ON ENERGY CONVERSION CALCULATIONS

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ABSTRACT

It is shown that calculations of the energy conversion from available potential energy to kinetic energy based on vertical velocities obtained by the so-called adiabatic method may lead to erroneous results. An analysis of the method shows that it measures the difference between the energy conversion from available potential energy to kinetic energy, and the generation of available potential by diabatic heating. This result holds for both zonal and eddy available potential energy.

The main conclusion is tested using numerical results from several energy conversion and energy generation studies.

The adiabatic method is compared with other methods to estimate vertical velocities.

1. INTRODUCTION

Several attempts have been made in the last few years to estimate the many energy conversions which can take place in the atmosphere. Some of the energy conversions involve only quantities such as temperature and the horizontal components of wind which are readily available from the routine observations. Other energy conversions include the vertical component of the wind which must be obtained indirectly from other observations, because direct observations of the vertical velocity, applicable to the large-scale flow, are impossible to obtain.

The meteorological literature contains several approximative methods for an indirect calculation of the vertical velocity (see Panofsky [6], [7] and Petterssen [8]). It is generally agreed that a calculation of the vertical velocity directly from the divergence of the wind field through the continuity equation is possible only in regions of dense data networks. Furthermore, the accuracy of the wind observations has to be very high in order to estimate the divergence of the horizontal wind and, therefore, the vertical velocity, with a reasonable accuracy. Such calculations have nevertheless been attempted (Palmén [5] and Holopainen [2]), but since an energy conversion calculation in order to be representative for the general circulation of the atmosphere must include data from as large a fraction of the globe as possible, it is impossible to employ this method at the present time.

Two other methods have been used to compute vertical velocities. The first is the so-called adiabatic method, hereafter referred to as method A, in which the vertical velocity is computed from the thermodynamic equation in the case of no heat sources by estimating the local and advective changes of temperature. The second method, method B, computes the vertical velocity as a by-product from a baroclinic model used for numerical prediction purposes. Both kinds of vertical velocities have been used in calculations of the energy conversion between available potential energy and kinetic energy. Method A has been used most extensively by Jensen [3], but is also used in studies by White and Nolan [15], and Reed, Wolfe, and Nishimoto [12] concerned with stratospheric energy conversions. Method B has been used by White and Saltzman [16], Wiin-Nielsen [17], and Saltzman and Fleisher [13], [14] who applied vertical velocities obtained at the initial time from a two-parameter, quasi-geostrophic model.

It is well known (Panofsky [6], [7]) that method A, when applied above the frictional layer, quite often will give reliable estimates of vertical velocities for certain purposes. It is, however, the purpose of this paper to demonstrate that the vertical velocities computed by method A will not give an estimate of the energy conversion from available potential to kinetic energy when they are substituted in the integral which defines this energy conversion. A similar conclusion has been reached by Holopainen [2], but since the method has been widely used, and further use may be planned for the future, it seems worthwhile to analyze the situation a little more closely and find a more nearly correct interpretation of the results. At the same time, we shall make some comments on the use of vertical velocities from method B in energy conversion calculations.

[1] Paper No. 80 from Department of Meteorology and Oceanography, University of Michigan.

2. ENERGY CONVERSIONS AND VERTICAL VELOCITIES

To bring out the main point in this discussion, we shall restrict ourselves to the quasi-geostrophic formulation of the energetics of the atmosphere. The rate of change of available potential energy, A, can be expressed (Lorenz [4])

$$\frac{dA}{dt}=G(A)-C(A, K) \tag{2.1}$$

in which $G(A)$ is the generation of available potential energy by diabatic processes, and $C(A, K)$ is the conversion of available potential energy to kinetic energy.

The available potential energy is defined by the integral

$$A=\frac{1}{2g}\int_0^{p_0}\int_S \frac{1}{\bar{\sigma}}\alpha'^2 dSdp \tag{2.2}$$

in which g is the acceleration of gravity, p_0 is the surface pressure considered constant, $\bar{\sigma}=\bar{\sigma}(p)$ is an average value of the static stability measure $\sigma=-\alpha(\partial \ln \theta/\partial p)$, α' is the deviation from the area mean of the specific volume, and dS is the area element. We assume for simplicity that S is the total area of the sphere. The expression (2.2) is in agreement with Lorenz's [4] approximative expression for available potential energy

$$A=\frac{1}{2}\int_0^{p_0}\int_S \frac{1}{\overline{T}(\gamma_d-\bar{\gamma})} T'^2 dSdp \tag{2.3}$$

In (2.3) $\overline{T}=\overline{T}(p)$ is the mean temperature over the isobaric surface, γ_d is the dry adiabatic lapse rate, $\bar{\gamma}$ an averaged value of the lapse rate, and T' the deviation from the area average of the temperature.

The formulas for $G(A)$ and $C(A,K)$ can be written

$$G(A)=\frac{1}{g}\int_0^{p_0}\int_S \frac{R}{c_p}\frac{1}{\bar{\sigma}p}\alpha' H' dSdp \tag{2.4}$$

and

$$C(A, K)=-\frac{1}{g}\int_0^{p_0}\int_S \omega\alpha' dSdp \tag{2.5}$$

H is the amount of heating per unit mass and unit time, H' the deviation from the area average, and $\omega=dp/dt$ is the vertical velocity.

The numerical evaluation of the integral in (2.5) requires a knowledge of the vertical velocity. It is the purpose of the following development to investigate this integral in the case where ω is estimated from the adiabatic method, method A. The formula for the evaluation of ω is obtained from the thermodynamic equation under the assumption that $H=0$:

$$\frac{\partial\alpha}{\partial t}+\mathbf{V}\cdot\nabla\alpha-\sigma\omega=0 \tag{2.6}$$

We shall, for simplicity, assume that the horizontal velocity $\mathbf{V}=\mathbf{V}_\psi$ in (2.6) is nondivergent and that $\sigma=\bar{\sigma}(p)$. These assumptions are in agreement with the quasi-geostrophic theory. For easy reference, we mark ω determined by (2.6) by an asterisk. Equation (2.6) then takes the form:

$$\frac{\partial\alpha}{\partial t}+\mathbf{V}_\psi\cdot\nabla\alpha-\bar{\sigma}\omega^*=0 \tag{2.7}$$

By forming the area average over the area S, it is seen that $\partial\bar{\alpha}/\partial t=0$, and we can therefore also write (2.7) in the form

$$\omega^*=\frac{1}{\bar{\sigma}}\left(\frac{\partial\alpha'}{\partial t}+\mathbf{V}_\psi\cdot\nabla\alpha'\right) \tag{2.8}$$

When ω^* is determined from observations, (2.8) is usually written in terms of temperature, $\partial\alpha/\partial t$ is determined by finite differences over a period of 12 hr., and the gradient of α is related to the vertical wind shear through the geostrophic thermal wind relation:

$$\nabla\alpha=f\mathbf{k}\times\frac{\partial\mathbf{V}_g}{\partial p} \tag{2.9}$$

Disregarding the finite difference aspects of the calculations, we shall see the effects of substituting (2.8) in (2.5) for an evaluation of $C^*(A,K)$. We find

$$C^*(A, K)=-\frac{1}{g}\int_0^{p_0}\int_S \frac{1}{\bar{\sigma}}\alpha'\left(\frac{\partial\alpha'}{\partial t}+\mathbf{V}_\psi\cdot\nabla\alpha'\right)dSdp \tag{2.10}$$

which also may be written:

$$C^*(A, K)=-\frac{1}{2g}\int_0^{p_0}\int_S \frac{1}{\bar{\sigma}}\left(\frac{\partial\alpha'^2}{\partial t}+\nabla\cdot(\alpha'^2\mathbf{V}_\psi)\right)dSdp \tag{2.11}$$

The second term in the integrand integrates to zero, and we find therefore that

$$C^*(A, K)=-\frac{1}{2g}\int_0^{p_0}\int_S \frac{1}{\bar{\sigma}}\frac{\partial\alpha'^2}{\partial t}dSdp \tag{2.12}$$

The local time derivative in the equations (2.8) to (2.12) is determined from observations. A comparison between (2.2) and (2.12) therefore shows that when we employ method A, we are not making an estimate of the energy conversion $C(A, K)$, but *we are estimating the rate of decrease of the available potential energy*, because (2.12) also may be written

$$C^*(A, K)=-\frac{dA}{dt} \tag{2.13}$$

We conclude, therefore, from the case treated above, that the measurement is an estimate of the change of available potential energy or, in terms of energy conversions and generations, the difference between the energy conversion $C(A, K)$ and the generation $G(A)$, since a comparison between (2.13) and (2.1) gives:

$$C^*(A, K)=C(A, K)-G(A) \tag{2.14}$$

The arguments above have been presented using the assumptions which are consistent with the quasi-geostrophic formulation. It is true that additional problems appear if we go beyond this formulation. However, the geostrophic relation is used in the evaluation of the gradient of α. It is therefore consistent to use this assumption throughout the calculation. It might nevertheless be of interest to evaluate the additional terms which appear in (2.11) if we use observed winds, and a horizontal variation of the static stability factor in the evaluation of ω^* from (2.6). We are still going to use the definition (2.2) of the available potential energy. In order to make this investigation, we divide the quantities α and σ into the area average denoted by a bar and the deviation from the area mean. We note that $\bar{\omega}=0$ and therefore $\omega=\omega'$. Equation (2.6) can be written in the form

$$\frac{\partial\bar{\alpha}}{\partial t}+\frac{\partial\alpha'}{\partial t}+\mathbf{V}\cdot\nabla\alpha'-(\bar{\sigma}+\sigma')\omega^*=0 \qquad (2.15)$$

It follows that

$$\frac{\partial\bar{\alpha}}{\partial t}+\overline{\mathbf{V}\cdot\nabla\alpha'}-\overline{\sigma'\omega^*}=0 \qquad (2.16)$$

Subtraction of (2.16) from (2.15) gives

$$\frac{\partial\alpha'}{\partial t}+\mathbf{V}\cdot\nabla\alpha'-\overline{\mathbf{V}\cdot\nabla\alpha'}-\bar{\sigma}\omega^*-(\sigma'\omega^*-\overline{\sigma'\omega^*})=0 \qquad (2.17)$$

The equation corresponding to (2.12) is obtained from (2.17) by multiplying by $\alpha'/\bar{\sigma}$ and integrating. We get:

$$C^*(A,K)=-\frac{1}{2g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\frac{\partial\alpha'^2}{\partial t}dSdp$$
$$+\frac{1}{g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\frac{\alpha'^2}{2}\nabla\cdot\mathbf{V}dSdp+\frac{1}{g}\int_0^{p_0}\int_S\frac{\sigma'}{\bar{\sigma}}\alpha'\omega^*dSdp \qquad (2.18)$$

The last two terms on the right hand side of (2.18) are the correction terms resulting from the divergent part of the wind and from the horizontal variation of stability. From the definition of σ it is easily seen that

$$\sigma'=-\left(\frac{\partial\alpha'}{\partial p}+\frac{c_v}{c_p}\frac{1}{p}\alpha'\right) \qquad (2.19)$$

When we substitute (2.19) in the last term of (2.18), and make use of the continuity equation, we get for the sum of the two correction terms:

$$\frac{1}{g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\left(\frac{1}{2}\alpha'^2\nabla\cdot\mathbf{V}+\sigma'\alpha'\omega^*\right)dSdp$$
$$=-\frac{1}{g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\left[\frac{1}{2}\frac{\partial(\omega^*\alpha'^2)}{\partial p}+\frac{c_v}{c_p}\frac{1}{p}(\omega^*\alpha'^2)\right]dSdp \qquad (2.20)$$

which shows that the correction terms combine to an integral whose value depends on the triple correlation between ω^* and α'^2. Such triple correlations are generally assumed to be small (Lorenz [4]). Correlations of this type are furthermore neglected in arriving at the definition (2.2) of the available potential energy, and should therefore also be neglected in the evaluation. Even if we therefore use non-geostrophic winds and a static stability variable in the horizontal, we find, with the minor correction given in (2.20), the results stated in (2.13) and (2.14).

The available potential energy is frequently divided into the available potential energy of the zonal mean, and the available potential energy of the deviation from the zonal mean, the eddies:

$$A=A_Z+A_E \qquad (2.21)$$

The two energies are defined by the expressions

$$A_Z=\frac{1}{2g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\alpha_Z'^2dSdp \qquad (2.22)$$

and

$$A_E=\frac{1}{2g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\alpha_E'^2dSdp \qquad (2.23)$$

in which the zonal mean values are obtained from the definition

$$\alpha_Z=\frac{1}{2\pi}\int_0^{2\pi}\alpha d\lambda \qquad (2.24)$$

and the eddies are defined by the expression

$$\alpha_E=\alpha-\alpha_Z \qquad (2.25)$$

A similar division can be made of the kinetic energy of the horizontal flow

$$K=K_Z+K_E \qquad (2.26)$$

Following Lorenz [4], it can be shown that the equations corresponding to (2.1) now can be written:

$$\frac{dA_Z}{dt}=G(A_Z)-C(A_Z,K_Z)-C(A_Z,A_E) \qquad (2.27)$$

and

$$\frac{dA_E}{dt}=G(A_E)-C(A_E,K_E)+C(A_Z,A_E) \qquad (2.28)$$

The definitions of the quantities appearing in (2.27) and (2.28) are given below for easy reference

$$\left.\begin{aligned}
G(A_Z)&=\frac{1}{g}\int_0^{p_0}\int_S\frac{R}{c_p}\frac{1}{\bar{\sigma}p}\alpha_Z'H_Z'dSdp\\
G(A_E)&=\frac{1}{g}\int_0^{p_0}\int_S\frac{R}{c_p}\frac{1}{\bar{\sigma}p}\alpha_E'H_E'dSdp\\
C(A_Z,K_Z)&=-\frac{1}{g}\int_0^{p_0}\int_S\omega_Z\alpha_Z'dSdp\\
C(A_E,K_E)&=-\frac{1}{g}\int_0^{p_0}\int_S\omega_E\alpha_E'dSdp\\
C(A_Z,A_E)&=\frac{1}{g}\int_0^{p_0}\int_S\frac{1}{\bar{\sigma}}\alpha_Z'\frac{\partial(v\alpha')_Z}{\partial y}dSdp
\end{aligned}\right\} \qquad (2.29)$$

The problem is now to investigate the implications of using the adiabatic method A in the evaluation of

$C(A_Z, K_Z)$ and $C(A_E, K_E)$. It will suffice to show the development for the first of the quantities. We obtain the zonal average of the adiabatic vertical velocity from (2.8):

$$\omega_Z^* = \frac{1}{\bar{\sigma}}\frac{\partial \alpha_Z'}{\partial t} + \frac{\partial(\alpha' v)_Z}{\partial y} \tag{2.30}$$

Substitution of (2.30) in the definition of $C(A_Z, K_Z)$ gives us the estimate

$$C^*(A_Z, K_Z) = -\frac{1}{2g}\int_0^{p_0}\int_S \frac{1}{\bar{\sigma}}\frac{\partial \alpha_Z'^2}{\partial t}\,dSdp - \frac{1}{g}\int_0^{p_0}\int_S \frac{1}{\bar{\sigma}}\alpha_Z'\frac{\partial(\alpha' v)_Z}{\partial y}\,dSdp \tag{2.31}$$

A comparison of (2.31) with (2.27) shows that

$$C^*(A_Z, K_Z) = C(A_Z, K_Z) - G(A_Z) \tag{2.32}$$

By a similar calculation, it can easily be shown that an analogous expression applies for $C^*(A_E, K_E)$, i.e.

$$C^*(A_E, K_E) = C(A_E, K_E) - G(A_E) \tag{2.33}$$

Equations (2.32) and (2.33) show that the general result stated in (2.14) also holds when we divide the energy into the zonal and eddy available potential energy.

The main reason for the results given in (2.14), (2.32), and (2.33) is naturally that we, in the evaluation of ω^* by method A, satisfy the thermodynamic equation only in the adiabatic case ($H=0$), but make no use of the equations of motion. The fact that we obtain the difference between $C(A, K)$ and $G(A)$ in (2.14) is due to the estimate of $\partial\alpha/\partial t$ from *observations*. This procedure gives us no possibility of separating the influence of the vertical velocities and the diabatic heating on the local temperature changes.

The method B which has been used by White and Saltzman [16], Wiin-Nielsen [17], and Saltzman and Fleisher [13], [14] makes use of vertical velocities which also are computed under the assumption of adiabatic (and frictionless) motion. It is therefore of importance to investigate if these vertical velocities suffer from the same deficiencies as those computed from method A when applied in energy conversion calculations. The baroclinic models which were used in the studies referred to above are special cases of the general quasi-geostrophic models reduced to two parameters. The equations for the quasi-geostrophic models may be written in the form (Phillips [10])

$$\frac{\partial\zeta}{\partial t} + \mathbf{V}_\psi\cdot\nabla(\zeta+f) = f_0\frac{\partial\omega^{**}}{\partial p} \tag{2.34}$$

$$f_0\left[\frac{\partial}{\partial t}\left(\frac{\partial\psi}{\partial p}\right) + \mathbf{V}_\psi\cdot\nabla\left(\frac{\partial\psi}{\partial p}\right)\right] + \sigma\omega^{**} = 0,\ \sigma=\sigma(p) \tag{2.35}$$

in the case of frictionless and adiabatic motion. The vertical velocity [1] can be obtained by eliminating the time derivatives from (2.34) and (2.35) leading to the so-called ω-equation

$$\sigma\nabla^2\omega^{**} + f_0^2\frac{\partial^2\omega^{**}}{\partial p^2} = f_0\left[\frac{\partial}{\partial p}\{\mathbf{V}_\psi\cdot\nabla(\zeta+f)\} - \nabla^2\left\{\mathbf{V}_\psi\cdot\nabla\left(\frac{\partial\psi}{\partial p}\right)\right\}\right] \tag{2.36}$$

which may be solved by three-dimensional relaxation techniques with proper boundary conditions from a knowledge of the stream function.

Another computational technique consists of eliminating ω^{**} from (2.34) and (2.35) leading to the "potential" vorticity equation

$$\frac{\partial}{\partial t}\left\{\nabla^2\psi + \frac{f_0^2}{\sigma}\frac{\partial^2\psi}{\partial p^2}\right\} + \mathbf{V}_\psi\cdot\nabla\left\{\nabla^2\psi + \frac{f_0^2}{\sigma}\frac{\partial^2\psi}{\partial p^2} + f\right\} = 0 \tag{2.37}$$

Equation (2.37) is solved by relaxation techniques for the tendency $\partial\psi/\partial t$ which, substituted in (2.35), makes it possible to compute ω^{**} from (2.35).

It is easy to see from (2.34) and (2.35) that the energy relations for the model are:

$$\frac{dK^{**}}{dt} = -\frac{1}{g}\int_0^{p_0}\int_S \omega^{**}\alpha\,dSdp = C^{**}(A, K) \tag{2.38}$$

$$\frac{dA^{**}}{dt} = -C^{**}(A, K) \tag{2.39}$$

These relations have been proved by Phillips [9] for the two-parameter case, but follow also directly from (2.34) and (2.35) when the kinetic energy is defined as

$$K = \frac{1}{2g}\int_0^{p_0}\int_S \nabla\psi\cdot\nabla\psi\,dSdp \tag{2.40}$$

The main difference between methods A and B is that although in method B we apply assumptions which are similar to those in method A, we do not make use of observed tendencies, and we compute vertical velocities, which in addition to the thermodynamic (adiabatic) equation, also satisfy a simplified form of the vorticity equation derived from the equations of motion. As the energy relations (2.38) and (2.39) show, we know that the vertical velocities, computed from the model, will, when substituted in (2.5), measure the energy conversion $C(A, K)$ *in the model.* If such calculations shall be estimates of the energy processes which take place *in the atmosphere* it is naturally required that the model is a good approximation of the atmosphere. Although the baroclinic models which have been used in the calculations of $C(A, K)$ leave much to be desired in terms of accuracy, there is little doubt that they are good first approximations

[1] The notation ω^{**} indicates a vertical velocity computed from some form of a quasi-geostrophic model.

to the large-scale flow of the atmosphere (Phillips [11]). Furthermore, the models are used only in a diagnostic sense. We therefore need not be concerned with the inaccuracies caused by extended numerical integrations in time.

On the basis of the previous discussion, it is thus plausible that calculations of the energy conversion $C(A, K)$ should be based on vertical velocities computed from a quasi-geostrophic model. The model should naturally be as realistic as possible, having a larger vertical resolution than before and including frictional effects and a better lower boundary condition. In a previous study of the ω-equation including the diabatic heating (Wiin-Nielsen [17]) it was concluded that the vertical velocities generated by heating on the large scale are of sufficient magnitude to alter the conversion $C(A, K)$. The indications are therefore that the heating, at least on certain scales, is important. It is difficult to see at the present time how this effect can be incorporated.

3. SOME NUMERICAL ESTIMATES

The results obtained in section 2 can be tested in a preliminary way by using some numerical values obtained by different investigations. The main results of our analysis are summarized in (2.32) and (2.33).

We shall first investigate the energy conversion for the eddies by taking the numerical values obtained by Jensen [3]. He used method A for the two months January and April 1958 and has therefore estimated $C^*(A_E, K_E)$. For January 1958 we can compute $C^*(A_E, K_E)$ from his table 6. By forming a weighted sum of the total energy conversion for the different layers we find a value of 4242.3 erg cm.$^{-2}$ sec.$^{-1}$, representing the combined effects of the transient and standing (zonal) eddies. This value is converted to the unit kj. m^{-2} sec.$^{-1}$ used by Brown [1] in his study of $G(A_Z)$ and $G(A_E)$. We find $C^*(A_E, K_E)=42.4\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$. An estimate of $G(A_E)$ does not exist for this month (Jan. 1958). Instead we take a mean value for the months January 1959, 1962, and 1963 as computed by Brown [1]. We find $G(A_E)=-23.8\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$ which gives the estimates $C(A_E, K_E)=C^*(A_E, K_E)+G(A_E)=18.6\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$. This value should be compared with a value of $C(A_E, K_E)$ computed by method B. Such a value does not exist either, but we may compare with the values of 14.6×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ obtained by Wiin-Nielsen [17] for January 1959, or the value of 26.8×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ obtained by Saltzman and Fleisher [13] for February 1959.

The corresponding value for April (1958) of $C^*(A_E, K_E)$ is 27.1×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ (Jensen [3]). The value for $G(A_E)$ is -8.3×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ as an averaged value obtained from the months of April 1961 and 1962. The corresponding value for $C(A_E, K_E)$ computed from (2.33) is 18.8×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ which may be compared with the value of 11.0×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ computed for April 1959 by method B (Wiin Nielsen [17]).

There are great annual and seasonal variations in the energy conversions computed as averaged monthly values. The comparisons made above should therefore be considered only as rough estimates. However, the comparison, combined with the previous analysis, indicates that the values estimated from method A are reduced to the same magnitude as those obtained by method B when the results of the analysis are applied to them.

It is unfortunately not possible to make a similar test of the energy conversion $C(A_Z, K_Z)$ because Jensen [3] does not give the values of $C^*(A_Z, K_Z)$. However, he reproduces the zonal and time averages of temperature for the months as a function of latitude and pressure (his tables 3 and 4) and the corresponding values of the vertical velocities, w^* (his tables 1 and 2). It is possible to make an evaluation of $C^*(A_Z, K_Z)$ from these time-averaged values. The results of the calculation will give the energy conversion $C^*_s(A_Z, K_Z)$ carried out by the so-called meridional "standing" eddies. Such a calculation has been made for the months January and April 1958 to test the order of magnitude of $C^*(A_Z, K_Z)$ assuming that the greater part of the conversion is accounted for by the "standing" eddies.

The calculation which we want to perform is given by the formula:

$$\frac{1}{S}C^*(A_Z, K_Z)=-\frac{1}{Sg}\int_0^{p_0}\int_{\phi_1}^{\phi_2}\int_0^{2\pi}\omega_Z\alpha_Z' a^2\cos\phi d\lambda d\phi dp \quad (3.1)$$

The data given for the calculation are the values of the temperatures T_Z and the vertical velocities w_Z. We can introduce these quantities in (3.1) by using the gas equation

$$\alpha_Z'=\frac{R}{p}T_Z' \quad (3.2)$$

and the conversion formula from which the values of w were obtained

$$\omega_Z=-g\bar{\rho}w_Z=-g\frac{p}{R\overline{T}}w_Z \quad (3.3)$$

in which $\bar{\rho}$ and $\overline{T}$ are area averages of density and temperature, respectively.

The area, S, of the region is computed as the area between the latitude circles $\phi_1=15$°N. and $\phi_2=85$°N. giving

$$S=2\pi a^2(\sin\phi_2-\sin\phi_1) \quad (3.4)$$

The area average of the temperature can be determined from the formula

$$\overline{T}=\frac{1}{S}\int_{\phi_1}^{\phi_2}\int_0^{2\pi}Ta^2\cos\phi d\lambda d\phi=\frac{1}{(\sin\phi_2-\sin\phi_1)}\int_{\phi_1}^{\phi_2}T_Z\cos\phi d\phi \quad (3.5)$$

TABLE 1.—*The area mean temperature for the different layers for January and April, 1958 and the pressure differences in cb. for the different layers*

Layer (mb.)	$\overline{T}_l$ (Jan.)	$\overline{T}_l$ (Apr.)	Δp (cb.)
1000–850	272.9	278.3	15
850 –700	269.1	273.4	15
700 –500	258.9	261.9	20
500 –300	239.0	241.7	20
300 –200	221.7	224.1	10
200 –100	213.3	215.4	10
100 –50	212.7	214.9	10

TABLE 2.—*The contributions for the different layers,* $C_l^*(A_Z, K_Z)$ *to the total value of* $C_s^*(A_Z, K_Z)$. *Units:* 10^{-4} *kj. m.*$^{-2}$ *sec.*$^{-1}$

Layer (mb.)	Jan. 1958	Apr. 1958
1000–850	−10.53	−13.89
850 –700	−11.23	−12.16
700 –500	−8.04	+3.13
500 –300	−1.81	−4.36
300 –200	+0.22	−0.47
200 –100	−0.23	+0.75
100 –0	+0.40	+0.78
Total	−31.22	−26.22

The integral in (3.5) was computed replacing it by a finite sum

$$\int_{\phi_1}^{\phi_2}(\quad)d\phi=\Delta\phi\sum_{j=1}^{7}(\quad)_j \tag{3.6}$$

in which $\Delta\phi$ corresponds to 10° of latitudes or $\Delta\phi=\pi/18$.

When these results are introduced in (3.1) we find

$$\frac{1}{S}\,C^*(A_Z,K_Z)=\frac{1}{(\sin\phi_2-\sin\phi_1)}\int_0^{p_0}\int_{\phi_1}^{\phi_2}\frac{w_Z T_Z'}{\overline{T}}\cos\phi\, d\phi dp \tag{3.7}$$

The integration with respect to pressure has also been carried out using a finite sum. We may summarize the procedure by writing

$$C^*(A_Z,K_Z)=\sum_{l=1}^{7}C_l^*(A_Z,K_Z) \tag{3.8}$$

corresponding to the seven layers represented in the data tables. The expression for $C_l^*(A_Z,K_Z)$ is given by

$$C_l^*(A_Z,K_Z)=\frac{\Delta p\cdot\Delta\phi}{(\sin\phi_2-\sin\phi_1)}\,\frac{1}{\overline{T}_l}\sum_{j=1}^{7}(w_Z T_Z')_j\cos\phi \tag{3.9}$$

In the numerical integration we have taken the values in the layer between 100 and 50 mb. to represent the total layer from 100 mb. to the top of the atmosphere, but the contribution from this layer is insignificant for the results in any case.

Table 1 gives the values of $\overline{T}_l$ for the two months, January and April 1958, together with the values of Δp in the different layers.

With the information in table 1 combined with tables 1–4 in the paper by Jensen [3], it is possible to compute $C_s^*(A_Z,K_Z)$ for the two months. The results of the calculations are given in table 2 showing the contributions from the different layers in the unit 10^{-4} kj. m.$^{-2}$ sec.$^{-1}$

The results in table 2 may be substituted in (2.32). Since it is known from calculations using method B that $C(A_Z,K_Z)$ is very small compared to other energy conversions, we have approximately

$$C^*(A_Z,K_Z)\simeq -G(A_Z) \tag{3.10}$$

The estimate of $G(A_Z)$ obtained by Brown [1] in the average for the months January 1959, 1962, and 1963 is 37.9×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$, while the averaged value for the months of April 1962 and 1963 is $+14.3\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$ These values are at least of comparable magnitude to $-C_s^*(A_Z,K_Z)$. The differences may be ascribed to the facts that we have only the contributions from the standing eddies to $C^*(A_Z,K_Z)$, that the vertical resolution in Brown's [1] investigation is much smaller, and that Brown, in the numbers quoted here, has included the effects of friction and mountains. The latter factors are not considered in Jensen's study.

If we include only the contribution from the layer 850–500 mb. for January 1958 we get $C_s^*(A_Z,K_Z)=-55.0\times10^{-4}$ kj. m.$^{-2}$ sec.$^{-1}$, which then should be compared with the value of $G(A_Z)$ obtained by Wiin-Nielsen and Brown [18] for January 1959 without the effects of friction and mountains. This value is 50.0×10^{-4} kj. m.$^{-2}$ sec.$^{-1}$ which is in good agreement with the value for $-C_s^*(A_Z,K_Z)$.

4. CONCLUDING REMARKS

The main conclusion from the paper is that the so-called adiabatic method for calculations of vertical velocities will give erroneous results if it is applied to the calculations of the energy conversion between available potential energy and kinetic energy. It is shown that the method gives the difference between this energy conversion and the generation of available potential energy by diabatic processes. The conclusion holds for both zonal and eddy available potential energy.

Additional complications appear if observed winds and horizontal variations of static stability are used in the calculations of vertical velocities by the adiabatic method. It is, however, shown that these factors result in only three factor correlations which supposedly are small.

The results of the analysis are tested by applying them to a comparison of energy conversions and generations computed by different methods. Although the comparison is made difficult by the fact that the different calculations are not based on the same data, we obtained general agreement at least in orders of magnitude.

REFERENCES

1. J. A. Brown, Jr., "A Diagnostic Study of Tropospheric Diabatic Heating and the Generation of Available Potential Energy," 1963 (submitted for publication in *Tellus*).
2. E. O. Holopainen, "On the Dissipation of Kinetic Energy in the Atmosphere," *Tellus*, vol. 15, No. 1, Feb. 1963, pp. 26–32.
3. C. E. Jensen, "Energy Transformation and Vertical Flux Processes over the Northern Hemisphere," *Journal of Geophysical Research*, vol. 66, No. 4, Apr. 1961, pp. 1145–1156.
4. E. N. Lorenz, "Available Potential Energy and the Maintenance of the General Circulation," *Tellus*, vol. 7, No. 2, May 1955, pp. 157–167.
5. E. Palmén, "Vertical Circulation and Release of Kinetic Energy During the Development of Hurricane Hazel into an Extratropical Storm," *Tellus*, vol. 10, No. 1, Feb. 1958, pp. 1–24.
6. H. A. Panofsky, "Methods of Computing Vertical Motion in the Atmosphere," *Journal of Meteorology*, vol. 3, No. 2, June 1946, pp. 45–49.
7. H. A. Panofsky, "Large-Scale Vertical Velocity and Divergence," *Compendium of Meteorology*, American Meteorological Society, Boston, 1951, pp. 639–647.
8. S. Petterssen, *Weather Analysis and Forecasting*, vol. 1, 2d Ed., McGraw-Hill Book Co., Inc., New York, 1956, 428 pp. (See pp. 292–299.)
9. N. A. Phillips, "Energy Transformations and Meridional Circulations Associated with Simple, Baroclinic Waves in a Two-Level, Quasi-Geostrophic Model," *Tellus*, vol. 6, No. 3, Aug. 1954, pp. 273–286.
10. N. A. Phillips, "Geostrophic Errors in Predicting the Appalachian Storm of November 1950," *Geophysica*, vol. 6, No. 3/4, 1958, pp. 389–405.
11. N. A. Phillips, "Geostrophic Motion," *Reviews of Geophysics*. vol. 1, No. 2, 1963, pp. 123–176.
12. R. J. Reed, J. L. Wolfe, and H. Nishimoto, "A Spectral Analysis of the Energetics of the Stratospheric Sudden Warming of Early 1957," *Journal of Atmospheric Sciences*, vol. 20, No. 4, July 1963, pp. 256–275.
13. B. Saltzman and A. Fleisher, "The Modes of Release of Available Potential Energy in the Atmosphere," *Journal of Geophysical Research*, vol. 65, No. 4, Apr. 1960, pp. 1215–1222.
14. B. Saltzman and A. Fleisher, "Further Statistics on the Modes of Release of Available Potential Energy," *Journal of Geophysical Research*, vol. 66, No. 7, July 1961, pp. 2271–2273.
15. R. M. White and G. F. Nolan, "A Preliminary Study of the Potential to Kinetic Energy Conversion Process in the Stratosphere," *Tellus*, vol. 12, No. 2, May 1960, pp. 145–148.
16. R. M. White and B. Saltzman, "On Conversions Between Potential and Kinetic Energy in the Atmosphere," *Tellus*, vol. 8, No. 3, Aug. 1956, pp. 357–363.
17. A. Wiin-Nielsen, "A Study of Energy Conversion and Meridional Circulation for the Large-Scale Motion in the Atmosphere," *Monthly Weather Review*, vol. 87, No. 9, Sept. 1959, pp. 319–332.
18. A. Wiin-Nielsen and J. A. Brown, Jr., "On Diagnostic Computations of Atmospheric Heat Sources and Sinks and the Generation of Available Potential Energy," *Proceedings of the International Symposium on Numerical Weather Prediction, Tokyo, November, 1960*, Meteorological Society of Japan, Tokyo, Mar. 1962, pp. 593–613.

[*Received December 13, 1963; revised February 10, 1964*]