Evaluation of the Fitch Wind-Farm Wake Parameterization with Large-Eddy Simulations of Wakes Using the Weather Research and Forecasting Model

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ABSTRACT: Wind-farm parameterizations in weather models can be used to predict both the power output and farm effects on the flow; however, their correctness has not been thoroughly assessed. We evaluate the wind-farm parameterization of the Weather Research and Forecasting Model with large-eddy simulations (LES) of the wake performed with the same model. We study the impact on the velocity and turbulence kinetic energy (TKE) of inflow velocity, roughness, resolution, number of turbines (one or two), and inversion height and strength. We compare the mesoscale with the LES by spatially averaging the LES within areas correspondent to the mesoscale horizontal spacing: one covering the turbine area and two downwind. We find an excellent agreement of the velocity within the turbine area between the two types of simulations. However, within the same area, we find the largest TKE discrepancies because in mesoscale simulations, the turbine-added TKE has to be highest at the turbine position to be advected downwind. Within the downwind areas, differences between velocities increase as the wake recovers faster in the LES, whereas for the TKE both types of simulations show similar levels. From the various configurations, the impact of inversion height and strength is small for these heights and inversion levels. The highest impact for the one-turbine simulations appears under the low-speed case due to the higher thrust, whereas the impact of resolution is low for the large-eddy simulations but high for the mesoscale simulations. Our findings demonstrate that higher-fidelity simulations are needed to validate wind-farm parameterizations.

KEYWORDS: Boundary layer; Wind; Large eddy simulations; Mesoscale models; Model evaluation/performance; Numerical weather prediction/forecasting

1. Introduction

The Weather Research and Forecasting (WRF) mesoscale model is a real-time numerical weather prediction model, in which wind-farm parameterizations are being implemented (Fitch et al. 2012; Volker et al. 2015). The main purpose of these parameterizations is to account for the impact of wind farms and wind-farm clusters, not only on the local and regional wind and turbulence fields, but also on temperature and fluxes. These parameterizations combine resolved and subgrid-scale effects, and, as the wind-farm power output can be computed from the simulated velocity field, they are also used for feasibility studies in wind resource assessments.

The WRF Model is nowadays the preferred atmospheric modeling system within the wind energy community to assess the regional, and in many cases, the local climatology of the wind (Hahmann et al. 2020), including wind over complex terrain (Solbakken et al. 2021). A wind-farm parameterization, the Fitch scheme (Fitch et al. 2012), has been part of the WRF Model for nearly 10 years and is therefore often used to evaluate wind-farm impacts widely (Feroz et al. 2020; Tomaszewski and Lundquist 2021). The Fitch scheme in its current open-community version only works with the MYNN2 (Nakanishi and Niino 2009) planetary boundary layer (PBL) scheme, although there are efforts in integrating it with other PBL schemes (Rybchuk et al. 2022). The Fitch scheme is frequently used to parameterize the wind-farm effects; however, its ability to simulate wind speed and turbulent kinetic energy (TKE) fields is yet not well assessed. Evaluation of its correctness is therefore of great value and this can be carried out by comparison with high-fidelity and detailed wake simulations.

Also within the WRF Model, a generalized actuator disk (GAD) model was implemented to simulate the effect of wind turbines at higher fidelity. In an actuator disk (AD), the rotor is represented by a permeable disk that allows fluid to pass through the rotor, while subjected to surface forces (Sörensen 2022). Higher fidelity can be accomplished by decreasing the grid spacing, turning off PBL parameterizations, and activating the large-eddy simulation (LES) capability of the model (Mirocha et al. 2014). Tangential and normal forces are computed in this GAD at different radii along the blade via lookup tables with precomputed aerodynamic specifications. Thus, this is also commonly referred to as an AD model based on airfoil data (Réthoré et al. 2014; van der Laan et al. 2015). These forces are then converted into local forces, which are aligned with the coordinate system of the WRF Model. The local forces are last incorporated into the tendencies of the simulated velocity components of the WRF Model. As such, WRF-LES using the GAD explicitly resolves the 3D turbulent mixing responsible for the heat and energy transport germane to climatological impact assessment. Therefore, the computationally more expensive LES with the GAD in the WRF Model can thus also be used to evaluate the
computationally cheaper wind-farm parameterizations and provide the means for their refinement and improvement; this is a need as neither annual energy production computations for large wind farms are affordable using the LES with the GAD nor long-term studies of the impact of wind farms on the regional climatology or on the future impact of wind farms using climate model output.

Here, our aim is the evaluation of the Fitch wind parameterization with the GAD. The Fitch scheme is run in a mesoscale/PBL fashion, i.e., with horizontal grid spacings on the order of kilometers. The GAD is run using the LES capability of the WRF Model, i.e., with grid spacings on the order of meters/tens of meters. A similar evaluation was performed by Archer et al. (2020) but compared to their study, ours has three main distinctions. First, both PBL and LES runs are performed with the same modeling system, i.e., fundamental physics and numerics are the same. Thus, a fairer comparison between both types of models can be achieved and, thus, turbine impacts may be more straightforwardly assessed. Second, when simulating the turbines using the wind-farm parameterization, we use the power and thrust-coefficient curves, which are derived with the same GAD methods that are applied in the LES, further increasing the fairness of the comparison. Third, we analyze the sensitivity of the results to inflow wind conditions, mainly mean wind speed at hub height, horizontal grid spacing of both the LES and PBL runs, number of turbines within the mesoscale grid cell, surface roughness length, and inversion height and strength.

The work is organized as follows. Section 2 introduces the Fitch scheme and the GAD, as well as the performance of the GAD when compared to other high-resolution flow and aerelastic models. There, we also introduce the LES and PBL simulations and the approaches to compare the results between them. Section 3 presents the results. We start by analyzing the spatial-average inflow conditions of both LES and PBL simulations, just before the turbine is introduced. Then, we analyze time-averaged hub-height cross sections and time-averaged vertical cross sections of the simulated velocity and TKE differences, as these are the variables that the Fitch scheme parameterizes. Finally, we analyze spatial- and temporal-averaged vertical profiles of the velocity and TKE differences between both types of simulations within different areas for the LES and grid points for the PBL. The last section provides the main conclusions of the study.

2. Background and methods

We perform idealized simulations using the WRF Model version 3.7.1. The Fitch scheme is used for the PBL simulations and the GAD for the LESs. For both types of simulations, we use the same wind turbine: the DTU 10-MW reference turbine (Bak et al. 2013), which has a rotor diameter $D$ of 178.3 m and a hub height of 119 m. Table 1 provides relevant information with regards to the simulated turbine. Specific details on the Fitch scheme, the GAD, and the simulations are given in the following subsections.

Table 1. Relevant parameters of the DTU 10-MW reference turbine.

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Attribute/value</th>
</tr>
</thead>
<tbody>
<tr>
<td>Cut-in wind speed</td>
<td>4 m s$^{-1}$</td>
</tr>
<tr>
<td>Cut-out wind speed</td>
<td>25 m s$^{-1}$</td>
</tr>
<tr>
<td>Rated wind speed</td>
<td>11.4 m s$^{-1}$</td>
</tr>
<tr>
<td>No. of blades</td>
<td>3</td>
</tr>
<tr>
<td>Rotor diameter</td>
<td>178.3 m</td>
</tr>
<tr>
<td>Hub diameter</td>
<td>5.6 m</td>
</tr>
<tr>
<td>Hub height</td>
<td>119 m</td>
</tr>
<tr>
<td>Max tip speed</td>
<td>90 m s$^{-1}$</td>
</tr>
</tbody>
</table>

a. The Fitch scheme

In the Fitch scheme, wind turbines are modeled as sinks of momentum and sources of TKE. Considering a single wind turbine, the scheme computes the power output $P$ of the turbine from the simulated wind speed at hub height $U_h$ by using the power curve, which is normally provided by the turbine manufacturer. The power curve together with the thrust coefficient curve of the turbine are inputs to the scheme. The power coefficient $C_p$ is computed as

$$C_p = \frac{P}{\frac{1}{2} \rho A U_h^3},$$

where $A$ is the swept rotor area of the turbine and $\rho$ the air density. In the current distribution of the WRF Model, $\rho$ is assumed constant, $\rho = 1.225 \text{ kg m}^{-3}$. A TKE coefficient is also computed:

$$C_{\text{TKE}} = c_j (C_T - C_p),$$

where $c_j$ is a correction factor, which was introduced into the scheme by Archer et al. (2020) to account for mechanical and electrical losses, and $C_T$ the thrust coefficient at the given $U_h$. $C_{\text{TKE}}$ positively contributes to the TKE tendency, whereas $C_T$ negatively contributes to the tendencies of the two horizontal velocity components. In Eq. (2) $C_{\text{TKE}}$ assumes that the wind turbine TKE is the difference between the turbine extracted and the turbine generated electrical energy. This assumption can be questioned, since it is mainly the thrust force that generates additional TKE by converting mean kinetic energy of the flow to TKE via turbulent shear production. The use of $C_T$ in the TKE source is also discussed and applied in the wind-farm parameterization of Abkar and Porté-Agel (2015b). In addition, it should be noted that one could also write $C_T - C_p$ in terms of $C_T$ only, following 1D momentum theory:

$$C_T - C_p = C_T a_k = \frac{1}{2} C_T \left(1 - \sqrt{1 - C_T^2}\right),$$

for $C_T \leq 1$, with $a_k$ as the axial induction. The latter quantifies the influence of a rotor on the flow velocity.

b. Generalized actuator disk

The GAD implemented in the WRF Model combines 1D momentum theory and blade element theory (BEM) as in, e.g.,
Mikkelsen (2003). In the original implementation by Mirocha et al. (2014), normal and tangential induction factors were computed iteratively and the tip loss correction of Glauert (1935) was used to modify the tangential induction factor only. Here, we modify this implementation by using the tip correction by Shen et al. (2005) with constants \((c_1, c_2) = (0.125, 29, 0.1)\), which corrects the normal and tangential coefficients instead. The latter two coefficients are then used to compute both the normal and tangential induction factors within the iterative procedure. The accuracy of this new procedure is illustrated in Fig. 1 by comparison of the normal and tangential forces at different sections along the wind turbine blade with those computed from simulations of a uniform inflow, 8 m s\(^{-1}\), using the EllipSys3D finite-volume flow solver (Michelsen 1992; Sørensen 1995). EllipSys-RANS-AD is based on a Reynolds-averaged Navier–Stokes simulation (RANS), where the wind turbine is modeled as an AD including airfoil data (Réthoré et al. 2014) with same tip correction as the GAD model, and the \(k–\omega\) SST turbulence model by Menter (1993) is used; the numerical setup is described in detail by Pirrung et al. (2020). EllipSys-DES-FR represents a detached eddy simulation (DES), where the blade geometry is resolved in the numerical grid (Bak et al. 2013).

The force distributions of the three models compare quite well as shown in Fig. 1. The largest differences between the AD models and the rotor resolved model are found at the radii, where the gradient of the force with respect to the radius is the largest, i.e., at the blade tip and at the inboard part of the blade. The AD models compare well at the blade root but show differences at the tip and around \(r = 25\) m. We also use the BEM code HAWC2 (Hansen 2012) to calculate the aerodynamic coefficients as function of wind speed (Bak et al. 2013). In terms of integrated aerodynamic coefficients, we obtain \(C_T = 0.815, 0.832, 0.814, 0.834\) and \(C_p = 0.476, 0.516, 0.511, 0.491\) for HAWC2, WRF-GAD, EllipSys-RANS-AD, and EllipSys-DES-FR, respectively. It should be noted that the constant \(c_2\) is calibrated following Réthoré et al. (2014) such that the below-rated \(C_T\) of the EllipSys-RANS-AD model equals the value predicted by BEM-HAWC2.

Figure 2 shows that both the power and the thrust coefficients, which are integrated over the turbine rotor, for a number of wind speeds using the BEM methods of the GAD implemented in the WRF Model compare well with results from HAWC2. The largest differences are obtained at 12 m s\(^{-1}\), where the GAD model computes the turbine power output beyond rated power. This can be solved by adjusting the pitch angle table obtained from HAWC2, but this is beyond the scope of this study, since our focus is on winds below that where the turbine achieves rated power. The integrated values are those we use as inputs for the mesoscale simulations using the Fitch scheme.

c. Simulations setup

All PBL and LES runs use the same vertical grid and levels with 120 grid points; within the first 250 m, the vertical spacing is constant and close to 5 m with the model top at 2000 m. Also, for both types of runs, the PBL is assumed dry and neutral; the runs are initialized with a virtual sounding with an initial potential temperature of 289.5 K that is constant up to the inversion height \(z_i\). This type of atmosphere is also known as conventionally neutral. The sensitivity of the results to the inversion height and strength is considered; Table 2 provides the overview of the inflow simulation cases. As shown in the same Table, the simulations aim at generating the same inflow hub-height wind speed for the different cases, i.e., \(U_h = 10\) m s\(^{-1}\), except for a case in which we test the sensitivity to a lower value, i.e., \(U_h = 6\) m s\(^{-1}\). Also, we aim at having a hub-height inflow as parallel as possible to the grid, i.e., that the hub-height direction is 270°, which is parallel to the \(x\) direction. Therefore, each simulation is initialized differently, with a sounding that describes the initial PBL vertical profiles of the \(u\) and \(v\) velocities, which are in the \(x\) and \(y\) horizontal directions, respectively. Here, both velocity vertical profiles are initialized with constant values and Table 2 shows both the initial

![Fig. 1](image1.png)

**Fig. 1.** (a) Normal and (b) tangential forces as function of the radius for a wind speed of 8 m s\(^{-1}\) using the GAD in WRF, and RANS and DES computations using EllipSys3D.

![Fig. 2](image2.png)

**Fig. 2.** Integrated rotor thrust coefficient and power as function of wind speed using the BEM-based HAWCStab2 and the GAD in WRF for the DTU 10-MW reference wind turbine.
The 100 m × 100 m horizontal grid points. We test the sensitivity of the results to the horizontal grid spacing Δx, so we use 1 and 2 km, i.e., typical spacings when running wind-farm parameterizations, as well as to simulating one or two turbines at the mesoscale grid point. We refer to these sensitivity cases as turbine simulation cases (see Table 3).

As mentioned, the PBL simulations are performed with the MYNN2 PBL scheme; we first run the model without the turbine during 10 h under periodic lateral boundary conditions to develop the PBL. We then perform two restart runs: one without and one with the turbine in the middle of the domain. These two runs are performed using open lateral boundary conditions and carried out for 6 h; 1-min simulated outputs within the last hour are here time averaged.

We apply the bug fix in the WRF code to enable TKE to be horizontally advected (see Archer et al. 2020, for details), as only the last WRF Model distributions solve the issue. The problem does not originate from the Fitch scheme but is due to bypassing of the TKE advection by the WRF module updating the TKE. Within the Fitch scheme, we also implement the TKE correction factor in Eq. (2) and; we use cf = 0.25 as suggested by Archer et al. (2020). As mentioned, we use the power and thrust coefficient curves derived from the GAD BEM methods in Fig. 2 as inputs to the Fitch scheme.

2) LARGE-EDDY SIMULATIONS

The LES runs are performed using two domains with a 1:3 grid aspect ratio. The outer and the inner one-way nested domains have 500 × 500 and 502 × 502 horizontal grid points, respectively. As with the PBL runs, we test the sensitivity of the results to both the horizontal grid spacing, i.e., Δx = 5, 10, and 20 m in the inner domain, as well as to simulating one or two turbines (see Table 3). The inner domain has its origin at the outer domain grid point (166, 166) from the left-bottom corner. Figure 3 shows the domain configuration for the case

<table>
<thead>
<tr>
<th>Case</th>
<th>Horizontal grid spacing (Δx; m)</th>
<th>No. of turbines</th>
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<tbody>
<tr>
<td>PBL-1WT</td>
<td>1000</td>
<td>1</td>
</tr>
<tr>
<td>PBL-2WT</td>
<td>1000</td>
<td>2</td>
</tr>
<tr>
<td>PBL-1WT-coarse</td>
<td>2000</td>
<td>1</td>
</tr>
<tr>
<td>LES-1WT</td>
<td>10</td>
<td>1</td>
</tr>
<tr>
<td>LES-2WT</td>
<td>10</td>
<td>2</td>
</tr>
<tr>
<td>LES-1WT-coarse</td>
<td>20</td>
<td>1</td>
</tr>
<tr>
<td>LES-1WT-fine</td>
<td>5</td>
<td>1</td>
</tr>
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As mentioned, the PBL simulations are performed with the MYNN2 PBL scheme; we first run the model without the turbine during 10 h under periodic lateral boundary conditions to develop the PBL. We then perform two restart runs: one without and one with the turbine in the middle of the domain. These two runs are performed using open lateral boundary conditions and carried out for 6 h; 1-min simulated outputs within the last hour are here time averaged.

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<tr>
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<td>20</td>
<td>1</td>
</tr>
<tr>
<td>LES-1WT-fine</td>
<td>5</td>
<td>1</td>
</tr>
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where $\Delta x = 10$ m in the inner domain; this serves as baseline for the LESs.

As for the PBL simulations, we run the outer domain without the turbine for 10 h under periodic lateral boundary conditions to develop the PBL. In this domain, turbulence is therefore self-triggered after $\approx 2$ h and the speed of the jet shows maxima within 10–12 h, similarly to the WRF-based LES runs under neutral atmospheric conditions in Peña et al. (2021). Although, in this work we use two domains, model options with regards to both physics and dynamics are identical to the single domain runs in Peña et al. (2021). The latter study showed the ability of WRF-LES to represent atmospheric turbulence under a range of stability conditions by comparison of mean wind and turbulence characteristics, which included both turbulence fluxes and spectra, against high-quality measurements from sonic anemometers covering the first 241 m from the ground.

After the 10-h spin up, we perform two restart runs: one without and one with the turbine in the middle of the inner domain using open lateral boundary conditions, while keeping periodic lateral boundary conditions in the outer domain. The two restart runs are carried out for 2 h and we time average the 10-s LES outputs within the last hour. The LES runs are carried out using the subgrid-scale model of Deardorff (1980) with the prognostic equation for the subgrid TKE. The WRF-GAD includes a yaw controller, which turns the wind turbine rotor toward the wind. For these simulations, we turn off the yaw controller to fairly compare the LES with the PBL outputs.

Similarly to Archer et al. (2020), we define three areas to spatially average the LES time averages. We aim at intercomparing the LES spatiotemporal averaged vertical profiles of both the velocity and TKE differences against the temporal averages of the PBL runs at the grid point correspondent to those areas. That is the grid point in the middle of the domain of the PBL run and the two grid points downwind of this. For the velocity field, the difference is between the run without and with the turbine, whereas for the TKE, it is between the run with and without the turbine. For the $\Delta x = 5$ and 10 m cases, these LES areas are each of 1 km $\times$ 1 km, as shown in Fig. 3. When simulating two turbines, we place them at (2500, 2300) m and (2500, 2700) m, i.e., within the same 1 km $\times$ 1 km area in the middle of the inner domain. They are as separated as much as possible to ensure the same inflow at both rotors and as close as possible to avoid the wakes to meander to the areas above and below the middle area.

As shown in Table 2, we have five different inflow cases: baseline, low $z_i$, low inversion, low $U$, and high $z_0$. These are combined with the first wind turbine (WT) case, which can be either a PBL or LES run as shown in Table 3. Further, we add the combination of the inflow baseline case with, first, the two turbine cases using a PBL and an LES run, i.e., PBL-2WT and LES-2WT, the single wind turbine cases with a coarser
horizontal grid spacing, i.e., PBL-1WT-coarse and LES-1WT-coarse, and the single with turbine case with a finer horizontal grid spacing, i.e., LES-1WT-fine. All these combinations result in seven different types of simulations for the PBL runs and eight for the LES runs. As we run both with and without the turbine, we run in total 28 simulations; baseline-PBL-2WT and baseline-LES-2WT only need to be run with the turbine.

3. Results

Here we show results of the intercomparisons between the LES and the PBL runs in four different steps. First we look at the inflow conditions, i.e., vertical profiles of a number of variables before the turbine is simulated, of all runs in section 3a. Then we look at hub-height cross sections of both the horizontal velocity difference $\Delta U$ between the simulation without and with the turbine and the horizontal TKE difference $\Delta$TKE between the simulation with and without the turbine for a PBL and an LES run in section 3b. We also look at vertical cross sections along the inflow hub-height wind direction of both $\Delta U$ and $\Delta$TKE for a PBL and an LES run in section 3c. Finally, in section 3d we show the main intercomparisons between the PBL and LES runs by looking at the vertical profiles of both $\Delta U$ and $\Delta$TKE for all the simulations. Note that for the LES cases, the TKE is the total TKE, i.e., we add the sub-grid to the resolved part.

a. Inflow conditions

Figure 4 shows spatial-averaged vertical profiles over the whole domain of a number of variables within the first 900 m for the PBL runs at 10 h, i.e., just before the turbine is simulated. As illustrated in Fig. 4a, the potential temperature is nearly the same for all PBL runs within the rotor area; inversions are located where targeted, i.e., at 350 or 700 m, with strengths well defined except for the high $z_0$-PBL run, where the profile wiggles the most above the inversion. As shown, for all runs except low $U$-PBL, we match a hub-height wind speed of 10 m s$^{-1}$ (Fig. 4b) and a hub-height direction of 270° (Fig. 4c) so that the flow is nearly parallel to the grid. When looking at the TKE profiles in Fig. 4d within the rotor area in particular, we observe clearly both the effect of the higher roughness, close to 1 m$^2$ s$^{-2}$ at hub height, and the lower wind speed, which has nearly half the value of the runs with the same low roughness.

Similarly, Fig. 5 shows spatial-averaged vertical profiles of the same variables but for the LES runs. As for the PBL runs and within the rotor area, the potential temperature is nearly the same and both the hub-height wind speed (Fig. 5b) and direction (Fig. 5c) match the targeted values. None of the simulations present wiggles and the largest differences, when compared to the PBL runs, are found above the rotor area; the high $z_0$-LES run shows a much lower vertical shear than the high $z_0$-PBL run, which for the latter results in...
wind speeds higher than 15 m s$^{-1}$ around the inversion. We can also see that the resolved TKE (Fig. 5d) by the LES runs is in good agreement with the TKE of the PBL runs with a peak in the profile very close to the lower-tip height, slightly higher for the baseline-LES-coarse run. When looking at the total TKE that accounts for resolved and subgrid terms (Fig. 6a), the TKE matches well the hub-height TKE value of the high-roughness PBL run, but we also see larger differences when decreasing horizontal grid spacing in the LES runs compared to those in the PBL runs. This, as illustrated in Fig. 6b, is partly due to a larger fraction of the subgrid TKE on the total TKE. The effect of increasing grid spacing in the LES is only noticeable in the TKE; the vertical profiles of the other variables barely change (see Fig. 5). Although the effect of grid spacing is visible in the LES, all LES runs show the same behavior for the ratio of subgrid to total TKE with the high $z_0$-LES run showing the largest ratios very close to the ground.

b. Horizontal cross sections

We illustrate horizontal-cross sections at hub height of both $\Delta U$ and $\Delta TKE$, which are time averaged within the last hour of a PBL run baseline-PBL-1WT (Fig. 7) and an LES run baseline-LES-1WT (Fig. 8). As shown in Fig. 7, values of $\Delta U$ and $\Delta TKE$ are different from zero within one/two grid points upwind and a number of grid points downwind the grid point where the turbine is placed, which is within $(x, y) = 49\sim50$ km. The turbine causes a velocity deficit and sinks some TKE at the upwind grid point. This velocity deficit is mainly driven by pressure differences and might be not an artifact of the PBL simulation. However, as shown latter when describing the LES results, a TKE sink should not appear in the PBL runs. The TKE is highest at the turbine grid point, whereas the highest velocity differences appear at the two downwind grid points. One can also see that, although the hub-height direction at 10 h is 270$^\circ$, the mesoscale wake is slightly deflected. However, note that the cross-section shows the time average between 15 and 16 h where the hub-height direction ranges between 274.1$^\circ$ and 274.6$^\circ$ for the simulation without turbine. Although not shown in Fig. 7, the presence of the turbine appears to affect these simulated fields for some kilometers downstream; at hub height, the velocity deficit is around 5.3% at 2.5 km ($\sim$14D) and below 2% at 10 km ($\sim$56D) downstream of the turbine.

Figure 8 shows that in the LES and at hub height, the turbine affects the area where it is placed and the downwind areas mostly. The $\Delta U$ and $\Delta TKE$ values are much higher than those of the PBL run, $\geq$4 m s$^{-1}$ and $>1$ m$^2$ s$^{-2}$, respectively, because the LES results are not spatially averaged. At hub height, there is a small area just after the rotor, where the TKE is lower for the simulation with than without the turbine and we can clearly notice the induction zone where the velocity decreases in front of the turbine. At 2.5 km ($\sim$14D) downstream of the turbine, the maximum velocity deficit at hub height is 4.1%.

c. Vertical cross sections

Here, we illustrate vertical cross sections throughout the $x$ direction in the middle of the domain, of both $\Delta U$ and $\Delta TKE$. As with the horizontal cross sections, the values are time averages.
within the last hour and we show both the result for a PBL run baseline-PBL-1WT (Fig. 9) and for an LES run baseline-LES-1WT (Fig. 10). As shown in Fig. 9, the largest velocity differences are at the rotor disk but one can notice a slightly skewed wake toward the upper-tip height. Negative velocity differences appear below the lower-tip height, which can occur because of speed up effects around the turbine (Abkar and Porté-Agel 2015a). However, these are exaggerated as they appear up to 4–5 km downstream of the rotor, which contrast with the LES results analyzed below. The induction in front of the turbine is also present. We also see a clearly skewed $\Delta TKE$ field with higher values above hub height and significant TKE values above the upper-tip height for a couple of kilometers downwind the turbine. We can also see a region about 1 km ($\approx5.6D$) upstream of the turbine that extends well within the heights covered by rotor, which is actually just a vertical column at the grid point at $x = 48$ km, where $\Delta TKE$ is negative. The negative sign is caused by the simulation with the turbine; the TKE is slowly increasing when approaching the turbine and at the grid cells just in front of the rotor, the TKE shows a sudden dubious reduction. As we will show below, such a TKE reduction does not appear in the LES results. Close to hub height, the TKE is about a third of the value of the upstream grid point.

**Fig. 7.** Time-averaged hub-height horizontal cross section of (a) the velocity difference $\Delta U$ between the simulation without and with the turbine, and (b) the TKE difference $\Delta TKE$ between the simulation with and without the turbine for the baseline-PBL-1WT run. The grid cell where the turbine is located is highlighted in green.
Figure 10a shows the effect of the turbine for the I1-LES1 run, which decreases the velocity ($\Delta U > 0$ m s$^{-1}$), particularly downwind but also upwind the rotor. There is also a clear increase of velocity ($\Delta U < 0$ m s$^{-1}$) due to the turbine below the lower-tip height extending 780 m ($\sim 4.4D$) downstream of the turbine. The velocity difference behind the turbine and within the rotor area appears rather symmetric with a slight inclination toward the ground, which is probably due to ground clearance (van der Laan 2017), i.e., the vertical spacing between the ground and the start of the rotor. This is not captured by the PBL results. When looking at the $\Delta$TKE field behind the turbine in Fig. 10b, we observe higher values over the areas above hub height than those below hub height, particularly when comparing the regions at the top and bottom part of the rotor. This is expected because of the role of the vertical wind shear. Close to hub height at $x = 2600$–$3000$ m, we find other regions downwind the turbine with high TKE values. Below the rotor and downstream of the turbine near the ground, TKE decreases. This sink in TKE is not captured by the PBL results and has an important impact on the warming and cooling near the ground (Wu and Archer 2021).

d. Vertical profiles

Direct intercomparisons between all the PBL and LES runs are shown in Figs. 11–13 for the study areas 1–3, which are described in Fig. 3. For the PBL runs, the vertical profiles of $\Delta U$ and $\Delta$TKE correspond to the time averages within the last simulation hour, whereas for the LES runs, these are vertical profiles that are time averaged within the last simulation hour and spatial averaged within each of the areas.

Starting with area 1 (Fig. 11a) in which the turbine is placed, we notice that the agreement for the $\Delta U$ vertical profiles between the PBL and LES results is excellent for all inflow and turbine cases. Cases simulating one turbine with either averaging over the 1 km$^2$ areas or with 1-km grid spacing, i.e., all PBL-1WT and LES-1WT cases and the baseline-LES-1WT-fine case, lie all very close to each other except for the low $U$-PBL-1WT and low $U$-LES-1WT cases, as expected, as the turbine experiences the highest thrust under the low speeds from all simulations. The closeness between the baseline/low $\Delta U$-PBL/LES-1WT cases shows the low impact of both inversion height and strength for this turbine. Also, as expected, the largest $\Delta U$ values are clearly seen for the baseline-PBL-2WT and baseline-LES-2WT cases, and the lowest $\Delta U$ values for the case where LES results are averaged over a 4-km$^2$ area corresponding to the PBL 2-km runs, i.e., the baseline-PBL-1WT-coarse and the LES-1WT-coarse runs. The LES results with the highest horizontal resolution baseline-LES-1WT-fine are nearly on top of those with the 10-m grid spacing, i.e., all LES-1WT cases.

When looking at the vertical profiles of $\Delta$TKE from the PBL runs (Fig. 11b), we observe nearly the same pattern with regards to the order in which the vertical profiles appear in the plot as that for the PBL $\Delta U$ profiles, except that here it is the low $U$-PBL-1WT run that shows the lowest $\Delta$TKE values...
at most vertical levels. The LES runs, however, show much lower TKE values than the PBL runs; when spatially averaging the LES results over the 1- and 4-km² areas, we do distinguish TKE highs close to both hub height and upper-tip height but these are far from the values from the PBL runs. The highest $D_TKE$ LES values are found for the two-turbine case, i.e., baseline-LES-2WT, and the lowest for the low $U$-LES-1WT case, as expected. Interestingly, the high $z_0$-LES-1WT run shows relatively high $D_TKE$ values above hub height when compared to the other LES cases, but most $D_TKE$ values below hub height are negative. This means that the TKE added by the turbine depends on the TKE level of the inflow, which for this case is the highest (see Fig. 5d); shear production is reduced for the turbine simulation compared to that without a turbine.

Within area 2, which is the first downwind area, we find the largest $\Delta U$ values for all the PBL cases (Fig. 12a). As within area 1, PBL cases simulating one turbine with $\Delta z = 1$ km show similar $\Delta U$ values for those simulations in which we decrease the inversion strength and height, i.e., baseline-PBL-1WT, low $z_0$-PBL-1WT, and low inv.-PBL-1WT, thus confirming the low impact of these changes for a single turbine wake operating over heights well below the inversion. The high $z_0$-PBL-1WT case becomes more isolated within this area than within area 1, showing the influence of high turbulence background on the wake recovery. The LES and PBL $\Delta U$ profiles are also in good agreement; however, PBL $\Delta U$ values tend to be higher than LES values in the upper part of the rotor, whereas the opposite behavior is generally observed for the lower part of the rotor. The double-peak in the $\Delta U$ profiles observed in all LES cases within area 1 disappears in the profiles within area 2.

In contrast to the results within area 1, there is a fairly good agreement between PBL and LES $\Delta TKE$ profiles within area 2 (Fig. 12b). For all inflow and turbine cases, the shape of the vertical profiles of both types of simulations resemble each other well; they show a clear skewness toward the upper-tip height. The skewness is more pronounced in the two-turbine cases, baseline-PBL-2WT and baseline-LES-2WT, and the high $z_0$-PBL-1WT and high $z_0$-LES-1WT cases, as expected due to the larger vertical wind shear. $\Delta TKE$ values tend to be higher for the PBL runs compared to the LES runs from the upper tip height down to the surface, whereas above the upper tip height both LES and PBL results match relatively well. The impact of inversion height and strength within area 2 is as low as that within area 1. Within area 2, we observe the largest $\Delta TKE$ values from the LES among the three areas,
whereas the maximum $\Delta TKE$ values from the PBL runs are nearly as large as those found within area 1.

Finally, within area 3, we find the largest differences between the LES and PBL $\Delta U$ profiles (Fig. 13a); the PBL values being much higher than the LES ones, particularly within the vertical levels where the turbine operates. The shape of the vertical profiles is also different for both type of simulations: PBL cases show a quasi-Gaussian shape and some skewness toward the upper part of the rotor, whereas for the LES cases the skewness is toward the surface and in the low-resolution, low-speed and high-roughness cases, i.e., the low $U$/high $z_0$-LES-1WT and baseline-LES-1WT-coarse runs, the $\Delta U$ values do not change much within the rotor levels. The overall picture is, however, the same; the impact of inversion strength and height is very low both in the LES and PBL results, whereas the impact of roughness, mainly through inflow

![Figure 10](image.png)

**Fig. 10.** As in Fig. 9, but for the run baseline-LES-1WT.

![Figure 11](image.png)

**Fig. 11.** Vertical profiles of time-averaged outputs of the PBL runs (solid lines), and spatial- and time-averaged outputs of the LES runs (markers) for area 1. (a) Velocity difference $\Delta U$ and (b) TKE difference ($\Delta TKE$). For simplicity, the legend only contains seven of the PBL cases, but the analogous LES cases are displayed with same colors and in markers.
turbulence, and inflow velocity is much higher. The PBL $\Delta U$ profiles are very similar to those within area 2, as expected when looking at the hub-height cross sections where one could anticipate the rather extended impact of the turbine on all grid points downwind that where the turbine is placed (see Fig. 7a). As within area 2, below lower-tip height all PBL runs show negative $\Delta U$ values, i.e., the speed up of the flow is exaggerated below the turbine. Due to the spatial-averages performed with the LES output, this effect nearly disappears, although present as shown for the results where the LES output are only time averaged (Fig. 10a).

Similarly to the results within area 2, PBL and LES $\Delta TKE$ profiles show good agreement (Fig. 13b). Both PBL and LES clearly show skewness toward the upper-tip height but, in contrast to the results within area 2, $\Delta TKE$ values tend to be higher for the LES compared to the PBL runs. The high $z_0$-PBL-1WT case, in particular, shows negative $\Delta TKE$ values, i.e., lower TKE levels below hub height compared to the simulation without turbine. In analogy to the high roughness LES case within area 1, here shear production is reduced for the turbine simulation compared to that without a turbine. This also occurs for the corresponding LES case, high $z_0$-LES-1WT, but at the vertical levels close to the surface. As for the other areas and for the $\Delta U$ results, inversion strength and height do not have an important role on the $\Delta TKE$ profiles.

4. Conclusions

From the intercomparison of LES and PBL simulations within the three different areas, we observe a very good match of the velocity differences within the turbine operating area. However, larger discrepancies are observed when analyzing the

FIG. 12. As in Fig. 11, but for area 2.

FIG. 13. As in Fig. 11, but for area 3.
velocity differences within the two next downwind areas. This is mainly because the wake does not recover fast enough in the PBL simulations. For the TKE differences, a somewhat opposite behavior is observed; within the turbine operating area, TKE differences are highest, whereas there is a relatively good agreement between LES and PBL results within the two downwind areas. For the PBL simulations, the added TKE by the turbine has to be highest at the turbine grid point in order to be advected downwind as the PBL TKE directly depends on the slowly decreasing downwind wind speed. However, in the LES, the added TKE appears mostly at the tip of the blades with spatial averages of the TKE field close to the turbine will inevitably show only a fraction of the impact of the turbine. Also, in LES some distance is required for the shear induced by the flow around the rotor edges to generate TKE as well.

From both the LES and PBL simulations’ results, we observe that when varying the inversion height and, particularly, the inversion strength, the vertical profiles of $\Delta U$ and $\Delta$TKE do not significantly change. This is observed at the three areas of study. However, it is important to note that the turbine is operating at heights well below the inversion level. The impact of inversion height and strength is higher in the LES than in the PBL runs and is higher for the $\Delta$TKE than for the $\Delta U$ vertical profiles.

From the one-turbine cases, the largest impact is found for the low-speed case both in the velocity and TKE differences, for both LES and PBL simulations and within all three analyzed areas. Not that high, but also important is the impact when increasing roughness; this is particularly noticed when looking at the TKE differences and for the PBL results. Doubling the grid resolution in the LES has negligible impact on both TKE and velocity differences within the turbine operating area. Doubling the grid spacing for the PBL simulations shows the high sensitivity of the Fitch scheme to horizontal grid spacing. Both LES and PBL simulations using two turbines show, as expected, the highest velocity and TKE differences of all cases with no particular improvement/deterioration of the PBL–LES differences within any of the three analyzed areas when compared to the other PBL/LES cases. However, it should be noted that for the latter simulations, the turbines are not aligned with the mean flow and that for more aligned cases, the differences between the PBL and LES simulations become dependent on the flow alignment and turbine interspacing. For cases in which the turbines are more aligned with the mean flow, i.e., partial or total waked turbines, one alternative is to choose the resolution of the mesoscale run in such a way that the turbines can be placed each in a grid cell. Also, one could model the effect of a turbine layout by adapting the drag force using an equivalent wind-farm $C_D$ instead of the turbine-specific $C_D$. This equivalent $C_D$ is a function of wind speed and direction. Abkar and Porté-Agel (2015) showed this using a correction factor for $C_D$.

Our findings demonstrate that mesoscale wake simulation outputs must be used with caution and that, if possible, results from high-resolution wake simulations should be used to validate the mesoscale outputs. We therefore recommend to use the Fitch scheme for wind-farm planning purposes but to turn into detailed wake simulations when designing and predicting the power output and climatological impacts of wind farms. In the near future, we will evaluate mesoscale wake simulations with LES wake simulations in the WRF Model for a large wind farm under different atmospheric stability conditions.

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Data availability statement. The numerical model simulations upon which this study is based are too large to archive. We therefore provide information needed to replicate the simulations using the WRF Model, version 3.7.1, which is available at https://www2.mmm.ucar.edu/wrf/users/download/get_source.html; initial conditions (input_soundings), configuration and settings (namelist.input), and turbine specific files are available at https://figshare.com/s/bd5f0f6a5223740e8e8e (Peña 2022). Due to its current proprietary nature, the modifications to the WRF Model source code that include the GAD can be requested to Jeffrey D. Mirocha (mirocha2@llnl.gov).

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