16	
	16	1	2	4	8	
	32	—	1	2	4	

log law. While native WRF overestimates the velocity in the surface layer, SR-IB substantially underestimates it. Conversely as the aspect ratio increases to $\alpha = 8$ and 16, VR-IB performance decreases, substantially underestimating the velocity (note that the perceived log-law agreement is misleading due to a highly underestimated value of $\langle \bar{u}_* \rangle_S$). While SR-IB captures the bulk drag in the surface layer accurately, closely matching the native WRF solution, there is a discontinuity (or “kink”) in the velocity profile at the first grid point above the surface. This issue is evident when $\alpha \geq 4$, and is exacerbated as α (or Δx) increases. Although the specific cause of the discontinuity is unknown, it is related to the interpolation and extrapolation of the shear stress in the region around the IB. This will be investigated further in future studies.

Because the hybrid RANS/LES scheme has been shown to improve velocity predictions relative to the log law, a subset of the cases in Table 3 are tested with this scheme. In particular, the $\Delta x = 64$ m cases with aspect ratios of 2, 4, and 8, are simulated with a blending height $h_B = 100$ m, and the results are shown in Fig. 7. For all tested cases, the hybrid RANS/LES scheme generally improves model performance. The use of RANS near the surface alleviates issues resolving near-surface eddies, which is the ultimate source of α variability (Brasseur and Wei 2010; Ercolani et al. 2017; Arthur et al. 2019). It should be noted that more advanced LES closure schemes that more accurately account for the effects of near-surface SGS turbulence have also been shown to reduce α variability in WRF (Mirocha et al. 2010; Kirkil et al. 2012; Arthur et al. 2019), as well as in other codes, and the application of these schemes to the present IB method framework is a topic of future work.

4. Application to Askervein Hill

To better understand IB method performance over topography, each method is applied to the well-known case study of Askervein Hill. The Askervein Hill field campaign (Taylor 1983; Taylor and Teunissen 1985, 1987) observed flow over a small, moderately sloped hill

off the coast of Scotland. The hill is 116 m tall with a 2 km major axis and a 1 km minor axis, and the flow was measured along several transects (A and AA are included here; see Fig. 8b).

Due to its moderate scale and slope, Askervein Hill facilitates direct comparison of simulations using both terrain-following and nonconforming grids with observations, and has therefore been used extensively in previous IB method (Bao et al. 2018; DeLeon et al. 2018) and other ABL modeling studies (Castro et al. 2003; Chow and Street 2009; Golaz et al. 2009). However, to the knowledge of the authors, the present study is the first application of the SR-IB algorithm to flow over Askervein Hill.

Here, following Chow and Street (2009) and Bao et al. (2018), simulations are compared to observations averaged over the period of 3 October 1983 1200–1700 (UTC+1 h). During this window, the predominant wind direction was 210° (clockwise from north) with roughly neutral stability (Taylor and Teunissen 1985).

a. Model setup

The present computational setup closely follows that of Bao et al. (2018). To provide a turbulent inflow condition for flow over Askervein Hill, a nested configuration is employed with flat terrain ($h = 10$ m) on the outer domain and the Askervein topography on the inner domain (Fig. 8a). The outer domain has periodic lateral boundary conditions and is forced by a geostrophic velocity $u_1^g = 18 \text{ m s}^{-1}$ with small initial near-surface velocity perturbations to trigger turbulence. The inner domain is forced at its lateral boundaries by the outer domain via one-way nesting. Askervein Hill topography at 25 m resolution (Walmsley and Taylor 1996) is used on the inner domain and is rotated 60° clockwise to provide the appropriate incoming wind direction with only positive u_1^g forcing (see Bao et al. 2018). The Smagorinsky turbulence closure model is used with a surface roughness $z_0 = 0.03$ m and the Coriolis parameter $f = 10^{-4} \text{ s}^{-1}$.

Two grid resolutions are tested with this configuration, a high-resolution case with $\Delta x = 5$ m (following Bao et al. 2018) and a lower-resolution case with $\Delta x = 30$ m (following Golaz et al. 2009). The surface scheme is varied for each case as well, using either the LES closure only or the hybrid RANS/LES scheme. Although the hybrid scheme assumes attached flow, flow separation can still occur (e.g., DeLeon et al. 2018). In what follows, a velocity profile with an inflection point denotes separated flow, while a velocity profile with no inflection point denotes attached flow. By applying RANS over the blending height h_B and thus encouraging attached flow, the hybrid scheme improves performance over the

FIG. 6. (left) Planar- and time-averaged velocity profiles $\langle \bar{V}_{\text{horz}} \rangle$, as well as (right) profiles normalized by the surface friction velocity $\langle \bar{u}_* \rangle_s$ and compared to the theoretical log law, for flat-terrain LES-only cases with varying grid resolution, as summarized in Table 3. Cases are grouped by their grid aspect ratio: (a),(b) $\alpha = 1$, (c),(d) $\alpha = 2$, (e),(f) $\alpha = 4$, (g),(h) $\alpha = 8$, and (i),(j) $\alpha = 16$. The terrain height $h = 0$ m for all cases.

LES-only surface treatment in some cases. Blending height values and additional simulation details are shown in Table 4. Each simulation is run for a total of 19 h and 30 min, which includes 18 h (roughly one inertial period) of spinup on the outer domain to generate a quasi-steady turbulent inflow, followed by 30 min of additional spinup on the inner domain. Finally, results are time averaged (denoted by $\langle \cdot \rangle$, with output every minute) over the final hour of the simulation and compared to observations from the Askervein experiment.

b. Comparison to observations

WRF and the two IB method approaches are tested for both high- and low-resolution cases, both with and without the hybrid RANS/LES scheme, for a total of 12 simulations (see Table 5). As in previous studies, model results are compared to Askervein observations using the fractional speedup,

$$\Delta S_{z_{\text{AGL}}} = \frac{S(z_{\text{AGL}}) - S_{\text{RS}}(z_{\text{AGL}})}{S_{\text{RS}}(z_{\text{AGL}})}, \quad (15)$$

FIG. 7. As in Fig. 6, but for select cases both with and without the hybrid RANS/LES scheme. Included are the $\Delta x = 64$ m cases from Table 3, with grid aspect ratios $\alpha = 2, 4$, and 8.

where S is the horizontal wind speed along transect A or AA and S_{RS} is the horizontal wind speed at the reference site, located upwind of the hill along line A. To calculate ΔS for the model results, $\langle \bar{V}_{\text{horz}} \rangle$ is interpolated to transect A or AA on the inner domain and used for S . Because the real reference site is outside of the inner domain, the model reference velocity is calculated as the time-averaged wind speed $\langle \bar{V}_{\text{horz}} \rangle$ at the point RS* in Fig. 8b. This point is on flat terrain upwind of where the topography begins to affect the flow, and thus provides a reasonable estimate of S_{RS} . While Bao et al. (2018) used the spatial- and time-averaged flow on the outer domain for the model reference velocity, the high aspect ratio on that domain leads to poor prediction of the velocity profile in the present cases ($\alpha = 6$ and 15 for the high- and low-resolution cases, respectively, see Figs. 6g–j).

Askervein Hill results are summarized in Table 5, which shows the RMSE of modeled velocity and speedup compared to observations. For the high-resolution cases, the present values range from 0.74–1.57 (RMSE $_{\langle \bar{V}_{\text{horz}} \rangle}$ at the reference site RS*) to 0.12–0.35 (RMSE $_{\Delta S_{10}}$ along transects A/AA). For the low-resolution cases, the ranges are 0.34–3.07 (RMSE $_{\langle \bar{V}_{\text{horz}} \rangle}$ at the reference site RS*) and 0.14–0.36 (RMSE $_{\Delta S_{10}}$ along transects A/AA). These values are similar to those found in previous studies. For example, using a similar WRF setup with a $\Delta x = 5$ m grid but finer vertical resolution ($\Delta z = 1$ m), Bao et al. (2018)

achieved RMSE values of 0.8 m s^{-1} ($\langle \bar{V}_{\text{horz}} \rangle$ RS*), 0.25 (ΔS_{10} A), and 0.18 (ΔS_{10} AA) for native WRF and 0.4 m s^{-1} ($\langle \bar{V}_{\text{horz}} \rangle$ RS*), 0.11 (ΔS_{10} A), and 0.09 (ΔS_{10} AA) for VR-IB. Using the Advanced Regional Prediction System (ARPS) model with a $\Delta x = 35$ m grid, Chow and Street (2009) achieved RMSE $_{\Delta S_{10}}$ values along transect A of 0.22 using the TKE 1.5-order subgrid model and 0.09 using the more robust dynamic reconstruction model (Chow et al. 2005).

While the RMSE facilitates comparison to previous results, it is limited as a metric of model performance due to the small sample size of available observations. For this reason, full transects of fractional speedup are shown relative to the observed values in Figs. 9 and 10. The reader is referred to Figs. 4–7 in Chow and Street (2009), Fig. 9 in DeLeon et al. (2018), and Fig. 6 in Bao et al. (2018) for comparison to transects of fractional speedup from other studies. The fractional speedup provides a good measure of model performance over topography because it is normalized by the inflow velocity. However, comparisons of velocity magnitude are also important for the evaluation of overall model performance. For this reason, modeled velocities at 10 m AGL are also compared to observations in Figs. 11 and 12. Raw velocities were not reported along transects A and AA for the field experiment, so these values were calculated from the reported speedup and inflow velocity values.

FIG. 8. (a) The nested domain configuration for the high-resolution Askervein Hill cases (that of the lower-resolution cases is qualitatively similar) showing topography contours at 10 m intervals rotated 60° clockwise. (b) Time-averaged flow on the inner domain at 10 m AGL for the high-resolution WRF with LES only case. Also included are the locations of observational transects A and AA with center points HT and CP, respectively, as well as the point at which the model reference velocity is calculated, RS*.

Based on the results shown in Table 5 and Figs. 9–12, IB method performance is overall quite similar to that of WRF. It should be noted that IB methods are expected to behave equivalently to WRF over gently sloping terrain, such as Askervein Hill, but are expected to outperform WRF for cases with steep terrain. Variability among the methods depends on the grid resolution/aspect ratio and the surface-layer turbulence scheme, as discussed below. This variability is most evident in the lee of the hill, where flow separation may occur. As noted in DeLeon et al. (2018), the MOST parameterization upon which the WRF and IB method boundary conditions are based is not necessarily applicable to separated flow, but is nevertheless used widely due to the absence of an accepted theory for complex terrain. The goal of the present work is to explore IB method performance using existing surface parameterizations, while the development of new parameterizations is left for future study. Moreover, the true amount of flow separation is unknown because observations were taken only at 10 m AGL. Comparisons between WRF and the IB methods should be considered in this context.

1) WRF RESULTS

In the present study, WRF performs well at high resolution ($\Delta x = 5$ m on the inner domain) using the LES-only surface treatment. Although the inflow velocity is slightly overestimated due to the low aspect ratio ($\alpha = 2$, see Figs. 6c,d), the flow over the hill is well captured. When the hybrid RANS/LES scheme is used with WRF, the inflow velocity prediction is improved, reducing the RMSE from 1.33 to 0.79 m s^{-1} . Additionally, by encouraging attached flow in the lee, the hybrid scheme

slightly modifies WRF performance over the hill. Although the RMSE values in Table 5 are similar to those without the hybrid scheme, the change can be seen in the velocity profiles in Fig. 11b, especially the third profile, where the flow remains attached. This results in slightly less slowdown in the lee (Fig. 9). However, WRF's overall performance at this resolution is similar both with and without the hybrid scheme.

As the grid resolution is coarsened (to $\Delta x = 30$ m on the inner domain), native WRF performance remains relatively consistent. The inflow velocity prediction actually improves from the high-resolution LES-only case (with the RMSE reduced from 1.33 to 0.79 m s^{-1}) due to the increase in aspect ratio from $\alpha = 2$ to 5 , which is closer to the optimal value of 4 . When the hybrid RANS/LES scheme is used with WRF, the inflow velocity RMSE is further reduced to 0.54 m s^{-1} . The primary difference between the high- and low-resolution WRF results is the amount of separation in the lee of the hill. Flow separation is no longer predicted in the low-resolution LES-only case (Fig. 12b), resulting in slightly less slowdown (Fig. 10) and a slight increase in the speedup RMSE values (from 0.18 and 0.13 to 0.26 and 0.15 , see Table 5). Furthermore, because the flow is attached in the LES-only case, adding the hybrid scheme has a minimal effect on the results.

2) IMMersed BOUNDARY METHOD RESULTS

The performance of the IB methods is evaluated not only relative to observations, but also relative to WRF, which is used here as a baseline. In the high-resolution case, the VR- and SR-IB methods capture the

TABLE 4. Simulation details for Askervein Hill cases, including the domain size L , the number of grid points N , and the grid spacing. The vertical grid spacing Δz is constant up to $z_{\Delta z}$, above which it is stretched by a factor r until $\Delta z \approx 100$ m, above which it remains constant. Two numbers are reported for L_z and N_z (terrain-following and IB), which are larger for IB method cases to allow for three grid cells beneath the terrain. Rayleigh damping is applied over the top z_{damp} of the domain. The blending height h_B is included for cases that use the hybrid RANS/LES scheme.

Domain	(L_x, L_y, L_z) (km)	(N_x, N_y, N_z)	$\Delta x = \Delta y$ (m)	Δz (m)	$z_{\Delta z}$ (m)	r	z_{damp} (m)	h_B (m)
High-resolution outer	(6, 5.5, 2.25/2.2575)	(400, 367, 134/137)	15	2.5	225	1.1	250	20
High-resolution inner	(4, 3.5, 2.25/2.2575)	(798, 699, 134/137)	5	2.5	225	1.1	250	10
Low-resolution outer	(10, 9.6, 2.25/2.268)	(112, 106, 75/78)	90	6	226	1.1	250	75
Low-resolution inner	(4, 3.5, 2.25/2.268)	(132, 114, 75/78)	30	6	226	1.1	250	35

inflow velocity well, both with and without the hybrid RANS/LES scheme (see Table 5 and Fig. 11a). However, differences again arise in the lee of the hill. The VR-IB method predicts attached flow both with and without the hybrid scheme, leading to less slowdown than WRF or the observations. Conversely, the SR-IB method predicts separated flow both with and without the hybrid scheme, leading to more slowdown. This is related to the grid aspect ratio $\alpha = 2$, for which the near-surface VR-IB velocity tends to be faster, and the SR-IB slower, than WRF (see Figs. 6c,d). Due to the large amount of flow separation predicted by the SR-IB method, use of the hybrid RANS/LES scheme tends to improve its prediction by encouraging additional flow attachment (see Figs. 9 and 11b). Whether this combination is robust to a variety of flow scenarios warrants further study.

Changes in IB method performance for the low-resolution case depend in part on the change in grid aspect ratio from $\alpha = 2$ to 5. The VR-IB method with LES-only surface treatment underpredicts the inflow velocity substantially (see VR-IB cases for $\alpha > 4$ cases in Figs. 6g,i); however, the hybrid RANS/LES scheme greatly improves this prediction, reducing the RMSE from 3.07 to 0.34 m s^{-1} (see Table 5 and Fig. 12a).

The SR-IB method performs better at high aspect ratios (again, see Figs. 6g,i), with inflow velocity RMSE values of 0.65 m s^{-1} with LES only and 0.41 m s^{-1} with the hybrid scheme.

IB method performance for the low-resolution case also varies over the hill. While the validity of the VR-IB method with LES only result is called into question by the underestimated inflow velocity, the VR-IB method with hybrid RANS/LES result approaches both the observations and coarse-resolution WRF results. Moreover, it is similar to the high-resolution VR-IB results (see Table 5 as well as Figs. 10 and 12), showing consistency over a range of grid resolutions. As in the high-resolution case, the SR-IB method overpredicts slowdown in the lee of the hill in the low-resolution case (Fig. 10). However, because the LES-only simulation does not predict much flow separation at this resolution, the hybrid scheme does not substantially affect performance (Fig. 12b).

An issue that becomes evident in the low-resolution SR-IB cases is poor prediction of the velocity at the first grid point above the surface, as was seen in flat-terrain tests in section 3. Here, a discontinuity in the velocity profile is present in the inflow profile (Fig. 12a) and also leads to jaggedness in the speedup along transect A

TABLE 5. RMSE results for Askervein Hill cases relative to mean observed values. At RS*, the RMSE is calculated over the observed vertical $\langle V_{\text{horz}} \rangle$ profile. Along transects A and AA, the RMSE is calculated at 10 m AGL for both $\langle V_{\text{horz}} \rangle$ and ΔS .

Method	Δx (m)	Surface scheme	RMSE($\langle V_{\text{horz}} \rangle$) RS* (m s^{-1})	RMSE($\langle V_{\text{horz}} \rangle_{10}$) A (m s^{-1})	RMSE($\langle V_{\text{horz}} \rangle_{10}$) AA (m s^{-1})	RMSE ΔS_{10} A	RMSE ΔS_{10} AA
WRF	5	LES only	1.33	2.30	2.11	0.19	0.14
WRF	5	RANS/LES	0.79	1.99	1.90	0.16	0.15
VR-IB	5	LES only	0.89	3.79	2.96	0.31	0.22
VR-IB	5	RANS/LES	1.57	3.90	3.02	0.28	0.17
SR-IB	5	LES only	0.87	3.23	1.73	0.35	0.18
SR-IB	5	RANS/LES	0.74	1.48	1.33	0.15	0.12
WRF	30	LES only	0.79	1.71	1.00	0.26	0.14
WRF	30	RANS/LES	0.54	2.09	1.53	0.25	0.20
VR-IB	30	LES only	3.07	2.53	2.78	0.36	0.32
VR-IB	30	RANS/LES	0.34	2.22	1.36	0.24	0.14
SR-IB	30	LES only	0.65	2.63	2.11	0.24	0.16
SR-IB	30	RANS/LES	0.41	2.37	1.82	0.25	0.18

FIG. 9. Comparison of modeled and observed speedup ΔS_{10} for high-resolution ($\Delta x = 5$ m on the inner domain) Askervein Hill cases both with (dashed) and without (solid) the hybrid RANS/LES scheme. Results are shown for transects (a) A and (b) AA. HT and CP are the locations of the peak elevation along transects A and AA, respectively (see Fig. 8b).

FIG. 10. As in Fig. 9, but for low-resolution ($\Delta x = 30$ m on the inner domain) Askervein Hill cases. The high-resolution native WRF with LES only case is also included for reference.

FIG. 11. Comparison of modeled velocity profiles ($\langle V_{\text{horz}} \rangle$) to observed velocity data for high-resolution ($\Delta x = 5$ m on the inner domain) Askervein Hill cases both with (dashed) and without (solid) the hybrid RANS/LES scheme. Results are shown for (a) the reference site RS* and (b) the lee side of transect A. HT is the location of the peak elevation along transect A (see Fig. 8b).

(around -0.25 km in Fig. 10a). This occurs both with and without the hybrid scheme, and limits the consistency of the SR-IB method relative to WRF and VR-IB.

Based on the full suite of Askervein Hill cases presented here, the hybrid RANS/LES scheme generally improves both WRF and IB method results. By applying RANS in the near-surface region, the scheme reduces the inherent variability of the IB methods, which stems from interpolation near the surface. For example, the hybrid scheme was especially useful for the high-resolution SR-IB case, for which slowdown was overpredicted with the LES-only surface treatment, and for the coarse-resolution VR-IB case, for which the inflow velocity was underpredicted with the LES-only surface treatment. It is important to note that the hybrid scheme depends on the blending height h_B , and care must be taken to limit the value of h_B such that it does not smooth out the turbulence that should be resolved in an LES (Senocak et al. 2007).

5. Summary and conclusions

Immersed boundary methods have been employed successfully to simulate flows in the atmospheric boundary layer, yet their performance over a range of model configurations has not been fully explored. Most importantly, previous work has focused on IB method simulations at microscale resolutions that are computationally restrictive and thus finer than those used in many ABL

studies. In this work, two test cases were used to evaluate IB method performance across a range of grid resolutions and aspect ratios. Following on the work of Lundquist et al. (2010a, 2012), Bao et al. (2018), and Wiersema et al. (2020), tests were performed in the Weather Research and Forecasting model with a goal of enabling multiscale atmospheric simulations over complex terrain. Although the results herein confirm the applicability of IB methods with surface stress parameterizations in WRF, important sensitivities to model configuration have been revealed.

The velocity reconstruction immersed boundary (VR-IB) method, which sets the velocity and eddy viscosity at grid nodes just above the immersed boundary by assuming surface-tangential log-law flow, showed relatively consistent performance over the range of cases tested. Although the method is inherently sensitive to the location at which the immersed boundary intersects the model grid, this sensitivity was reduced by applying the hybrid RANS/LES surface scheme of Senocak et al. (2007) and by moving the reconstruction point up one grid level when it is too close to the boundary. Although it was not a focus of the present study, the VR-IB method has the added benefit of not requiring grid points underneath the terrain. Thus, domains using the IB method can be nested within domains using terrain-following WRF grids, making VR-IB a natural choice for multiscale, nested simulations (e.g., Wiersema et al. 2020).

FIG. 12. As in Fig. 11, but for low-resolution ($\Delta x = 30$ m on the inner domain) Askervein Hill cases. The high-resolution native WRF with LES only case is also included for reference.

The shear stress reconstruction immersed boundary (SR-IB) method, which sets the shear stress within a defined region above and below the immersed boundary, showed less consistent performance than WRF or the VR-IB method for the cases tested. Because the surface shear stress is diagnosed using the velocity at a constant height above the boundary, the SR-IB method shows limited sensitivity to the location at which the boundary crosses the model grid. However, because of sensitivity to interpolation and extrapolation of the near-surface shear stress, the SR-IB method does show variation with grid resolution. At coarser resolutions, a discontinuity can appear in the near-surface velocity profile; future work should examine the cause of this error as well as potential solutions. In the case of flow over a hill, the SR-IB method consistently overpredicts slowdown in the lee, regardless of grid resolution. While the hybrid RANS/LES scheme counteracted this tendency at high resolution by encouraging attached flow, it was less effective at coarser resolution. Solutions to this issue should also be explored if the SR-IB method is to be used at resolutions typical of many ABL LES studies.

Based on the results shown here, future work on immersed boundary methods is needed to assess their performance at intermediate-to-coarse LES resolution. Due to grid aspect ratio sensitivity, care should be taken to choose an appropriate grid based on the desired method and grid resolution, using vertical grid nesting (e.g., Daniels et al. 2016) when necessary. Additionally, a new immersed

boundary method that combines the features of the VR- and SR-IB methods has shown promise at the coarser resolutions used here (Bao et al. 2016) and is currently under development. Because large-eddy simulations depend on adequate resolution of turbulence structures, model performance generally decreases as grid resolution coarsens regardless of the surface boundary condition.

More sophisticated turbulence closure schemes have been shown to improve LES performance in WRF (Mirocha et al. 2010; Kirkil et al. 2012; Khani and Porté-Agel 2017), and more extensive testing with IB methods (as in Ma and Liu 2017, for the SR-IB method) is necessary. Surface boundary conditions that are applicable to separated flow or generally complex terrain should also be used as they become available. For grids that approach the gray zone (Wyngaard 2004), the development of new turbulence parameterizations is needed (e.g., Zhong and Chow 2013; Kosovic et al. 2016, 2017; Goger et al. 2018; Chow et al. 2019). Such work has the potential to improve model performance regardless of whether IB methods or standard boundary conditions with terrain-following coordinates are used.

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