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UTILIZATION OF HAIL-DAY DATA IN DESIGNING AND EVALUATING HAIL SUPPRESSION PROJECTS

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ABSTRACT

Historical hail-day records of U.S. Weather Bureau first-order stations and cooperative substations are the only long, objective records of hail occurrence available throughout the United States. Although hail-day data are limited in areal density and are not necessarily the most desired measure of seeding effects, they are the only data available to obtain a measure of the areal-temporal variability of hail for most areas of the United States. Consequently, hail-day data from Illinois have been employed in a pilot project to determine the time required to obtain statistically significant changes in hail-day frequencies over various sized areas. Four statistical designs were investigated using the historical hail-day data for five areas in Illinois. The results show that the optimum design for hail-day data is the continuous seeding (seeding on all days likely to have hail) over an area. The optimum test is the sequential test involving the Poisson and Negative Binomial distributions. Detection of a 20-percent reduction in summer hail days would require, on the average, a continuous seeding program ranging from 13 to 37 yr, depending on the level of precision desired, and the size and location of the seeded area. Major reductions, those in excess of 60 percent, would require experiments of only 1- to 3-yr length.

1. INTRODUCTION

Recent hail modification suppression experiments in Russia (Atlas, 1965, Battan, 1965, and Sulakvelidze, 1967) have been instrumental in developing more scientific interest in hail suppression in the United States. In May 1965, the Interdepartmental Committee on Atmospheric Sciences asked the National Science Foundation to prepare plans for a national program of hail suppression. Subsequently, the Foundation held the First Symposium on Hail Suppression, a meeting of leading hail scientists, at Dillon, Colorado, in October 1965. The results of this meeting led to Project Hailswath, a pilot 1-mo field experiment in southwestern South Dakota (Schleusener, 1966), and to the formation of the National Hail Suppression Committee (Goyer et al., 1966). The goal of this committee was to evaluate the national hail problem and to propose plans and recommendations concerning further hail modification activities.

Most prior and current hail suppression activities in the United States have often been plagued with controversies and questionable results common to rain-enhancement efforts (Hagen and Butchbaker, 1967, and Stout, 1961). One scientifically oriented project in Colorado did show reduction in hail intensity over a 5-yr period (Schleusener and Auer, 1964). However, the relative infancy of hail suppression activities indicates that preliminary statistical studies in regard to data collection, size of study area, statistical design, and duration of hail suppression experiments should be performed prior to actual experimentation. Such studies should serve to eliminate some of the problems that have plagued many rain-modification experiments. To this end, a 2-yr project designed to study techniques for evaluating hail suppression activities was begun during 1966 with partial support from the National Science Foundation and the Crop-Hail Insurance Actuarial Association (Changnon, 1967*a*, 1968).

One phase of this project concerned the study of all available historical hail data with the primary purpose of using these data to choose the optimum type of statistical design for field projects and to define the duration of an experiment needed to detect various degrees of change that might be produced by suppression efforts. It was ascertained that there are only two types of historical hail data available in Illinois and in most other States—the U.S. Weather Bureau point (station) records of hail days, and the crop insurance records of monetary loss and areal extent of damage.

The insurance data are a more direct measure of the condition to be suppressed (loss or damage) than are hail-day occurrences. However, insurance data suffer from shortness of record (only 19 yr in Illinois), lack of areal availability (covers only 60 percent of Illinois), changes in target (liability and dollar value) with time, and changes in the crop's susceptibility to hail damage during the growing season (Changnon, 1967*d*). For instance, a hailstorm in July with an intensity identical to that of a storm in June will cause three times more damage to a corn crop.

A preliminary limitation of the hail-day data for use in designing a hail-suppression project or in evaluating the results is that hail days are only an indirect measure of the most desired change, a reduction in crop and property damage. Their employment must be based on the supposition that a reduction in hail days would be accompanied by a comparable reduction in losses from hail. Previous research in Illinois has shown that there is a good relationship between the annual amount of crop loss and the extent of area experiencing an extensive number of summer hail days (Chagnon, 1959).

Another limitation inherent in the hail-day data is that there are relatively few points (stations) with quality data in areas of potential seeding activities ranging from 500 to 3,000 sq mi in size. Thus, the true areal frequency of hail days cannot always be established from available data. Nevertheless, the results from the hail-day data can provide useful information as to site selection, choice of area size, and length of experimentation.

This paper contains the results of the research involving the U.S. Weather Bureau hail-day data for five areas in Illinois ranging in size from 500 to 3,000 sq mi. The data, their natural variability, and the statistical techniques used to analyze them are described. Results pertaining to length of time necessary to verify different levels of reduction (suppression) for summer and annual seeding periods, and for different sized areas and statistical designs are presented in the final section.

2. DATA

Research into the hail climatology of Illinois revealed 85 cooperative substations in the State with quality hail-day records of at least 15-yr duration in the 1901–1963 period (Changnon, 1967*c*). Also, there were data available for 10 first-order stations in and adjacent to Illinois. Examination of all stations with quality hail data during the 1934–1963 period, when the greatest density of stations was available, indicated that there were three regions with relatively high station densities. Five stations in three regions of the State occupied areas of nearly equal size, and a boundary was constructed for each which formed an area of 1,000 sq mi and was generally oriented southwest-northeast. The names and locations of the stations in these three areas (Areas 1, 2, and 3) are depicted in figure 1. When three other stations north of Area 1 were combined with those in Area 1, a 3,000-sq mi area was formed (Area 4, fig. 1). Four stations in central Illinois with records for the 1944–1963 period were used to define a 500-sq mi area, labeled Area 5.

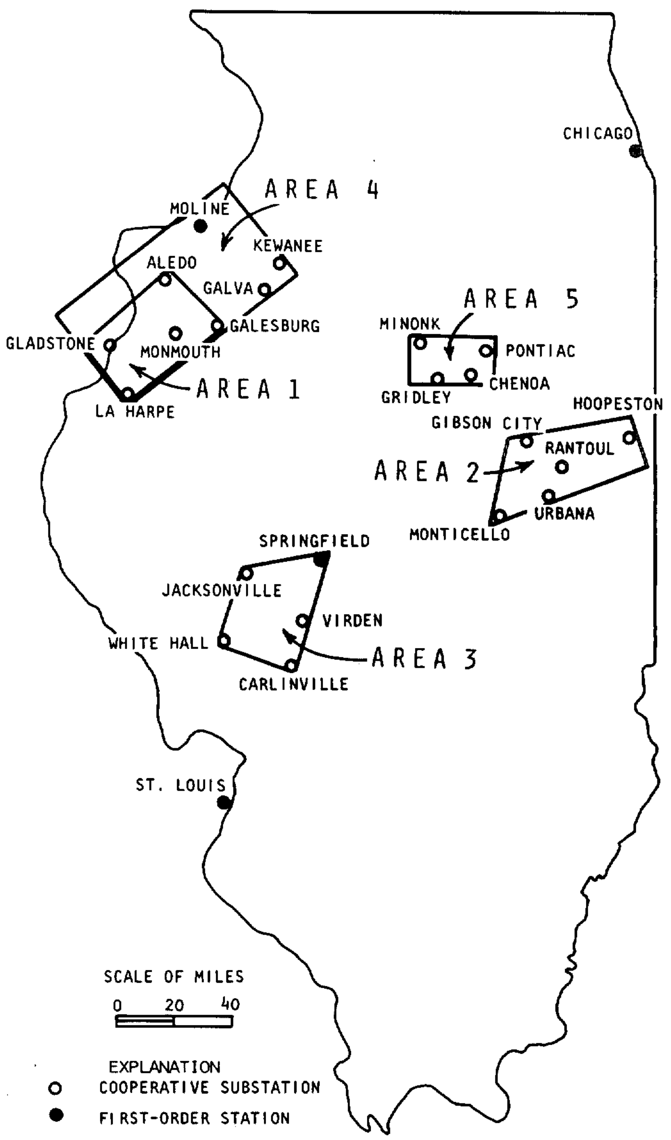


FIGURE 1.—Location of hail data stations and study areas.

TABLE 1.—*Number of hail days per area in 5-yr periods during 1934–1963*

	Area 1		Area 2		Area 3		Area 4		Area 5	
	≥1S*	≥2S*	≥1S	≥2S	≥1S	≥2S	≥1S	≥2S	≥1S	≥2S
Summer										
Average	12	2−	12	1+	11	2−	17	4−	8	2−
Maximum	25	6	28	4	15	6	28	6	12	3
Minimum	6	0	3	0	3	0	8	0	4	0
Total										
Average	39	7+	41	8−	43	11−	55	13	28	6
Maximum	56	12	77	19	64	25	69	21	39	11
Minimum	24	3	14	0	31	1	46	7	17	1

*S=station(s).

The dates of hail at each station in an area were combined to develop a list of hail days for each area. The area data were summarized for each season and on an annual basis. However, the data presented are for the summer (June–August) because that is the crop-damage season, and for the entire year to provide results relating to crop and property damage throughout the year.

TABLE 2.—*Frequency distributions of summer and annual hail days during 1934–1963*

	Number of years for given numbers of hail days per year																			
	0	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19
Summer																				
Area 1	6	4	5	7	5	1	1	0	1	0										
Area 2	8	7	5	3	2	0	1	1	1	2										
Area 3	4	9	5	4	6	2	0	0	0	0										
Area 4	4	0	8	4	7	3	1	0	2	1										
Area 5	3	6	7	4	0	0	0	0	0	0										
Annual																				
Area 1		1	0	2	1	3	5	2	4	3	3	2	2	2	0	0	0	0	0	0
Area 2		3	0	4	6	0	1	1	0	2	2	1	4	0	2	1	1	0	0	2
Area 3		0	0	1	3	1	0	6	4	2	4	3	0	1	4	0	0	0	1	0
Area 4		0	0	1	0	1	2	0	4	6	2	1	1	2	2	2	4	2	0	0
Area 5		0	1	4	3	3	0	4	2	2	1	0	0	0	0	0	0	0	0	0

3. NATURAL VARIABILITY

Various point and regional frequencies of hail days were determined, and selected results are presented to illustrate the natural variability of hail. Average and extreme values for the five areas and the summer and total hail-day data during 5-yr periods are shown in table 1. The differences in the averages for hail days, as defined by one station or more in the area, and those for hail days defined by hail occurrence at two or more stations per area (more widespread hailfall condition) reveal that most of the hail in an area occurred only at one station. Note that the 5-yr average values for summer hail days in the three 1,000-sq mi areas (Areas 1, 2, and 3) are 11 or 12 days, whereas the summer averages defined by hail occurrences at two or more stations are 1 or 2 days in these three areas. Comparison of either the summer or annual average values for the three 1,000-sq mi areas reveals little significant difference. All three areas are located in relatively high hail-frequency areas of Illinois (Changnon, 1967*c*). As expected, the differences in area size affect the averages with the lowest averages from the 500-sq mi area (Area 5) and the highest from the largest area (Area 4).

The frequency distributions of hail days per year are shown for the five areas in table 2. The summer distributions for Areas 1, 2, and 3 (each=1,000 sq mi) are not alike with Areas 1 and 2 having some years with six or more hail days in summer, whereas Area 3 did not experience more than 5 hail days. This lack of extremes in Area 3 is reflected in the summer data in table 1, which show that Area 3 had a 5-yr maximum of 15 summer hail days, whereas Area 1 had a maximum of 25 days and Area 2 had 28 days. Thus, although the three equal-sized areas had similar averages, the distributions of days and their extremes were considerably different.

The frequency distributions of summer hail days were used to construct recurrence interval graphs for each area and station. Those for overlapping Areas 1 and 4 and for three stations within these areas (fig. 1) are portrayed in figure 2. At least once in 10 yr, each of the stations will experience two or more hail days in summer, whereas Area 1 will have five or more hail days and Area 4 will have six or more hail days in a given summer.

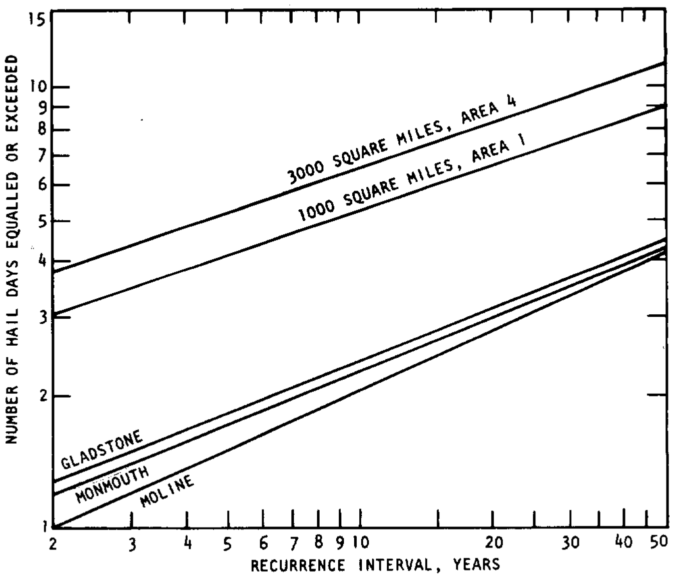


FIGURE 2.—Frequency distribution of summer hail days.

The temporal variability of the area hail days is displayed in figure 3, which has curves based on non-overlapping 5-yr totals. The curves for Areas 1–4 all display low values for the period ending in 1938, and these low values are related to the statewide low incidences of hail accompanying the droughts of this period (Huff and Changnon, 1959). The curves exhibit considerable fluctuation after 1948 after being somewhat homogeneous from 1934 through 1948. Area 1 shows an almost constant decrease in hail days reaching a minimum in 1958, whereas Area 2 shows a constant increase through 1958. Interestingly, the curve of Area 3 (the other 1,000-sq mi area) exhibits a compromise, trending downward with Area 1 from 1948 through 1953, but with a shape similar to that

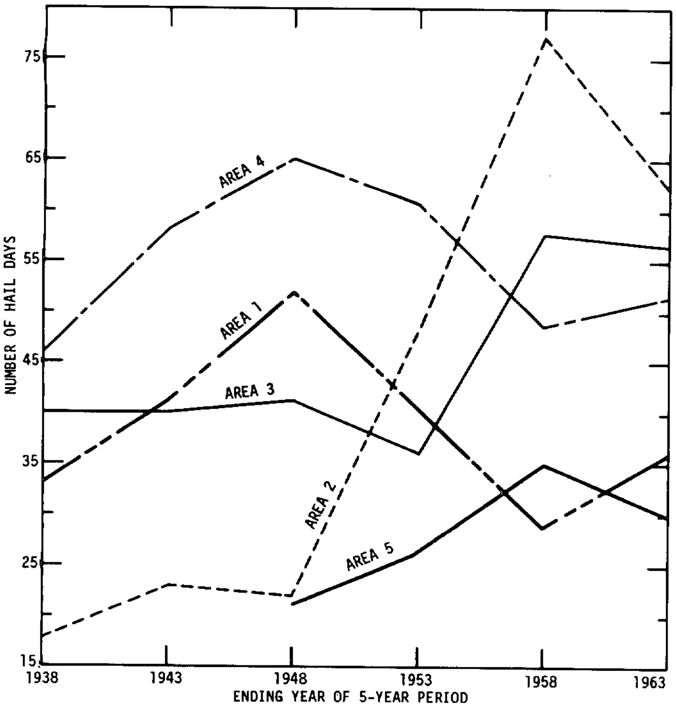
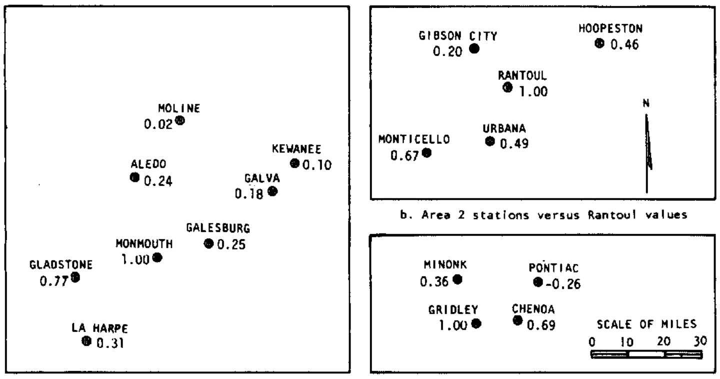


FIGURE 3.—Five-year totals of annual hail days in five areas.



a. Area 4 station values versus Monmouth values

b. Area 2 stations versus Rantoul values

c. Area 5 stations versus Gridley values

FIGURE 4.—Correlation coefficients for annual numbers of hail days between pairs of stations.

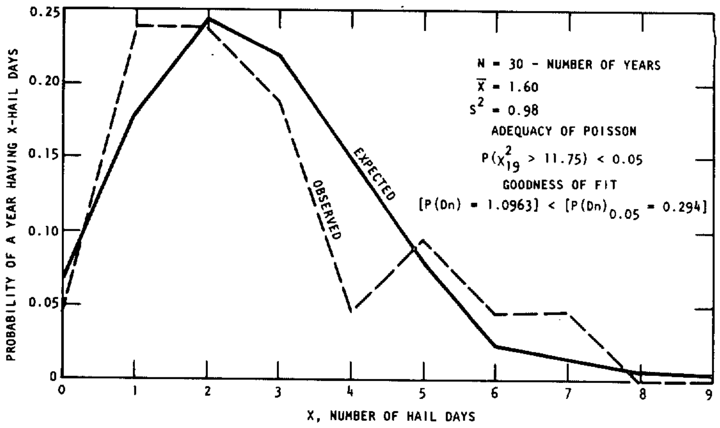


FIGURE 5.—Distribution of observed summer hail days in Area 5 and that expected (fitted) by the Poisson distribution.

of Area 2 after 1953. Area 4, which is 200 percent larger than Areas 2 and 3, had lower hail-day values than these two smaller areas during the 1954–1963 period. The variations in curve shapes, rapid shifts with time, and the 10-yr trends of increase or decrease indicate the magnitude of the problem associated with assigning statistical significance to a change in hail days during a modification project.

A measure of the natural variability of hail days within the areas is exhibited in figure 4, which depicts the linear correlation coefficients achieved between the annual hail-day frequencies of stations in three different areas. The coefficients at most stations surrounding a chosen base station were less than 0.5, indicating very little relationship between their hail-day frequencies. The few higher coefficients were found at stations located to the west-southwest or east-northeast of the base stations, in agreement with the prevalent motion of most hailstorms in Illinois (Changnon et al., 1967).

The Poisson and Negative Binomial distributions were fitted to the hail-day data to further describe their variability. First, the Poisson distribution was fitted to the data and the adequacy of the distribution was determined by testing for equality of the sample mean and variance. If this fit was inadequate, the data were fitted by the moment estimates of the Negative Binomial distribution. If the efficiency of the moment estimates was unsatisfactory, then the maximum likelihood estimates of the Negative Binomial distribution were used as suggested by Thom (1957*b*). Of the 20 distributions fitted (spring, summer, fall, and annual for the five regions), 13 were fitted by the Poisson; two by the moment estimates of the Negative Binomial distribution; and five by the maximum likelihood estimates of the Negative Binomial distributions. The Kolmogorov-Smirnov "goodness of fit" test was applied to the distributions and a good fit was obtained at the 0.05 level of significance. An example of the Poisson distribution fitted to summer data from Area 5 appears in figure 5. The "goodness of fit" is illustrated by the observed and theoretical curves, as well as the probability values for the Kolmogorov-Smirnov statistic, D_n.

The Poisson distribution is one in which the mean is the parameter of the distribution. The Negative Binomial distribution is a two-parameter distribution, with K and P being the parameters of the distributions. The sample estimates of K and P for the regional data fitted by the Negative Binomial distribution and the sample means for all regions are shown in table 3. A tendency is shown for the summer data to be fitted by the Negative Binomial and the annual data to be fitted by the Poisson. This occurs because summer data are more likely to be a series of dependent events and hence a distribution such as the Negative Binomial, which allows for dependence, is required. This tendency was hypothesized by Thom and was shown in his data (1957*b*).

The adequacy of the data from a few point records to represent all the regional hail days was investigated.

TABLE 3.—*Hail-day averages and parameters of the best fit distributions for each area*

	Areas				
	1	2	3	4	5
Summer					
Average hail days	2.5	2.4	2.2	3.4	1.6
Distribution of best fit	*NB**	*NB**	*Po*	*NB*	*Po*
K	4.2	1.10		6.14	
P	.59	2.20		.55	
Annual					
Average hail days	7.7	8.3	9.1	11.0	5.7
Distribution of best fit	*Po*	*NB**	*Po*	*Po*	*Po*
K		3.03			
P		2.75			

NB: Negative Binomial distribution with moment estimate of parameters.

NB:* Negative Binomial distribution with maximum likelihood estimate of parameters.

Po: Poisson distribution.

Initially, the sampling adequacy of the eight stations in Area 4 was checked by a process of data deletion (Changnon, 1967*b*). Area-mean averages of summer hail days were developed using combinations of any two stations, any three stations, and on through the eight possible stations. These averages displayed a curvilinear trend, and a quadratic equation was developed from them. Its solution showed that the highest (true) summer average for a 5-yr period was 19 hail days (two more than that from the eight stations, table 1), and that 12 stations in the area were necessary to achieve true sampling of hail days within the 3,000-sq mi area. This station frequency indicated a density of one station per 250 sq mi. The five stations in Areas 1, 2, and 3 represented densities of one station per 200 sq mi, and the four in Area 5 represented a density of one station per 125 sq mi. Thus, it appears that the sampling densities in these four areas were adequate to define all the hail days in these areas, whereas that in Area 4 underestimated the frequency of summer hail days by about 12 percent.

4. STATISTICAL DESIGNS AND TESTS OF HYPOTHESES

In choosing the optimum design and estimating the minimum duration of a hail modification experiment, the first necessity is to obtain an estimate of the distribution parameters for days which would have been seeded. This can be accomplished by one of the following methods (Schickedanz, 1967): 1) simulate the seeded sample using the Monte Carlo technique with various changes in the distribution parameters; 2) assume that the number of hail days for each period (summer or annual) were decreased a certain percentage each year; or 3) present the number of hail days in terms of a particular statistical test. In this last method, it is sufficient to first compute the components of the test for the sample from the nonseeded sample of the nonseeded distribution. Then, with certain assumed changes in the distribution parameters, the differences required for significance can be obtained through algebraic relations. Since the only effect that can be tested with hail-day data is a reduction in the total or average number of days, method 3 was chosen for this study. Once this was chosen, four different seeding designs were considered.

One possible design using the hail-day data is that in which individual storms are seeded. However, the hail-day data in any area are too sparse to define characteristics (areal size or intensity) of individual hailstorms, and thus cannot be used in evaluating modification project designs involving seeding of individual storms.

Another design is one in which the seeded days are selected at random. However, this design reduces the sample size to one-half if the randomization factor is one-half. A hail day is an event too infrequent in most areas, including Illinois, to squander approximately half of the potential (forecasted) hail days to "no-seed" trials. A design in which the yearly unit (total or average number of hail days per year) is randomized would be completely erroneous.

The third type of design considered was the "target-control" continuous seed regression approach wherein the data from the seeded days are compared with the data from a nearby control area. A small correlation coefficient eliminates the effectiveness of a target-control approach. The quantity $1-r^2$ is the percentage of the total variation in target variable that is unexplained when a control area is employed. A correlation coefficient on the order of 0.60 indicates that 64 percent of the target variable is unexplained. Therefore, a correlation coefficient of 0.60 indicates that the unexplained variation is only reduced 36 percent when an areal control is employed. The correlation coefficient between the annual hail days of Areas 2 and 5, the two nearest non-overlapping areas, was only 0.57, and those between all other possible areas were less. Hence, the target-control design was discarded for hail-day data. A similar conclusion was reached from the Colorado hail suppression experiments, which employed other means of measuring of hail (Schleusener et al., 1965).

The final design considered in reference to hail-day data was a continuous seeding design on a single area without any control area. In the continuous seed design all days with potential hailstorms in the study area are seeded. The analysis of the data was based on the assumption that all hail days in the historical record would have been designated as potential hail days. First, the classical non-sequential test was employed between the historical data and data from the seeding period. For those distributions fitted by the Poisson distribution (table 3), a Poisson test was used. Since there is no non-sequential test for Negative Binomial distributions available in the literature, the Positive Binomial non-sequential test was used. It is believed that the error involved in applying the Positive Binomial in this case, for illustrative purposes, is not large enough to warrant concern.

For the Poisson test, the basic method described by Johnson and Leone (1964) was used. The method was modified so that assumed seeding effects could be tested.

TABLE 4.—*Years needed to detect 20- and 60-percent reductions in summer and annual hail days with continuous seeding design and the sequential test*

	Number of years for 20% reductions* for given areas					Number of years for 60% reductions* for given areas				
	1	2	3	4	5	1	2	3	4	5
Summer	14.4	28.8	10.6	10.3	18.5	1.2	2.3	1.0	1−	1.3
Annual	3.0	9.8	2.5	2.1	4.1	1−	1.8	1−	1−	1−

*Based on error levels of 0.05 for type 1 and 0.50 for type 2.

TABLE 5.—*Years needed to detect 40-percent reductions in summer hail days in Area 1 for different error levels*

	Combination of type 1 and type 2 error levels			
Type 1 (α) error	0.01	0.01	0.05	0.05
Type 2 (β) error	0.20	0.50	0.20	0.50
Years needed for each combination	10	4	9	3

For the Positive Binomial test, the equation given by Dixon and Massey (1951) was solved for n, and the Z value of 1.65 (corresponding to a one tail probability at $\alpha = 0.05$) was used. The results indicated that it is doubtful that the application of the classical tests of hypothesis to this particular design is better than the target-control approach.

Thom (1957*a*) has suggested that the lack of an areal control can be compensated for by the favorable aspects of the sequential analysis approach. Thus, this final statistical design involving continuous seeding on all potential forecasted hail days in an area was considered in the context of a sequential analysis. This design can be based on hail-day frequencies, and potential frequency changes from seeding can be evaluated with a statistical technique suggested by Thom (1957*a*). This approach is based on the sequential analysis test procedure (Wald, 1947). In the non-sequential analysis, the α (type 1 error) and N (number of samples) are fixed, and β (type 2 error) is the dependent variable. In the sequential test, α and β are fixed and the observations are tested sequentially. With each new sample, one of the following decisions is made: 1) accept the hypothesis, 2) reject the hypothesis, or 3) continue taking observations. For the details of applying the test, one is referred to Thom (1957*a*). Before the experiment is conducted, the *ASN* (average sampling numbers required to reach a decision) can be determined from theoretical *ASN* equations. On the average, the sequential method of testing requires less observations than non-sequential methods (Wald, 1947). The *ASN* were computed for the various areas using the summer and annual data. The results indicated that this test and the continuous seed design are the optimum ones to use with hail-day data.

TABLE 6.—*Years needed to detect various reductions in summer and annual hail days with continuous seeding design using a sequential test*

Area	20%*	40%*	60%*	80%*
Summer				
1	14.4	3.2	1.2	1−
2	28.8	6.2	2.3	1.0
3	10.6	2.4	1.0	1−
4	10.3	2.3	1−	1−
5	14.4	3.3	1.3	1−
Annual				
1	3.0	1−	1−	1−
2	9.8	2.1	1−	1−
3	2.5	1−	1−	1−
4	2.1	1−	1−	1−
5	4.1	1−	1−	1−

*Based on error levels of 0.05 for type 1 and 0.50 for type 2.

5. RESULTS

Results for 20- and 60-percent reductions in hail days from continuous seeding designs (table 4) reveal that it would be easier to detect comparable reductions with annual data than for summer data in all areas. For instance, to prove a 20-percent reduction in the summer data from Area 5 (500 sq mi) would require, on the average, 18 yr, whereas proof of the same reduction in the annual data (seeding on all hail days) would require only 4 yr. Results in table 4 also reveal significant geographical differences at the 20-percent reduction level. For instance, the 1,000-sq mi areas (Areas 1, 2, and 3) require 14, 29, and 11 yr, respectively. Geographical differences in these same-sized areas are also shown by the number of years to detect 60-percent reductions.

The effect of different α and β levels on the test of significance is depicted in table 5 for data from Area 1. For a very conservative test, $\alpha=0.01$ and $\beta=0.2$, 10 yr of experimenting are required to obtain significance. For type 2 errors of 0.5, which is a common level for the 0.01 and 0.05 significance levels, 4 and 3 yr, respectively, are required to obtain significance for a 40-percent decrease in summer hail days. Results chosen for presentation in this paper are based on the $\alpha=0.05$ and $\beta=0.5$ levels.

Changes in Areas 3 and 4 would require less time to detect than comparable ones in the other areas (table 6), and changes in Area 2 would require considerably more time than in the other areas. The lowest values in table 6 are for the largest area (Area 4) which has the largest averages (table 1), and this suggests that as the average of an area is increased, less time is required to obtain significance. Also, the Negative Binomial test requires more observations than the Poisson test. When the maximum likelihood solution is required, more time is required to obtain significance.

The considerably higher values for Area 2 (table 6) were carefully investigated. The summer data for this area had an upward trend in the 1945–1960 period, although the trend was not significant at the 0.10 signifi-

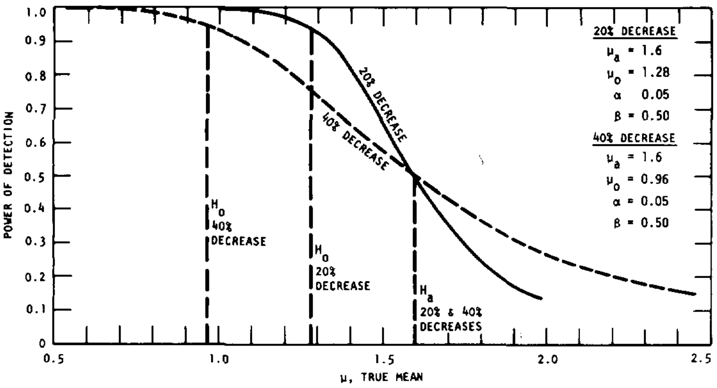


FIGURE 6.—Poisson *OC* function for summer hail days in Area 5.

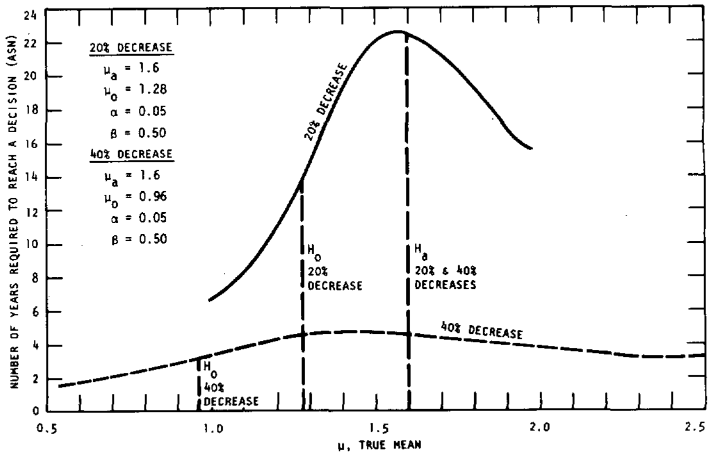


FIGURE 7.—Poisson *ASN* function for summer hail days in Area 5.

TABLE 7.—*Comparison of the number of years required to obtain significance for the sequential and non-sequential tests* (α=.05, β=0.5)

Area	Test		40%		60%		80%	
	s	*ns*	*s*	*ns*	*s*	*ns*	*s*	*ns*
				Summer				
1	*NB**	*PB*	3.2	6.7	1.2	2.8	.6	1.7
2	*NB**	*PB*	6.2	6.7	2.3	2.8	1.0	1.7
3	*P*	*P*	2.4	10.0	1.0	4.9	.5	2.8
4	*NB*	*PB*	2.3	4.8	.9	2.1	.4	1.2
5	*P*	*P*	3.3	15.1	1.3	6.6	.6	3.7
				Annual				
1	*P*	*P*	.7	3.2	.3	1.4	.1	.8
2	*NB**	*PB*	2.1	7.9	.8	3.5	.3	2.0
3	*P*	*P*	.6	2.7	.2	1.2	.1	.7
4	*P*	*P*	.5	2.2	.2	1.0	.1	.5
5	*P*	*P*	.9	4.3	.4	1.9	.2	1.1

s: sequential.
ns: non-sequential.
PB: Positive Binomial.
Po: Poisson.
NB: Negative Binomial, moment estimates.
NB:* Negative Binomial, maximum likelihood estimates.

cance level. This level was chosen since it is the most common level to use for significance tests of random climatological series (Thom, 1966). The annual series for Areas 2 and 3 were found to have significant upward trends (fig. 3). The sequential test is known to be sensitive to non-randomness in the climatological data series. Therefore, it is possible that the larger number of years in Area 2 can be attributed to these trends. The fact that an upward trend is present and a downward effect is being tested implies that the test is conservative. That is, the numbers in table 6 represent the maximum number of years required to obtain significance. It is very likely that the Area 2 figures are in reality less than indicated.

Values presented in table 6 show dramatic decreases in the number of years required to obtain significance as hail-day reductions change from 20 to 40 percent. If hail suppression projects could produce 60-percent or greater reductions in Illinois or other areas with comparable climate and topography, then proof can be shown in relatively short periods of time, 3 yr or less.

Up to this point, it was assumed that the true values of μ_0 (the reduced mean due to seeding) and μ_a (the historical mean) were the "true values" of the parameter μ of the Poisson distribution. What happens at other values of μ, if the estimates of μ_0 and μ_a are not the "true" values? The function $L(\mu)$, or the operating characteristic (the probability of accepting H_0 when μ is the true value), gives insight into this aspect. The *OC* (operating characteristic) curves for the summer data in Area 1 are shown in figure 6. The *OC* curves show that for a given hypothesized decrease, large effects are easier to detect than small effects. For the 40-percent curve, and for a mean of 1.0 hail days, the ability to detect is very good, power = 0.94, whereas for a mean of 2.0 hail days it is very low, power = 0.27. Hence, if seeding could reduce the true mean to 1.0 it would very likely be detected.

The relationship between the true mean and the *ASN* is shown in figure 7. The *ASN* represents the number of years required to reach a decision. The maximum point of each curve represents the center of the interval of indifference, which is the area where the decision would be to continue the testing procedure. This illustrates that if the true mean for the 20-percent hypothesized decrease is close to the center (1.45 hail days), it will take a relatively long time to detect it (20.6 yr). On the other hand, if the true mean is reduced by hail suppression to 1.0 (1 hail day), it would be detected in less than 7 yr.

In table 7 there is a comparison between the sequential and non-sequential tests, and the superiority of the sequential method is further demonstrated. On the average, the sequential method reduced the number of required observations by 60 percent for the summer data and 80 percent for the annual data. A comparison between the sequential and non-sequential tests for Area 2 shows that the sequential method is only 22 percent better, on the average, than the non-sequential test. This implies that even though the assumptions of the sequential analysis were violated when the sequential test was applied to Area 2 data, the sequential method was still superior. The data from table 7 also imply that as the average number of hail days in an area increases, the superiority of the sequential method also increases. Also, the sequential test is 78 percent better than the non-sequential test for the Area 2 annual data. This implies that the errors involved when the assumptions of the sequential method are violated decrease as the average number of hail days increases.

6. CONCLUSIONS

The results of this research indicate that the historical Weather Bureau hail-day data in Illinois exhibit a great deal of areal and temporal variability, but they could be used in planning and designing hail modification experiments. It is believed that the results for Illinois are applicable to other areas in the United States with a similar hail climate. The optimum design using hail-day data is one in which every potential hail day is seeded. The natural distribution of hail days is well described by the Poisson and Negative Binomial distributions in Illinois. The optimum test found in this research is the sequential test involving the above-mentioned distributions. In all cases, the Negative Binomial test required more observations (samples) than did the Poisson test, and when the maximum likelihood solution was required, the number of observations needed exceeded those of the other two tests. If 40-percent reductions can be effected in the number of annual hail days, the reduction can be detected in a reasonable amount of time, 3 yr or less, depending on the size of area and the amount of statistical risk one is willing to accept.

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