SOME CHARACTERISTICS OF THE CALLENDAR PYRHELIOMETER.

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SYNOPSIS.

Theory of the Callendar automatic pyrheliometer. The indications of the Callendar pyrheliometer differ from the calculated intensities of radiation upon a horizontal surface on account of (1) greater sensitiveness for low intensity than for high; (2) selective absorption of short-wave radiation by platinum compensating grids; (3) internal reflection of light from glass cover to grids; (4) selective absorption by cover glass of ultra-violet and infra-red radiation from sun, and total absorption of radiation from grids; (5) grid surfaces not geometrical planes; (6) lag of registration behind radiation.

The data in the accompanying paper[1] on "Measurements of Solar Radiation at Madison, Wis., with the Callendar pyrheliometer" are entirely uncorrected for instrumental error. While such data may be used for comparison with data obtained with the same type of instrument elsewhere, it is obviously desirable to know the probable relation of the data to absolute heat units. Study of the apparatus has also suggested the possibility of eliminating two of the most obnoxious errors.

The "bolometric sunshine receiver" of Prof. H. L. Callendar, the eminent English physicist, is related structurally to both the bolometer and the electrical resistance thermometer. Its electrical circuits are those of the four-lead compensated electrical resistance thermometer, but with the compensating leads extended to include compensating grids of the same dimensions as the thermometer grids. The thermometer grids are blackened with enamel, the compensating grids are bare, bright, platinum wire. The receiver at Madison, like that described by Kimball (1) has four square grids—two black thermometer grids, two bright compensating grids—arranged checkerboard fashion in a horizontal plane, and inclosed in a vacuum bulb of glass.

This apparatus is intended to record continuously, with the aid of a Callendar Recorder, Wheatstone Bridge Type, the intensity of the radiation from the sun and sky as it would be received upon a horizontal surface. This datum, the vertical component of sun and sky radiation, is of fundamental importance in all large-scale climatological studies, where the local irregularities of the surface become negligible, and the total amount of energy delivered to the earth's surface is required.

The comparison instruments used at Madison in testing the Callendar apparatus are the Marvin pyrheliometer and the Smithsonian pyranometer, the former from the beginning of observations in 1911, the latter from 1917. The Marvin pyrheliometer, which has been described by Kimball (2) and Foote (3), measures only the intensity of direct solar radiation at normal incidence. The Callendar pyrheliometer is compared with that of Marvin by shading the Callendar receiver from the direct rays of the sun with the Kimball screen (1, p. 477), measuring the drop in the ordinate on the recorder thereby produced, and dividing this by the sine of the sun's altitude. The Smithsonian pyranometer, described by Abbot and Aldrich (4) measures the same component of sun and sky radiation as the Callendar apparatus, and is supplied with attachments to adapt it to measure the sun's radiation alone, or the sky radiation alone. It can therefore be compared with the Marvin pyrheliometer, and with the residual ordinate of the Callendar pyrheliometer when shaded from the sun.

[1] This REVIEW, pp. 338–343.

Although the Marvin pyrheliometer has been established as a primary pyrheliometer by the experiments of Marvin, Kimball, and Foote, it is used at Madison as a secondary pyrheliometer based on the Smithsonian Standard, and has been intercompared at approximately biennial intervals. The pyranometer is also based upon the Smithsonian Standard, but pyranometer No. 1 at Madison has given indications slightly below those of the Marvin pyrheliometer, and shows a progressive change of ratio with increasing altitude of the sun, ranging from 91 per cent of the Marvin at 20° to 99 per cent at 60°. Pyranometer No. 2 at Washington shows a similar change of ratio, but its indications are higher. The difference is not due to the ammeters[1] used with the two instruments. The two pyranometers were carefully intercompared during their standardization at the Smithsonian Institution.

The factor for receiver No. 9864, at Madison, supplied by Prof. Callendar is equivalent to 0.3505 gram calories per square centimeter of horizontal surface per minute, for each inch of ordinate on the trace sheet. This factor was determined by Prof. Callendar by comparisons with an Ångström pyrheliometer. The Ångström standard gives results 3.23 per cent below the Smithsonian Standard (5) and other Callendar instruments have had to have their constants increased by this much or more to agree with the Smithsonian standard, but No. 9864 gives results agreeing on the average very closely with the Smithsonian Standard for solar altitudes above 20° without such a correction. The instrumental characteristics of the Callendar pyrheliometer will now be systematically discussed.

SCALE ERROR.

The Callendar pyrheliometer depends upon the static method of pyrheliometry, (6) in which is determined the maximum excess of temperature over that of the surrounding medium that is attained by a body exposed to solar radiation. Inasmuch as the radiation and the rate of loss by conduction and radiation enter the equation exponentially, it is not to be expected that the relation will be linear, as is assumed by the employment of a uniform reduction factor. The temperature attained by the black grid under intense radiation will not be as high in proportion to the radiation as under less intense radiation.

Experimenting with a rotating disk diaphragm, Kimball (1, p. 475, Table 3) found that the sensitiveness of the Callendar pyrheliometer compared with the Marvin pyrheliometer by pointing both directly at the sun decreases with increasing intensity of radiation in the following proportion:

TABLE 1.

Intensity of radiation.	10 per cent.	20 per cent.	30 per cent.	50 per cent.	100 per cent.
Ratio (Callendar/Marvin)	1.18	1.13	1.10	1.05	1.00

The Callendar receiver at Madison was not subjected to this experiment, but the phenomenon, partly offset by other errors, clearly appears in the following table based on comparisons, under working conditions, with the pyranometer.

[1] The mil-ammeter employed at Washington has had its scale errors determined at the Bureau of Standards, and its corrected readings can not be in error by more than ±1 per cent; that at Madison has been compared with precise instruments in the University of Wisconsin with a similar result.

TABLE 2.

	No. of observations.	Mean radiation (cal.)	Callendar constant (cal./in.).	Radiation intensity, per cent of:					Sensitiveness, per cent of:				
				A.	B.	C.	D.	E.	A.	B.	C.	D.	E.
A	25	1.334	.359	....	....	....	....	....	....	....	....	....	....
B	38	.918	.352	69	....	....	....	....	102 (102)	....	....	....	....
C	82	.879	.351	66	96	....	....	....	102 (103)	100 (100)	....	....	....
D	21	.516	.342	39	56	59	....	....	105 (107)	103 (104)	103 (103)	....	....
E	36	.362	.355	27	39	41	75	....	107 (111)	105 (107)	105 (107)	102 (102)	....
F	39	.153	.333	13	17	17	32	51	108 (117)	106 (114)	105 (114)	103 (109)	101 (105)

A—Radiation from sun and sky, midsummer, within 2½ hours of noon.
B—Sun alone, all altitudes of sun included.
C—Sun and sky, all altitudes of sun.
D—Sun and sky, autumn, within 2½ hours of noon.
E—Overcast skies.
F—Sky alone, clear blue skies.
Calculated sensitiveness, in parentheses, from Table 1.

PLATINUM SELECTIVE ABSORPTION ERROR.

Researches on the reflecting power of metals by Rubens and Hagen (**7**) and by Coblentz (**8**) have shown that all metals vary in their reflecting power for different wave lengths of the spectrum, tending to reflect less and absorb more at short wave lengths than at long. Platinum, which reflects 70 to 80 per cent of the energy in wave lengths of $.8\mu$ to 1.5μ, absorbs 50 to 66 per cent of the energy between $.25\mu$ and $.40\mu$. The latter region of the spectrum of sunlight is not only the region of greatest intensity, but also the region of greatest change of intensity from high sun to low; furthermore, it is the region of greatest intensity in the spectrum of sky light. Curves representing the reflecting power of platinum throughout the spectrum, as determined by Rubens, Hagen, and Coblentz, the distribution of energy in the solar spectrum for high and low sun, according to Abbot (**9**) and the spectrum of sky light at dawn and midday, according to Nichols (**10**) appear in figure 1. Figure 2, drawn to the same scale, shows the proportion absorbed by platinum when exposed to these several types of spectral distribution. Graphical integration indicates that platinum absorbs 30 per cent of the light from a high sun (30° or higher, air mass not greater than 2) and 26 per cent of the light from low sun (10° or less, air mass equals or exceeds 6). Of sky radiation about 45 per cent is absorbed at both dawn and midday.

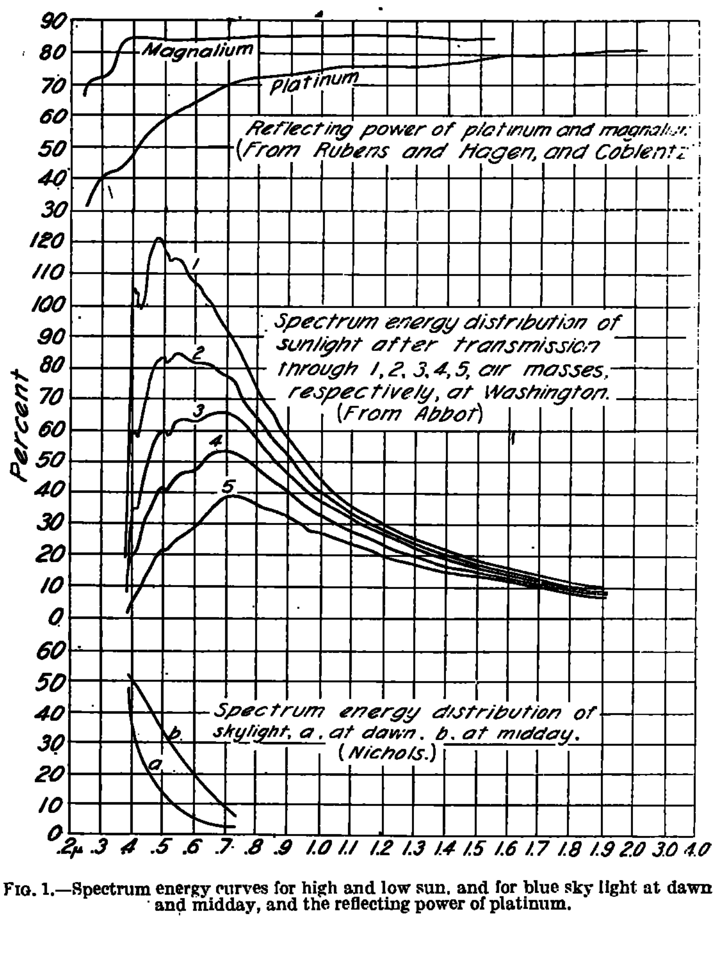

FIG. 1.—Spectrum energy curves for high and low sun, and for blue sky light at dawn and midday, and the reflecting power of platinum.

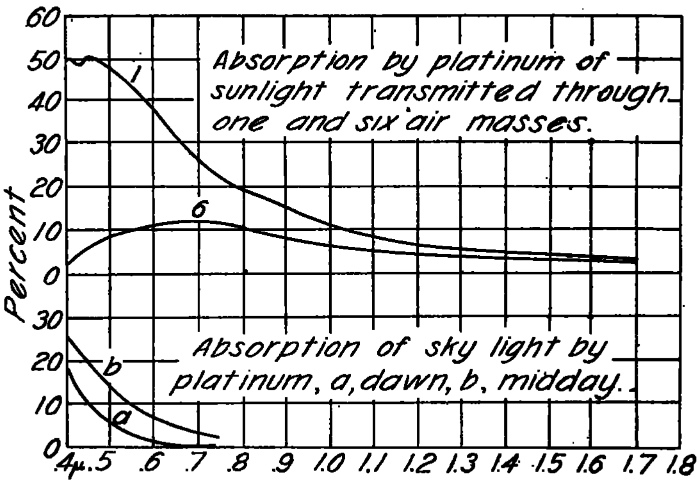

FIG. 2.—Absorption by platinum of sun and sky radiation, same scale as figure 1.

Theoretically, then, the Callendar pyrheliometer should show a decrease of sensitiveness of 5 per cent as the sun passes from 10° to 30° in altitude, and should be less sensitive to sky radiation than to solar radiation by at least 21 per cent.

Kimball ((**1**) p. 476, table 4, pt. 2) experimented with a ray filter designed to reduce the solar spectrum energy curve to approximately that of sky light. Exposed alternately to full sun light, and through this filter, the Callendar pyrheliometer showed a drop of 10 per cent in sensitiveness. But the ray filter cut off 87½ per cent of the energy of the sun, which, according to Table 1, should have increased the sensitiveness of the Callendar pyrheliometer by 17 per cent. Hence we may conclude that the Callendar is actually 27 per cent less sensitive to sky light than to sun light. The excess of this observed value over the calculated value is explainable by the absence of data as to the intensity of sky light at wave lengths less than $.385\mu$, where the platinum absorption is known to be greatest.

The white pigments, lead carbonate, and the oxides of magnesium, zirconium, and zinc not only have the advantage of high reflection in this and other regions of the visible spectrum, but are good radiators in the long wave heat spectrum so that they would lag less than the platinum in cooling. It seems probable, therefore, that this error could be eliminated by coating the compensating grids with a white matt surface of one of these substances (**12**).

THE INTERNAL REFLECTION ERROR.

Parallel rays of light reflected from the interior surface of a cylinder or sphere, form a brightly illuminated, cusped curve that is often seen on the table cloth within a napkin ring, or water glass, and which is known as the "caustic." Such an illuminated curve is formed within the spherical cover of the Callendar pyrheliometer, and causes an error in its indications that varies with the altitude of the sun above the horizon, and according to the grid, whether thermometer or compensating, upon which the intensely localized caustic falls. Illustrations of the form of the caustic at different elevations of the sun have been given in an earlier number of the Monthly Weather Review (11), and a typical example of its effect on the registration of the Callendar pyrheliometer has been given by Kimball (1), page 479, figure 6, where the curve is depressed in the morning when the caustic fell upon the compensating grid, raised in the afternoon when it fell upon the black grid.

The Callendar pyrheliometer is exposed at Madison with the bright grids in the east and west, the black grids in the north and south corners of a diagonal square. Consequently the caustic falls upon the compensating grids both morning and evening. The effect of this is shown in the following table of the "constant" obtained by comparison with the Marvin pyrheliometer for various elevations of the sun.

TABLE 3.

Sun's altitude.	Over 55°.	55°–35°.	35°–25°.	25°–20°.	20°–15°.	Under 15°.
Instrumental constant, Callendar pyrheliometer calories per cm² per min. per inch of ordinate	.351	.345	.361	.350	.408	.502
Sensitiveness relative to Callendar's constant of .350	100	102	97	100	86	70

It is much more desirable to eliminate than to correct for so variable an error. Theoretically the caustic extends only half the radius from the spherical surface toward the center. The receiving grids should therefore be confined to an inner circle of half the radius of the inclosing glass.

COVER GLASS ABSORPTION ERROR.

The glass cover selectively absorbs both the ultra violet and the infra red portions of the spectrum of the incoming radiation from the sun and the sky. It also absorbs all of the energy radiated by the grids. In consequence of these facts, Kimball (1, p. 475) found that removal of the cover increased the sensitiveness of the instrument by 10 per cent and Ångström (12) found that when the ventilation of the glass cover is poor, as in calm, sunny weather, the grids do not cool, because the glass cover, to which they must radiate, remains hot, and the instrument may on this account be in error as much as 10 per cent. Another consequence of the opacity of the glass for long-wave radiation is that the grids remain at equal temperatures all through the night from sunset to sunrise.

GRID FORM ERROR.

The measurement of the vertical component of sun and sky radiation implies its interception upon a horizontal plane surface. Neither pair of grids in the Callendar pyrheliometer precisely satisfies this condition. The surface of the black grid is wavy and only roughly approximates a plane surface, while the bright grids are made up of cylindrical wires.

LAG.

The indications of the Callendar apparatus are affected by thermometric lag in the receiver and by mechanical lag in the recorder. The definition of thermometric lag by Harper (13) may be restated for the pyrheliometer in the following form:

If a pyrheliometer has been exposed for a long time to a stream of radiation whose intensity is rising at a uniform rate, the lag is the number of seconds between the time when the radiation stream attains any given intensity, and the time when the pyrheliometer indicates this intensity. In other words, it is the number of seconds that the pyrheliometer lags behind the radiation.

The quantitative determination of the lag is more easily arrived at by using another interpretation, viz: If the pyrheliometer be exposed to constant radiation, after having been exposed to radiation of a different intensity, the lag is the number of seconds in which the difference between the indication of the pyrheliometer and the intensity of the radiation to which it is newly exposed is reduced to ϵ^{-1} times its initial value. The value of ϵ^{-1} being approximately .368, the lag of a pyrheliometer is easily found by alternately shading and exposing it, and counting the number of seconds that elapse from the moment the intensity of the radiation is changed until the indicated radiation reaches 63 per cent of the whole change of ordinate. Experiments with the pyrheliometer at Madison in August, 1919, gave a value of two minutes and sixteen seconds for the total lag. The galvanometric and mechanical lag of the recorder, determined separately by displacing the pen carriage accounts for 20 seconds of this. The remainder is due to the thermal capacity of the receiving grids, and of the glass cover, to the extent that it acts as a secondary source of radiation. Study of the traces in experiments in which the Callendar pyrheliometer has been suddenly shaded, or exposed, (such experiments have been made at intervals since March, 1913) shows that the lag has always been about 2 minutes.

The lag error of the pyrheliometer at any time can be determined from the slope of the trace, by making use of the fundamental equation of thermometric lag, viz:

$$\frac{\partial\theta}{\partial t}=\frac{1}{\lambda}(\mu-\theta)$$

where μ = intensity of vertical component of radiation from sun and sky.
θ = intensity indicated by pyrheliometer.
λ = constant of lag.
t = time.

$\frac{\partial\theta}{\partial t}$ is the slope of the trace, while $\mu-\theta$ is the difference between the true and the indicated values of the radiation. The error of the Callendar pyrheliometer at Madison corresponding to various angles of slope is as follows:

TABLE 4.

Angle of slope of trace.	Error due to lag.
°	*Calory per cm² per min.*
5.4	.001
43.6	.010
84.0	.100
89.6	1.000

While these departures from the true value affect the indicated value at any instant, and are therefore of great importance in making comparisons, yet the fluctuations of the stream of radiation are so continual that the pen of the instrument keeps somewhere near the true mean value, integrating the minor fluctuations so that the indicated sum total for an hour or a day is not appreciably affected by lag.

TOTAL OR RESULTANT ERROR.

For solar radiation, the constant supplied by Prof. Callendar for Receiver No. 9864 is shown by comparison with the Marvin pyrheliometer, to be approximately true for solar altitudes above about 20°. For sky light the sensitiveness of the instrument is very much lower, but the proportion of sky light in the total at noon on very clear days is only about 10 per cent. The proportion of sky light in total radiation increases as the sun descends. The average net error for blue sky (selective absorption error minus scale error) is estimated by Kimball (1, page 480) to have the effect of diminishing the indicated radiation below the true radiation by about 2 per cent when the sky is cloudless, by 1 per cent when the sky is half clouded. On overcast days the internal reflection of sun light, and the selective absorption of sky light are both in abeyance, and the intensities are low, so that the scale error is large. The results of comparative observations between the Callendar pyrheliometer, and the Smithsonian pyranometer (uncorrected for difference between its scale and that of the Marvin pyrheliometer) are shown in figure 3 for a clear day, September 16, 1919, and a very cloudy day, September 18, 1919. It will be seen that on the clear day, from 2½ to 5½ hours after noon, the Callendar indications are depressed below the pyranometer by about 10 per cent, but that the difference diminished as the sun went down. On the overcast day, the scale error is seen to raise the Callendar indications above the pyranometer by 15 or 20 per cent.

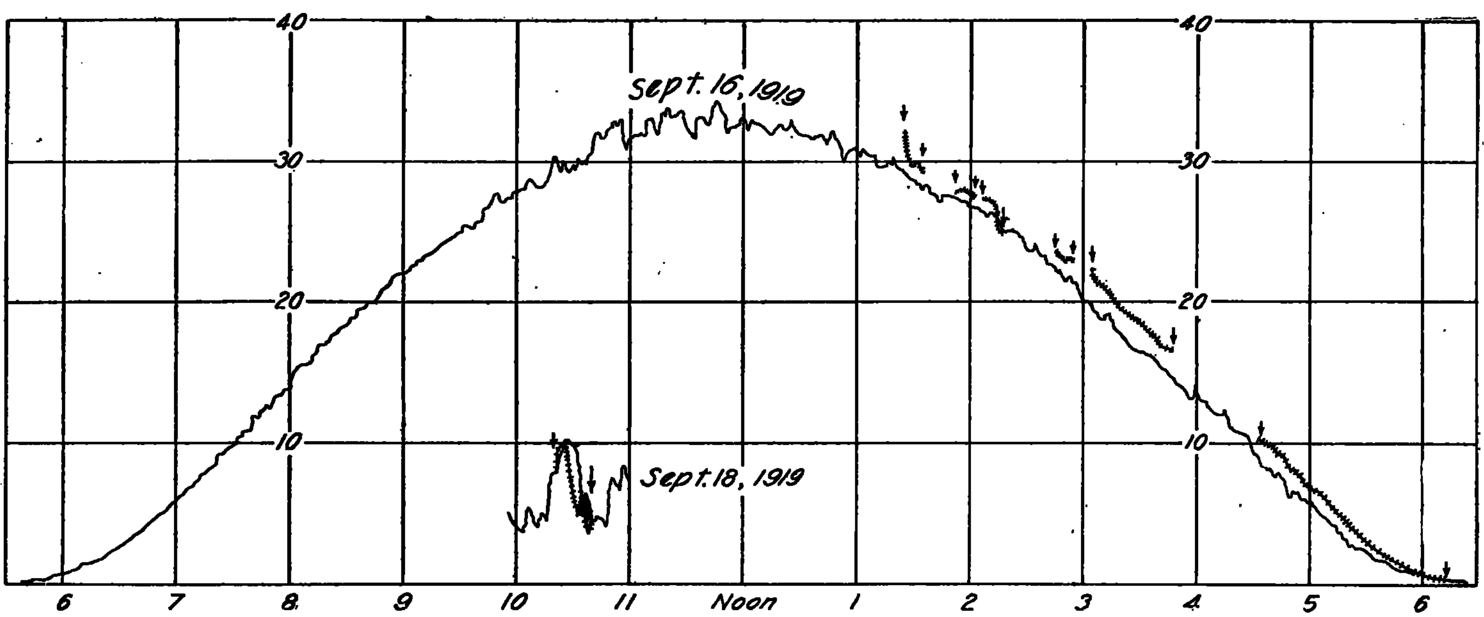

FIG. 3.—Comparative observations, Callendar pyrheliometer, and the Smithsonian pyranometer, on a clear day (September 16, 1919) and on a densely cloudy day (September 18, 1919).

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