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DISTRIBUTION FUNCTION FOR SEASONAL AND ANNUAL RAINFALL OVER INDIA

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ABSTRACT

Distribution functions for seasonal (southwest monsoon) and annual rainfall at 53 long-record stations in India have been obtained. It was found that the frequency distributions are right skewed. Tests for normality show that while normal distribution gives a good fit to seasonal and annual rainfall at stations in some parts of India it does not give a good fit to seasonal and annual rainfall at stations over the major portion of the country. Tests of goodness of fit of the Gamma distribution, however, clearly indicate that this distribution provides a good fit to seasonal and annual rainfall at stations in different parts of the country.

1. INTRODUCTION

Two studies have investigated the normality aspect of seasonal or annual rainfall over India. Sankaranarayanan (1933) studied the frequency distribution of southwest monsoon precipitation at 68 representative stations over India, Pakistan, and Burma. He concluded that there is very little justification for assuming a non-normal distribution but neither can one say that the curves are "necessarily" normal. The other study, by Pramanik and Jagannathan (1953), examined annual rainfall series at 30 well-distributed stations over India and Pakistan for any significant departures from Gaussian distribution. They found that some of the g's (i.e., g_1 and g_2, measures of skewness and kurtosis, respectively) are more than twice their standard errors and can be considered significant. Table 2 of their paper indicates that the annual rainfall distribution at 13 stations of 30 is significantly different from normal.

Barger and Thom (1949) originally found that the Gamma distribution provides a good fit to the precipitation series under a wide range of conditions in the United States. Mooley and Crutcher (1968) have shown that monthly rainfall in India during the summer monsoon can be described by a Gamma distribution. Suzuki (1964) fitted a hyper-Gamma distribution to the monthly and annual rainfall of Tokyo and Niigata and found the fit to be good.

In this paper, it is proposed to examine the fit of the normal and Gamma distributions to the summer monsoon (i.e., June through September) and annual rainfall at a large number of stations covering the different parts of India. Hereafter, summer monsoon rainfall will be referred to as seasonal rainfall.

2. DATA

The 53 stations from which rainfall data have been utilized in the present study are shown in figure 1. These stations cover various climatic regimes. All available data up to and including those of 1960 have been used. The number of years of data utilized for each station is given below the name of the station.

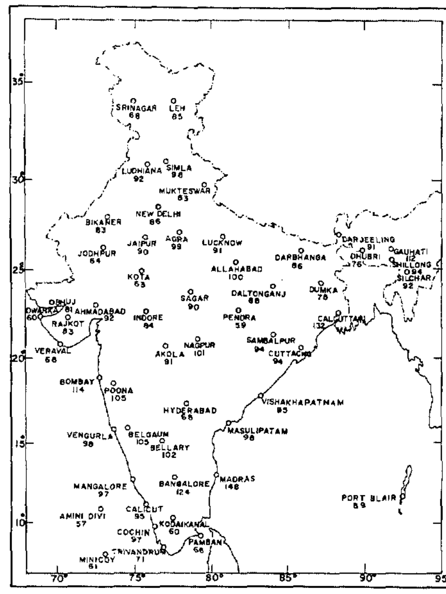

FIGURE 1.—Stations from which rainfall data was obtained for this study.

Seasonal and annual rainfall data for each of the stations were obtained from the monthly values given in "World Weather Records" and from relevant publications of the India Meteorological Department. For most stations the rainfall record covers 60–100 yr.

3. TESTS FOR DEPARTURE FROM NORMALITY

Rao (1952) has stated that a goodness-of-fit test can be applied to the observed frequency distribution to test normality, but it is quite insensitive in testing for some specific aspects of the distribution, such as symmetry and kurtosis. Hence, g_1 and g_2 departures from zero were tested for significance and the chi-square test for frequency distribution was applied, as was done by Mooley and Appa Rao (1970) in their study of pentad rainfall distribution. The results of these tests are listed in table 1. If g_1, g_2, or chi-square is significant at a station, then the rainfall distribution at that station is taken to be significantly different from normal. It is seen from table 1

TABLE 1.—*Normality test for rainfall over India*

Serial no.	Station	Seasonal g_1	Seasonal g_2	Seasonal χ^2	Annual g_1	Annual g_2	Annual χ^2
1	Agra	0.11	−0.15	8.35	0.11	−0.32	4.44
2	Ahmadabad	0.62**	1.69**	4.36	0.67**	1.73**	6.12
3	Akola	0.80**	1.77**	14.40*	0.71**	1.10*	27.31**
4	Allahabad	0.50*	0.37	8.54	0.62**	0.64	11.78*
5	Amini Divi	0.31	−0.15	3.83	0.41	−0.25	4.84
6	Bangalore	0.60**	−0.07	24.49**	0.35	0.34	5.39
7	Belgaum	1.08**	1.83**	27.54**	0.82**	1.34**	13.66*
8	Bellary	0.45	0.10	7.02	0.40	0.01	4.82
9	Bhuj	1.75**	4.13**	19.67**	1.84**	4.72**	20.20**
10	Bikaner	0.68**	−0.03	12.48*	0.86**	0.96*	14.75*
11	Bombay (Colaba)	0.49*	1.61**	6.51	0.36	0.98**	8.12
12	Calcutta (Alipore)	0.74**	0.82*	7.90	0.30	−0.01	38.13**
13	Calicut	0.23	0.26	4.64	0.30	0.00	3.54
14	Cochin	0.56*	0.65	4.78	0.21	−0.56	4.76
15	Cuttack	0.46	−0.14	6.82	0.44	−0.13	5.51
16	Daltonganj	0.36	0.08	8.78	0.24	−0.11	5.16
17	Darbhanga	0.06	−0.58	5.67	0.19	−0.46	11.05
18	Darjeeling	0.67**	0.29	14.29*	0.49*	−0.26	11.57*
19	Dhubri	0.21	−0.34	2.49	0.24	−0.54	6.01
20	Dumka	0.02	−0.39	2.98	−0.11	−0.56	4.30
21	Dwarka	0.91**	1.06*	10.06	0.75*	0.63	11.13*
22	Gauhati	0.67**	1.12**	6.33	0.03	0.42	3.19
23	Hyderabad	0.99**	0.44	17.30**	1.18**	1.28*	13.00*
24	Indore	0.61*	0.27	4.62	0.62*	0.47	6.50
25	Jaipur	0.65**	0.86	4.01	0.75**	1.00*	3.23
26	Jodhpur	1.02**	1.36**	9.88	1.45**	3.73**	13.82*
27	Kodaikanal	0.00	−0.31	5.13	−0.10	−0.27	6.49
28	Kota	0.27	−0.81	9.99	0.41	−0.38	11.40*
29	Leh	1.45**	3.76**	18.75**	0.99**	1.65**	5.58
30	Lucknow	0.31	0.02	4.40	0.37	−0.04	16.72**
31	Ludhiana	0.67**	0.32	17.82**	0.77**	0.59	10.69
32	Madras	0.50*	0.42	8.22	0.46*	−0.31	8.78
33	Mangalore	−0.03	0.03	14.47*	0.31	−0.22	3.71
34	Masulipatam	0.76**	0.00	19.98**	0.79**	0.20	9.49
35	Minicoy	−0.15	−0.80	12.11*	0.07	−0.07	7.38
36	Mukteswar	0.66*	0.70	5.47	0.57	0.48	13.06*
37	Nagpur	−0.10	0.22	10.43	−0.26	−0.17	3.84
38	New Delhi	0.54*	1.42**	10.00	0.49	0.95*	5.44
39	Pamban	1.61**	3.34**	18.62**	0.68*	0.81	7.16
40	Pendra	0.02	0.66	7.08	−0.19	0.49	6.22
41	Poona	1.03**	2.11**	9.09	0.80**	1.49**	12.55*
42	Port Blair	0.92**	1.82**	9.81	0.22	−0.21	2.95
43	Rajkot	0.57*	0.38	9.71	0.74**	0.44	12.14*
44	Sagar	0.41	0.23	4.38	0.31	−0.03	0.82
45	Sambalpur	0.01	−0.53	2.59	0.01	−0.72	6.00
46	Shillong	0.31	−0.26	4.07	0.41	0.07	10.80
47	Silchar	0.77**	2.29**	15.07**	0.79**	20.7**	5.06
48	Simla	0.91**	1.05*	11.19*	1.05**	1.67**	20.05**
49	Srinagar	0.64*	0.21	6.48	0.55	0.49	5.26
50	Trivandrum	1.16**	2.26**	15.50**	0.67*	0.59	14.11*
51	Vengurla	−0.03	0.19	7.08	0.18	0.35	6.60
52	Veraval	0.78**	0.43	7.67	0.75**	0.17	10.79
53	Vishakhapatnam	0.38	−0.74	8.73	0.72**	1.16**	4.97

* and ** show significance at the 5-and 1-percent levels, respectively; there are five degrees of freedom for χ^2.

that, of 53 stations, the seasonal rainfall at 34 stations is significant at the 5-percent level; and at 25 stations, it is significant at the 1-percent level. The corresponding figures for annual rainfall are 30 and 21 percent, respectively. The tests thus show that at the 5-percent level the distributions of seasonal and annual rainfall are not Gaussian over the major portion of India.

Sankaranarayanan (1933) drew inference about the nature of the monsoon rainfall distribution for nine homogeneous regions into which he divided India, Pakistan, Burma, and Ceylon, on the basis of the g_1 and g_2 values of individual stations within these groups. He made two assumptions: (1) stations in a group form random selections from the same population and (2) this population is normally distributed. Consistent with assumption (1), he computed the highest 5-percent significance value of g_1 for related distributions that may be expected in the sample from a particular group. He tested the significance of g_1 for all stations in each of the groups by utilizing this "5 percent highest value for related distributions" for each of the groups. His table of g_1 and g_2 values for 68 stations shows that, at the 5-percent level, g_1 and g_2 are significantly different from zero at 34 and 15 stations, respectively. It is thus seen from his study that, at the 5-percent level, the monsoon rainfall distribution is not Gaussian at stations over a large portion of India and neighboring regions.

TABLE 2.—*Chi-square test for Gamma distribution of rainfall over India*

	Frequencies for different ranges of $P(\chi^2 \geq \chi_0^2)$: <0.01	≥0.01 but <0.05	≥0.05 but <0.10	≥0.10 but <0.25	≥0.25 but <0.50	≥0.50 but <0.75	≥0.75	Total
Seasonal rainfall	0	1	7	17	12	6	10	53
Annual rainfall	0	1	4	8	14	15	11	53

4. TESTS FOR GAMMA DISTRIBUTION

Since the normality assumption does not appear to hold, the rainfall distribution has been tested for a Gamma distribution, which is next to the normal distribution in simplicity. The normal distribution is an asymptotic case of the Gamma distribution as the shape parameter tends toward infinity. The chi-square test was applied to test the null hypothesis of the Gamma distribution against the alternative hypothesis that the rainfall frequency distribution is different from the Gamma distribution. Table 2 gives the number of cases for different ranges of $P(\chi^2 \geq \chi_0^2)$ where χ_0^2 is the actual chi-square value obtained. The table reveals that at none of the stations is the seasonal and annual rainfall distribution significantly different from the Gamma distribution at the 1-percent level, and that at only one station are the seasonal and annual rainfall distributions significantly different from Gamma distribution at the 5-percent level. It is thus clear that the null hypothesis of Gamma distribution is not contradicted and the seasonal and annual rainfall at stations in different parts of India can be taken to be Gamma distributed.

To strengthen the above conclusion, the Kolmogorov-Smirnov test (hereafter referred to as K-S test) was applied to test the hypothesis that rainfall is Gamma distributed. This test is applied to the cumulative distribution and consists of finding the value of $D_N = \text{Max}|S_N(x) - F_N(x)|$, where $S_N(x)$ and $F_N(x)$ are respectively empirical and theoretical cumulative distributions and the latter is completely specified. The distribution of D_N is independent of $F_N(x)$ provided $F_N(x)$ is continuous. Massey (1951) has tabulated critical values of D_N for levels of significance 0.01, 0.05, 0.10, 0.15, and 0.20. In the present study, estimates of the parameters have been obtained from the sample. In such situations, Massey (1951) has stated that (1) when the K-S test strongly implies rejection of the null hypothesis (i.e., when D_N is much greater than the critical value), the hypothesis should be rejected, (2) when D_N is near the critical level there is some uncertainty as to the decision not to reject the null hypothesis, and (3) when rejection of the null hypothesis is not implied and D_N is not near the critical level, then the nonrejection decision is correct.

In this study, D_N has been found to be less than the critical values at the 0.20 level of significance in all the cases; hence, at the 5-percent level, the null hypothesis cannot be rejected. It may be mentioned that in most cases, the ratio $D_N/D_N^{0.20}$ is less than 0.5, $D_N^{0.20}$ being the critical value of D_N at the 0.20 level of significance. Thus, the actual cumulative probability distributions of seasonal and annual rainfall are not different from the cumulative Gamma probability distributions.

Table 3 gives $\bar{x}$, the mean; s, the estimate of standard deviation of seasonal and annual rainfall (in mm); and $\hat{\gamma}$ and $\hat{\beta}$, the maximum likelihood estimates of Gamma distributions fitted to the seasonal and annual rainfall.

TABLE 3.—*Rainfall parameters for India*

Serial no.	Station	Seasonal $\bar{x}$ (mm)	Seasonal s (mm)	Seasonal $\hat{\gamma}$	Seasonal $\hat{\beta}$ (mm)	Annual $\bar{x}$ (mm)	Annual s (mm)	Annual $\hat{\gamma}$	Annual $\hat{\beta}$ (mm)
1	Agra	629	215	7.11	88.4	707	217	9.69	73.0
2	Ahmadabad	746	303	5.37	139.1	779	304	5.94	131.1
3	Akola	692	213	10.54	65.6	808	245	11.01	73.4
4	Allahabad	873	256	11.31	77.2	990	278	12.82	77.2
5	Amini Divi	1011	286	12.20	82.9	1445	283	26.65	54.3
6	Bangalore	486	142	12.08	40.2	895	192	21.45	41.7
7	Belgaum	1009	263	16.09	62.8	1310	279	23.17	56.6
8	Bellary	260	108	5.22	49.9	492	154	9.91	49.6
9	Bhuj	322	183	3.20	100.5	350	190	3.58	97.7
10	Bikaner	249	129	3.45	72.0	299	139	4.32	69.1
11	Bombay (Colaba)	1768	469	13.57	130.4	1869	479	14.57	128.4
12	Calcutta (Alipore)	1209	257	23.31	51.9	1618	295	30.16	53.7
13	Calicut	2258	469	22.59	100.0	3053	544	31.71	96.3
14	Cochin	1924	411	22.36	86.0	2973	4807	37.15	80.0
15	Cuttack	1166	280	17.69	65.9	1536	337	21.16	72.6
16	Daltonganj	1028	249	16.92	60.7	1193	267	19.73	60.5
17	Darbhanga	1061	240	18.91	56.1	1254	254	24.14	52.0
18	Darjeeling	2356	415	33.88	69.5	2887	451	42.32	68.2
19	Dhubri	1792	429	17.09	104.9	2561	487	28.08	91.6
20	Dumka	1179	250	19.61	60.1	1471	263	28.01	52.5
21	Dwarka	349	215	2.27	153.7	376	219	2.49	150.9
22	Gauhati	1057	241	19.93	53.0	1642	287	31.58	52.0
23	Hyderabad	588	168	13.72	42.8	797	220	14.89	53.5
24	Indore	828	242	11.92	69.4	914	263	12.27	74.5
25	Jaipur	564	225	5.82	96.9	628	241	6.59	95.3
26	Jodhpur	323	176	3.07	105.5	361	196	3.27	110.5
27	Kodaikanal	556	138	15.33	36.3	1162	279	34.44	48.3
28	Kota	725	281	6.16	117.7	785	294	6.76	116.0
29	Leh	42	29	2.03	20.7	93	43	4.79	19.5
30	Lucknow	874	309	7.07	123.6	988	317	9.29	106.4
31	Ludhiana	540	200	7.43	72.7	706	231	9.70	72.8
32	Madras	382	128	8.63	44.2	1253	374	11.22	111.8
33	Mangalore	2842	471	35.24	80.7	3353	506	44.48	75.4
34	Masulipatam	599	181	11.70	51.2	1039	286	14.17	73.3
35	Minicoy	877	191	20.13	43.6	1613	248	41.82	38.6
36	Mukteswar	979	259	14.88	65.8	1329	292	21.38	62.2
37	Nagpur	1055	247	16.50	64.0	1188	280	16.09	73.8
38	New Delhi	556	221	5.70	97.6	668	229	8.15	82.8
39	Pamban	54	49	1.13	47.5	937	249	14.65	64.0
40	Pendra	1188	249	21.38	55.6	1416	281	23.25	60.9
41	Poona	520	172	9.50	54.8	703	195	13.44	52.3
42	Port Blair	1770	353	26.86	65.9	2970	390	58.51	50.8
43	Rajkot	607	242	5.79	104.8	643	252	6.45	99.7
44	Sagar	1138	307	13.54	84.1	1251	322	14.74	84.9
45	Sambalpur	1443	288	24.13	59.8	1623	304	27.69	58.6
46	Shillong	1486	330	20.31	73.2	2201	431	26.47	83.2
47	Silchar	1974	377	29.34	69.7	3236	578	32.82	98.6
48	Simla	1205	281	19.43	62.0	1610	321	27.40	58.8
49	Srinagar	198	77	6.69	29.6	674	152	20.27	33.3
50	Trivandrum	824	249	12.12	68.0	1763	378	22.88	77.1
51	Vengurla	2549	606	16.06	158.7	2750	658	16.55	166.1
52	Veraval	521	280	3.26	159.5	559	300	3.27	170.8
53	Vishakhapatnam	534	179	8.74	61.0	1006	286	12.72	79.2

5. COMPUTATION OF RAINFALL PROBABILITIES

Utilizing the result that the Gamma distribution is a good fit to the seasonal and annual rainfall distribution, probabilities of rainfall not exceeding or exceeding specified precipitation levels can be computed by using the tables given by Pearson (1934) or Wilk et al. (1962).

The arguments in Pearson's (1934) tables are $p=\hat{\gamma}-1$ and $u=x/(\hat{\beta}\sqrt{\hat{\gamma}})$, x being the precipitation level; $\hat{\beta}$ has the same unit as x.

In the tables by Wilk et al. (1962) and Thom (1968), the arguments are $\eta=\hat{\gamma}$ and probability expressed as percentage. The quantities contained in the table are values of $(x/\hat{\beta})$. Hence, for a precipitation level x, $x/\hat{\beta}$ has to be computed first and then the probability in percentage can be interpolated from the table. This table, however, does not extend beyond $\eta=22.0$. In the tables by Thom (1968), $t=x/\hat{\beta}$, and $\hat{\gamma}$ varies from 0.5 to 36.0.

Each rainfall recording station represents a certain surrounding area. Its representativeness depends on the nature and time scale of rainfall considered. For convective-type precipitation, single station rainfall may not be representative; but for monsoon rain, it is representative of rainfall over a sizable area around the station. As the period of rainfall considered increases from an hour to a day, 1 day to 5 days, 5 days to a month, etc., the representativeness of a rainfall recording station increases markedly. Sajnani (1964) has shown that Bombay (Colaba) station rainfall for 5-day periods during the different months of the southwest monsoon is representative of that over the entire Colaba district. Its representativeness for monthly, monsoon, and annual rainfall is, therefore, expected to be much larger. Hence, it is felt that the results derived for individual stations could be applied to areas much larger than that of a district.

In most parts of India, the economy depends heavily on the rain which falls during the 4-mo period June through September and water needs for crops and for several other purposes have to be balanced against the monsoon rainfall. Water resource planning, therefore, assumes considerable importance. The rainfall probabilities computed by utilizing the parameters of the Gamma distribution given in table 3 can be used for planning water resources. Depending on the stakes involved, a suitable level of rainfall probability may be chosen for planning purposes.

6. CONCLUSIONS

1. Gamma distribution provides a good fit to the seasonal and annual rainfall at stations in different parts of India. This distribution can be applied to obtain requisite rainfall probabilities for planning purposes.

2. While normal distribution gives a good fit to seasonal and annual rainfall at stations in some parts of India, it does not give a good fit to seasonal and annual rainfall at stations over the major portion of the country.

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