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NORMAL DEGREE DAYS BELOW ANY BASE*

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ABSTRACT

Heating engineers sometimes have need for normal degree days for bases other than 65° F. Further analysis of the relationship between mean temperature and mean degree days for base 65° F. showed that the form of this relationship is independent of the base. This makes it possible to vary the base to any value and hence to compute degree-day normals for any base from normal temperature. Results are presented for selected stations for bases ranging from 35° to 70° F.

INTRODUCTION

Although 65° F. is by far the most used base below which degree days are measured, other bases have also been occasionally employed [1]. For example 70° is preferred by at least one large user of degree days. Bases lower than 65° are frequently needed in special applications where inside temperatures need only be maintained at values considerably below that required for human comfort. Examples are application to warehouses, automatic substations of various kinds, etc. Unfortunately the lack of statistics for bases other than 65° has discouraged the use of the degree-day concept in many applications where it might have been used to considerable advantage. There are also indications that in some instances other bases between 60° and 70° might be more satisfactory than 65°. It is the purpose of this discussion to extend the method previously developed [2] so that degree days below any base may be readily computed from temperature statistics alone.

THE DISTRIBUTION FOR ANY BASE

In the previous paper [2] on the rational relationship it was shown that the frequency distribution of degree days below base 65° is related to the temperature distribution with the part above 65° cut off or truncated. In the ordinary truncated distribution the values in the cut-off portion are usually ignored or treated in terms of the original scale. This is different from the situation in the degree-day problem, for here all temperatures above 65° have degree-day values of zero. Hence as shown in [2] the degree-day distribution is a mixture of the temperature distribution on a scale measured downward from 65° with the zeros occurring when the temperature is above 65°. If F is the cumulative distribution of the *daily* average temperature t, D the degree days below 65°, and p and q the probabilities of temperatures above and below 65°, respectively; then the distribution function of degree days may be expressed in the form (see [2]):

$$G(D|D\geq 0)=p+qF(65-t|t\leq 65). \tag{1}$$

Since the development leading to equation (1) is clearly independent of the degree-day base because 65 enters only as a parameter, the general base b may be substituted for 65 giving

$$G(D|D\geq 0)=p+qF(b-t|t\leq b). \tag{2}$$

This is the distribution of degree days to any base.

THE GENERAL RELATIONSHIP OF DEGREE DAYS TO TEMPERATURE

Since there can be no parameters in degree-day statistics which are not in the distribution of degree days,

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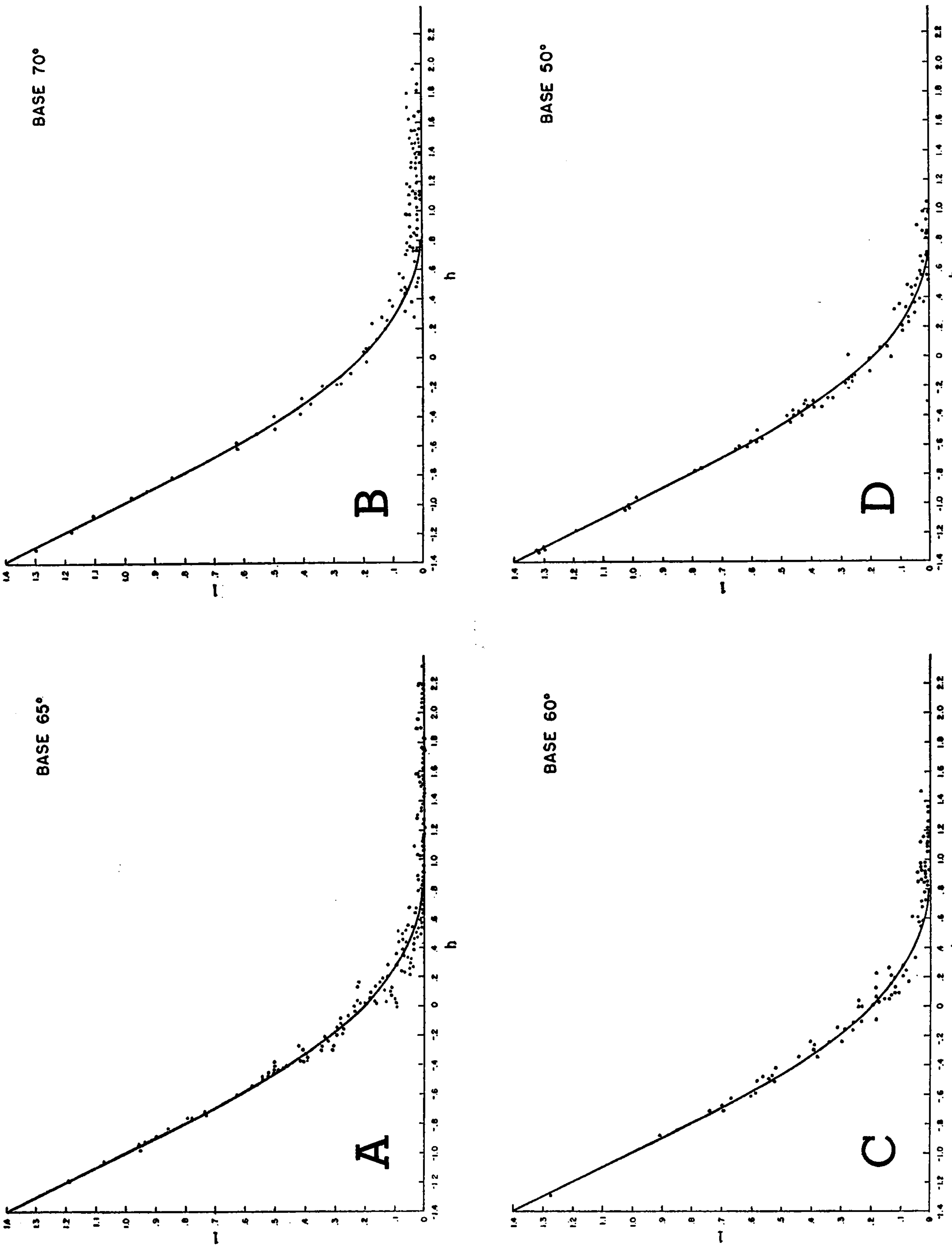

FIGURE 1.—l vs h data for various degree-day bases. The l curve for base 65° (A) is superimposed on the data for the other bases for comparison of fit.

the derivation of the relationship of mean degree days to mean temperature will be identical with the development for 65° in reference [2], except that b will be substituted for 65. Hence the rational relationship becomes

$$E(D)=q[b-E(t)+\lambda\sigma]. \tag{3}$$

where E is the population mean degree day, λ is the ratio of the ordinate of the frequency curve at b to the area under this curve below b, and σ is the standard deviation of t. Rearrangement of equation (3) immediately gives

$$\lambda-\frac{E(D)}{\sigma}\frac{(1-q)}{q}=\frac{E(D)-b+E(t)}{\sigma}. \tag{4}$$

This relationship is for a single day, and since σ for each day is not available, it must be adjusted for use with monthly data. It was shown in reference [2] that the monthly standard deviation σ_m is proportional to $\sigma/\sqrt{N}$ for an average day during a month with N days. Hence substituting $\sqrt{N}\sigma_m$ for σ and l for the left member in equation (4) gives the required empirical generalization

$$l=\frac{E(D)-b+E(t)}{\sqrt{N}\sigma_m}. \tag{5}$$

$E(D)$, $E(t)$, and σ_m are population values which are estimated by the statistics $\overline{D}$, $\bar{t}$, and s_m. Substituting these in equation (5) gives

$$l=\frac{\overline{D}-b+\bar{t}}{\sqrt{N}s_m}. \tag{6}$$

l is a function only of the parameters in the left member of equation (4) which are in turn functions of σ and $E(t)$ and hence of $\sqrt{N}s_m$ and $\bar{t}$. Therefore l is a function of the normalized value of b, $h=(b-\bar{t})/\sqrt{N}s_m$. It is to be expected if the theory is general, that the l curves will be identical for all bases. Figures 1B, C, D show the original l-curve obtained for the 65° base superimposed on l vs h data for three different bases. These data were obtained from all months with degree days at 12 stations well distributed over the United States. They may be compared to figure 1A for 65° data. There is clearly little difference in the fit of the l-curve among the data for the various bases and therefore the theory seems to be general. In an application to automatic substations, bases were required down to 0°. For this purpose the theory was checked for 30° and 0° bases and found to be as good as shown for the higher bases in the figures. The l table (table 1 of reference [2]) may therefore be used for all bases. The computation formula for monthly degree-day means to any base is now easily obtained by solving equation (6) giving

$$N\overline{D}=N(b-\bar{t}+l\sqrt{N}s_m). \tag{7}$$

EXAMPLES

From a manuscript set of charts of monthly standard deviation we find s_m for April at Minneapolis to be 4.0°. The normal temperature for April is 46.0°. Hence for base 50°, $h=(50-46.0)/(5.477)(4.0)=0.18$. From table 1 of reference [2], l is found to be 0.11. Using equation (7) gives

$$30\overline{D}=30[50-46.0+(0.11)(5.477)(4.0)]=192.$$

This is the normal degree days to base 50° for April at Minneapolis.

In table 1, the tabulation for selected cities in the United States and Canada gives an idea how the degree-day normals vary for several bases.

ACKNOWLEDGMENTS

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2. H. C. S. Thom, "The Rational Relationship between Heating Degree Days and Temperature," *Monthly Weather Review*, vol. 82, No. 9, Jan. 1954, pp. 1–6.

TABLE 1.—*Degree-day normals for several bases for selected cities in the United States and Canada*

Base	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Annual
SAINT LOUIS, MO., CITY OFFICE													
70° F.	0	9	90	318	720	1,048	1,138	932	766	405	175	38	5,639
65	0	0	38	202	570	893	983	792	620	270	94	7	4,469
60	0	0	0	119	426	738	828	652	483	156	41	0	3,443
55	0	0	0	56	292	591	683	520	365	77	6	0	2,590
50	0	0	0	19	176	452	547	394	255	29	0	0	1,872
45	0	0	0	0	93	321	422	283	174	7	0	0	1,300
40	0	0	0	0	38	221	306	187	110	0	0	0	862
35	0	0	0	0	5	137	220	119	56	0	0	0	537

TABLE 1.—*Degree-day normals for several bases for selected cities in the United States and Canada*—Continued

Base	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Annual
NEW YORK, N. Y., CITY OFFICE													
70° F	0	31	117	403	711	1,063	1,150	1,044	908	606	289	68	6,390
65	0	0	39	263	561	908	995	904	753	456	153	18	5,050
60	0	0	5	148	411	753	840	764	598	311	63	0	3,893
55	0	0	0	68	269	598	685	624	458	183	10	0	2,895
50	0	0	0	18	144	443	538	484	325	90	0	0	2,042
45	0	0	0	0	60	305	406	356	213	29	0	0	1,369
40	0	0	0	0	17	183	282	239	131	0	0	0	852
35	0	0	0	0	0	100	189	145	70	0	0	0	504
CLEVELAND, OHIO, AIRPORT													
70° F	27	53	164	484	849	1,212	1,287	1,159	1,029	681	349	102	7,396
65	0	10	75	340	699	1,057	1,132	1,019	874	531	223	46	6,006
60	0	0	20	212	549	902	977	879	719	386	125	0	4,769
55	0	0	0	118	404	747	822	739	582	258	55	0	3,725
50	0	0	0	51	270	592	676	599	446	152	20	0	2,806
45	0	0	0	12	157	452	530	466	327	77	0	0	2,021
40	0	0	0	0	80	318	402	346	227	25	0	0	1,398
35	0	0	0	0	30	207	293	232	155	5	0	0	922
PHILADELPHIA, PA., CITY OFFICE													
70° F	0	13	94	357	666	1,011	1,088	977	822	519	207	38	5,792
65	0	0	33	219	516	856	933	837	667	369	93	0	4,523
60	0	0	0	114	366	701	778	697	518	232	29	0	3,435
55	0	0	0	44	224	546	623	557	377	120	0	0	2,491
50	0	0	0	6	111	396	484	423	257	47	0	0	1,724
45	0	0	0	0	37	257	353	294	157	8	0	0	1,106
40	0	0	0	0	0	150	246	188	92	0	0	0	676
35	0	0	0	0	0	70	154	104	40	0	0	0	368
PITTSBURGH, PA., CITY OFFICE													
70° F	7	33	125	437	762	1,079	1,147	1,019	880	546	249	56	6,340
65	0	0	56	298	612	924	992	879	735	402	137	13	5,048
60	0	0	16	175	462	769	837	739	590	268	60	0	3,916
55	0	0	0	90	322	614	692	599	465	163	16	0	2,961
50	0	0	0	37	198	473	556	474	349	86	0	0	2,173
45	0	0	0	5	104	340	430	356	243	36	0	0	1,514
40	0	0	0	0	45	221	314	246	167	0	0	0	993
35	0	0	0	0	8	139	226	165	110	0	0	0	648
WASHINGTON, D. C., CITY OFFICE													
70° F	0	10	90	365	660	986	1,039	910	753	459	175	24	5,471
65	0	0	32	231	510	831	884	770	606	314	80	0	4,258
60	0	0	7	127	360	676	729	630	459	187	24	0	3,199
55	0	0	0	55	222	521	582	496	336	92	0	0	2,304
50	0	0	0	14	113	371	454	369	228	34	0	0	1,583
45	0	0	0	0	41	244	325	255	145	0	0	0	1,010
40	0	0	0	0	0	139	232	159	78	0	0	0	608
35	0	0	0	0	0	67	147	89	42	0	0	0	345
CHICAGO, ILL., AIRPORT													
70° F	24	43	171	493	915	1,302	1,398	1,193	1,023	657	353	119	7,691
65	0	0	90	350	765	1,147	1,243	1,053	868	507	229	58	6,310
60	0	0	34	224	615	992	1,088	913	713	368	131	14	5,092
55	0	0	7	128	465	837	933	773	576	241	69	0	4,029
50	0	0	0	61	332	682	778	633	439	141	25	0	3,091
45	0	0	0	17	211	544	632	500	320	69	0	0	2,293
40	0	0	0	0	119	406	495	374	228	20	0	0	1,642
35	0	0	0	0	55	293	367	263	144	0	0	0	1,122
BOSTON, MASS., AIRPORT													
70° F	19	52	181	465	768	1,153	1,268	1,142	1,004	684	378	120	7,234
65	0	7	77	315	618	998	1,113	1,002	839	534	236	42	5,791
60	0	0	16	188	468	843	958	862	694	384	118	5	4,536
55	0	0	0	89	323	688	803	722	539	242	46	0	3,452
50	0	0	0	31	196	533	648	582	398	129	0	0	2,517
45	0	0	0	7	96	391	500	448	278	53	0	0	1,773
40	0	0	0	0	38	260	368	320	171	10	0	0	1,167
35	0	0	0	0	0	155	250	209	99	0	0	0	713
DETROIT, MICH., CITY AIRPORT													
70° F	28	50	193	530	897	1,256	1,358	1,212	1,082	708	384	128	7,826
65	0	8	96	381	747	1,101	1,203	1,072	927	558	251	60	6,404
60	0	0	35	249	597	946	1,048	932	772	414	145	17	5,155
55	0	0	5	145	447	791	893	792	626	281	73	0	4,053
50	0	0	0	75	307	636	738	652	488	171	23	0	3,090
45	0	0	0	23	182	487	591	519	360	90	0	0	2,252
40	0	0	0	0	96	345	452	386	258	38	0	0	1,575
35	0	0	0	0	34	229	321	272	164	0	0	0	1,020

TABLE 1.—*Degree-day normals for several bases for selected cities in the United States and Canada*—Continued

Base	July	Aug.	Sept.	Oct.	Nov.	Dec.	Jan.	Feb.	Mar.	Apr.	May	June	Annual
MINNEAPOLIS, MINN., AIRPORT													
70° F	49	70	262	608	1,110	1,569	1,717	1,450	1,212	720	378	145	9,290
65	8	18	157	459	960	1,414	1,562	1,310	1,057	570	259	80	7,854
60	0	0	79	332	810	1,259	1,407	1,170	902	433	170	28	6,590
55	0	0	30	212	660	1,104	1,252	1,030	747	303	95	5	5,438
50	0	0	8	126	517	949	1,097	890	601	192	41	0	4,421
45	0	0	0	67	381	794	942	750	464	115	9	0	3,522
40	0	0	0	23	258	648	799	620	345	57	0	0	2,750
35	0	0	0	6	164	511	655	498	244	18	0	0	2,096
CALGARY, ALBERTA, CANADA													
65° F	136	175	420	713	1,110	1,426	1,612	1,344	1,209	750	465	279	9,639
60	55	75	285	558	960	1,271	1,457	1,204	1,054	600	320	155	7,994
55	12	20	166	403	810	1,116	1,302	1,064	899	450	189	72	6,503
50	0	0	87	269	669	961	1,147	924	744	317	92	24	5,234
45	0	0	29	150	528	818	992	784	597	202	33	0	4,133
40	0	0	6	73	405	687	850	654	458	109	0	0	3,242
35	0	0	0	26	300	556	720	533	327	51	0	0	2,513
EDMONTON, ALBERTA, CANADA													
65° F	133	209	450	744	1,200	1,612	1,829	1,512	1,302	750	434	222	10,397
60	53	106	316	589	1,050	1,457	1,674	1,372	1,147	600	289	111	8,764
55	9	44	194	448	912	1,302	1,519	1,232	992	457	167	40	7,316
50	0	5	103	313	775	1,163	1,364	1,092	837	326	80	5	6,063
45	0	0	45	200	637	1,023	1,209	952	691	216	27	0	5,000
40	0	0	9	121	524	884	1,069	823	545	125	0	0	4,100
35	0	0	0	63	411	744	928	694	417	67	0	0	3,324
VANCOUVER, BRITISH COLUMBIA, CANADA													
65° F	78	91	243	465	660	806	899	728	682	510	341	175	5,678
60	16	21	116	314	510	651	744	588	527	360	208	78	4,133
55	0	0	39	181	360	496	589	454	372	222	100	27	2,840
50	0	0	0	82	228	346	448	326	230	113	36	0	1,809
45	0	0	0	26	120	216	313	215	118	41	6	0	1,055
40	0	0	0	0	53	116	200	129	45	0	0	0	543
35	0	0	0	0	8	46	121	66	0	0	0	0	241
WINNIPEG, MANITOBA, CANADA													
65° F	68	100	341	744	1,290	1,829	2,108	1,764	1,519	810	409	157	11,139
60	18	36	219	589	1,140	1,674	1,953	1,624	1,364	660	279	80	9,636
55	0	6	125	449	990	1,519	1,798	1,484	1,209	517	174	36	8,307
50	0	0	59	322	840	1,364	1,643	1,344	1,054	381	93	0	7,100
45	0	0	15	211	700	1,209	1,488	1,204	899	258	38	0	6,022
40	0	0	0	128	570	1,054	1,333	1,064	755	164	7	0	5,075
35	0	0	0	68	439	899	1,178	924	610	96	0	0	4,214
SASKATOON, SASKATCHEWAN, CANADA													
65° F	95	136	426	806	1,290	1,798	2,046	1,736	1,488	840	448	192	11,301
60	35	55	292	651	1,140	1,643	1,891	1,596	1,393	690	313	100	9,739
55	5	12	176	503	990	1,488	1,736	1,456	1,178	548	200	45	8,337
50	0	0	93	370	852	1,333	1,581	1,316	1,023	423	121	5	7,117
45	0	0	38	251	713	1,178	1,426	1,176	880	306	63	0	6,031
40	0	0	5	154	587	1,023	1,271	1,049	737	205	18	0	5,049
35	0	0	0	86	472	880	1,132	909	606	129	0	0	2,214
DAWSON, YUKON TERRITORY, CANADA													
65° F	181	310	690	1,209	1,920	2,449	2,666	2,156	1,891	1,080	589	254	15,395
60	82	175	540	1,054	1,770	2,294	2,511	2,016	1,736	930	434	141	13,683
55	26	75	390	899	1,620	2,139	2,356	1,876	1,581	780	289	64	12,095
50	0	20	255	744	1,470	1,984	2,201	1,736	1,426	638	167	20	10,661
45	0	0	144	598	1,320	1,829	2,046	1,596	1,271	497	80	0	9,381
40	0	0	68	460	1,170	1,674	1,891	1,456	1,116	373	27	0	8,235
35	0	0	22	339	1,020	1,519	1,736	1,316	961	266	0	0	7,179