A Multiscale Dynamical Model in a Dry-Mass Coordinate for Weather and Climate Modeling: Moist Dynamics and Its Coupling to Physics

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ABSTRACT

A multiscale dynamical model for weather forecasting and climate modeling is developed and evaluated in this study. It extends a previously established layer-averaged, unstructured-mesh nonhydrostatic dynamical core (dycore) to moist dynamics and parameterized physics in a dry-mass vertical coordinate. The dycore and tracer transport components are coupled in a mass-consistent manner, with the dycore providing time-averaged horizontal mass fluxes to passive transport, and tracer transport feeding back to the dycore with updated moisture constraints. The vertical mass flux in the tracer transport is obtained by reevaluating the mass continuity equation to ensure compatibility. A general physics–dynamics coupling workflow is established, and a dycore–tracer–physics splitting strategy is designed to couple these components in a flexible and efficient manner. In this context, two major physics–dynamics coupling strategies are examined. Simple-physics packages from the 2016 Dynamical Core Model Intercomparison Project (DCMIP2016) experimental protocols are used to facilitate the investigation of the model behaviors in idealized moist-physics configurations, including cloud-scale modeling, weather forecasting, and climate modeling, and in a real-world test-case setup. Performance evaluation demonstrates that the model is able to produce reasonable sensitivity and variability at various spatiotemporal scales. The consideration and implications of different physics–dynamics coupling options are discussed within this context. The appendix provides discussion on the energetics in the continuous- and discrete-form equations of motion.

1. Introduction

For atmospheric models, height and mass are two major choices for the vertical coordinate. Mass-based coordinates are widely used in the global modeling community that conventionally adopts a hydrostatic formulation. While a mass-based coordinate is widely referred to as a

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pressure coordinate (Eliassen 1949), the air pressure does not actually possess any monotonic relation with height according to its first definition, that is, the gas-state law. It is the pressure pressed by the vertical weight of air molecules that can be used as a vertical coordinate, which results from the definition of fluid mass within a vertical layer. This point was first generalized in Laprise (1992) by theoretically extending such a coordinate to nonhydrostatic compressible equations, and later realized by Bubnová et al. (1995) in practical modeling. To date, mass-based systems have been widely used for global or limited-area nonhydrostatic atmospheric modeling (e.g., Janjić et al. 2001; Skamarock et al. 2008; Wedi et al. 2009; Lin et al. 2017; Cheng et al. 2018; Peng et al. 2019; Taylor et al. 2020). However, the underlying techniques behind these models are quite different.

For a mass-based moist atmospheric model, a branch is formed because one may choose to use either a moist-mass or a dry-mass vertical coordinate. For a moist-mass system, the relation for setting up the generalized vertical coordinate relies on

$$\frac{1}{g} \frac{\partial \pi_m}{\partial \eta} = -\rho_m \frac{\partial z}{\partial \eta}; \quad (1)$$

and for a dry-mass system:

$$\frac{1}{g} \frac{\partial \pi}{\partial \eta} = -\rho_d \frac{\partial z}{\partial \eta}. \quad (2)$$

Here, η represents the generalized vertical coordinate (Kasahara 1974; Simmons and Burridge 1981). The meaning of the other mathematical symbols (and those used in the rest of the paper) can be found in Table 1, and some acronyms are further explained in Table 2. For both systems, Eq. (1) or (2) is related to the physical definition derived from the equivalent mass of moist or dry air per unit area. So, both π_m and π are feasible for setting up the vertical coordinate, regardless of the specific governing equations. The hydrostatic constraint (i.e., $dw/dt = 0$) leads to an additional assumption that $p \approx \pi_m$ or $p \approx \pi$ for a moist or dry shallow atmosphere such that the air pressure obtained from the gas-state law is identical to that used for vertically integrating the hydrostatic equation. For a moist atmosphere with a dry-mass system, it is still $p \approx \pi_m$ that exerts the hydrostatic constraint (if such a constraint is needed), which further constrains ϕ by a vertical integration, while $\partial \pi / \partial z = -\rho_d g$ is used as a definition for dry air density. Clearly, a diagnostic relation between moist and dry mass within a layer can be found for such a system.

For a full atmospheric model with a moist-mass coordinate, π_m is essentially a prognostic variable, and the effect of moisture is implicitly included in the definition

TABLE 1. Major symbols used in this paper and their definitions.

Symbol	Definition
π, π_m	Equivalent mass for dry and full/moist air
α_d, α_m	Specific volume for dry and full/moist air
τ_d, τ_t, τ_m	Dycore time step, tracer time step, and model time step
τ_{rk}, τ_p	RK sub-time step and physics time step
ρ_d, ρ_v, ρ_m	Dry-air density, water vapor density, and full/moist-air density
q_v, q_c, q_r	Dry mixing ratio for water vapor, cloud water, and rainwater
T, T_v, T_m	Temperature, virtual, and modified temperature
$\theta, \theta_v, \theta_m$	Potential temperature and virtual and modified potential temperature
$\Theta_m, \mathbf{V}$	$\delta \pi \theta_m$ and horizontal velocity vector
η, η_s, η_t	Coefficient of the vertical coordinate, the surface value, and the top value
$\mathcal{H}$	Generic variable that denotes any physical variable
u_n, u_t	Normal and tangent horizontal velocity component
U, V	Zonal and meridional horizontal velocity component
KE	Horizontal kinetic energy
$\zeta_a, \zeta_r, \zeta_p$	Absolute vorticity, relative vorticity, and potential vorticity
$\dot{\eta}$	Generalized vertical velocity
p, z	Full pressure and geometric height
w, ϕ	Vertical velocity and geopotential
R_v, R_d	Gas constants for water vapor and dry air
c_{pd}, c_{vd}	Heat capacities at constant pressure and volume for dry air
c_{pv}, c_{vv}	Heat capacities at constant pressure and volume for water vapor
∇	Horizontal gradient (aka, lambda, del) operator
P_0	1000 hPa
$\delta \mathcal{H}$	Vertical increment of a general variable within one layer
γ	The ratio of the heat capacities: $\gamma = c_{pd}/c_{vd}$
g	Gravity parameter
$S(\mathcal{H})$	Source/sink terms including physics forcing and numerical diffusion (if any)
$\partial \mathcal{H} / \partial t$	Time derivative
$\partial \mathcal{H} / \partial n$	The gradient component along the normal direction of the primal cell's edge

of the model layer. This leads to some theoretical and technical inconvenience when considering physics-dynamics coupling, for which Lauritzen et al. (2018) has provided a detailed illustration, and Peng et al. (2019) demonstrated the numerical modeling differences between the dry- and moist-mass coordinate using an idealized supercell test. A dry-mass vertical coordinate exactly conserves the dry air mass and leads to a cleaner separation of the dry dynamical core (hereafter, dycore), tracer transport, and model physics (hereafter, physics). The wave dynamics and passive advection in the system can be treated in a more isolated manner. The potential inconvenience compared to a moist-mass coordinate is that a dry-mass coordinate would require some additional treatments for model initialization

TABLE 2. Acronyms/abbreviations used in this paper.

Acronym/ abbreviation	Definition
DCMIP	Dynamical Core Model Intercomparison Project
FCT	Flux-corrected transport
FFSL	Flux-form semi-Lagrangian
HDC	Hydrostatic dynamical core
MITC	moist idealized test case
NDC	Nonhydrostatic dynamical core
PPM	Piecewise parabolic method
RK	Runge–Kutta
RK3O3	Third-order upwind flux operator with a third-order RK integrator
TSPAS	Two-step shape-preserving advection scheme

because the dry air mass is only a theoretical definition for the vertical coordinate, and is typically not provided by (re)analysis datasets or analytic testing scripts.

A new nonhydrostatic dynamical framework for a global-to-regional integrated forecast system (GRIST) has been recently established. As an icosahedral/Voronoi mesh model, this model differs from other existing counterparts in the combination of an unstructured C-grid horizontal discretization and a vertically layer-averaged approach (cf. Zhang et al. 2019; hereafter, Z19), with enhanced treatment for vorticity dynamics (Zhang 2018; Wang et al. 2019). Based on a mass vertical coordinate, the formulation and evaluation of the dycore was presented, while the model performance under a moist environment remains unknown. The primary motivation of this study is to extend the dycore to moist dynamics in a dry-mass coordinate and to test and understand its multiscale behaviors with a minimum list of simple-physics parameterizations (Reed and Jablonowski 2012; Klemp et al. 2015; Thatcher and Jablonowski 2016; Ullrich et al. 2016). The model behaviors forced in this way share important physical relevance to that obtained by using a full-physics package, thus helping to focus on different multiscale modeling issues. A discrete moist model with physics creates a more complex environment than a dry model, as more assumptions and choices are typically required, and these branches are further entangled with the numerical discretization. This probably constitutes a reason why moist models, even if combined with the same simple physics, may generate larger differences than their dry counterparts (e.g., Zarzycki et al. 2019). Moreover, understanding the behaviors of the moist model is more difficult because reference solutions are rarely available. Some solutions might look different but can be reasonable for the respective model, further complicating model intercomparison. Thus, examining model sensitivity and variability becomes more helpful to verify that the moist model behaves properly.

Another motivation is to establish a general physics–dynamics coupling (PDC) workflow with a robust and efficient dycore–tracer–physics (DTP) splitting function. In the dycore, no time-scale separation is made within the integration; the time step of the dycore is limited by the horizontally propagating fast waves. When moist species and physics are incorporated, for efficiency considerations it is generally beneficial to have different integration time steps for the dycore, tracer, and physics, which necessitates a robust DTP splitting strategy. Moreover, as a basis for multiscale modeling, some ongoing work has been carried out to test different-type physics packages. For typical global climate modeling applications (e.g., horizontal resolution of ~ 25 – 100 km), it is a common and effective practice that the physics package provides a total tendency from various processes (while these processes may be called at different frequencies, or with subcycles) for a prognostic variable (e.g., Lauritzen et al. 2018; Rasch et al. 2019). This total forcing may be coupled to the dynamical equations via different PDC strategies. Both theoretical considerations (e.g., Caya et al. 1998; Staniforth et al. 2002) and modeling studies (e.g., Zhang et al. 2018; Lauritzen and Williamson 2019) have suggested that the model behaviors are sensitive to these choices. On the other hand, for applications that target higher resolution like current global weather forecasting (e.g., ~ 15 km or higher), sometimes it is useful to separate the physics into slow and fast processes. These processes may be organized into several subpackages, called at different frequencies and even at different places in the model, and their tendencies may be assimilated by the dynamical equations at different timing during the integration (e.g., Wedi 1999; Skamarock et al. 2008; Zängl et al. 2015). Previous studies (e.g., Dubal et al. 2005; Termonia and Hamdi 2007) have also suggested that choosing appropriate processes and adding their tendencies at an appropriate place in the model integration may result in improved stability and accuracy. Based on these general considerations, a multipurpose PDC workflow would be valuable such that the dynamical model can manage the physics tendencies in a more flexible manner.¹

This study presents an extension of the previously developed dycore by coupling it with tracer transport and establishing a general PDC workflow. While a mass-based coordinate is used, the PDC workflow and the

¹ Certainly, both approaches can be used for weather and climate modeling. What we are emphasizing here is that in a relative sense, there should be an optimal model configuration for each target application.

DTP splitting are not tied to the specific vertical coordinate. Two major PDC options, including sudden and incremental adjustment, are explored. In this context, several tracer transport options and two dynamical solvers [hydrostatic dynamical core (HDC) and non-hydrostatic dynamical core (NDC)] are tested to verify the robustness of the model, and also to explore possible sensitivity. We are aware that the model performance under different PDC options is affected by both dynamics and physics. The simple physics offers an environment that can produce some well-expected model behaviors. Thus, the main focus of the paper is twofold: (i) to confirm that the dynamical model can simulate these expected results under the proposed PDC workflow with a DTP splitting function (e.g., reasonable sensitivity and variability) and (ii) to explore several PDC choices and understand their possible impact. These endeavors essentially create a prototype global model with a more complete architecture, providing a common basis for model development and applications in future.

The remainder of this paper is organized as follows: [section 2](#) describes the model formulation. [Section 3](#) provides an assessment of the model behaviors. [Section 4](#) presents a summary.

2. Model formulation

a. Moist governing equations

The continuous-form moist governing equations are basically similar to the dry shallow-atmosphere non-hydrostatic compressible equations in [Laprise \(1992\)](#), and a discussion of its energetics is detailed in [appendix A](#). For discretization consideration, an alternative set of the continuous-form equations is derived in [appendix B](#). This section thus only describes the parts that are closely related to the model discretization and the DTP coupling. The discretization and numerical technique closely follows the layer-averaged unstructured-mesh dycore (cf. [Z19](#) and references therein). In particular, potential temperature, which is a conserved variable for dry air in the adiabatic process, is used as a prognostic variable in the dycore. For moist atmosphere, there are various moist potential temperature definitions available in the literature ([Marquet 2011](#)). In this model, a modified potential temperature $\theta_m = \theta(1 + \varepsilon q_v)$ is defined ([Klemp et al. 2007](#)), and details related to this definition are given in [appendix B](#); $\varepsilon = R_v/R_d \approx 1.608$. If $\varepsilon = R_v/R_d - 1$ and q_v becomes specific humidity $s_v = \rho_v/\rho_m$, then θ_m becomes virtual potential temperature θ_v . All temperature-like and potential-temperature-like fields are related by $T/\theta = T_v/\theta_v = T_m/\theta_m = (p/p_0)^{R_d/c_{pd}}$, where

$(p/p_0)^{R_d/c_{pd}}$ is the Exner function. The definition of T_v and T_m lies in the different formulations of the moist gas-state law; that is, $p = \rho_m R_d T_v = \rho_d R_d T_m$, which can be rewritten to a θ_m form

$$p = p_0 \left(\frac{R_d \theta_m}{p_0 \alpha_d} \right)^\gamma, \quad (3)$$

where $\gamma = c_{pd}/c_{vd}$ is the ratio of the heat capacities. A scaling factor $\mathcal{L}$ is defined such that

$$\mathcal{L} = \frac{\rho_d}{\rho_m} = \frac{1}{1 + q_v + q_c + q_r + \dots}, \quad (4)$$

where q_c and q_r are the dry mixing ratios for cloud water and rainwater, respectively. Note that q_v is necessary for a moist model, while the use of q_c , q_r , and their additional counterparts depends on the specific physics package. The mixing ratios q_c and q_r are given here because they are used in the simple-physics package. The semidiscrete layer-averaged nonhydrostatic moist governing equations read

$$\frac{\partial(\pi_s - \pi_t)}{\partial t} + \int_{\eta_t}^{\eta_s} \nabla \cdot (\delta \pi \mathbf{V}) = 0, \quad (5)$$

$$\frac{\partial(\delta \pi \theta_m)}{\partial t} + \nabla \cdot (\delta \pi \mathbf{V} \theta_m) + \delta \left(\frac{\partial \pi}{\partial \eta} \dot{\eta} \theta_m \right) = \delta \pi S(\theta_m), \quad (6)$$

$$\begin{aligned} \frac{\partial u_n}{\partial t} + \zeta_p \delta \pi u_t + \frac{\partial(\text{KE})}{\partial n} + \dot{\eta} \frac{\partial u_n}{\partial \eta} = \mathcal{L} \left[-\frac{1}{\rho_d} \frac{\partial p}{\partial n} - \frac{\delta p}{\delta \pi} \frac{\partial \phi}{\partial n} \right] \\ + S(u_n), \end{aligned} \quad (7)$$

$$\frac{\partial w}{\partial t} + \mathbf{V} \cdot \nabla w + \dot{\eta} \frac{\partial w}{\partial \eta} = \overline{\mathcal{L} g \frac{\delta p}{\delta \pi}} - g + S(w), \quad (8)$$

$$\frac{\partial \phi}{\partial t} + \mathbf{V} \cdot \nabla \phi + \dot{\eta} \frac{\partial \phi}{\partial \eta} = \overline{w g} + S(\phi), \quad (9)$$

$$\frac{\delta \phi}{\delta \pi} = -\alpha_d, \quad (10)$$

$$p = p_0 \left(-\frac{R_d \delta \pi \theta_m}{p_0 \delta \phi} \right)^\gamma, \quad (11)$$

$$\frac{\partial(\delta \pi q_i)}{\partial t} + \nabla \cdot (\delta \pi \mathbf{V} q_i) + \delta \left(\frac{\partial \pi}{\partial \eta} \dot{\eta} q_i \right) = \delta \pi S(q_i). \quad (12)$$

In Eq. (5), π_t is actually a time-invariant constant that does not generate time tendency. The overbars in Eqs. (8) and (9) denote terms associated with the vertically semi-implicit computation. From Eq. (5), the dry-mass continuity equation exactly conserves the dry air mass. The change of $\delta \pi$ is due to the horizontal mass flux

divergence, the conservation law, and the setup of the Eulerian vertical coordinate. For the tracer transport equation, Eq. (12), q_i denotes the dry mixing ratio of each tracer, and $i \in [v, c, r, \dots]$ represents different moist species. Equations (5) through (11) basically maintain their mathematical formulations as in the dycore, except that some variables (e.g., θ_m) now possess different physical meanings, and a scaling factor $\mathcal{L}$ is required in front of the horizontal and vertical pressure gradient force (PGF) terms. Hence, the numerical solution procedure of the dycore is largely retained, with some minor modifications to include moist variables. The tracer transport module is independent from the dycore, and the interplay between the dycore and tracer components leads to a moist adiabatic model. As $\mathcal{L}$ is determined by the mixing ratio, this establishes a connection between tracer transport and the PGF term.

If a moist hydrostatic model is needed, the hydrostatic equation is based on Eq. (1):

$$\frac{\delta\phi}{\delta\pi_m} = -\frac{R_d T_v}{\pi_m}. \quad (13)$$

The diagnostic relation between π and π_m is

$$\frac{\delta\pi_m}{\delta\pi} = \frac{\alpha_d}{\alpha_m} = \frac{1}{\mathcal{L}}. \quad (14)$$

Equation (14) is used for evaluating π_m at the face level, and the full-level π_m value is an average of two neighboring face-level values, which are used in the vertical integral of Eq. (13). Equation (14) also suggests that, as long as $\mathcal{L}$ remains unchanged, $\delta\pi_m$ and $\delta\pi$ should have a fixed relation regardless of the instantaneous state of α_d and α_m . Therefore, α_d for the moist system is diagnosed as $\alpha_d = -\delta\phi/\delta\pi$, which is then used as an input for the horizontal PGF. For the HDC, in the absence of condensates, the partial pressure of dry air p_d may be physically regarded as the dry air mass π , and α_d may also be expressed as $R_d T/\pi$. However, α_d written in this way would violate the constraint exerted by $1/\mathcal{L}$, because $R_d T/\pi$ is hardly equal to $-\delta\phi/\delta\pi$ during the model integration because Eq. (2) is not used to constrain the moist hydrostatic system, and moisture is always present. Note that p_d should not be physically equal to π provided that condensate is present (cf. Lauritzen et al. 2018).

b. A unified workflow

Before going into the details of the DTP coupling, the general workflow of the model is first shown to facilitate the following description. As shown in Fig. 1, the entire runtime modeling environment is separated into four

parts, including a model control unit and three sub-components: dycore, tracer, and physics. The time scales for these components include the dycore_step (τ_d), the tracer_step (τ_t), and the model_step (τ_m), and $\tau_d \leq \tau_t \leq \tau_m$; τ_t/τ_d and τ_m/τ_t are integers in the current version; τ_{rk} denotes each RK substep within τ_d . During the model integration, the dycore component occupies the inner cycle, the tracer component occupies the intermediate cycle, and the model control unit occupies the outer cycle. The time scale for physics depends on the cycle in which it is called. $\tau_p = \tau_m$ is used for all results shown in this paper. For a complete τ_m from time level N to $N+1$, the control unit arranges the sequence and frequency of calling different subcomponents, and performs model data diagnostics and input/output.

c. Tracer transport and its coupling to dycore

The horizontal and vertical transport terms in the tracer transport equation are evaluated in a time-split manner. For horizontal transport, the basic operator is a discrete divergence operator based on the Gauss theorem (nominally second-order accurate), while the flux operators may have more flexible choices. Some specific numerical methods for the flux operators have been described and evaluated in several earlier studies (Z19; Wang et al. 2019; Zhang 2018; Zhang et al. 2017). A list of the horizontal tracer transport methods and their acronyms is given in Table 2. In this paper, we use RK3O3, a purely upwind high-order flux formulation (Skamarock and Gassmann 2011) combined with a third-order Runge–Kutta (RK) integrator (Wicker and Skamarock 2002), an incremental-remapping style flux-form semi-Lagrangian (FFSL) based on a quadratic reconstruction [Miura and Skamarock 2013; their first upwind-biased quadratic approximation (UQA-1)], and an unstructured-mesh version of the two-step shape-preserving advection scheme (TSPAS; Zhang et al. 2017; Yu 1994). The flux-corrected transport (FCT) limiter (Zalesak 1979) is used to preserve the monotonicity for RK3O3 and FFSL. RK3O3 and FFSL are two high-order flux operators of similar numerical accuracy (RK3O3 is slightly more accurate if its upwinding effect is reduced); TSPAS is a low-order shape-preserving operator but it has the lowest cost. RK3O3 and TSPAS perform scalar reconstruction without information on local wind magnitudes (they only need the sign), while FFSL needs both normal and tangent velocities at the edge point for approximating the upstream swept parallelogram area. Here, this time-averaged normal velocity is obtained by dividing the time-averaged normal mass flux with the time-averaged layer thickness, both of which are obtained from the dycore cycle. The tangent velocity is reconstructed from the time-averaged normal

FIG. 1. Schematic view of a unified modeling workflow, including four major components (control unit, dycore, tracer transport, and physics), and their layout in the model integration, where the dycore, tracer transport, and physics (DTP) are controlled via a splitting approach; this figure gives an example with a DTP splitting number of $2\tau_d = \tau_t = \tau_m(\tau_p)$.

velocity using the C-grid vector reconstruction method (Thuburn et al. 2009) to ensure its compatibility with the normal velocity. Thus, while FFSL only needs to evaluate the normal flux once per tracer_step, this additional vector reconstruction procedure somewhat affects its efficiency for a C-grid model.

For vertical transport, two choices are considered in this paper for approximating the vertical flux at the layer

interface. The first one is a standard line-based third-order flux operator with a third-order RK integrator (Wicker and Skamarock 2002; RK3O3), which is also used by the dycore component. The second one is a piecewise parabolic method (PPM; Colella and Woodward 1984), and the implementation here follows that given in the appendix of Skamarock (2006), adapted to the mass coordinate:

$$\left\{ \begin{array}{l} q_{ii+1/2} = \hat{q}_{ii+1/2} - \text{Cr}_{ii+1/2}(\hat{q}_{ii+1/2} - q_{ii+k}) - |\text{Cr}_{ii+1/2}|(1 - |\text{Cr}_{ii+1/2}|)(\hat{q}_{ii-1/2+k} - 2q_{ii+k} + \hat{q}_{ii+1/2+k}) \\ \hat{q}_{ii+1/2} = \frac{[7(q_{ii} + q_{ii+1}) - (q_{ii-1} + q_{ii+2})]}{12} \\ \text{Cr}_{ii+1/2} = \frac{\left(\frac{\partial \pi}{\partial \eta} \dot{\eta}\right)_{ii+1/2} \Delta t}{\delta \pi} \\ k_{ii+1/2} = \frac{\text{Cr}_{ii+1/2} - |\text{Cr}_{ii+1/2}|}{2\text{Cr}_{ii+1/2} + \mathcal{J}} \end{array} \right. \quad (15)$$

where $\mathcal{J}$ is a very small number (e.g., 10^{-80}) such that $k_{ii+1/2} = 0$ when $\text{Cr}_{ii+1/2} \geq 0$, and $k_{ii+1/2} = 1$ when $\text{Cr}_{ii+1/2} < 0$, $ii \pm 1/2$ denotes the face level and ii denotes the full level. The vertical limiter is also FCT.

Before the dycore–tracer coupling, we tested the isolated tracer module using the Dynamical Core Model Intercomparison Project (DCMIP) 3D passive transport test case (Kent et al. 2013). Here, the results of the convergence test from the Hadley-like circulation test (DCMIP1–2) are shown in Fig. 2. The resolutions are G5L30, G6L60, and G7L120, and the vertical resolutions follow the recommended choices in Kent et al. 2013.

The time steps are 600 s (G5L30), 300 s (G6L60), and 150 s (G7L120). For this test, the PPM-based solver has three subcycles for vertical transport within a full tracer time step, leading to the same iterative count as the RK3O3-based solver. Besides being a convergence test, this test mainly challenges the horizontal–vertical coupling, so it is expected that the solutions are not very sensitive to different horizontal flux operators (as shown). By examining l_1 , l_2 , and l_∞ error norms, it is found that the PPM-based 3D solvers generally generate smaller absolute errors than the RK3O3-based solvers at each resolution, but the RK3O3-based solvers exhibit higher

FIG. 2. The 3D passive transport test (Hadley-like meridional circulation): (a),(b) numerical solution errors in terms of l_1 , l_2 , and l_∞ norms for the RK3O3- and PPM-based (vertical transport) solvers, with horizontal transport performed by three different schemes, where the gray lines denote the ideal first- and second-order convergence rate. (c),(d) Sample solutions at 12 and 24 h for the RK3O3-based solver with RK3O3 as horizontal transport. (e),(f) As in (c) and (d), but horizontal transport is done via TSPAS.

convergence rates (approaching second-order from G6L60 to G7L120). The solution errors are comparable to those shown in Kent et al. (2013), and generally indicate the robustness of the passive transport solver. The supercell test will examine both solvers, while other tests only use the RK3O3-based solver.

In terms of the dycore–tracer coupling, the dycore loop is first performed, and the normal mass flux after the final RK step of each dycore_step is accumulated during the dycore–tracer subcycle. The time-averaged normal mass flux $(\delta\pi\mathbf{V})_t$ during τ_t is used for horizontally transporting the tracer mass $\delta\pi q_i$ with one τ_t . The layer thicknesses

before and after the tracer integration are saved as $\delta\pi^N$ and $\delta\pi^{N+\tau_t}$, both of which are controlled by the dycore update. The vertical mass flux $[(\partial\pi/\partial\eta)\hat{\eta}]_t$ is obtained by reevaluating the mass continuity equation based on $(\delta\pi\mathbf{V})_t$, as in Eqs. (26) and (27) in Z19. After each tracer_step, the tracer transport module updates the tracer mass, and diagnoses the tracer mixing ratio based on the updated tracer mass and $\delta\pi^{N+\tau_t}$. The updated dry mixing ratio is used to evaluate the $\mathcal{L}$ needed by the next dycore subcycles. Thus, for a tracer_step from time level N to $N + \tau_t$, the horizontal transport equation is first integrated as (in forward-in-time notation)

$$(\delta\pi q)^h = (\delta\pi q)^N - \tau_t \nabla \cdot \left[\overline{(\delta\pi \mathbf{V})}_t q^N \right], \quad (16)$$

$$(q)^h = \frac{(\delta\pi q)^h}{\delta\pi^{N+\tau_t}}. \quad (17)$$

The vertical transport solver updates the tracer mass after the horizontal update:

$$(\delta\pi q)^{N+\tau_t} = (\delta\pi q)^h - \tau_t \delta \left[\overline{\left(\frac{\partial \pi}{\partial \eta} \right)}_t (q)^h \right], \quad (18)$$

$$(q)^{N+\tau_t} = \frac{(\delta\pi q)^{N+\tau_t}}{\delta\pi^{N+\tau_t}}. \quad (19)$$

As already mentioned, the horizontal and vertical transport can also be subcycled by choosing different time steps to complete their own integration, and the calling sequence may also be reversed (i.e., horizontal transport follows vertical transport) to allow more flexible configurations. These variations are not considered in the following moist model test.

d. Physics packages

The simple-physics packages of the 2016 DCMIP (DCMIP2016) were used to examine the model performance under multiscale scenarios. These include a Kessler warm-rain microphysics scheme (cf. Klemp et al. 2015), a simplified physics package from Reed and Jablonowski (2012), and an extended simplified physics package for climate modeling [moist idealized test case (MITC); Thatcher and Jablonowski 2016]. The latter two packages contain several different physical processes, which are coupled in a time-split manner. The variables used for physics are collocated at the cell center, and some additional treatments are needed for the horizontal wind field. The zonal and meridional winds (U , V) at the cell center used for physics are reconstructed based on the edge-based normal velocities using the Perot reconstruction (cf. Z19, their appendix B). The cell-based physical tendencies for U and V at two neighboring cells are averaged to obtain the edge-based $(\partial U/\partial t)_e$ and $(\partial V/\partial t)_e$, which are then converted to the edge-based normal velocity tendency $(\partial u_n/\partial t)_e$ using a vector conversion procedure. These physical tendencies are then integrated to the dynamical equations depending on the specific PDC option (see section 2e). Meanwhile, it should be pointed out that the pressure field used by the physics packages is π_m , which is in tune with the assumptions in the physics. For the NDC, this helps to eliminate the acoustic perturbation in the full pressure field, while for the HDC, $p = \pi_m$ is full pressure due to the hydrostatic shallow-atmosphere constraint.

As in the dycore test, a flow-dependent Smagorinsky horizontal diffusion is activated. It is evaluated at the

beginning of each model step, and the tendencies are integrated within the RK integration (for the dycore variables), and after the passive transport processes (for the tracer variables). Note that in the splitting supercell test, a constant-coefficient second-order diffusion replaces the Smagorinsky-style diffusion.

e. Dynamics–physics coupling

The general equation for describing a PDC procedure is

$$\frac{d\mathcal{H}(t)}{dt} = D(\mathcal{H}) + P(\mathcal{H}), \quad (20)$$

where $d\mathcal{H}(t)/dt$ represents a Lagrangian derivative for a time-dependent variable, $D(\mathcal{H})$ denotes the dynamical (resolved scale) processes other than advection (if any), and $P(\mathcal{H})$ denotes the physical (underresolved dynamics or nondynamical) processes. Because the integration is done by the model dynamics, the PDC procedure may be viewed as the way how the model dynamics assimilates the tendencies generated by the physics. PDC depends on both the model dynamics and physics (see Gross et al. 2018 for a review). In this study, we only focus on limited options and their possible differences. For illustrative purposes, we decompose the PDC procedure to two components: operator and workflow. A PDC operator represents how the physics tendency is exerted to the state variables, and a PDC workflow represents how the PDC operators are managed, that is, how to fill them with appropriate physics tendencies and how to use them to update the model state.

1) PDC OPERATORS

As shown in Fig. 1, the current model has three PDC operators—ptend_f1, ptend_f2, and ptend_rk—but they are not necessarily used simultaneously. Both ptend_f1 and ptend_f2 update the state variables in a sequential operator splitting approach, which is theoretically first-order accurate unless special manipulation is taken. In ptend_f1, the local copies of the physical tendencies for the dycore [$S(u_n)$, $\delta\pi S(\theta_m)$] and for the tracer [$\delta\pi S(q_i)$] are used to update their respective prognostic variables:

$$\mathcal{H}^{\text{dycore}} \xrightarrow{\text{ptend_f1} \times \tau_d} \mathcal{H}^*, \quad (21)$$

$$\mathcal{H}^{\text{tracer}} \xrightarrow{\text{ptend_f1} \times \tau_t} \mathcal{H}^*. \quad (22)$$

In this approach, a PDC update of the dycore and tracer variables can be done before or after the dycore/tracer integration; we choose to use them after the integration in this paper. Ptend_f2 updates all dynamical state variables at the same time step:

$$\mathcal{H}^{\text{dynamics}} \xrightarrow{\text{ptend_f2} \times \tau_x} \mathcal{H}^{N+\tau_x}, \quad (23)$$

which may be further organized into ptend_f2_sudden ($\tau_x = \tau_m$) and ptend_f2_dribbling ($\tau_x = \tau_t$); see [section 2e\(2\)](#) for details. For all these operator splitting approaches, the model dynamics is solved without any physical forcing (except explicit diffusion, which may sometimes be viewed as a physical process). In contrast, ptend_rk, which is also known as RK physics or the tendency method (e.g., [Skamarock et al. 2008](#); [Husain et al. 2019](#)), includes the physical forcing into the discrete dycore integration:

$$\mathcal{H}^{**} \xrightarrow{\text{ptend_rk} \times \tau_{\text{rk}}} \mathcal{H}^{***}, \quad (24)$$

where $\mathcal{H}^{**}$ and $\mathcal{H}^{***}$ are the dycore state variables before and after this RK substep. The physical tendency is written on top of the arrow to indicate that the dycore update has been blended with the physical forcing.

Previous studies (e.g., [Caya et al. 1998](#); [Dubal et al. 2005](#); [Termonia and Hamdi 2007](#)) have suggested that the operator splitting approach is able to allow larger dynamics–physics coupled time steps and is thus theoretically more stable. The truncation error is generally acceptable, except when the time step becomes too large. A time-splitting update also has an advantage insofar as the model-generated steady state is independent of the time step ([Beljaars et al. 2018](#)). The tendency method (ptend_rk) is more appropriate for filling with physics forcing that does not challenge the numerical stability (e.g., [Termonia and Hamdi 2007](#); [Zängl et al. 2015](#)), because it is usually more restricted in terms of large–time step stability. Moreover, we have also found that for GRIST-NDC, it is important to carry the diabatic heating over the acoustic integration. These three PDC operators can be flexibly organized into various PDC workflows. Since only simple physics is considered here, all physics packages are called once at the end of each model step (or based on the initial state), and they provide a total forcing. The next sections describe two major PDC workflows.

2) PDC WORKFLOW I: SUDDEN ADJUSTMENT

The sudden adjustment only uses ptend_f2. The two variants, ptend_f2_dribbling and ptend_f2_sudden, differ in the frequency and time increment of adding the physical tendencies. Ptend_f2_dribbling splits the physics update into several small sudden steps within a model step, while ptend_f2_sudden only updates once per model step. They loosely correspond to the options of ftype=0 and ftype=1 in the Community Atmosphere Model (CAM) or its independent branch, but note

that there are some minor differences (e.g., the calling sequence of physics and dynamics; where to call the explicit diffusion procedure). In this paper, the ptend_f2_dribbling option is called before each subcycled dynamical integration (dycore+tracer), while ptend_f2_sudden is used immediately after the evaluation of the physics tendencies (except the initial evaluation). This means that when $\tau_p = \tau_t$, ptend_f2_dribbling remains a time-splitting approach. For more details about the process-splitting versus the time-splitting approach, see, for example, [Williamson \(2002\)](#) and [Dubal et al. \(2004\)](#).

The pros and cons of the sudden and dribbling approaches have been discussed. For example, it has been shown that for CAM-SE (SE indicates spectral element), the sudden approach tends to trigger undesired large-scale gravity waves, while the dribbling approach does not ([Thatcher and Jablonowski 2016](#); [Lauritzen and Williamson 2019](#)). This seems to be a model-dependent feature. [Lauritzen and Williamson \(2019\)](#) suggested that the dribbling approach may lead to spurious sources or sinks of mass and total energy. [Zhang et al. \(2018\)](#) showed that a change of the tracer-physics coupling from the dribbling to the sudden approach leads to a large reduction in water conservation errors. In general, we prefer to use ptend_f2_sudden for our model, while the ptend_f2_dribbling option is also tested in this paper [[section 3b\(2\)](#)].

3) PDC WORKFLOW II: INCREMENTAL ADJUSTMENT

The incremental adjustment splits the PDC update into a number of smaller-step adjustments that conform with the time scale of the dycore and tracer, respectively. In this paper, the physical tendencies are added after the dycore or tracer integration. This means that when $\tau_d = \tau_t = \tau_p$, this approach leads to a process-splitting between dynamics and physics (when physics is called at the end of each model step, as in this study). If the time steps of these subcomponents are not equal, then although physical tendencies remain unchanged during a model step, the model states that work with these tendencies will be altered at each substep, leading to time-split coupling.

The dycore update has two variants. The physical tendencies $\delta\pi S(\theta_m)$ and $S(u_n)$ may be added by using either ptend_rk or ptend_f1. When using ptend_f1, both the dycore and tracer state variables are updated in an operator splitting manner. This PDC option is referred to as ptend_f1_f1. When using ptend_rk, both $\delta\pi S(\theta_m)$ and $S(u_n)$ are added in the dynamical integration, and the details are given in [Fig. 3](#). This option is referred to as ptend_f1_rk, and it is the only PDC option in this

FIG. 3. Flowchart comparing the two PDC options: (left) ptend_f2_sudden (based on the modified operator splitting approach) and (right) ptend_f1_rk (based on the tendency method).

paper that treats the HDC and NDC in exactly the same manner.

4) MODIFIED OPERATOR SPLITTING

We have found that for GRIST-NDC, when using the operator splitting approach (both ptend_f1 and ptend_f2), it is necessary to include the diabatic heating over the acoustic integration, or erroneous results will be generated. Hence, the following procedure has been taken and found to alleviate this problem well. We use ptend_f2_sudden to illustrate this modified operator splitting (Fig. 3). The diabatic heating $\delta\pi S(\theta_m)$ generated by the physics package in the last model step (if this is the first model time step, then the tendencies come from the initial state) is added to the dynamical integration of the thermodynamic equation as a forward step within each RK loop. After all RK loops of a dycore_step are finished, this diabatic heating tendency is subtracted from the model state with τ_d . When both the dycore and tracer cycles are finished, the latest prognostic variables, which have been updated by the dycore and tracer components, are used for evaluating the new physical tendencies, and then perform an update. With such an additional treatment

of diabatic heating, the NDC can share the same operator splitting workflow with the HDC. This also implies that in the ptend_f1_f1, the diabatic heating is actually added by a tendency method instead of an operator splitting method.

This modified approach is similar to that used by the WRF-ARW model (Skamarock et al. 2008) when dealing with diabatic heating from non-RK physics. The major difference is that, in the operator splitting approach, there is no RK physics except the explicit horizontal diffusion (if activated). If this modification is turned off, the NDC will produce computational noise [see section 3b(2) for details]. The robustness of the modified operator splitting approach has been largely examined by all test cases in this paper. That said, some MITC tests have also suggested that when using ptend_f2_sudden for long-term NDC integration, a more careful time step configuration is needed to ensure stability [e.g., see Fig. S10 in the online supplemental material; this issue is somewhat different from the stability issue as we discuss in section 3b(2)]. Though we currently do not have a theoretical analysis, we view this issue and the necessity to use a modified operator splitting as related to the interaction between the acoustic solver and the physics

TABLE 3. Model grids and their nominal resolutions used in this study.

Notation	Averaged spherical grid distance	Averaged spherical grid distance on a reduced-radius Earth
G4	~480 km	~4 km
G5	~240 km	Not tested
G6	~120 km	~1 km
G7	~60 km	~0.5 km
G8	~30 km	~0.25 km
G8B3	~10 km	Not tested

forcing. The tendency method does not show such sensitivity and is more robust with this regard.

f. Model configuration and initialization

As this paper focuses on the coupling of model components and the performance of a moist model, the configuration for the dycore is held fixed. The third-order RK time integrator is used, and all nonmass numerical fluxes (ζ_p , θ_m , w , and ϕ) are approximated in a purely upwind third-order flux operator, which only needs to reconstruct the local second-order derivatives at the upwind cell (Skamarock and Gassmann 2011; Zhang 2018). The mass flux is approximated with a two-point average. Other dycore configurations are similar to those used in Z19 for a given resolution. Meanwhile, we have summarized all the grid resolutions used in this paper in Table 3, and all the investigated model options are given in Table 4. Other specific configurations will be mentioned along with each test case.

As mentioned in the introduction, models with a dry-mass coordinate require some additional procedures for extracting dry air mass. For idealized DCMIP2016 test cases (sections 3a–3c) with analytic initial conditions based on the geometric height, an approach similar to that in Lauritzen et al. (2018) is used. Assuming $p = \pi$ at the model top, and π_t is known from the vertical coordinate configuration, then z_t can be obtained based on π_t and the DCMIP testing scripts (based on a fixed-point iterative procedure). Next, π_s at the surface can be evaluated by numerically integrating (a 20-point Gauss integral),

$$\frac{\partial \pi}{\partial z} = -\rho_d g = -\frac{p}{R_d T_v} g(1 - s_v), \quad (25)$$

from top to surface:

$$\pi = \pi_t - \int_{z_t}^z \frac{p}{R_d T_v} g(1 - s_v) dz. \quad (26)$$

Then, π_s leads to π at the face and full levels. The value of π at the face level and the numerical integral relation,

TABLE 4. Perturbed model options in this paper (for tests in section 3).

Component	Options
Dycore	NDC, HDC
PDC	ptend_f1_rk, ptend_f2_sudden, ptend_f1_f1, ptend_f2_dribbling
DTP splitting	$m\tau_d = n\tau_t = \tau_m(\tau_p)$, where m , n , and m/n are integers
Horizontal transport	RK3O3, TSPAS
Vertical transport	RK3O3, PPM

Eq. (26), are then used in a fixed-point iterative procedure (similar to those in the DCMIP testing scripts) to evaluate z at the face level. At the full level, z is an average of two neighboring face-level values. The height is used as the final input for the DCMIP scripts to produce the analytic model state.

We have also performed tests based on realistic initial and topographic data (section 3d). We use the ERA-Interim model-level dataset (60 full levels and 0.75° horizontal resolution) as the initial condition. The surface topography is generated using the NCAR topo software (Lauritzen et al. 2015). The dry air mass at the surface is extracted based on Eq. (14) with $\pi = \pi_m$ at the model top. We can then obtain the model-level dry hydrostatic pressure for our model and for the ERA-Interim dataset. Then, a linear interpolation procedure is used to interpolate the ERA-Interim-based initial conditions to GRIST-based initial conditions. For NDC, the vertical speed is initialized as zero and the geopotential is obtained based on the hydrostatic equation. The horizontal remapping from a regular grid to an unstructured grid is done via an interpolation function of the climate data operators (cf. Schulzweida 2019).

3. Model behaviors

a. Splitting supercell thunderstorms

The splitting supercell test (Klemp et al. 2015) adopts the small-planet testing framework (Kuang et al. 2005; Wedi and Smolarkiewicz 2009) such that an examination of global modeling under the typical nonhydrostatic regimes becomes an economic practice. Earth's radius is reduced by a factor of 120. By using constant-coefficient explicit second-order diffusion ($500 \text{ m}^2 \text{ s}^{-1}$ for the momentum equations and $1500 \text{ m}^2 \text{ s}^{-1}$ for the scalar equations) and a Kessler warm-rain microphysics scheme as simplified physics, a global nonhydrostatic model is expected to produce converged solutions as the horizontal resolution is refined. Results of DCMIP2016 (Zarzycki et al. 2019) have demonstrated that the state-of-the-art

FIG. 4. Splitting supercell thunderstorm. Rainwater mixing ratios at four horizontal resolutions: (left to right) ~ 4 , ~ 1 , ~ 0.5 , and ~ 0.25 km. Results using (top) RK3O3-PPM and (bottom) TSPAS-PPM. The DTP splitting number is $2\tau_d = \tau_r = \tau_p$, and results are shown at 5 km after 7200 s.

global nonhydrostatic models tend to produce non-negligible differences in this test, and various factors in the model design may be the reasons.

For explicit modeling of the convection process, it is beneficial for the microphysics scheme to work as an

adjustment procedure for both moisture and temperature, which guarantees that the final saturation balance is accurate for the updated temperature and moisture (Skamarock et al. 2008). Therefore, only `ptend_f2_sudden` is considered as the PDC option here. [Figure 4](#)

FIG. 5. Simulated supercell thunderstorms with different DTP splitting numbers: (a) $4\tau_d = \tau_t = \tau_m$; (b) $6\tau_d = \tau_t = \tau_m$; (c) $8\tau_d = \tau_t = \tau_m$; (d) $12\tau_d = \tau_t = \tau_m$; (e) $12\tau_d = 3\tau_t = \tau_m$, with $\tau_d = 0.375$ s. The horizontal tracer transport scheme is RK3O3 and the vertical transport scheme is PPM. The horizontal resolution is 0.25 km. Results are shown for rainwater mixing ratio (kg kg^{-1}) at 5 km after 7200 s.

compares the 5-km q_r mixing ratios at four resolutions (4, 1, 0.5, and 0.25 km); the vertical resolution is ~ 500 m with a top at ~ 20 km (40 full vertical levels). Here, the vertical transport is done with PPM, and the horizontal transport uses both RK3O3 and TSPAS, leading to a comparison of two groups. The DTP splitting number is $2\tau_d = \tau_t = \tau_p$. At 7200 s, the supercell splits to two quasi-symmetric centers across the equator. There are small updraft structures in the equator, which are also found in some DCMIP models (Zarzycki et al. 2019). At 4 km, the model already has the ability to resolve the overall structure of the supercell, but the grid-scale rainfall is too high due to the relatively coarse grid space, implying that a 4-km resolution is effective but still not sufficient to fully resolve convection. The rainwater maximum magnitudes in the 1-, 0.5-, and 0.25-km runs are generally comparable, suggesting that a 1-km resolution is able to retain many features of finer-scale convection modeling. For a particular transport scheme, the difference from 0.5 to 0.25 km is smaller than the difference from 1 to 0.5 km, suggesting that the solutions almost converge at 0.5 km. At 0.25 km, results from RK3O3-PPM and TSPAS-PPM exhibit remarkable similarity, also implying that the supercell has been well resolved and the solution is converged at this resolution.

To examine possible sensitivity associated with the DTP splitting, we performed several additional tests at 0.25 km using the RK3O3-PPM option. Figures 5a and 5b show results from $4\tau_d = \tau_t = \tau_p$ and $6\tau_d = \tau_t = \tau_p$. At these not-small splitting numbers, the general structures of the supercell are similar to the default case ($2\tau_d = \tau_t = \tau_p$), while the maximum intensity in the supercell center tends to decrease as the splitting number increases. We then further increased the splitting number to some very large cases. In the $8\tau_d = \tau_t = \tau_p$ case (Fig. 5c), the supercell begins to show larger differences from the $2\tau_d = \tau_t = \tau_p$ case. In the $12\tau_d = \tau_t = \tau_p$ case (Fig. 5d), the shape of the supercell differs greatly from that in the default case; the maximum intensity is substantially reduced, and more updrafts are generated at the equator. Figure 5e shows the result from $12\tau_d = 3\tau_t = \tau_p$, where the time steps of the dycore and tracer are identical to those in the $4\tau_d = \tau_t = \tau_p$ case, but the physics step is identical to that in Fig. 5d. In this case, the shape and maximum intensity are closer to the default $2\tau_d = \tau_t = \tau_p$ case as compared to the $12\tau_d = \tau_t = \tau_p$ case, but still differs from the $4\tau_d = \tau_t = \tau_p$ case. These results suggest that both the dycore–tracer splitting number and the dynamics–physics splitting number should be carefully examined to ensure that a very large splitting number

does not significantly degrade the model performance. This point will be further discussed in [section 3b\(2\)](#).

Figure 6 compares the evolution of the area-integrated rainfall rate and domain maximum vertical speed at four resolutions (4, 1, 0.5, 0.25 km). For each resolution, the models are tested with six different tracer transport options (three horizontal operators and two vertical operators) using the $2\tau_d = \tau_t = \tau_p$ option. Results from the RK3O3-PPM option are plotted as lines, and the root-mean-square deviations ($\pm$) of other five options against the RK3O3-PPM option are plotted as the shaded range. Except for the 4-km tests, all tests generate comparable results overall with regard to these two metrics. Results from the two high-resolution tests (0.5 and 0.25 km) are a little bit closer, but the 1-km test can retain most features found in the higher-resolution tests. The root-mean-square deviation at each resolution is generally limited to a relatively small range, suggesting a robustness of the model. The magnitude of rainfall and domain maximum vertical speed are generally in accordance with those found in the DCMIP models.

b. Idealized tropical cyclone

This test is based on the analytic vortex initialization technique of [Reed and Jablonowski \(2011\)](#) and a simple-physics package described in [Reed and Jablonowski \(2012\)](#). The simulated tropical cyclone (TC) in this simple-physics package resembles that in the full-physics modeling well, leading to important implications for real-world weather modeling. This package contains three processes: large-scale condensation, surface flux, and boundary layer processes. These processes may be viewed as fast physics, which should ideally be added by an operator splitting approach ([Dubal et al. 2005](#); [Termonia and Hamdi 2007](#)). However, for testing purposes, several different PDC options were examined. In this section, we first present an overview of the model performance; then, we discuss the choice and consideration associated with different PDC options. Moreover, we will also examine the performance from two tracer transport options: RK3O3 and TSPAS. As mentioned in [section 2a](#), tracer transport in this model determines the scaling factor that the PGF terms need, and PGF is important in regulating the TC intensity. Thus, it will be useful to find whether such an impact is significant.

1) OVERALL PERFORMANCE

We first utilize global 10-km simulations with 30 full vertical levels to generate a reference for lower-resolution tests. While this TC test does not exhibit convergence as the horizontal resolution is refined, a high-resolution reference is helpful to understand the resolution sensitivity. This also helps us to examine the

FIG. 6. Supercell thunderstorm. The (a) area-integrated rainfall rate and (b) domain maximum vertical speed obtained from the supercell simulations. The lines denote the configuration using RK3O3-PPM as the tracer transport option, and the shaded area denotes the root-mean-square deviation of five other tracer transport configurations against the default configuration. See the main text for details.

possible differences between the HDC and NDC at a resolution at which the theoretical assumption of hydrostatic balance tends to break down. Some studies (e.g., [Qi et al. 2018](#)) have reported that the simulated TC in a real-world model is sensitive to the choice of the hydrostatic and nonhydrostatic options. The 10-km mesh is obtained by trisecting the G8 grid (G8B3, 5 898 242 primal cells), with 20 000 step spherical centroid Voronoi tessellation (SCVT) iteration. We have used the message passing interface (MPI) SCVT software ([Jacobsen et al. 2013](#)) to facilitate the generation of this high-resolution tessellation. The PDC option is `ptend_f2_sudden`. Both RK3O3 and TSPAS are used for NDC and HDC simulations, respectively. The time steps are $6\tau_d = 2\tau_t = \tau_p = 90\text{ s}$ for all four runs. The time steps can be larger, but we found in a previous test (HDC with TSPAS) that $\tau_p = 120\text{ s}$ with the same DTP splitting number tends to produce a slightly more oblate TC center. Although this does not seem to be a disadvantage, we opt for a more conservative choice.

Results from these four tests are shown in [Fig. 7](#). Generally, the features of the TC (including the center location and wind strength) are comparable overall. The minor difference lies in that the HDC tends to produce a

FIG. 7. Idealized TC. The simulated wind speed (m s^{-1}) at the G8B3 resolution for the NDC and HDC, including two tracer transport options (RK3O3 and TSPAS). Results are shown for (a)–(d) the vertical structures and (e)–(h) the horizontal structure at 1500 m. Note that results for the vertical structure have been interpolated to a 0.25° regular latitude–longitude grid, and the horizontal map is plotted on the raw unstructured mesh.

slightly larger area for the strong winds: for a wind contour that ranges from 20 to 25 m s^{-1} , the average distances in degrees from the TC centers are $\sim 2.45^\circ$ (NDC-RK3O3), $\sim 2.59^\circ$ (HDC-RK3O3), $\sim 2.60^\circ$ (NDC-TSPAS), and $\sim 2.81^\circ$

(HDC-TSPAS). While this contour is not perfectly round, it approximately indicates the area size of strong winds. The TSPAS runs also produced slightly larger sizes for this strong-wind circle, but tended to simulate lower wind

TABLE 5. Time steps (s) used in the 48 TC tests, for (row 1) non-splitting and (rows 2–4) DTP splitting options.

	HDC- G6	HDC- G7	HDC- G8	NDC- G6	NDC- G7	NDC- G8
Nonsplit	300	120	60	150	90	40
τ_m	1200	720	360	1200	720	320
τ_t	600	360	180	600	360	160
τ_d	300	120	60	150	90	40

maxima than the RK3O3 runs. The higher extreme probably reflects that RK3O3 (as a high-order flux operator) captures a sharper gradient of the moisture distribution, which may in turn affect the wind field via the PGF term.

Having these high-resolution references, we then examine the results from 48 tests by considering two PDC options (ptend_f2_sudden and ptend_f1_rk), two solvers (HDC and NDC), two horizontal tracer transport schemes (RK3O3 and TSPAS), and DTP splitting and nonsplitting options at three resolutions (G6, G7, and G8). The time steps used for the DTP splitting and nonsplitting options are given in Table 5. This time step configuration ensures that the dycore has the same time step in both options, while the influences of the transported moisture and parameterized physics on the dycore are exerted at different frequencies. We have examined the day-10 horizontal wind speed at 1500 m (Figs. S1 and S2). The TC intensity gradually increases as the resolution is refined. The locations of the TC centers (Fig. S3) are somewhat randomly distributed within a $3.5^\circ \times 4^\circ$ region, exhibiting consistent solutions. The vertical structure of the wind speed along a meridional transect ($\pm 5^\circ$ from the cyclone center) from 48 tests is given in Figs. S4 and S5. The maximum range of the vertical wind speed extends from ~ 1 to ~ 5 km, generally supporting the usefulness of choosing the 1500-m wind speed as a representative. Both the NDC and HDC simulate comparable results, while the wind maxima in the HDC tests tend to be slightly larger.

To summarize these features, Figs. 8 and 9 present the daily minimum surface pressure and the daily maximum wind speed at 1500 m for the NDC, respectively. Results for the HDC are shown in Figs. S6 and S7. With the increase of the horizontal resolution, the surface pressure at the final day decreases from ~ 980 hPa (G6) to ~ 960 hPa (G7) and ~ 940 hPa (G8); and the maximum wind speed increases from ~ 40 m s $^{-1}$ (G6) to ~ 55 m s $^{-1}$ (G7) and ~ 70 m s $^{-1}$ (G8), based on a visual comparison. Results from the DTP splitting and nonsplitting runs generally match closely, while the DTP splitting sometimes tends to produce slightly weaker wind and higher surface pressure (e.g., results of G8). However, as all

these differences seem to be limited, we may conclude that the DTP splitting does not degenerate the simulations when configured properly. Results from RK3O3 and TSPAS are generally consistent, but TSPAS tends to produce slightly weaker maximum wind intensity (or higher minimum surface pressure), especially in the DTP splitting case (e.g., Figs. 8c and 9c). This might be related to the fact that the solution of TSPAS is more sensitive to the time step, which impacts the tracer distribution, and thus the PGF and the TC strength.

2) PDC CONSIDERATIONS

In this section, we compare several PDC options with the aid of this test case. As mentioned, the simple physics of Reed and Jablonowski (2012) may be regarded as fast processes for a typical weather and climate model, which ideally should be coupled in an operator splitting manner. However, for the NDC, if diabatic heating is not included in the acoustic integration, erroneous results will be generated. Figure S8 presents such an example. In this case, the day-10 solution in the original ptend_f2_sudden test is dominated by computational noise. When the modified procedure is used, the NDC (Fig. S8b) simulates similar results as the HDC (Fig. S8c). As mentioned in section 2e(4), we view this point as an acoustic-solver-related issue (when interacting with physics) because the HDC does not have it. While the modified operator splitting solves this problem well, more tests are still needed in full-physics scenarios.

Another issue is related to the stability and accuracy, which may be examined by using different physical time steps. As discussed in earlier studies (Caya et al. 1998; Dubal et al. 2005), the operator splitting approach overall possesses higher stability and permits a larger time step; but if the time step becomes too large, the errors may be unacceptable. Based on the NDC at the G7 resolution, we have examined four PDC options (ptend_f1_f1, ptend_f1_rk, ptend_f2_dribbling, and ptend_f2_sudden) by varying τ_p by 360, 720, 1440, 1800, and 2700 s. Time steps τ_d and τ_t are 120 and 360 s, respectively. All these tests use the RK3O3 as the tracer transport option. For a 10-day forecast, it is found that only ptend_f2_sudden can survive from 2700 s; ptend_f2_dribbling and ptend_f1_rk are unstable at 2700 s; ptend_f1_f1 aborts at 1800 s. For all the stable runs, the maximum wind speeds at 1500 m are compared for four PDC options (Fig. 10). At day 10, ptend_f1_f1 and ptend_f1_rk overall generate consistent results that are not very sensitive to the time step. For two ptend_f2 cases, and especially for ptend_f2_sudden, a longer physical time step tends to decrease the maximum wind speed. The 2700-s run for ptend_f2_sudden, though

FIG. 8. Idealized TC. The evolution of domain minimum surface pressure (hPa) over time from (left) the `ptend_f2_sudden` and (right) the `ptend_f1_rk` runs for three resolutions: (top) G6, (middle) G7, and (bottom) G8. Only results from the NDC are shown, including two tracer transport options in both DTP splitting and nonsplitting modes, as indicated in the legend in (a).

stable, produces significantly lower maximum wind speeds, suggesting that the TC does not fully grow. This is similar to that shown in the supercell test in that a very large DTP splitting number may result in evidently

different solutions. This suggests that when an operator splitting approach is used, the maximum allowable time step should be carefully examined to ensure that the resultant splitting error is acceptable.

FIG. 9. As in Fig. 8, but for the domain maximum wind speed at 1500 m.

c. Moist Held–Suarez test in an aquaplanet scenario (MITC)

The MITC examines the long-term statistical behaviors of a moist model in an aquaplanet scenario (Thatcher and Jablonowski 2016). It completes the simple-physics package

of Reed and Jablonowski (2012) with modifications to allow long-term climate modeling—in particular, a radiation-like temperature-relaxation process and a low-level damping of winds (mimicking boundary layer mixing), as in the dry Held–Suarez test (Held and Suarez 1994, hereafter HS94).

FIG. 10. Idealized TC. Daily maximum wind speed (m s^{-1}) at 1500 m when the model uses different physics time steps. Four PDC options are compared: (a) ptend_f1_f1; (b) ptend_f1_rk; (c) ptend_f2_dribbling; (d) ptend_f2_sudden. Missing lines for each plot denotes unstable runs.

The DTP splitting option is used for all MITC tests. A 1200-day integration is done for the HDC and NDC using the G6 ($\tau_m = 1200\text{ s}$) and G7 ($\tau_m = 720\text{ s}$) resolutions, with initial conditions given by the moist baroclinic wave (Ullrich et al. 2014). The model is sampled at the end of each day based on the instantaneous state (which is also true for the precipitation rate).

Figures 11a–d show the NDC-generated climate statistics (zonal wind, eddy heat flux, temperature, and eddy kinetic energy) during the last 1000 days (201–1200), for both G6 and G7. A more complete figure including the results from the HDC is given in Fig. S9. These results are based on the ptend_f1_rk option and RK3O3; sensitivity tests based on the ptend_f2_sudden option (Fig. S10; for

NDC and HDC at G6) produce quite similar climate statistics. In general, these climate metrics are symmetric across the equator, and the differences between the HDC and NDC are small (Fig. S9). Sensitivity tests with TSPAS as a tracer transport option have also been examined (Fig. S11, only for the NDC), and are generally consistent with the results in Figs. 11a–d and Fig. S9.

Compared to the dycore test in Z19, the MITC test uplifts the locations of the maximum centers of the zonal wind, and also increases the wind magnitude (the vertical coordinate setup is as same as in Z19). A noteworthy difference is that the maximum intensity of the eddy kinetic energy in the MITC test is smaller than that in the dycore test (see Fig. 14 of Z19). We have found

FIG. 11. The MITC. Climatological mean (days 201–1200) of (a) zonal wind (U , m s^{-1}), (b) eddy heat flux (EHF, K m s^{-1}), (c) temperature (T , K), and (d) eddy kinetic energy (EKE, $\text{m}^2 \text{s}^{-2}$). The colored parts denote solutions at the G6 resolution, and the contoured parts denote solutions at the G7 resolution. (e), (f) As in (c) and (d), but only for the G6 solution from a test using the HS94 temperature parameters (solver = NDC; PDC = ptend_f1_rk; tracer transport = RK3O3).

that this is related to the modification of two temperature-related parameters as documented in Thatcher and Jablonowski (2016). If these two parameters are regressed to their original values as in the dry HS94 forcing, the eddy kinetic energy will exhibit the same maximum magnitude ($\sim 400\text{--}450 \text{ m}^2 \text{s}^{-2}$; Fig. 11f; see also Fig. S13 for more complete results) as in our dycore test. This modification results in a smaller near-surface meridional temperature gradient (see Figs. 11c,e). Such temperature-related variability is consistent with previous studies (e.g., O’Gorman and Schneider 2008) in that an increase of the pole-to-equator surface temperature contrast tends to strengthen the eddy kinetic energy, confirming the reasonable behavior of the moist model.

Increasing the model resolution from G6 to G7 tends to adjust the centers of the eddy activities poleward—a feature that is also found in the vertical maximum centers of the zonal winds. Unlike other gridscale climate statistics (zonal wind, temperature, eddy kinetic energy, eddy heat flux), eddy momentum flux shows a substantial increase in maximum intensity and spatial range from the low to high resolution. Based on RK3O3 and ptend_f2_sudden, Fig. 12 shows the eddy momentum flux in the G6, G7 and G8 runs (using the HDC). As the resolution is refined, the peaks centered at 30°N/S substantially increase their intensity and spatial range, while

the peaks that are closer to the two poles only decrease their intensity slightly. As discussed in Held and Phillips (1993) and references therein, such sensitivity is due to the relation between the eddy momentum flux and meridional wave propagation: a high-resolution model possesses the ability to resolve more small-scale structures that allow the Rossby waves that propagate from midlatitudes to the equator to adjust their meridional scale. While such resolution sensitivity is consistent with earlier studies, it is valuable to see that the magnitude of amplification does not reduce from G6 to G7 to G8 in the MITC test.

In terms of the precipitation field (Fig. 13), its maximum center is located at the equator. From G6 to G7, the rainfall peak increases its maximum by an amount of $\sim 12 \text{ mm day}^{-1}$. Two rainfall subpeaks are located at 40°N/S , corresponding to the centers of midlatitude eddy activities. A shift in the subpeak locations from G6 to G7 is also evident, corresponding to the eddy activities. Using HDC or NDC, and different tracer transport schemes, does not lead to evident sensitivity with regard to the mean state of the rainfall amount.

To better examine the differences in the rainfall simulation, we have decomposed the mean state into frequency and intensity (cf. Zhang and Chen 2016). As shown in Fig. 14a, the differences in the rainfall frequency

FIG. 12. The MITC. Simulated eddy momentum flux (EMF, $\text{m}^2 \text{s}^{-2}$) at (left) G6, (center) G7, and (right) G8 (solver = HDC; PDC = ptend_f2_sudden; tracer transport = RK3O3).

are only found in extratropical regions, which are typical baroclinic areas. All the G7 tests produce lower frequencies than the G6 runs from $\pm 20^\circ \text{N/S}$ poleward. For a given resolution, TSPAS tends to produce slightly higher frequency values than RK3O3, but the results are not sensitive to the hydrostatic option. We believe that such sensitivity reflects that atmospheric baroclinity is more sensitive to the horizontal resolution and the PGF (PGF is indirectly related to tracer transport in this model), resulting in different performance in the rainfall frequency, a metric that may reflect frequency of disturbance because only gridscale saturation can trigger precipitation in this test (see also Z19 for resolution sensitivity in the dry baroclinic wave test). The climatological rainfall amount (Fig. 13) does not show such sensitivity because it is determined by intensity (Fig. 14b). For the intensity field, all the G7 runs produce much higher values than the G6 runs in the tropics. In the MITC test, the model can neither properly resolve nor parameterize convection, and it can only eliminate instability via gridscale adjustment. A high-resolution model can generate stronger gridscale updrafts (figure not shown; see also Herrington and Reed 2018) in the tropics, and thus increases the resolved-scale precipitation intensity. While this resolution sensitivity is reasonable and is frequently found in real-world modeling (cf. O'Brien et al. 2016), one should be careful with the resultant high rainfall intensity/amount because, in some cases, overly strong updrafts (which ideally should be removed by convection parameterization) may lead to unrealistic gridpoint storms because the updrafts occur at the scale in which they should not reside. On the other hand, the intensity centers in the midlatitudes exhibit a poleward shift from G6 to G7, corresponding to the poleward shift of the rainfall amount (Fig. 13).

This is related to the increased eddy activity in the poleward direction as the resolution is refined.

d. Real-case modeling

Based on the simple-physics package of Reed and Jablonowski (2012), we have tested the model with realistic initial and topographic conditions. The surface parameterization has to be turned off to accommodate the surface elevation. We note that this might also affect the rationality of other components (e.g., the boundary layer process) in the simple-physics package, and a meaningful forecast is limited to a relatively short range. Here, a southwest vortex (SWV) case on the lee side of the Tibetan Plateau from 16 to 18 August 2015 is selected to examine the model performance. The SWV is a typical meso- α -scale system over southwestern China and is mainly influenced by the large-scale circulations and the topographic forcing of the Tibetan Plateau. The model is initialized at 0000 UTC 16 August 2015. Figure 15 shows the initial conditions (flow field and

FIG. 13. The MITC. Zonally averaged climatological gridscale daily instantaneous rainfall rate (mm day^{-1}) during days 201–1200. Results are shown for six configurations, as indicated in the legend.

FIG. 14. As in Fig. 13, but for the (a) frequency (%) and (b) intensity (mm day^{-1}) of rainfall rates.

specific humidity) and four 48-h forecasts produced by the HDC and NDC at two horizontal resolutions (G6 and G7), together with the ERA-Interim data at the corresponding date (at 700 hPa). On 16 August, the southwesterlies occupy the southeastern flank of the Tibetan Plateau, bringing abundant moisture. This southwestern flow then develops into a closed vortex system after two days on the eastern flank of the Tibetan Plateau. At the G6 resolution, both the HDC and NDC can simulate the closed vortex system, while the local maximum strength of the vortex is slightly weaker than that in G7 (based on a check of the relative vorticity field). The simulated vortex center is slightly eastward shifted compared to that in the reanalysis data. Besides the SWV, the anticyclonic system over the Bay of Bengal, which is not clearly shown in the initial field, can also be simulated by the model at two resolutions. The moisture content is generally distributed in accordance with the flow pattern at various locations.

We have also examined the possible reason for the eastward-shifted SWV pattern in the simulations. It is found that the SWV in the model grows somewhat faster than that in the ERA-Interim data. After 12 h from 0000 UTC 16 August, the model shows a pattern closer to the ERA-Interim data (Figs. S13a–c): an inverted-trough-style cyclonic circulation pattern is already generated, and a warm sector is found on the lee side of the Tibetan Plateau. After 24 h (Figs. S13d–f), ERA-Interim

still shows such a pattern, which develops slightly deeper than 12 h before. Meanwhile, the model has already produced a closed vortex, and the area clinging to the mountains is occupied by the northerly wind. Notably, the warm sector is still found on the lee side of the Tibetan Plateau; while in the ERA-Interim data, the magnitude of the warm sector has been largely reduced. After 48 h (Figs. S13g–i), the model-generated temperature field still supports a warm sector associated with the SWV, creating a larger temperature gradient in the north–south direction, which partly helps to strengthen the northerly in the west of the SWV. The existence of the warm area is probably due to the lack of diabatic cooling, and the faster growth of the SWV may be partly related to the insufficient boundary layer process. The incomplete physics might be a reason for such a model behavior, which deserves further exploration.

4. Summary and concluding remarks

In this study, a multiscale dynamical model for weather and climate modeling has been established by extending a dycore to moist dynamics and parameterized physics. A flexible coupling of the dycore, tracer, and physics is achieved to allow efficient DTP splitting and a multi-purpose PDC workflow. Tracer transport is consistently coupled with the dycore by using the time-averaged normal mass fluxes and feeds back to the dycore via a moisture constraint. To accommodate various physics packages, a general physics–dynamics coupling workflow is established. Both idealized and real-world tests have been used to examine the model performance. The major conclusions can be summarized as follows:

- (i) The model produces reasonable resolution sensitivity and variability for multiscale scenarios. In the supercell test, the model exhibits a converged behavior at ~ 0.25 km. Nonnegligible sensitivity to the DTP splitting can be found provided that the splitting number becomes very large. In the TC test, it is shown that while the HDC tends to produce a slightly stronger storm than the NDC, the general difference between the two solvers is limited in the simple-physics configuration. There is also certain sensitivity associated with the two tracer transport options, and such sensitivity tends to increase as the resolution is refined. In the MITC test, the model simulates an increase in the poleward midlatitude eddy activities and precipitation as the resolution increases. Sensitivity of rainfall frequency to resolution and tracer transport is found in extratropical regions because these two factors may impact atmospheric baroclinity, while sensitivity of rainfall

FIG. 15. Real-world case modeling, in which the model starts from (a) 0000 UTC 16 Aug 2015. Shown are results after 48 h for the flow field (streamlines) and the specific humidity (color shaded, kg kg^{-1}) at 700 hPa from (b) the ERA-Interim data, (c) HDC-G6, (d) NDC-G6, (e) HDC-G7, and (f) NDC-G7.

intensity to resolution is found for regions dominated by midlatitude eddy activities and tropical updrafts. The eddy momentum flux exhibits a significant increase to the horizontal resolution among several dynamical metrics. A real-world SWV case further demonstrates that the model in its current configuration is able to simulate the dynamically driven mesoscale cyclonic circulations under realistic initial and topographic conditions. These results confirm that the moist model behaves properly under a multiscale environment. The proposed PDC workflow with a DTP splitting function works well provided that the model is configured properly.

- (ii) We have compared several PDC options in the context of model development. It is found that a pure operator splitting approach (`ptend_f2_sudden`) is more stable for large time steps, but a careful examination of the model behaviors at the maximum allowable time step (or using a very large DTP splitting number) should be done to confirm that the

splitting error does not degenerate the modeling performance. For GRIST-NDC, the use of a pure operator splitting approach should be combined with a special treatment of diabatic heating. The tendency method (`ptend_f1_rk`), while more restricted under very large time steps, generates solutions that are consistent at various time steps, and can be used for both solvers without special treatment. In general, we suggest using a combination of the HDC and `ptend_f2_sudden` for long-term multiyear climate modeling at the typical hydrostatic scale and a combination of the tendency method and the operator splitting approach for the NDC that targets the nonhydrostatic regime. For the resolution ranges where both two solvers are justified (e.g., $\sim 7\text{--}25$ km), a decision should be made to follow either of the above two choices, which also depends on the target application and physics package.

Other model improvements include a substantial enhancement of the computational infrastructure and

a further exploration of variable-resolution modeling. Currently, based on the G8B3 mesh, from 160 to 320 nodes (32 cores per node), the present code achieves a parallel efficiency of $\sim 90\%$ at 320 nodes (Fig. S14). For higher-resolution G10-mesh runs, there are some signs that the cache memory access of indirect addressing may lead to less-efficient performance (e.g., superlinear speedup ratios are generated); the computational performance is also sensitive to different machines. Some ongoing optimization efforts have been undertaken to improve the computational efficiency. A deeper understanding of variable-resolution modeling has also been explored, including experiments based on various refinement styles (e.g., multiregion, hierarchical style). These issues will be reported in possible future publications. We also note that an enhanced treatment of the steep and irregular topography for global high-resolution modeling is important in future model development efforts. Testing of the full-physics package is on the way. The present model serves as a new starting point for these future endeavors.

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Data availability statement: The ERA-Interim data are available at <https://www.ecmwf.int/en/forecasts/datasets/reanalysis-datasets/era-interim>. The model data used for this paper are stored at a local cluster (10.32.12.74:/Volumes/ARECA0/2019/dtp/), which may be accessed via the internal internet of the CMA or a VPN account.

APPENDIX A

The Total Energy Formula for the Moist Shallow-Atmosphere Nonhydrostatic Equations of Motion

The global total energy (TE) formula for the continuous-form moist shallow-atmosphere nonhydrostatic equations under a generalized vertical coordinate is derived. To facilitate the derivation, a temperature-based thermodynamic equation and an advective-form horizontal momentum equation is used; frictionless and adiabatic conditions without source/sink terms are assumed. A set of alternative-form moist-atmosphere governing equations may be written as follows:

$$\frac{d\mathbf{V}}{dt} + f\mathbf{k} \times \mathbf{V} + \alpha_m \nabla p + \left(\frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial \pi_m} \right) \nabla \phi = 0, \quad (\text{A1})$$

$$\frac{dw}{dt} + g \left(1 - \frac{\partial p}{\partial \pi_m} \frac{\partial \eta}{\partial \eta} \right) = 0, \quad (\text{A2})$$

$$\frac{dT}{dt} - \frac{RT}{pc_p} \frac{dp}{dt} = 0, \quad (\text{A3})$$

$$\frac{\partial}{\partial t} \left(\frac{\partial \pi}{\partial \eta} q_i \right) + \nabla \cdot \left(\mathbf{V} \frac{\partial \pi}{\partial \eta} q_i \right) + \frac{\partial}{\partial \eta} \left(\dot{\eta} \frac{\partial \pi}{\partial \eta} q_i \right) = 0, \quad (\text{A4})$$

$$\frac{d\phi}{dt} = wg, \quad (\text{A5})$$

$$p = \rho_m RT = \frac{RT}{\alpha_m}, \quad (\text{A6})$$

$$\frac{\partial \phi / \partial \eta}{\partial \pi / \partial \eta} = -\alpha_d, \quad \frac{\partial \phi / \partial \eta}{\partial \pi_m / \partial \eta} = -\alpha_m, \quad (\text{A7})$$

where q_i denotes any of $q_d, q_v, \dots$, and note that $q_d = 1$. Equation (A3) applies the enthalpy-based first law of thermodynamics to a moist atmosphere: R and c_p are generalized to include moist species (we may write water vapor q_v , liquid q_l and ice q_{ic} here), that is, $R = (q_d R_d + q_v R_v) / (q_d + q_v + q_l + q_{ic})$, $c_p = (q_d C_{pd} + q_v C_{pv} + q_l C_l + q_{ic} C_{ic}) / (q_d + q_v + q_l + q_{ic})$, and $c_v = (q_d C_{vd} + q_v C_{vv} + q_l C_l + q_{ic} C_{ic}) / (q_d + q_v + q_l + q_{ic})$. Summing Eq. (A4) for all q_i , we obtain the total/moist-mass continuity equation:

$$\frac{\partial}{\partial t} \left(\frac{\partial \pi_m}{\partial \eta} \right) + \nabla \cdot \left(\mathbf{V} \frac{\partial \pi_m}{\partial \eta} \right) + \frac{\partial}{\partial \eta} \left(\dot{\eta} \frac{\partial \pi_m}{\partial \eta} \right) = 0. \quad (\text{A8})$$

Let $m = \partial \pi_m / \partial \eta$, $K = 0.5(\mathbf{V}^2 + w^2)$ is the 3D total kinetic energy, and $E_1 = (K + c_p T + \phi)$. With some manipulation, the left-hand sides of the above equations can be rearranged as follows:

$$m \frac{d\frac{\mathbf{V}^2}{2}}{dt} + \overline{m \mathbf{V} \alpha_m \nabla p} + m \mathbf{V} \left(\frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial \pi_m} \right) \nabla \phi = 0, \quad (\text{A9})$$

$$m \frac{d\frac{w^2}{2}}{dt} + \overline{mwg} \left(1 - \frac{\partial p}{\partial \pi_m} \frac{\partial \eta}{\partial \eta} \right) = 0, \quad (\text{A10})$$

$$mc_p \frac{dT}{dt} - \overline{m \alpha_m \frac{dp}{dt}} = 0, \quad (\text{A11})$$

$$m \frac{d\phi}{dt} - m \frac{\frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial \pi_m}}{\frac{\partial \eta}{\partial \eta}} - m \frac{\frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial \pi_m}}{\frac{\partial \eta}{\partial \eta}} \mathbf{V} \cdot \nabla \phi - m \frac{\frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial \pi_m}}{\frac{\partial \eta}{\partial \eta}} \dot{\eta} \frac{\partial \phi}{\partial \eta} - mwg \left(1 - \frac{\frac{\partial p}{\partial \eta}}{\frac{\partial \pi_m}{\partial \eta}} \right) = 0, \quad (\text{A12})$$

$$\left(m \frac{\frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial \pi_m}}{\frac{\partial \eta}{\partial \eta}} + m \dot{\eta} \frac{\frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial \pi_m}}{\frac{\partial \eta}{\partial \eta}} \right) + \left(m \alpha_m \frac{dp}{dt} - m \alpha_m \mathbf{V} \cdot \nabla p \right) = 0, \quad (\text{A13})$$

$$E_1 \left[\frac{\partial}{\partial t} (m) + \nabla \cdot (m \mathbf{V}) + \frac{\partial}{\partial \eta} (m \dot{\eta}) \right] = 0. \quad (\text{A14})$$

Summing all terms in Eqs. (A9)–(A13), and canceling the two terms with the same hat, leads to

$$\left(m \frac{d\frac{\mathbf{V}^2}{2}}{dt} + m \frac{d\frac{w^2}{2}}{dt} + m c_p \frac{dT}{dt} + m \frac{d\phi}{dt} - \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial t} + \frac{\partial p}{\partial t} \frac{\partial \phi}{\partial \eta} \right) = 0. \quad (\text{A15})$$

Thus, E_1 can be rearranged into flux form

$$\frac{\partial}{\partial t} (m E_1) + \nabla \cdot (m \mathbf{V} E_1) + \frac{\partial}{\partial \eta} (m \dot{\eta} E_1) - \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial t} + \frac{\partial p}{\partial t} \frac{\partial \phi}{\partial \eta} = 0. \quad (\text{A16})$$

Letting

$$E_2 = (K + c_v T + \phi) = E_1 - RT, \quad (\text{A17})$$

we may rearrange Eq. (A16) to

$$\frac{\partial}{\partial t} (m E_2) + \frac{\partial}{\partial t} (m RT) + \nabla \cdot (m \mathbf{V} E_1) + \frac{\partial}{\partial \eta} (m \dot{\eta} E_1) - \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial t} + \frac{\partial p}{\partial t} \frac{\partial \phi}{\partial \eta} = 0. \quad (\text{A18})$$

Based on Eqs. (A6) and (A7),

$$\begin{aligned} \frac{\partial}{\partial t} (m RT) - \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial t} + \frac{\partial p}{\partial t} \frac{\partial \phi}{\partial \eta} &= \frac{\partial}{\partial t} \left(-\frac{\partial \phi}{\partial \eta} p \right) - \frac{\partial p}{\partial \eta} \frac{\partial \phi}{\partial t} \\ &\quad + \frac{\partial p}{\partial t} \frac{\partial \phi}{\partial \eta} = -\frac{\partial}{\partial \eta} \left(p \frac{\partial \phi}{\partial t} \right), \end{aligned} \quad (\text{A19})$$

and we obtain

$$\frac{\partial}{\partial t} (m E_2) + \nabla \cdot (m \mathbf{V} E_1) + \frac{\partial}{\partial \eta} (m \dot{\eta} E_1) - \frac{\partial}{\partial \eta} \left(p \frac{\partial \phi}{\partial t} \right) = 0. \quad (\text{A20})$$

A three-dimensional global integral of Eq. (A20) leads to the conservation of E_2 in the continuous system, under the upper and lower boundary conditions: (i) $\dot{\eta}_s = \dot{\eta}_t = 0$; (ii) ϕ_s at the surface does not change over time; and (iii) at the top of the atmosphere, $p = \pi = \pi_m = 0$. If $p_{\text{top}} = \pi_{\text{top}}$ is a nonzero constant, then the residual after the vertical integration, $p_{\text{top}}(\partial \phi_{\text{top}}/\partial t)$, should be included in the TE diagnostic for the model dynamics, and the global integration of E_2 alone does not conserve. Moreover, for such a top boundary condition, an alternative TE formula may be obtained. $-(\partial/\partial \eta)[p(\partial \phi/\partial t)]$ may be rewritten as

$$-\frac{\partial}{\partial \eta} \left[(p - \pi_m + \pi_m) \frac{\partial \phi}{\partial t} \right] = -\frac{\partial}{\partial \eta} \left[(p - \pi_m) \frac{\partial \phi}{\partial t} \right] - \frac{\partial}{\partial \eta} \left[\pi_m \frac{\partial \phi}{\partial t} \right], \quad (\text{A21})$$

and the second term on the right-hand side of Eq. (A21) is further rewritten as

$$\begin{aligned} -\frac{\partial}{\partial \eta} \left[\pi_m \frac{\partial \phi}{\partial t} \right] + \frac{\partial}{\partial \eta} \left(\phi \frac{\partial \pi_m}{\partial t} \right) - \frac{\partial}{\partial \eta} \left(\phi \frac{\partial \pi_m}{\partial t} \right) \\ = \frac{\partial}{\partial \eta} \left(\phi \frac{\partial \pi_m}{\partial t} \right) - \frac{\partial}{\partial t} \left[\frac{\partial (\pi_m \phi)}{\partial \eta} \right]. \end{aligned} \quad (\text{A22})$$

The term $-(\partial/\partial t)[\partial(\pi_m \phi)/\partial \eta]$ can be combined with $(\partial/\partial t)[\phi(\partial \pi_m/\partial \eta)]$ in $(\partial/\partial t)m E_2$, leading to $(\partial/\partial t)(-\pi_m \partial \phi/\partial \eta) = (\partial/\partial t)(\pi_m \alpha_m \partial \pi_m/\partial \eta)$. This leads to a third definition of TE as $E_3 = K + C_v T + \pi_m \alpha_m$, and the energy equation reads

$$\begin{aligned} \frac{\partial}{\partial t} (m E_3) + \nabla \cdot (m \mathbf{V} E_1) + \frac{\partial}{\partial \eta} (m \dot{\eta} E_1) - \frac{\partial}{\partial \eta} \left[(p - \pi_m) \frac{\partial \phi}{\partial t} \right] \\ + \frac{\partial}{\partial \eta} \left(\phi \frac{\partial \pi_m}{\partial t} \right) = 0. \end{aligned} \quad (\text{A23})$$

The vertical integral of $-(\partial/\partial \eta)[(p - \pi_m) \partial \phi/\partial t]$ will vanish as long as $p = \pi_m$ at the model top, and ϕ_s is time invariant. The vertical integral of $(\partial/\partial \eta)(\phi \partial \pi_m/\partial t)$ from top to surface leads to $\phi_s \partial \pi_{ms}/\partial t = \partial(\phi_s \pi_{ms})/\partial t$, and the final global integral of TE after the vertical integral reads

$$\oint \left\{ \frac{\partial(\phi_s \pi_{ms})}{\partial t} + \int_{\eta_t}^{\eta_s} \left[\frac{\partial}{\partial t} (m E_3) + \nabla \cdot (m \mathbf{V} E_1) \right] d\eta \right\} dA = 0. \quad (\text{A24})$$

The global area integral of the flux-form divergence term is zero, leading to

$$\frac{\partial}{\partial t} \left\{ \oint \left[\phi_s \pi_{ms} + \int_{\eta_i}^{\eta_s} (mE_3) d\eta \right] dA \right\} = 0. \quad (\text{A25})$$

The global integration of E_3 conserves because $\oint [\partial(\phi_s \pi_{ms})/\partial t] dA = 0$. If moisture has an unresolved contribution, then $\partial(\phi_s \pi_{ms})/\partial t$ should be included in the TE diagnostic for the model dynamic, equivalent to the diagnostic based on $p_{\text{top}} \partial \phi_{\text{top}}/\partial t$ and E_2 . In the hydrostatic limit, E_1 , E_2 , and E_3 correspond to the energy forms that have been widely discussed (Williamson et al. 2015; Arakawa and Lamb 1977).

The discrete moist model does not conserve TE exactly to within the time truncation error. Conventionally, conserving a discrete integral invariant is used as a protocol for constructing physical-based spatial discretization, and an absolute conservation of the TE itself is not that important (Arakawa 2000). This is somewhat different from exact mass conservation: it is safe to assume that the dry air mass can only be transported; thus, contribution from the unresolved scale or from the surface exchange is zero. Even so, exact mass conservation can still be given up provided that the source/sink can be well controlled and the gain for such a move outweighs its loss. For TE conservation, even though the spatial discretization can be energetically neutral, some additional mechanisms besides the time discretization in the dynamics can still dissipate the TE (cf. Taylor et al. 2020 and references therein), as model physics typically does. For practical modeling, the TE conservation to a certain extent is still important to avoid spurious trends in long-term climate integration (Williamson et al. 2015; Lauritzen and Williamson 2019). Based on the moist baroclinic wave initial condition, we have examined the TE time tendency ($\text{TE}_r = (\partial/\partial t) \left\{ \oint \left[\int_{\eta_i}^{\eta_s} (mE_3) d\eta \right] dA \right\}$) in a 1200-day adiabatic integration. The resolution is G6, and both the HDC and NDC are tested at their normal operational time steps. After a spinup period, both solvers reach a state of balance, in which $(1/g)\text{TE}_r$ per unit area steadily oscillates around -0.005 W m^{-2} with no significant trend (Fig. A1). Such a magnitude is small enough for practical climate modeling, while of course the TE tendency from other sources (e.g., the parameterization suite) should also be limited.

APPENDIX B

Derivation of the Model-Version Moist Governing Equations

The model discretization utilizes a modified version of the equations in appendix A. The horizontal momentum equation, by using the vector identity,

FIG. A1. Temporal evolution of the area normalized $(1/g)\text{TE}_r$ (W m^{-2}) for the HDC (black, with the bottom axis for time) and NDC (red, with the top axis for time). Results are shown for a period of balance from days 600 to 1200.

$$(\mathbf{V} \cdot \nabla) \mathbf{V} = (\nabla \times \mathbf{V}) \times \mathbf{V} + \nabla \frac{\mathbf{V}^2}{2} = \zeta_r \mathbf{k} \times \mathbf{V} + \nabla \frac{\mathbf{V}^2}{2}, \quad (\text{A26})$$

is converted to a vector-invariant formulation,

$$\begin{aligned} \frac{\partial \mathbf{V}}{\partial t} + \zeta_r \mathbf{k} \times \mathbf{V} + \nabla \frac{\mathbf{V}^2}{2} + \dot{\eta} \frac{\partial}{\partial \eta} \mathbf{V} + f \mathbf{k} \times \mathbf{V} \\ = -\frac{1}{\rho_m} \nabla p - \left(\frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial \pi_m} \right) \nabla \phi, \end{aligned} \quad (\text{A27})$$

which facilitates the treatment of spherical geometry. For the horizontal pressure gradient force term, they are rewritten as

$$\frac{\rho_d}{\rho_m} \left[-\frac{1}{\rho_d} \nabla p - \left(\frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial \pi} \right) \nabla \phi \right], \quad (\text{A28})$$

which facilitates the dycore–tracer splitting in a dry-mass coordinate. Similarly, for the vertical momentum equation, we obtain

$$\frac{dw}{dt} + g - g \frac{\rho_d}{\rho_m} \frac{\partial p}{\partial \eta} \frac{\partial \eta}{\partial \pi} = 0. \quad (\text{A29})$$

The thermodynamic equation for moist air, Eq. (A3), is converted to an entropy-based formulation: $(1/T)(dT/dt) - (R/c_p)(1/p)(dp/dt) = (1/\theta)(d\theta/dt)$. We define θ_m based on θ as $\theta_m = \theta(1 + \varepsilon q_v)$. The equation for θ_m reads

$$\frac{1}{\theta_m} \frac{d\theta_m}{dt} = \frac{1}{\theta} \frac{d\theta}{dt} + \frac{\varepsilon}{(1 + \varepsilon q_v)} \frac{dq_v}{dt}, \quad (\text{A30})$$

which can be rearranged to flux form,

$$\begin{aligned} \frac{\partial}{\partial t} \left(\frac{\partial \pi}{\partial \eta} \theta_m \right) + \nabla \cdot \left(\mathbf{V} \frac{\partial \pi}{\partial \eta} \theta_m \right) + \frac{\partial}{\partial \eta} \left(\dot{\eta} \frac{\partial \pi}{\partial \eta} \theta_m \right) \\ = \frac{\partial \pi}{\partial \eta} \left[(1 + \varepsilon q_v) \frac{d\theta}{dt} + \varepsilon \theta \frac{dq_v}{dt} \right]. \end{aligned} \quad (\text{A31})$$

The change of θ_m is contributed by both θ and q_v . The definition of θ_m enables a reformulation of the ideal gas law,

$$p = \rho_d R_d T + \rho_v R_v T = \rho_d R_d T_m = \rho_m R T, \quad (\text{A32})$$

to θ_m form

$$p = p_0 \left(\frac{R_d \theta_m}{p_0 \alpha_d} \right)^{c_p/c_v}. \quad (\text{A33})$$

This version of the gas-state law has the same structure as that in the dycore, except that θ_m replaces θ , which facilitates the vertically implicit acoustic solver for the nonhydrostatic model.

Meanwhile, as discussed in appendix A, Eq. (A3) has R and c_p generalized for moist air. However, an approximation,

$$\left(\frac{p}{p_0} \right)^{R/c_p} \approx \left(\frac{p}{p_0} \right)^{R_d/c_{pd}}, \quad (\text{A34})$$

is used in the model, and θ and θ_m are also defined by R_d/c_{pd} . This approximation is accurate only for the Exner function, rather than for the original temperature equation [i.e., $[dT/dt - (RT/c_p)(dp/dt)] = 0$ still holds, and the approximation does not imply $R \approx R_d$ or $c_p \approx c_{pd}$ separately]. This leads to the only difference between the model-version moist continuous equations and those discussed in appendix A. This approximation is accurate even when condensate is present (c_p has more small terms). Thus, by only using dry constants (for convenience and flexibility), the entropy-based first law of thermodynamics can accurately recover the enthalpy-based first law under various environments (dry/moist/with condensate). Meanwhile, all mass-continuity related flux-form equations can be exactly converted to a vertically integrated form, leading to

$$\frac{\partial}{\partial t} (\delta \pi) + \nabla \cdot (\mathbf{V} \delta \pi) + \delta \left(\dot{\eta} \frac{\partial \pi}{\partial \eta} \right) = 0, \quad (\text{A35})$$

$$\frac{\partial}{\partial t} (\delta \pi \theta_m) + \nabla \cdot (\mathbf{V} \delta \pi \theta_m) + \delta \left(\dot{\eta} \frac{\partial \pi}{\partial \eta} \theta_m \right) = 0, \quad (\text{A36})$$

$$\frac{\partial}{\partial t} (\delta \pi q_i) + \nabla \cdot (\mathbf{V} \delta \pi q_i) + \delta \left(\dot{\eta} \frac{\partial \pi}{\partial \eta} q_i \right) = 0. \quad (\text{A37})$$

The difference between using an integrated and unintegrated form may be demonstrated by using a pure sigma coordinate, where we may let $\pi = \eta \pi_s$. For each model layer, $\partial \pi / \partial \eta = \pi_s$ is a layer-invariant external variable, while $\delta \pi$ is a layer-dependent internal variable. Viewed from a height-based space, a layer-averaged model predicts the control-volume quantities per unit area for each model layer. Thus, the model-version continuous-form equations of motion recover the dry dynamical equations in Z19 in the dry limit, and they recover the moist governing equations in appendix A under the Exner function approximation.

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