INVESTIGATIONS OF ATMOSPHERIC PERIODICITIES AT THE GEOPHYSICAL INSTITUTE, LEIPZIG, GERMANY

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Careful studies of atmospheric periodicities have been made at the Geophysical Institute of Leipzig since 1923 under the supervision of Dr. Weickmann, the director. The following paper gives a short account of the methods used and some of the results found. The study of a wave having a period of 24 days will be dealt with in some detail to illustrate the methods employed.

The starting point was the discovery that in barograms there sometimes appear points (the so-called points of symmetry) with respect to which pressure changes before and after are surprisingly symmetrical.[1] Figure 1 gives seven examples of such points of symmetry that occurred during winter months from 1923–24 to 1929–30, inclusive, in London. The *full* line is the pressure curve constructed from the morning observations. The time scale is indicated on the upper margin for each year. The *dotted* line is the reversion of the original curve from the point of symmetry whose date is indicated by the vertical line in the center of each pair of curves. One such point occurred on January 16, 1924 (fig. 1). The lower horizontal axis gives the time scale for the reflected curve. Thus each date on this axis, which is n days after the date of symmetry, corresponds to a date on the upper horizontal axis, n days before the point of symmetry, and similarly for the pressure values shown by the two curves. Figure 1 shows that the general type of a curve and its reversed image agree very well and that often even indentations of the two coincide.

In some cases it is necessary to contract or expand uniformly the parts of the curve remote from the point of symmetry. This is more often required when a long interval of time is involved. According to Weickmann it is not surprising that such a change in the time scale has to be made because we compare different seasons of the year having especially different temperatures when we compare a very extended pressure curve with its reversed image. It is not to be expected that the waves (which cause the points of symmetry, as we shall see) can have the same velocity of propagation in the different seasons. Therefore, this change in the time scale seems quite justified from the physical point of view.

We shall get a better conception of the conditions for the creation of these points of symmetry, if we remember that a simple sine or cosine curve is symmetrical about its maximum and minimum points. A curve composed of a number of such curves has its points of symmetry where extreme values of all the single components coincide. It is not necessary however that all the component curves have maxima or all minima at this point. Some may have maxima, some minima. It is very easy to visualize this condition by considering a curve composed of only two

harmonic components. For details we refer to the published papers.[2]

Since points of symmetry occur in a curve composed of harmonic terms, it is obvious that the points of symmetry in the pressure trace indicate that the pressure curve is composed of harmonic components, the extreme values of which coincide on the date of the point of symmetry (this statement is not exact, but sufficient for our purpose). On the other hand every curve which satisfies certain very general mathematical conditions can be decomposed into a series of harmonic terms. But this is of meteorological interest only when one, or but a very few components, have much higher amplitudes then the other terms, because only then may we suppose that these components represent real waves. Thus we see that the points of symmetry form excellent criteria of epochs of periodicity.

As an example we choose the results of the analysis of the air pressure curve from November 25, 1923, to February 22, 1924. The point of symmetry occurred on January 15. The harmonic analysis of this period of 90 days resulted for the first 20 components as follows:

Length of period (days)	90	45	30	22.5	18	15	13	11.3	10	9
Amplitude	1.68	1.17	1.07	6.73	2.22	2.21	.76	2.34	1.11	1.46
Phase (degrees)	202	353	69	171	150	318	294	286	172	205

Length of period (days)	8.2	7.5	6.9	6.4	6.0	5.6	5.3	5.0	4.7	4.5
Amplitude	2.81	1.00	1.59	.82	1.86	1.21	1.45	1.37	1.22	1.44
Phase (degrees)	23	95	121	212	89	294	75	317	279	191

The periods of 22.5, 18, 15, 11.3, and 8.2 days have amplitudes larger than 2 mm, and thus we may suppose that these periods, or at least some of them are real and not merely byproducts of calculation. The first trace of figure 2 shows the actual pressure curve (A), the second the sum of all 20 components given above (A*), the third the sum of the five components with amplitudes larger than 2 mm (A_1). This curve agrees quite well with the general features of the original pressure curve. Finally in figure 2 are shown the five single components with amplitudes larger than 2 mm. Note especially how the extreme values of all five components coincide at the date of symmetry.

For those who wish to search for points of symmetry it may be mentioned that the pressure curve in figure 2 was drawn from the morning and evening values. But later investigations showed that it is sufficient to use the morning observations only.

Further, we should note that points of symmetry appear mostly near the solstices. The points of symmetry

[1] L. Weickmann. Wellen im Luftmeer. I. Mitteilung. Sitz. -Ber. Sächs. Akad. d. Wiss. Math. -Phys.Kl. vol. 39, no. 2.

[2] Comp. in Weickmann, loc. cit. chap. 1 by L. Lammert. We have here mentioned only the single not the double points of symmetry, which are less important.

which occur during the wintertime are generally better pronounced than those which occur in summer, owing to
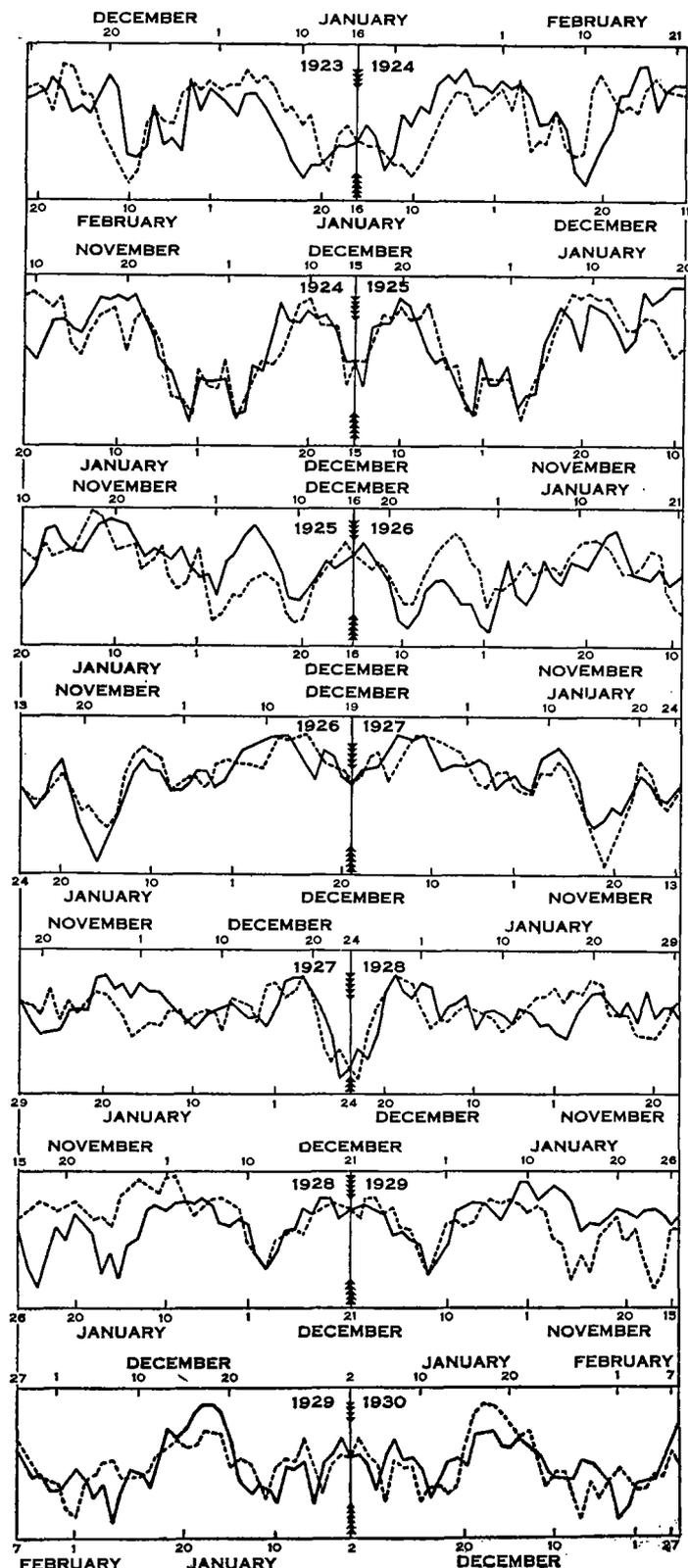

FIGURE 1.—Points of symmetry in the London winter barograms, 1923–30; full lines, actual curves; dotted, reversion curves. (Beitrage zur Geophysik, vol. 34, p. 244, fig. 1b.) 1931.

the fact that the barograms during the wintertime have more characteristic features than those of summer, when the pressure range is less pronounced.
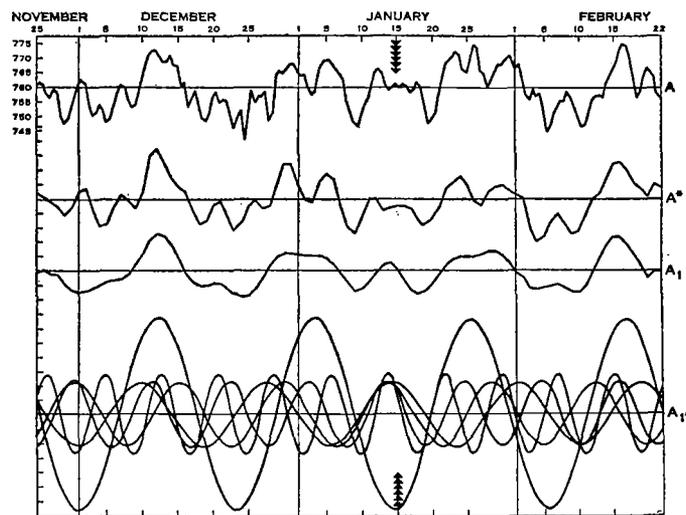
The investigations showed that a period of 20 to 24 days is very common in winter. We shall now illustrate the method of the further investigations on this particular

FIGURE 2.—Hamburg barograms and harmonics, November 25, 1923–February 22, 1924. Point of symmetry, January 15, 1924. (Met. Zeitz. vol. 44, p. 248. 1927.)

period which in some winters has great influence upon the European weather.

In order to get a better idea of the physical nature of the periods found it proved very useful to investigate these periods not only at a single place but to represent
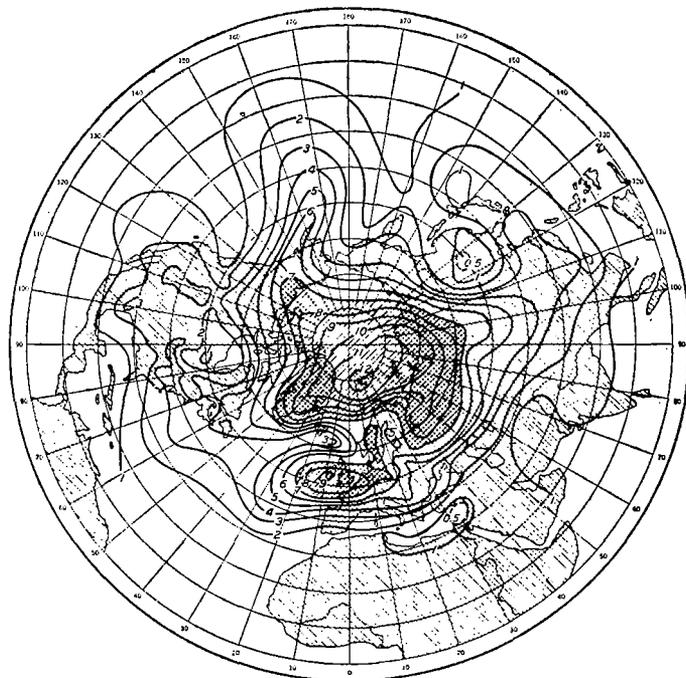

FIGURE 3.—Amplitude of the 24-day pressure wave, winter of 1923-24. (Met. Zeitz. vol. 44, p. 249. 1927.)

the distribution of their amplitudes and phases over a wider area by means of maps. That was first done by Mildner.[3] He dealt with the 24- and the 8-daily wave for the period from December 10, 1923, to February 19, 1924. (Point of symmetry Jan. 15, 1924.) The distribution of the amplitude of the 24-daily wave during

[3] P. Mildner, Ueber Luftdruckwellen. Veroeff. Geophys. Inst. Leipzig. II. Ser. vol. III, 3. Mildner has chosen an interval of 72 days instead of 90 days in order to obtain whole numbers for the shorter periods. Therefore we have now a period of 24 instead 22.5 days with a large maximum.

the winter 1923–24 over the northern hemisphere is shown in figure 3. This figure gives the result of Mildner's investigation for Europe, completed by Weickmann for the whole northern hemisphere.[4] The amplitudes surround the pole with maximum values at Spitsbergen. The observation material from these northern regions is of course rather scanty, but sufficient to draw the lines of equal amplitudes and phases (or more exactly the phases for the initial time $t=0$). The latter are given in figure 4. They are especially important since they inform us about the direction and velocity of the motion. We take for example the isophase of 90°. At the time $t=0$ all points connected by this line have maximum values. After $24/6=4$ days, all points of the isophase 30° will have maximum values because a time of 4 days corresponds to an angle of 60° and therefore the argument of the sine of points situated upon the isophase 30° is $30° + 60° = 90°$. The map of the isophases shows also that these are lines around the pole. It is characteristic of the lines of equal phase that they are extended farther south in those regions in which occur most of the cold air outbursts of the northern hemisphere. That gives the impression that the 24-daily period represents a pulsation of the cold polar air masses. This would also explain why its period is greatest in the polar regions.

This opinion is also confirmed by other investigations. Thus Weickmann[5] has shown that this 24-daily wave of the pressure during the winter 1923–24 corresponds to a period of the same length in the temperature. The lines of equal amplitude and phase of this temperature wave have the same features as those of the pressure wave.

Further, a comparison of the behavior of the 24-daily wave at mountain and neighboring valley stations showed that its amplitudes decrease rapidly with the elevation, more rapidly[6] than the ratio P_h/P_o (P_o air pressure at the valley station, P_h pressure at the mountain station). On the other hand it follows from statistical and theoretical investigations that the variability of the pressure decreases quickly with the elevation if the sign of the pressure change is opposite to the sign of the temperature change. In other words, the pressure change is to be explained by temperature changes in the intermediate layer. Thus the pronounced decrease of the amplitude of the 24-daily wave is a result of its thermal nature.

In conclusion we shall give a short account of the theory of the 24-daily wave, which was developed by Schwerdtfeger.[7] This 24-daily wave is according to him a sequence of rhythmic outbreaks of polar air masses as we have seen from figures 3 and 4. Following Margules's formula of the inclination angle of stable boundaries, we may consider the conditions under which a cold mass pushes forward and retires with respect to a neighboring warmer air mass. The main factor is here the temperature dif-

[4] L. Weickmann, Das Wellenproblem der Atmosphaere. Met. Zs. 1927, p. 241.
[5] L. Weickmann, Die thermische Wirkung der 24 tägigen polaren Druckwelle des Winters 1923–24. Beitr. Phys. fr. Atm. Hergesellfestband 1929, p. 226.
[6] B. Haurwitz, Luftdruckwellen auf Berg- und Talstationen. Ibid. p. 271.
[7] W. Schwerdtfeger, Zur Theorie polarer Temperatur- und Luftdruckwellen. Veröeff. Geophys. Inst. Leipzig, II. Ser. vol. IV, 5.

ference between the two sides of the surface of discontinuity. If we neglect differences of higher order we may write

$$\frac{\Delta T}{T_1} > \frac{2\omega \sin \phi}{g} \frac{/v_1/ + /v_2/}{\tan \alpha}$$

as the condition most favorable for a polar outbreak (ΔT temperature difference between both masses, T_1 temperature of the cold mass, $/v_1/$, $/v_2/$ absolute values of the wind velocities in the cold and warm mass respectively, α angle of inclination of the surface of discontinuity, g
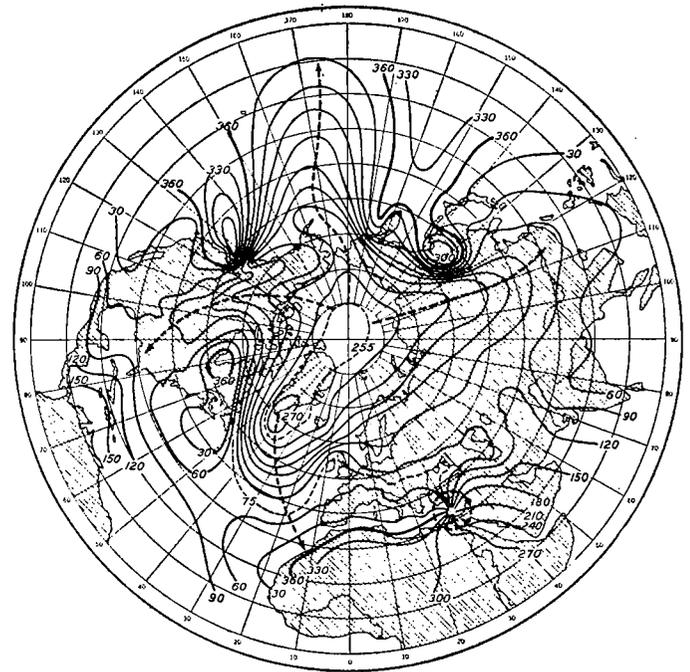

FIGURE 4.—Phase of the 24-day pressure wave, winter of 1923–24. (Met. Zeitz. vol. 44 p. 249. 1927.)

gravity acceleration, ω angular velocity of the earth, ϕ geographic latitude).

Since angles of the inclination $\frac{1}{100}$ and $\frac{1}{200}$ are to be assumed for the polar surface of discontinuity a temperature difference of between 15° and 25° C. might be the condition of release of a polar outbreak. This temperature difference is brought about mainly by the large scale horizontal heatflow (Austausch). Schwerdtfeger finds by discussion of other possible sources and losses of heat that a temperature difference of about 20° C. between cold and warm air masses up to 5,000 m height will be created during a lapse of 19 to 26 days. If this is so, the pressure and temperature period of about 24 days is explained.

The foregoing is only a very brief account of the investigations mentioned in the title. No reference has been made to work concerning longer periods which certainly will prove very useful for long range forecasting.