TABLE 6.—*Hourly values of* $(K+K_r)\times10^{-4}$ *for Drexel, Nebr., for the 750 m, m.s.l., level*

Time	Spring	Summer	Autumn	Winter
1 a.m.	1.1	3.2	1.6	1.5
2 a.m.	1.3	3.2	1.8	2.1
3 a.m.	1.6	3.3	1.6	2.6
4 a.m.	2.1	3.7	1.5	3.0
5 a.m.	2.4	3.8	1.5	2.9
6 a.m.	2.4	3.9	1.2	2.3
7 a.m.	1.7	----	0.7	1.4
8 a.m.	----	----	----	----
9 a.m.	----	13.0	----	----
10 a.m.	10.9	12.0	----	----
11 a.m.	9.1	11.3	6.4	----
12 m.	8.2	11.1	4.8	8.4
1 p.m.	7.4	11.4	4.1	5.6
2 p.m.	6.5	11.7	4.0	3.9
3 p.m.	5.2	12.5	3.8	2.8
4 p.m.	2.5	----	3.8	1.8
5 p.m.	----	----	----	----
6 p.m.	----	----	2.1	----
7 p.m.	7.6	7.7	2.3	2.3
8 p.m.	5.3	5.6	2.3	1.4
9 p.m.	3.8	4.7	2.1	0.9
10 p.m.	2.5	4.0	2.0	0.6
11 p.m.	1.6	3.5	1.8	0.6
12 p.m.	1.2	3.2	1.6	0.9

TABLE 7.—*Values of* $(K+K_r)\times10^{-4}$ *computed from the diurnal temperature range*

	Height above surface (meters)	Ellendale, N. Dak.	Drexel, Nebr.	Broken Arrow, Okla.	Groesbeck, Tex.
Spring	0–200	3.0	3.3	9.5	29.2
	200–400	4.7	4.1	16.2	29.2
	400–600	19.9	3.4	9.5	29.2
	600–800	13.4	4.4	7.1	21.1
	800–1,000	10.9	8.8	8.8	14.0
Summer	0–200	3.4	5.7	6.9	25.5
	200–400	7.8	7.4	9.5	27.2
	400–600	14.7	5.9	8.8	31.5
	600–800	15.4	5.5	7.4	15.4
	800–1,000	14.0	5.5	5.9	14.0
Autumn	0–200	3.0	2.1	3.2	6.2
	200–400	4.0	3.2	7.1	14.0
	400–600	3.4	4.8	6.8	18.9
	600–800	29.2	15.4	4.0	12.3
	800–1,000	17.9	36.7	8.8	8.8
Winter	0–200	1.2	2.7	3.3	4.1
	200–400	8.8	3.0	3.5	7.1
	400–600	42.9	4.6	2.3	14.7
	600–800	42.9	43.7	6.1	16.2
	800–1,000	31.7	----	----	11.4

THE USE OF GLASS COLOR SCREENS IN THE STUDY OF ATMOSPHERIC DEPLETION OF SOLAR RADIATION

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[Apr. 10, 1933]

There are two kinds of atmospheric depletion of solar radiation to be considered, namely, (1) scattering by the gas molecules and dust of the atmosphere, and (2) selective absorption by atmospheric gases, principally water vapor. The scattering by pure dry air may be computed by the use of equations developed by Lord Rayleigh and modified by King (1). Fowle (2) has shown the relation between the amount of water vapor in the atmosphere and the depletion of solar radiation in the great infrared water-vapor absorption bands of the solar spectrum. There remains, therefore, the absorption by gases other than water vapor, for which Fowle (3) estimates that by ozone to be from 0.2 to 0.4 percent, and that by the remaining permanent gases to be less than 1 percent of the solar constant of radiation. But the water-vapor content of the atmosphere can be obtained through measurements, other than by the spectroscope, only approximately. If, therefore, the reduction of solar radiation through scattering, in addition to the scattering by pure dry air, can be determined, the absorption by water vapor becomes the only unknown factor in atmospheric depletion, and since pyrheliometric measurements give the total reduction, we may determine the amount due to water vapor with a degree of accuracy that depends upon the accuracy with which the other factors are known. Then from this value and the relation between water-vapor content and water-vapor absorption of solar radiation in the atmosphere, as developed by Fowle (2), it is possible to determine the water-vapor content of the atmosphere, a matter of considerable importance to meteorologists.

Ångström (4) has shown that the depletion of solar radiation through scattering by dry air containing ordinary atmospheric dust (and he includes in this the scattering that Fowle (2) found associated with water vapor in the atmosphere) may be expressed by $(e^{-\beta/\lambda^{1.3}})^m$. Hence the intensity of solar radiation after depletion by scattering may be expressed by the equation

$$I_m=\int_{\lambda=0}^{\lambda=\infty} e_{o\lambda}\,(a_{a\lambda})^m\;(e^{-\beta/\lambda^{1.3}})^m\,d\lambda \qquad 1$$

in which

$e_{0\lambda}$ = the intensity of radiation of wave length λ before depletion by the atmosphere,

$a_{a\lambda}$ = the atmospheric transmission coefficient for radiation of this same wave length,

m = the air mass, approximately the secant of the sun's zenith distance,

e = the base of the Naperian system of logarithms, and

β = the coefficient of atmospheric turbidity as defined by Ångström.

Equation 1 has been solved for the 38 different values of λ given in table 111, Smithsonian Meteorological Tables, fifth revised edition, hereafter referred to as table 111, and corrected for the ultraviolet and the infrared radiation not measured. It also has been solved for the values of $e_{0\lambda}\ a_{a\lambda}$ given in the same table, for values of m = 0.0, 0.526, 1.0, 2.0, 3.0, and 4.0; and for β = 0.0, 0.025, 0.050, 0.075, 0.100, 0.150, and 0.200. The integration of equation 1 has been effected graphically by summing up values of I_m equally spaced with reference to the U.V. glass deviation from ω_1. (See table 111.) It will be noted that some of the intensity values near the extremes of the spectrum are twice as far apart on the deviation scale as those nearer the center of the spectrum. Such values were given double weight in the summation.

Figure 1 is a reproduction of a spectrobologram of solar radiation obtained by the Astrophysical Observatory of the Smithsonian Institution. It will be noted that while the wave lengths change relatively faster at the ultraviolet than at the infra-red end of the spectrum, the prismatic deviation is uniform throughout. It is for this reason that deviations instead of wave lengths are used in connection with computations of radiation intensities in this paper. The results obtained from equation 1 have been plotted on figure 2 and connected by curved lines.

In the integrations no attention has been paid to the water-vapor absorption bands in the infra-red. (See figure 1.) Therefore, the curved lines of figure 2 show variations in radiation intensity that would prevail in an atmosphere free from water vapor, with the air mass varying from 0.0 to 4.0, and the atmospheric turbidity from 0.00 to 0.200. The intensities are expressed as percentages of the mean value of the solar constant on the Smithsonian pyrheliometric scale of 1913.

Early in 1932 the United States Weather Bureau received one each of the glass screens OG1 (yellow) and RG2 (red) from Dr. Süring, then director of the Magnetic-Meteorological Observatory at Potsdam, Germany. They were designed to separate out sections of the solar spectrum free from atmospheric absorption bands, but especially from the great water-vapor absorption bands in the infra-red. The intensities measured through these screens have been published each month in the MONTHLY WEATHER REVIEW, beginning with February 1932, but only readings measured through the red screen have been used in determining the value of the turbidity factor, β. For this determination curves constructed by Ångström

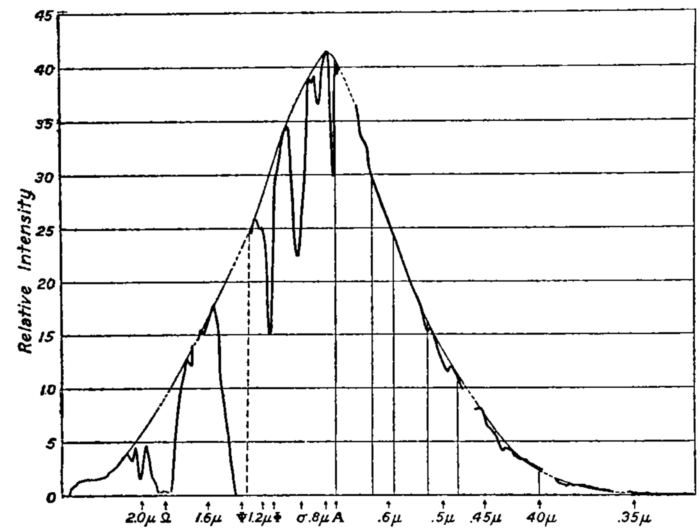

FIGURE 1.—Spectrobologram of solar radiation.

(4) have been employed, for which the abscissas represent the air mass and the ordinates the intensity $I_m - I_r$, or the difference between the intensity of the unscreened radiation and the radiation transmitted by the red-glass filter after the latter has been corrected for absorption and reflection at the screen.

Late in 1932 a second set of glass color screens was received from Potsdam, this time for use at the Blue Hill Meteorological Observatory of Harvard University. For their transmission coefficients at different wave lengths, reference was made to a paper by F. Feussner (Met. Zeit., 1932, Heft 6, S 242–244). The coefficients are given in table 1.

TABLE 1.—*Spectral transmission of Schott filter glasses*

[OG1 (yellow) and RG2 (red)]

Wave length (in $m\mu$)	511	518	522	526	530	535	540	550	560	577	617	622	627	632
Transmission (OG1 in percent)	----	6	29	55	68	78	83.3	87.0	88.2	89.2	89.8	----	----	----
Transmission RG2 in percent	----	----	----	----	----	----	----	----	----	0	(1)	5	18	38

Wave length (in $m\mu$)	637	642	647	652	657	678	698	800	990	1,200	1,370	1,450
Transmission (OG1 in percent)	89.9	----	----	----	89.9	89.9	89.8	89.4	88.6	88.6	88.9	89.3
Transmission RG2 in percent)	60	73	78.8	82.9	85.0	88.3	89.0	88.5	86.8	86.6	87.2	87.6

Wave length (in $m\mu$)	1,750	2,000	2,140	2,250	2,350	2,520	2,650	2,720	2,760	2,820	2,860
Transmission (OG1 in percent)	89.6	89.1	88.0	86.7	86.6	84.9	83.3	68	39	(26)	(23)
Transmission RG2 in percent)	88.3	87.7	86.7	85.0	84.5	83.2	81.0	75	39	(21)	(13)

It will be noted that both these screens drop from a rather high to a low, or even zero, transmission between narrow-wave-length limits. For the yellow screen it is between 518 and 535$m\mu$, and we have taken 526$m\mu$ as the mean point in this drop. For the red screen it is between 622 and 647$m\mu$, and we have taken 636$m\mu$ as the mean point in the drop. The corresponding prismatic deviations on the Smithsonian prismatic scale are 132.5′ and 108′ from ω_1, respectively. Over most of the scale the probable error of the transmission factors as given is stated to be

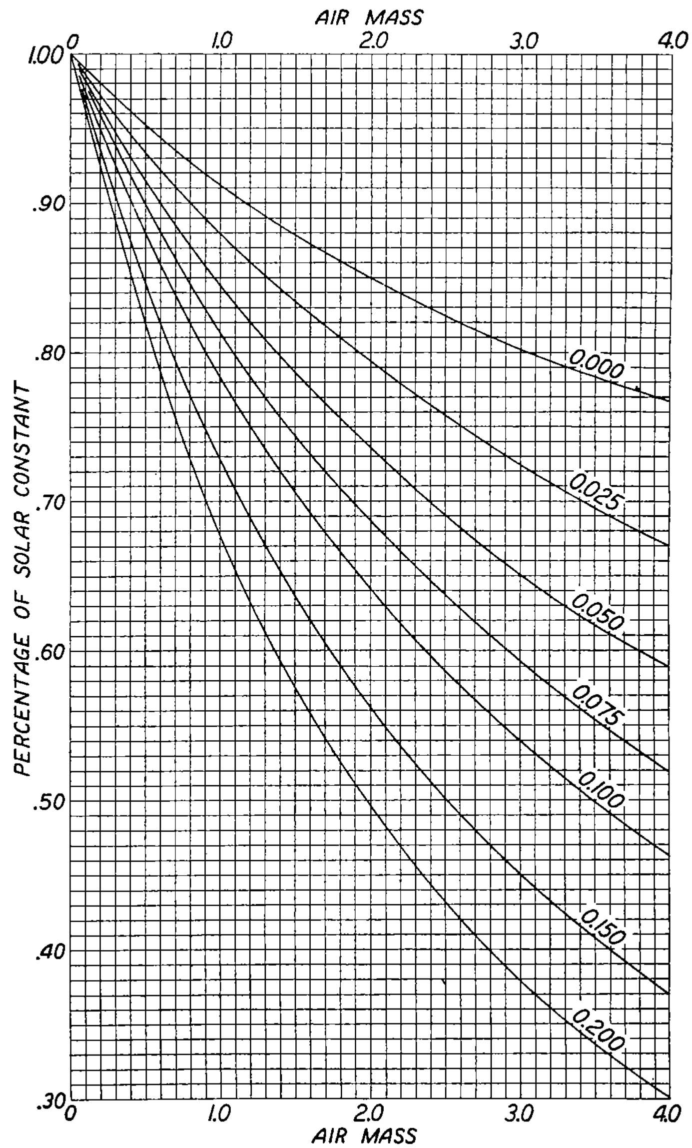

FIGURE 2.—Solar radiation intensity, I_m, after depletion by atmospheric scattering only, for different values of m and β.

±0.3 percent. For comparision of one filter with another it is placed at ±0.5 percent.

Unfortunately neither filter, OG1 nor RG2, cuts off at exactly the same wave length in different samples of the screens. Ångström suggests that this be taken care of by a correction to the transmission determined for an average screen. This would require a standardization of each screen received, at least for the shorter wave lengths. This has not been done either for the screens in use at the United States Weather Bureau or for those at Blue Hill.

Applying the transmission coefficients of the respective filters for given wave lengths to $e_{o\lambda}\ a_{a\lambda}$ (energy distribution for pure dry air, table 111) we have obtained for the transmission of the red screen, RG2, 87.8 percent, for

$\lambda > 526m\mu$, and for the yellow screen, OG1, 88.9 percent, for $\lambda > 636m\mu$.

It appears that for different spectral distributions of solar radiation, as for instance that given in the last column of table 111, slightly different transmission coefficients would be found.

If now we take the difference $I_m - (I_y/0.889)$ we have left $\Sigma I_m(\lambda < 526m\mu)$ which represents the intensity of the

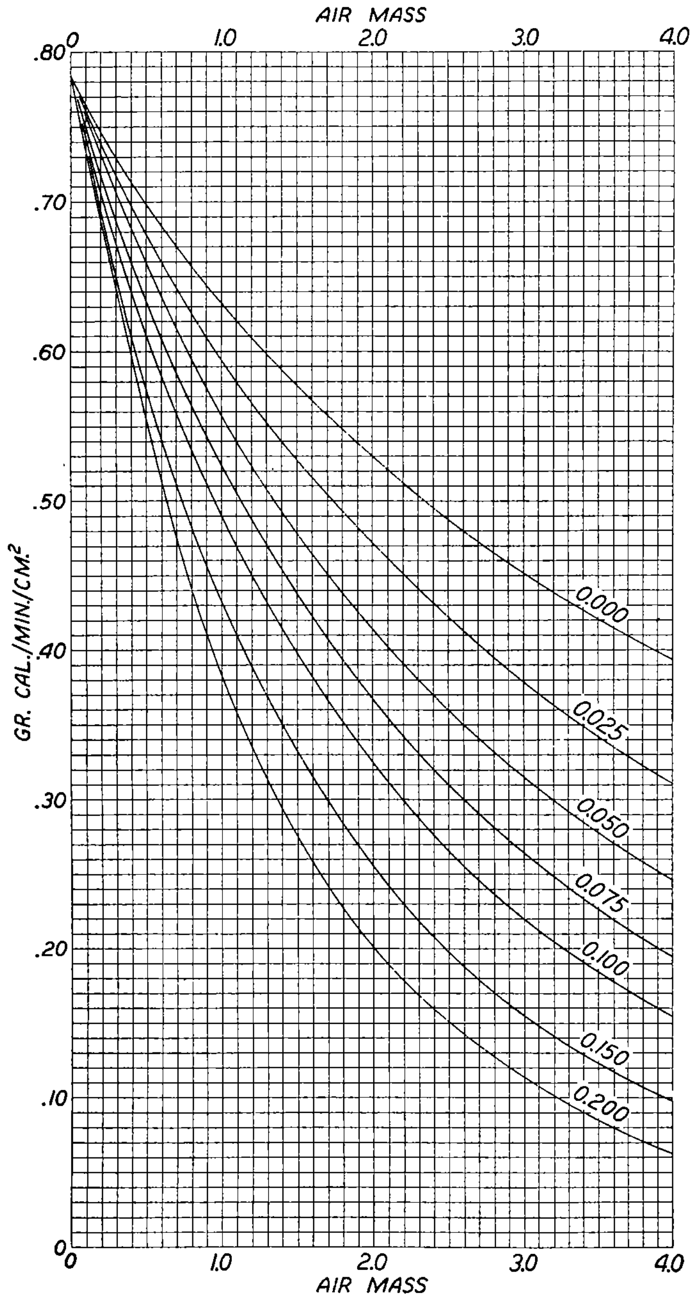

FIGURE 3.—Solar radiation intensity, $I_m - I_r$, for $\lambda < 636m\mu$ and different values of m and β.

radiation in a part of the spectrum where there are few absorption bands. Similarly, and perhaps preferably, if we take the difference $I_m - (I_r/0.878)$ we have left $\Sigma I_m(\lambda < 636m\mu)$, also in a part of the spectrum where there are few atmospheric absorption bands, and which contains a greater proportion of the total spectrum than does $\Sigma I_m(\lambda < 526m\mu)$.

Values of $\Sigma I_m(\lambda < 636m\mu)$, for different values of m and β are plotted in figure 3, with m as abscissas and $\Sigma I_m(\lambda < 636)$ as ordinates.

If we take the difference $\Sigma I_m \lambda < 636m\mu - \Sigma I_m \lambda < 526m\mu$, we have a measure of the intensity of solar radiation in a part of the spectrum free not only from depletion by atmospheric absorption, but also from the ultra-violet radiation where the intensity is not well known. These values have been plotted in figure 4, with m as abscissas and intensities ΣI_m $(636m\mu > \lambda > 526m\mu)$ as ordinates.

Evidently, having constructed figures 2, 3 and 4, we may determine the value of β at the time and for the value of m at which a measurement of $I_m - I_r$, or $I_y - I_r$, was obtained, by interpolating in figure 3 or figure 4, respectively. Having found β, interpolation in figure 2 will give the solar radiation intensity that would have been found for the same value of m with an atmosphere free from moisture, and I_m $(w=0) - I_m =$ the atmospheric absorption of solar radiation.

Finally, referring to Smithsonian Meteorological Tables, 1931, p. lxxxlv, figure 1 (see, also, this REVIEW, February 1930, p. 52), interpolation between curves 2 to

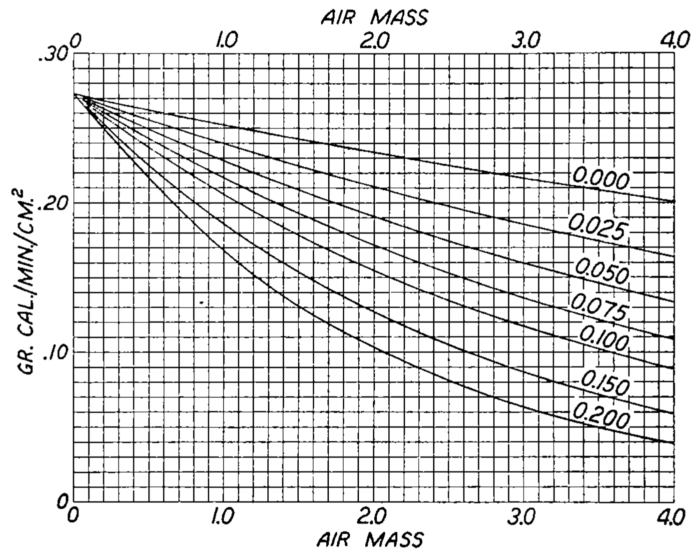

FIGURE 4.—Solar radiation intensity, $I_y - I_r$, for $526 < \lambda < 636m\mu$ and for different values of m and β.

8 and 9 to 15, respectively, as the value of $(I_m(w=0) - I_m)$ requires, will give the water-vapor content of the atmosphere w, expressed in centimeters of precipitable water. An example of these various determinations is given in table 2.

The values obtained for both β and w are smaller than we would expect, and for w they are smaller than are indicated by the psychrometically determined surface water-vapor pressure. It is suspected that the reason for this may be due to the fact that for the screens in use at Washington the value of λ at which the transmission coefficients become zero are somewhat lower than the values we have been led to adopt from the transmission coefficients given in table 1. Exactly what this should be can only be determined by a special calibration of our screens.

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TABLE 2.—*Computations of the atmospheric turbidity factor, β, and the water-vapor content of the atmosphere, w, from screened solar radiation measurements*

[From measurements at Washington, D.C. April 18, 1932]

Air mass m	Atmospheric turbidity β								Water-vapor content of the atmosphere, w (cm)					
	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)	(10)	(11)	(12)	(13)	(14)[b]	(15)[c]
	I_m	$\frac{I_r}{0.878}$	$\frac{I_y}{0.889}$	(2−3)[a]	(4−3)[a]	β from (5)	β from (6)	Mean (7)+(8)	I_m (w=0) Percentage of solar constant	I_m Percentage of solar constant	(10)−(11) Percentage of solar constant	w (Cm)	Mm	Mm
3.99	0.881	0.700	0.790	0.183	0.091	0.077	0.098	0.088	0.490	0.459	0.031	0.16	0.70	----------
3.23	1.002	.737	.870	.268	.133	.065	.072	.078	.592	.522	.070	.39	1.75	----------
3.10	1.027	.747	.881	.283	.135	.062	.074	.068	.602	.536	.066	.35	1.57	----------
2.47	1.118	.778	.939	.343	.161	.060	.064	.062	.668	.582	.086	.49	2.20	----------
2.37	1.137	.782	.943	.359	.161	.058	.071	.064	.672	.592	.081	.47	2.11	3.63
1.83	1.223	.834	1.016	.393	.182	.064	.061	.068	.719	.637	.082	.50	2.24	----------
1.79	1.229	.837	1.022	.396	.185	.072	.069	.070	.719	.640	.079	.49	2.20	----------
1.57	1.296	.856	1.048	.444	.193	.063	.077	.065	.753	.675	.078	.49	2.20	----------
1.52	1.303	.859	1.060	.448	.201	.065	.060	.066	.757	.679	.078	.50	2.24	----------
1.44	1.326	.862	1.065	.469	.203	.059	.060	.060	.785	.691	.094	1.00	4.48	----------
1.40	1.338	.863	1.069	.480	.206	.053	.061	.057	.785	.697	.088	.95	4.26	----------
1.32	1.354	.843	1.073	.516	.230	.043	.025	.034	.883	.705	.128	1.41	6.32	----------
1.15	1.428	.913	1.124	.520	.211	.056	.073	.064	.805	.744	.061	.69	3.09	3.15
1.46	1.284	.846	1.053	.442	.207	.074	.055	.064	.765	.669	.096	1.01	4.49	----------

[a] Corrected for mean solar distance.
[b] Surface water-vapor pressure $=\frac{w}{.223}$
[c] From psychrometer measurements.

CONSERVATION OF ANGULAR MOMENTUM, OR AREAS, AS APPLIED TO AN AIRPLANE EN ROUTE TO THE POLE

By W. J. HUMPHREYS

[Weather Bureau, Washington, April 1933]

When a freely moving object is held on its course by a pull or push continuously directed to the same point, as illustrated by a planet tracing its orbit about the sun under the force of gravity, the areas swept over by the straight line, or radius vector, connecting the center of attraction with the moving object during different equal intervals of time are equal to each other, however near to or far from that center the object may be. This is the law of the conservation of areas, or conservation of angular momentum. The same law applies to the atmosphere, barring the effects of friction and turbulence, when forced to change latitude. In this case the radius vector is the perpendicular from the place occupied onto the axis of the earth, or radius of the small latitude circle through the place in question.

Rigid proofs of these laws are well known, though few books contain them in detail. They are based on, or in keeping with, the conservation of energy, hence without exception and not in the least contravened by the fact that the air in high latitudes often is just as quiet as that of any other part of the world. However, one may accept the logical proofs of all these statements and still be puzzled by the fact that one can fly to either pole of the earth, as has been done, without being driven into a dizzy west-to-east spin about it.

If the law of the conservation of areas is true, and if the force driving the plane seems all the time directed strictly towards the pole, then why is it that the plane, instead of spinning around the world from west to east, at an ever-increasing speed, keeps to the same meridian?

The law, as stated, is true, and the plane is kept from speeding eastward by a counter force—the driving force is not strictly towards the pole.

How great then is this counter force?

Let the conditions be:

Latitude of plane, $\lambda=80°$.

West-to-east velocity of plane same as earth beneath, $u=\frac{2\pi R\cos\lambda}{T}=\frac{2\pi r}{T}$.

R = Radius of the earth.

T = Time of rotation of the earth (siderial day) = 86,164 seconds.

r = Radius of circle of latitude at latitude λ.

Velocity of plane towards adjacent pole, $v=200$ kilometers per hour.

By the law of the conservation of areas, ur = constant.

Hence $\frac{du}{u}=-\frac{dr}{r}$

Then, if s is a distance along a meridian (approach to pole positive) the west-to-east acceleration

$$\frac{du}{dt}=-\frac{u}{r}\frac{dr}{dt}=\frac{u}{r}\frac{ds}{dt}\sin\lambda=\frac{u}{r}v\sin\lambda=\frac{u}{R}v\tan\lambda$$

When u is equal to the west-to-east velocity of the surface of the earth at the place in question, that is, when the plane has no motion across the meridian, the last expression,

$$\frac{u}{R}v\tan\lambda=\frac{2\pi v\sin\lambda}{86164}$$

On substituting the value of v in terms of centimeters and seconds, and the value of sinλ for $\lambda=80°$, it appears that, under the conditions stated, $\frac{du}{dt}=0.4$ cm/sec.[2], nearly, or about 1/2450 part of gravity acceleration. The maximum value, as the pole is reached, is but little greater.

Now the ratio of thrust to the lift, in the case of an airplane, is, roughly, 1 to 8. Hence, in the above case, an east-to-west push equal to about one 300th that of the poleward thrust would fully counteract the effect of the law of the conservation of areas and keep the plane on the same meridian. This would be accomplished by heading the plane rather less than one fifth of a degree west of the true meridian, an amount that would seem to the aviator, if noticed at all, as a mere drift correction.

The law of the conservation of areas is true, nevertheless it does not perceptibly interfere with the interzonal travel of airplanes, even to the poles of the earth.